RG Fixed Point with a 4th Generation

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Outline

- SM with fundamental scalars: A brief review of the Naturalness and Hierarchy problems.

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- Implications

Naturalness and Hierarchy problem

Common objection to non-SUSY SM with fundamental scalars:

The need for fundamental scalar fields in the theory of weak and electromagnetic forces is a serious flaw. They require unnatural adjustments of bare constants (Wilson, Susskind,...)

- Recall: $V = \mu_0^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$
  $\lambda > 0$ and $\mu_0^2 < 0 \Rightarrow$ SSB of SM:
  $\langle \phi^0 \rangle = \sqrt{\frac{-\mu_0^2}{2\lambda}} \approx 174\, GeV$.

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  $\langle \phi^0 \rangle = \sqrt{\frac{-\mu_0^2}{2\lambda}} \approx 174\, GeV$.
- Quantum corrections to μ_0^2 can drastically alter that value.

Naturalness and Hierarchy problem

- Flaw: Corrections to the scalar mass e.g. from a fermion loop: $\mu^2 = \mu_0^2 - \frac{g_f^2}{16\pi^2}\Lambda_{max}^2 + ...$ (Λ_{max}: UV Cutoff) ("Quadratic divergence")

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- Why is it a flaw? If the physical cut-off $\Lambda_{max} \sim M_{Pl} \sim 10^{19}\,GeV$, one expects $\mu^2 \sim O(M_{Pl}^2)$ (assuming $g_f = O(1)$) unless...

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- One *fine-tunes* the cancellation to at least 34 decimal places to make $\mu \sim \Lambda_{EW} \sim O(100\, GeV) \Rightarrow$ Naturalness and Hierarchy (between M_{Pl} and Λ_{EW}) problems.

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- Each momentum slice represents a distinct set of physics, e.g. $\lambda\,\phi^4 \Rightarrow$ Same equation of motion but with different couplings for different slices.
- Divergence arises when $n \to \infty$: The physics contribution from each momentum slice is finite but there is an infinite number of such slices.

Various remedies

Large Λ_{max}

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- UV cutoff Λ_{max} large (e.g. M_{Pl}): Need to cancel the “quadratic divergence”. Very attractive symmetry: SUSY.
- Exact SUSY: Cancellation between bosons and fermions to all orders $\Rightarrow$ No “quadratic divergence”. However, the absence so far of SUSY partners of SM particles $\Rightarrow$ Broken SUSY. To avoid reintroducing the “quadratic divergence” $\Rightarrow$ Softly Broken SUSY $\Rightarrow$ $\delta\mu^2 \propto m^2_{soft}$. The requirement $m_{soft} \leq \Lambda_{EW} \Rightarrow$ SUSY partners < O(TeV).

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Large Λ_{max}

- Bright Side: A possible “solution” to the hierarchy problem; Dark Matter candidates; Lots of particles to search for;...
- Flip Side: Lots of parameters: More than one hundred; FCNC problems; How SUSY is broken is still unclear; Has its own fine-tuning problem: the μ-problem,...

Various remedies

$\Lambda_{max} \sim O(TeV)$: Composite Higgs

- One attractive possibility: New Strongly coupled gauge theories at $O(TeV)$ such as Technicolor $\Rightarrow$ Higgs scalar is not fundamental but is a composite of "Technifermions" .

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- Many interesting Top-quark-like Condensate Models discussed in the workshop

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- Even in a massless theory, scale invariance can be broken at the quantum level.
- Divergence of dilatation current = $\theta^{\mu}_{\mu} \propto \beta(g)$
- If $\beta(g) = 0$ and there are no explicit masses, scale invariance is valid $\Rightarrow$ Absence of quadratic and quartic divergences.

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- However, could some approximation such as $\beta_{2-loop}(g^*) = 0$ hint at the energy scale where scale invariance might be restored?
- In particular, under what conditions would that energy scale be $\Lambda_{max} \approx O(TeV)$ with minimal enlargement of the 3-generation SM?
- Would a heavy 4th generation do it and what are the implications? Work done with Chi Xiong.

Heavy 4th Generation's scenario

What can a heavy 4th generation do?

- A heavy Higgs-4th Yukawa system can give rise to condensates and bound states of 4th generation fermions $\Rightarrow$ Implications on the vacuum structure of the SM and the number of Higgs doublets (a mixture of fundamental and composites)

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- A heavy Higgs-4th Yukawa system can give rise to condensates and bound states of 4th generation fermions $\Rightarrow$ Implications on the vacuum structure of the SM and the number of Higgs doublets (a mixture of fundamental and composites)
- The appearance of quasi-fixed points in the 2-loop approximation at a scale $\Lambda_{FP} \sim O(TeV)$ hints at a possible scale-invariant theory above Λ_{FP}.

