Strongly coupled fourth family
and
order of Electroweak Phase Transition

Y. Kikukawa

Institute of Physics, University of Tokyo

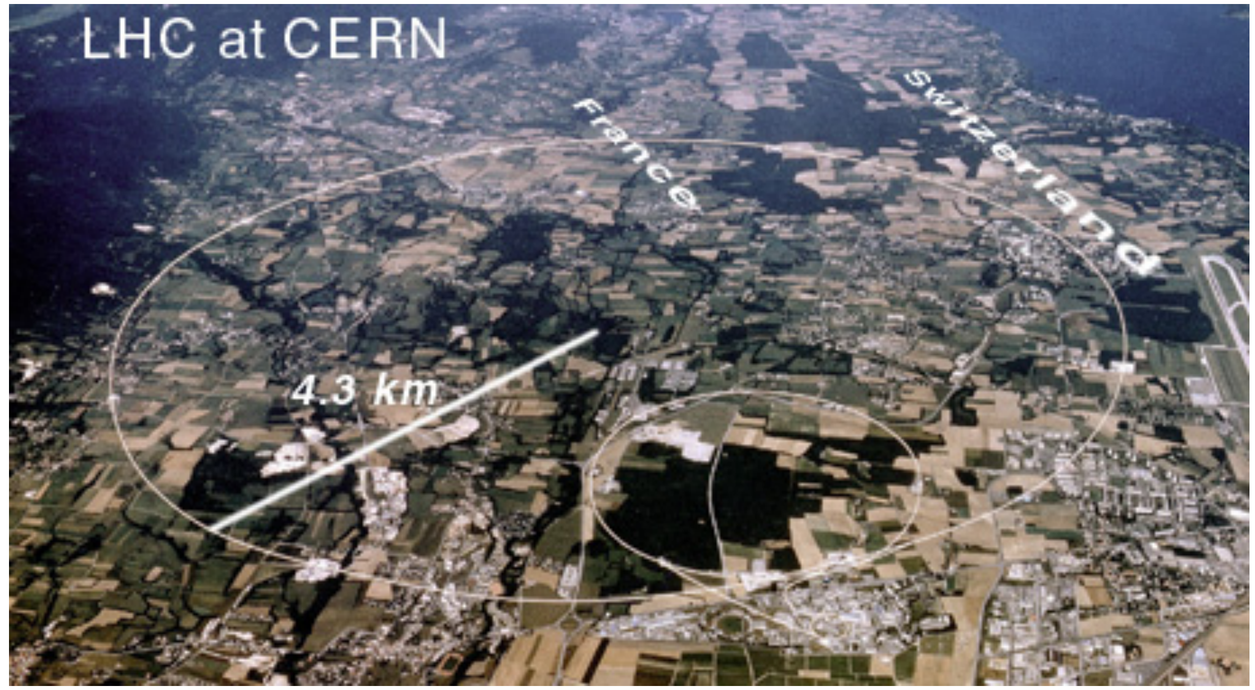
based on :

M . Kohda, J. Yasuda and Y.K.,

Prog. Theor. Phys. 122, No.2, 2009 (arXiv:0901.1962 [hep-ph])

Phys. Rev. D77, 015014 (2008) (arXiv:0709.2221[hep-ph])

the LHC era



Now, LHC is going to reveal

How the Electroweak Gauge Symmetry breaks

"the structure of the Higgs sector"

⇕

How the Electroweak Symmetry restores at finite temperature in the early Universe

"the behavior of the EW phase transition"

Baryogenesis

$$\eta \equiv \frac{n_B}{n_\gamma} = (4.7 - 6.5) \times 10^{-10}$$

- *required for big-bang nucleosynthesis*
- *measured thorough CMB (WMAP)*

Sakharov's three conditions

- *Baryon # non-conservation*
- *C, CP symmetry violation*
  (SU(2)xU(1) chiral int., Kobayashi-Maskawa Matrix)
- *Out of Equilibrium*
  (1st order restoration of EW symmetry at finite temp.)