Heavy 4th Generation's scenario

Higgs-Yukawa system with four generations

- Run the 2-loop RG equations:
  $16\pi^2 \frac{dY}{dt} = \beta_Y$
  from Λ_{EW} on, with $Y = \lambda, g_t^2, g_q^2, g_l^2$ (quartic, top, 4th quark, 4th lepton couplings).

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- Generically,
  $\beta/16\pi^2 = g(\beta_0(\alpha/4\pi) + \beta_1(\alpha/4\pi)^2 + ..)$ (see e.g.Abbott 1980)

Heavy 4th Generation's scenario

Higgs-top Yukawa system

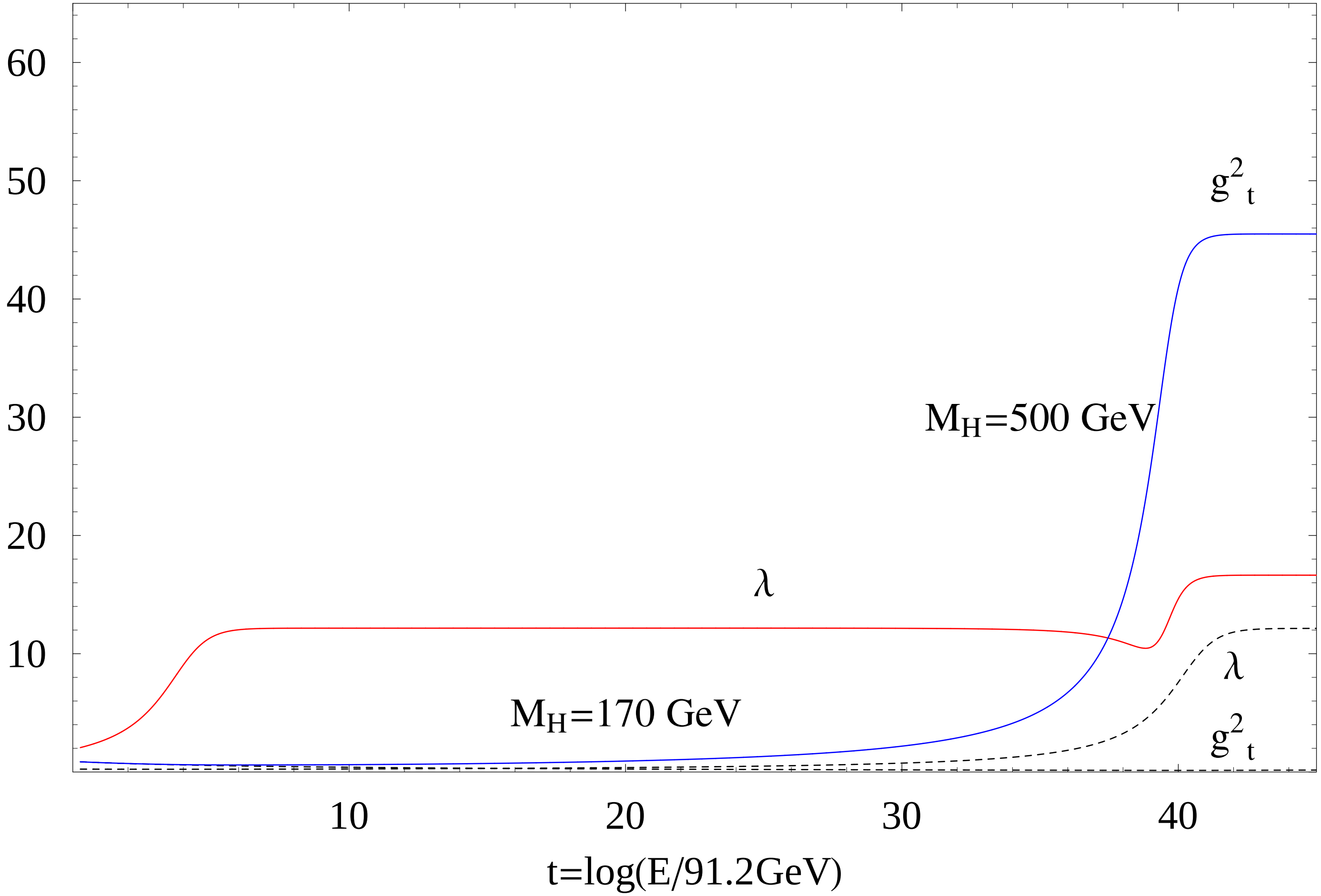

Fig. 1

Heavy 4th Generation's scenario

Higgs-4 generation Yukawa system

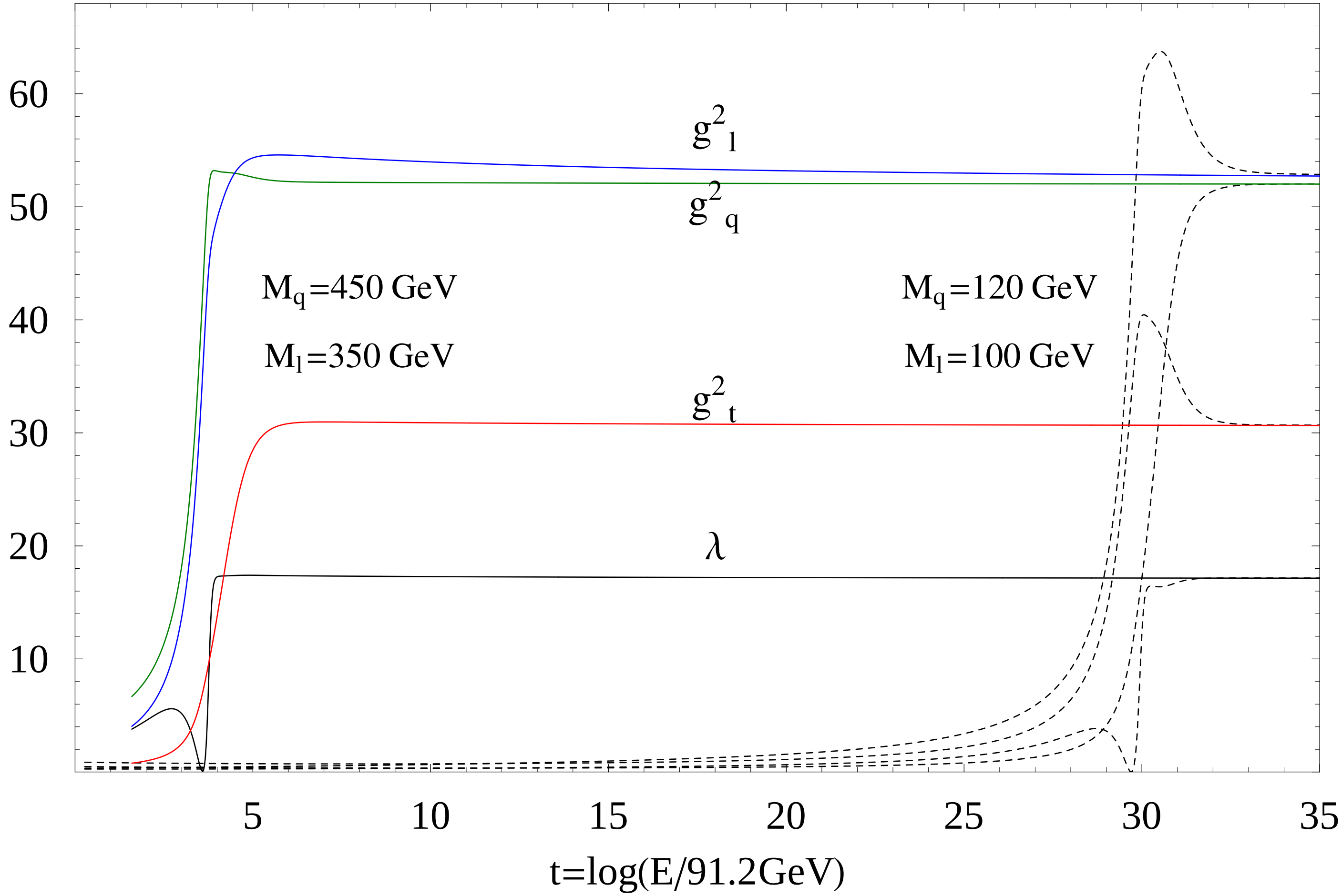

Fig. 2

Heavy 4th Generation's scenario

Some clarifications about the figures

- In Fig. 2, the light 4th generation (120 GeV and 100 GeV) case is shown only for comparison. It is excluded experimentally.

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- In Fig. 2, the light 4th generation (120 GeV and 100 GeV) case is shown only for comparison. It is excluded experimentally.
- The masses shown in Fig. 2 will be modified when extra dynamical Higgs doublets are included (discussed below)
- Notice the values of the quasi fixed point expansion parameters: $\alpha^*_{q,l}/4\pi \approx 0.3$, $\alpha^*_t/4\pi \approx 0.2$, $\lambda^*/16\pi^2 \approx 0.1$. More on this below.

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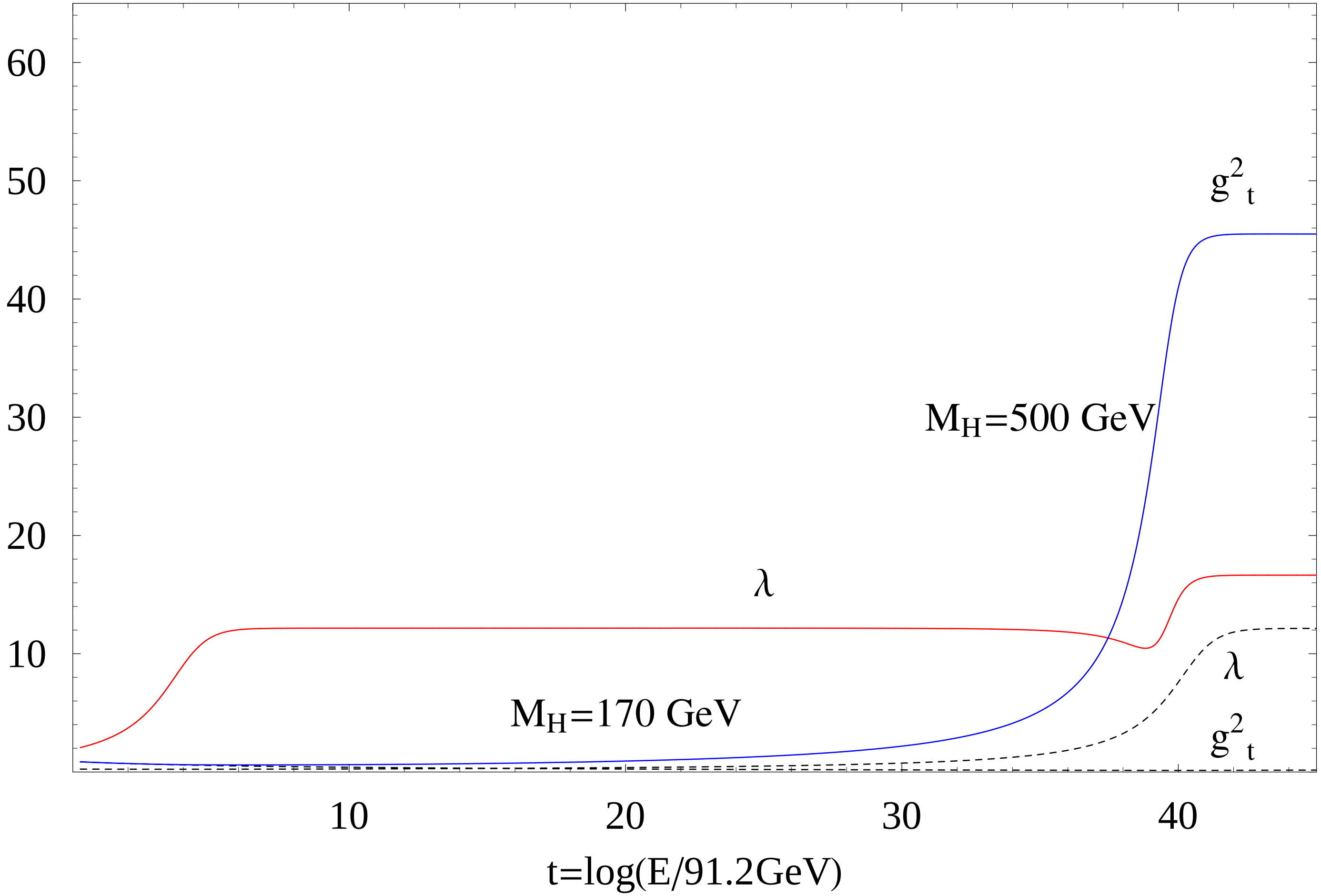

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- In Fig. 1, light Higgs (170 GeV) $\Rightarrow$ No quasi-fixed point below the Planck scale ($t_{pl} \approx 39$).
- For heavy Higgs (500 GeV) $\Rightarrow$ Quasi fixed point just above t_{pl}
- Notice at that quasi fixed point, $\alpha_t/4\pi \approx 0.2$