SM can in principle satisfy the three conditions !

But, in real life ...

$m_H > 114$ GeV **crossover or 2nd order EWPT**

Kajantie et al. (1996)
Csikor et al. (1999)

$$\epsilon_{CP} = \frac{1}{T_c^{12}} \prod_{i>j;u,c,t} (m_i^2 - m_j^2) \prod_{i>j;d,s,b} (m_i^2 - m_j^2)\, J_{CP} \simeq 10^{-19}$$

the effect of KM phase suppressed

Farrar and Shaposhnikov (1994)
Gavela, et al. (1994)
Huet and Sather (1995)

Baryogenesis

$$\eta \equiv \frac{n_B}{n_\gamma} = (4.7 - 6.5) \times 10^{-10}$$

- *required for big-bang nucleosynthesis*
- *measured thorough CMB (WMAP)*

Sakharov's three conditions

- *Baryon # non-conservation*
- *C, CP symmetry violation*
  (SU(2)xU(1) chiral int., Kobayashi-Maskawa Matrix)
- *Out of Equilibrium*
  (1st order restoration of EW symmetry at finite temp.)

SM can in principle satisfy the three conditions !

But, in real life ...

$m_H > 114$ GeV **crossover or 2nd order EWPT**

Kajantie et al. (1996)
Csikor et al. (1999)

$$\epsilon_{CP} = \frac{1}{T_c^{12}} \prod_{i>j;u,c,t} (m_i^2 - m_j^2) \prod_{i>j;d,s,b} (m_i^2 - m_j^2)\, J_{CP} \simeq 10^{-19}$$

Do not give up !

the effect of KM phase suppressed

Farrar and Shaposhnikov (1994)
Gavela, et al. (1994)
Huet and Sather (1995)

Electroweak Phase Transition

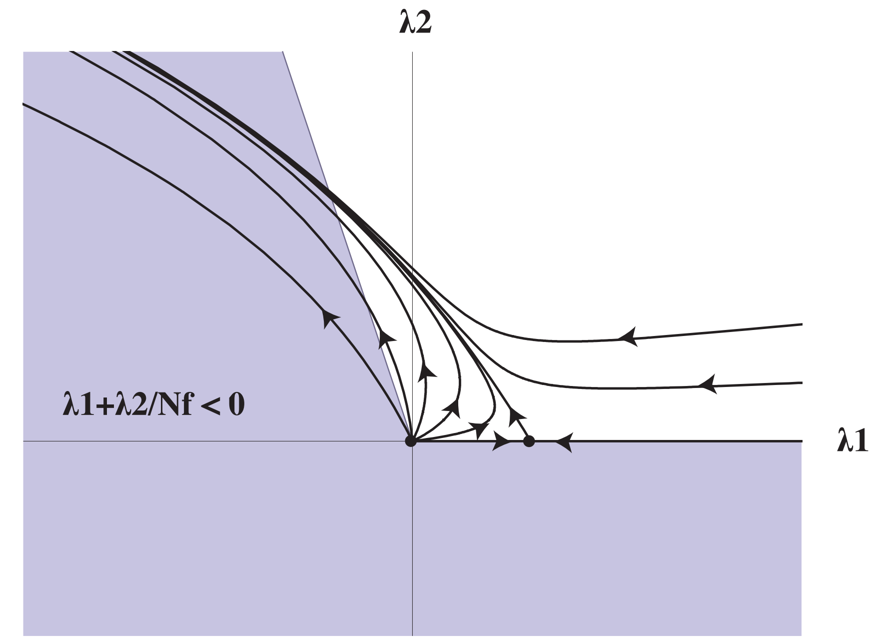
the Higgs sector in SM = O(4) linear sigma model

the finite-temperature PT governed by
"Wilson-Fisher IR-stable fixed point"
--> 2nd. order PT

Once the Higgs sector is extended to include multiple scalar fields

--> multiple quartic couplings (run away directions)