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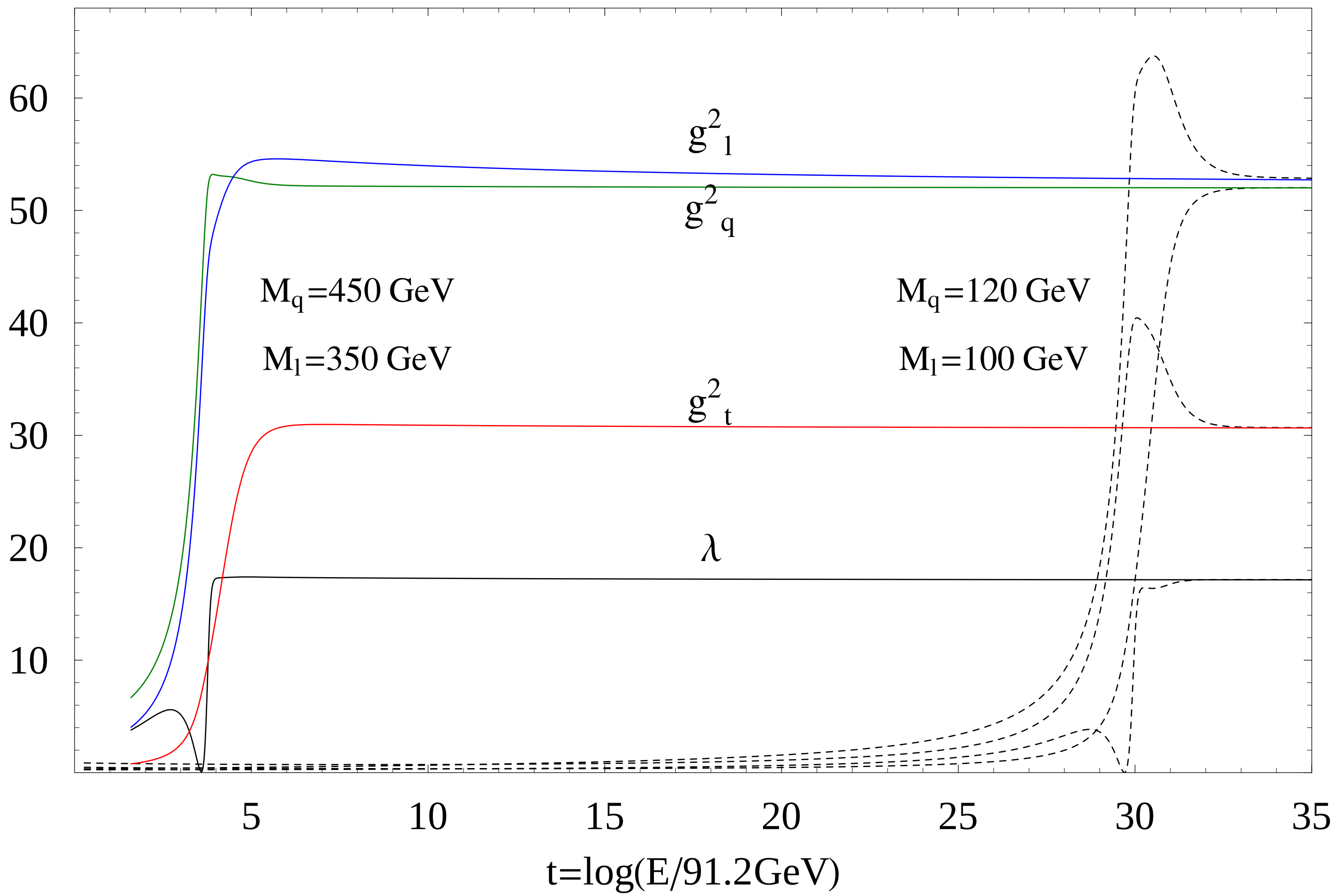

Fig. 2

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Comments on Higgs-4 generation Yukawa system: Fig. 2

- The light ($M_q = 120\,GeV$, $M_l = 100\,GeV$) and the heavy ($M_q = 450\,GeV$, $M_l = 350\,GeV$) reach the same quasi-fixed point but at different momentum scales $\Rightarrow$ Hint of some real fixed point so $\beta = 0 \Rightarrow$ Hint of scale invariance.

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- Vacuum stability requires $\lambda > 0 \Rightarrow$ Fig. 2 shows the minimum values of the initial λ which satisfy the stability criterion.

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- The initial choice of λ has to be fine-tuned to more than eight decimal places for the light case as compared with the heavy case $\Rightarrow$ unnatural to have a "light" 4th generation.

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- At the dip where $\lambda = 0$, the Yukawa coupling is large enough to have a "strong Coulomb-like" condensate formation of 4th generation fermions.