--> fluctuation-induced 1st. order PT



ex. massless QCD with Nf >= 3

SU(Nf) x SU(Nf) linear sigma model
--> no stable IR fixed point
--> chiral symmetry restoration cannot be 2nd. order PT

Pisarski-Wilczek (1984)

$$L = + \; \mathrm{tr}(\partial_\mu \Phi^\dagger \partial^\mu \Phi) - m_\Phi^2 \, \mathrm{tr}\, \Phi^\dagger \Phi - \frac{\lambda_1}{2}(\mathrm{tr}\, \Phi^\dagger \Phi)^2 - \frac{\lambda_2}{2}\, \mathrm{tr}(\Phi^\dagger \Phi)^2 + c(\det \Phi + c.c.)$$

walking Technicolor theories with large Nf (>= 3)

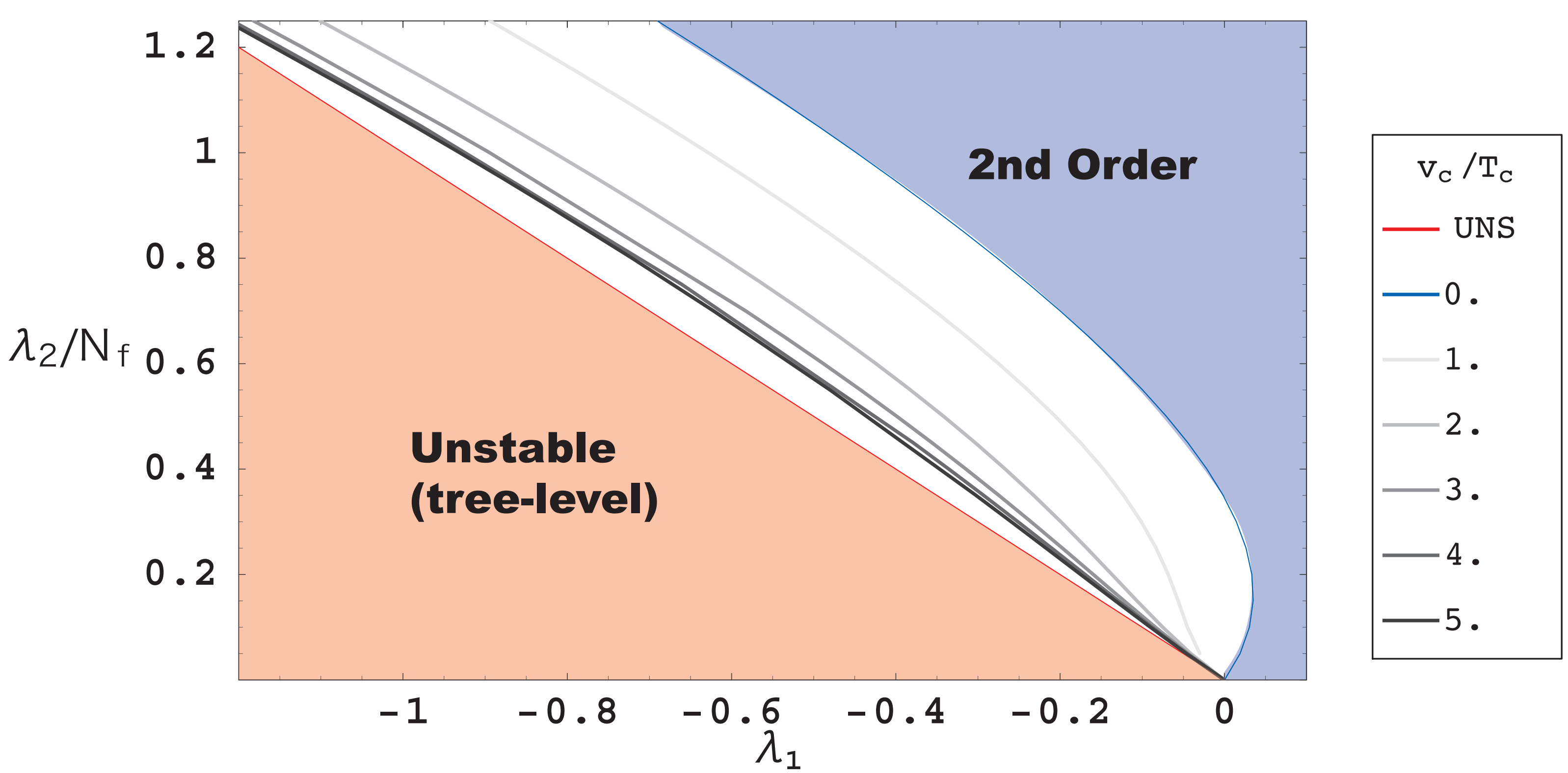
--> Chiral symmetry restoration,
i.e. Electroweak symmetry restoration,
cannot be 2nd. order PT !

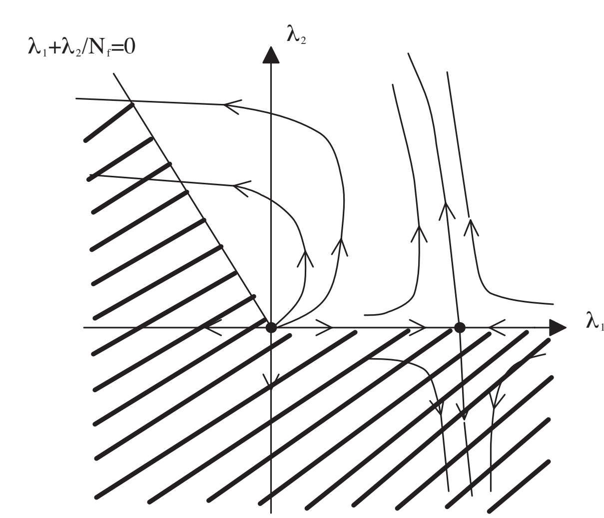
cf. M . Kohda, J. Yasuda and Y.K., PRD77, 015014 (2008) (arXiv:0709.2221)
"1st. order restoration of SU(Nf)$_L$xSU(Nf)$_R$ chiral symmetry
with large Nf and electroweak phase transition"

"semi-quantitative estimate of the strength of 1st. order PT" $\frac{\phi_c}{T_c} \gtrsim 1$ ⇐ *To avoid washout of BAU, B-violating sphaleron processes should decouple just after EWPT*

$$\Gamma_B \sim e^{-E_{sph}/T}|_{T=T_c}$$

$$E_{sph} = \frac{4\pi\phi}{g_2} \times (1.5 - 2.7)$$





Fourth Family

still consistent with experiments !

Holdom (2006)

Kribs , Plehn, Spannowsky and Tait (2007)

PDG 2008 .....

neutrino number (N_ν=3?), mass

direct searches(Tevatron, LEPII) $m_{t'}, m_{b'}$ > 256 GeV $m_{\tau'}, m_{\nu'}$ > 100 GeV

EW precision data $m_{t'} - m_{b'} \sim (1+ \ln(m_h/115\text{GeV})/5) \times 50$ GeV

$m_{\tau'} - m_{\nu'} >$ 30~60 GeV

CP asymmetry (anomaly) in B, K system

Hou et al (2006~)

Soni et al (2008)

If extremely heavy, close to Unitarity bound, couples strongly to the Higss sector

Holdom (1984)

cf. Nambu (1989); Miransky, Tanabashi , Yamawaki(1989)

$$\langle \bar{t}'t' \rangle, \ \langle \bar{b}'b' \rangle, \ \langle \bar{\tau}'\tau' \rangle, \ \langle \bar{\nu}'_\tau \nu'_\tau \rangle$$

order parameter of EW symmetry ⁉

extra CP violating phases in CKM scheme

Hou (2008)

large Jarlskog invariants possible ⁉

Fourth Family

still consistent with experiments !