Heavy 4th Generation's scenario

Landau Pole vs Quasi-fixed point

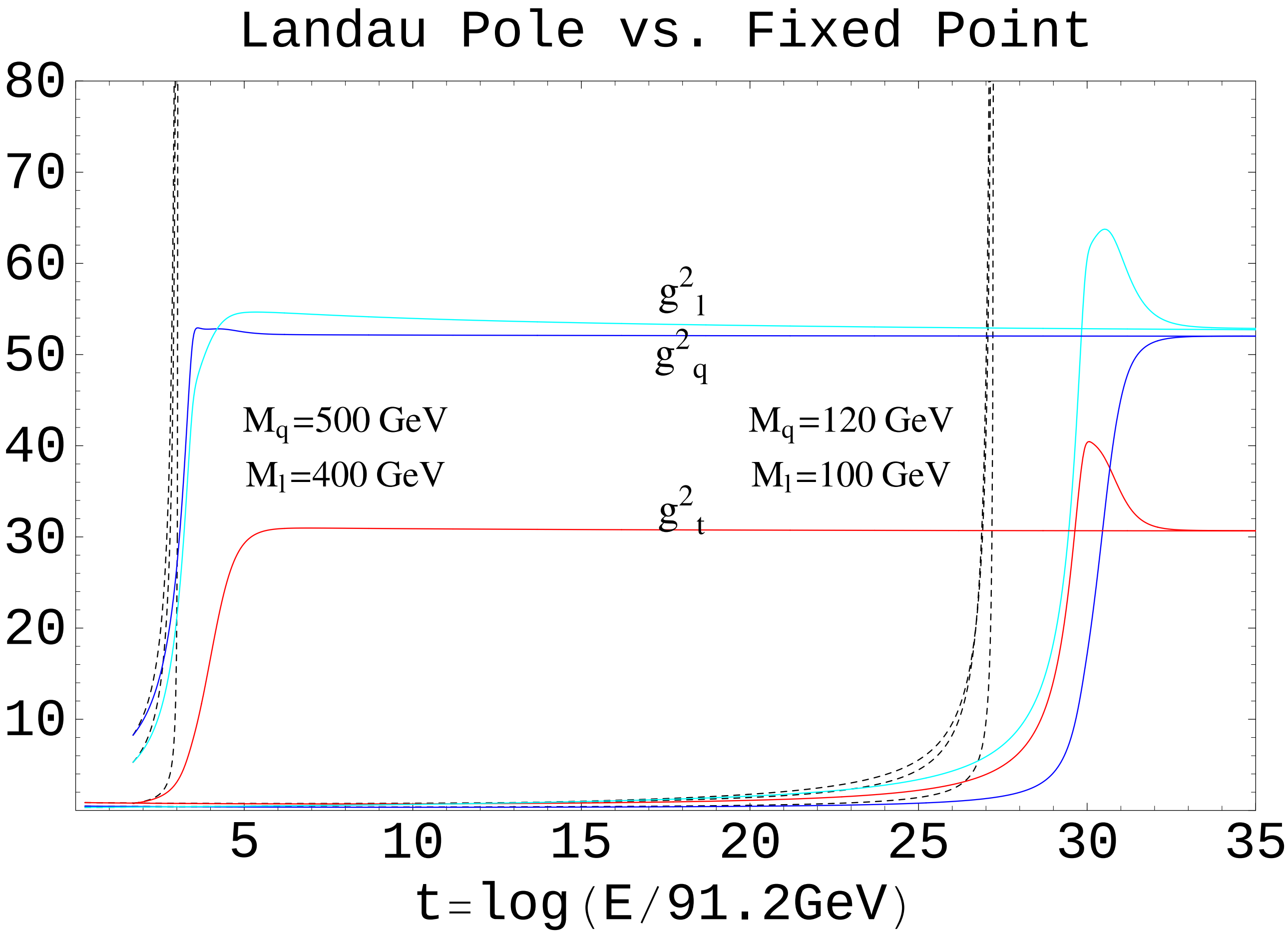

Fig. 2b

Heavy 4th Generation's scenario

Landau Pole vs Quasi-fixed point: Comments

- The Landau pole (one loop) and the Quasi-fixed point occur at around the same place Λ_{FP}.

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Landau Pole vs Quasi-fixed point: Comments

- The Landau pole (one loop) and the Quasi-fixed point occur at around the same place Λ_{FP}.
- Hint at something occuring at Λ_{FP} for a heavy 4th generation: perhaps a scale-invariant theory?

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Bound States and Condensates in Higgs-4 generation Yukawa system

- Bound states of 4th generation quarks and leptons can get formed by the exchange of the Higgs scalar.

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- Bound states of 4th generation quarks and leptons can get formed by the exchange of the Higgs scalar.
- For example, a non-relativistic short-range Yukawa potential for a $\bar{F}F$ bound state can be written as $V(r) = -\alpha_Y(r)\frac{e^{-m_H(r)r}}{r}$ with $\alpha_Y = \frac{m^2}{4\pi v^2}$

Heavy 4th Generation's scenario

Bound States and Condensates in Higgs-4 generation Yukawa system

- Numerical solution to the Schrödinger equation yields a constraint (negative bound state energy): $K_f = \frac{g_f^3}{16\pi\sqrt{\lambda}} > 1.68$.

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- Around the quasi-fixed point, $K_q = 1.82$, $K_l = 1.92$, $K_t = 0.82 \Rightarrow$ No $t\bar{t}$ bound state there.

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- Binding energies: $E_q \approx -4.9$ GeV, $E_q \approx -16$ GeV $\Rightarrow$ Very loose bound states in Region II of the following graph.

Heavy 4th Generation's scenario

Bound States and Condensates in Higgs-4 generation Yukawa system

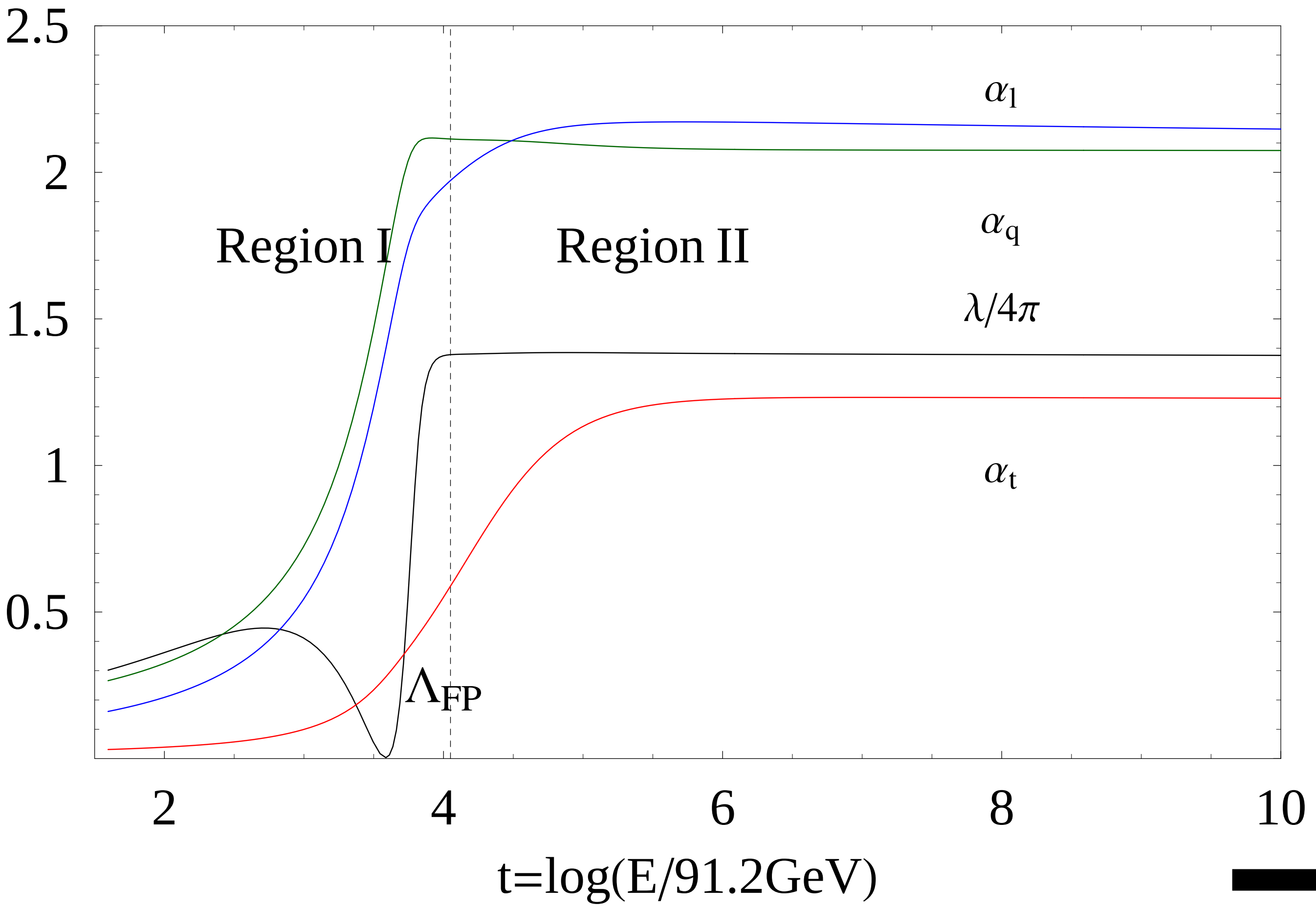

Fig. 8

Heavy 4th Generation's scenario

Bound States and Condensates in Higgs-4 generation Yukawa system

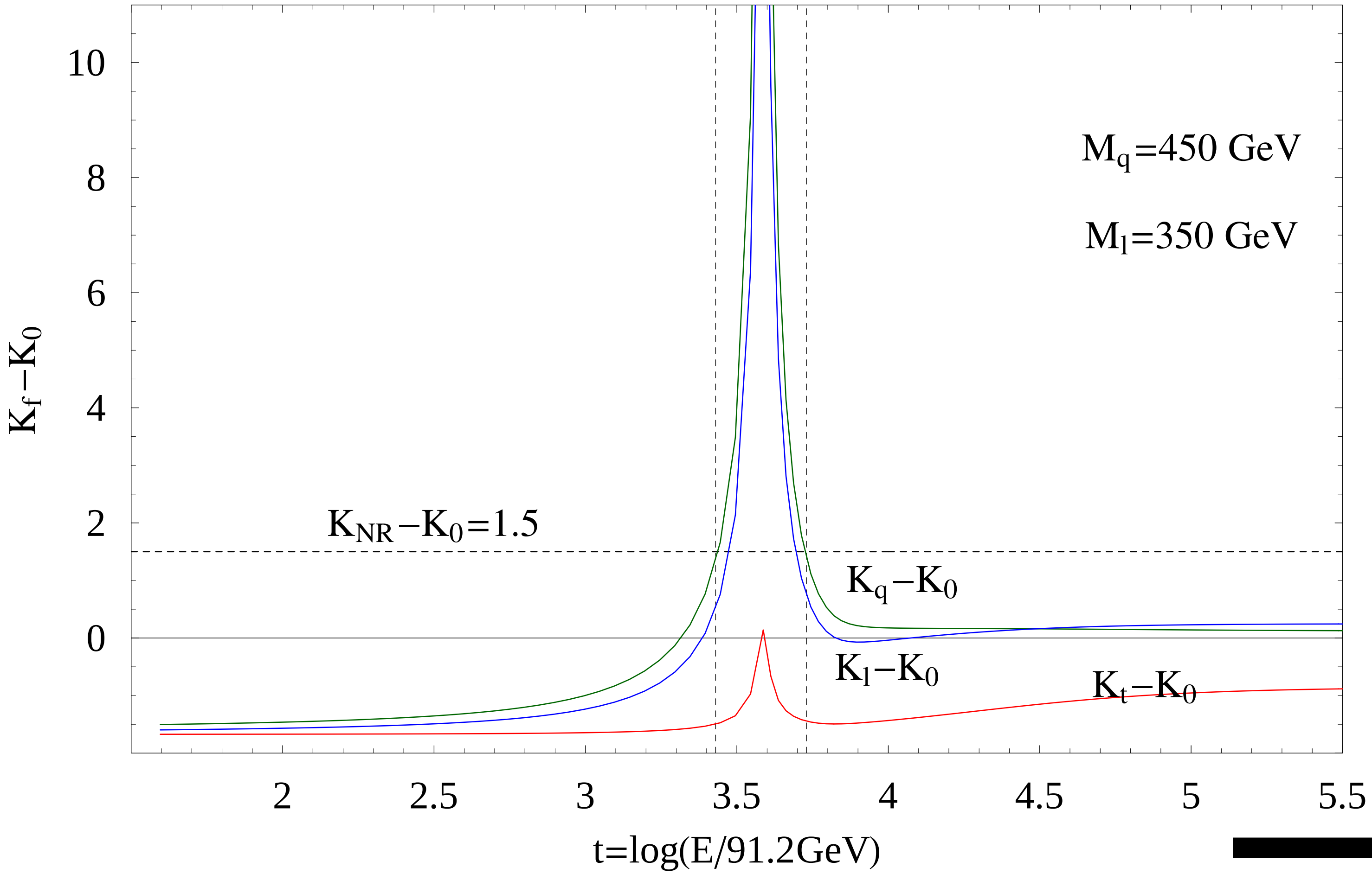

Fig. 4

Heavy 4th Generation's scenario

Implications of Fig. 3 and Fig. 4

- Region II shows the quasi fixed point $\Rightarrow$ Loose bound states of the 4th generation fermions. This corresponds to the right hand side of Fig. 4.

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- Region II shows the quasi fixed point $\Rightarrow$ Loose bound states of the 4th generation fermions. This corresponds to the right hand side of Fig. 4.
- Region I shows the "dip" where $\lambda \rightarrow 0$. As one approaches the dip, the correlation length $\xi_H \sim 1/m_H$ goes from a short-range correlation (small ξ) to an infinite-range correlation ($\xi = \infty$) with a "strong Coulomb-like potential" $V_{dip}(r) = -\frac{\alpha_Y}{r}$ $\Rightarrow$ Possibility of condensate formation.

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- A richer vacuum structure due to this possibility of condensate formation of 4th generation fermions.
- There could be an interesting phenomenology with these new composite Higgses.

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- The vacuum structure (and its phenomenological implications) of the SM with a 4th generation is richer and more interesting.
- Much more work needs to be done!