Holdom (2006)

Kribs , Plehn, Spannowsky and Tait (2007)

PDG 2008 .....

neutrino number (N_ν=3?), mass

direct searches(Tevatron, LEPII) $m_{t'}, m_{b'} > 256$ GeV $m_{\tau'}, m_{\nu'} > 100$ GeV

EW precision data $m_{t'} - m_{b'} \sim (1 + \ln(m_h/115\text{GeV})/5) \times 50$ GeV

$m_{\tau'} - m_{\nu'} > 30\sim60$ GeV

CP asymmetry (anomaly) in B, K system

Hou et al (2006~)

Soni et al (2008)

If extremely heavy, close to Unitarity bound, couples strongly to the Higss sector

Holdom (1984)

$$\langle \bar{t}'t' \rangle, \ \langle \bar{b}'b' \rangle, \ \langle \bar{\tau}'\tau' \rangle, \ \langle \bar{\nu}'_\tau \nu'_\tau \rangle$$

cf. Nambu (1989);
Miransky, Tanabashi , Yamawaki(1989)

order parameter of EW symmetry !?

How about EW phase transition?

cf. Carena, Megevand, Quiros, Wagner (2004)
Fok and Kribs(2008)

extra CP violating phases in CKM scheme

Hou (2008)

large Jarlskog invariants possible !?

Effective theory of Models of Dynamical EW Breaking with Fourth Family

(but, τ' ν' and top omitted; $SU(2)_L \times U(1)_Y$ switched off)

$SU(2)_L \times \underline{SU(2)_R}$ Linear sigma model + t' b'

$$\phi_1 = \begin{pmatrix} \phi_1^0 \\ \phi_1^- \end{pmatrix} \sim \begin{pmatrix} \bar{t}'_R t'_L \\ \bar{t}'_R b'_L \end{pmatrix} \quad \phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix} \sim \begin{pmatrix} \bar{b}'_R t'_L \\ \bar{b}'_R b'_L \end{pmatrix} \Rightarrow \Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix}$$

$$\begin{aligned} \mathcal{L} &= \bar{q}' i \partial\!\!\!/ q' - y(\bar{q}'_L \Phi q'_R + c.c.) \\ &+ \mathbf{Z}\, \mathrm{tr}(\partial_\mu \Phi^\dagger \partial^\mu \Phi) - m_\Phi^2 \,\mathrm{tr}\, \Phi^\dagger \Phi \\ &- \frac{\lambda_1}{2} (\mathrm{tr}\, \Phi^\dagger \Phi)^2 - \frac{\lambda_2}{2} \mathrm{tr}(\Phi^\dagger \Phi)^2 + c(\det \Phi + c.c.).(6) \end{aligned}$$

+ compositeness condition

$Z=0$ ⇒ $\bar{\lambda}_1(\mu) \to 0\,, \quad \bar{\lambda}_2(\mu) \to \infty\,, \quad \bar{y}(\mu) \to \infty \qquad \mu \to \Lambda_{4f}$ composite scale

+ mass term of pseudo NG bosons

$$\Delta V(\Phi) = -c(\det \Phi + h.c.)$$

Low energy properties of model

- mass spectra of Higgs bosons (tree)
  - **degeneracy** due to $SU(2)_R$ ($\phi_0 = 246\mathrm{GeV}$)

h : SM like Higgs

$$m_h = \sqrt{\lambda_1 + \lambda_2/N_f}\,\phi_0$$

ξ = (H⁰, H±): extra Higgs

$$m_\xi = \sqrt{2c + (\lambda_2/N_f)\phi_0^2}$$

η : pseudo scalar Higgs

$$m_\eta = \sqrt{2c}$$

⇒ pNGB of $U(1)_A$ breaking

π : would be NG bosons

$$m_\pi = 0$$

- experimental bound on Yukawa coupling (tree)

$$m_{q'} \gtrsim 256\mathrm{GeV} \quad \Rightarrow \quad y \gtrsim 2.1$$

cf. $m_{q'} = \dfrac{y}{\sqrt{2N_f}}\phi_0$

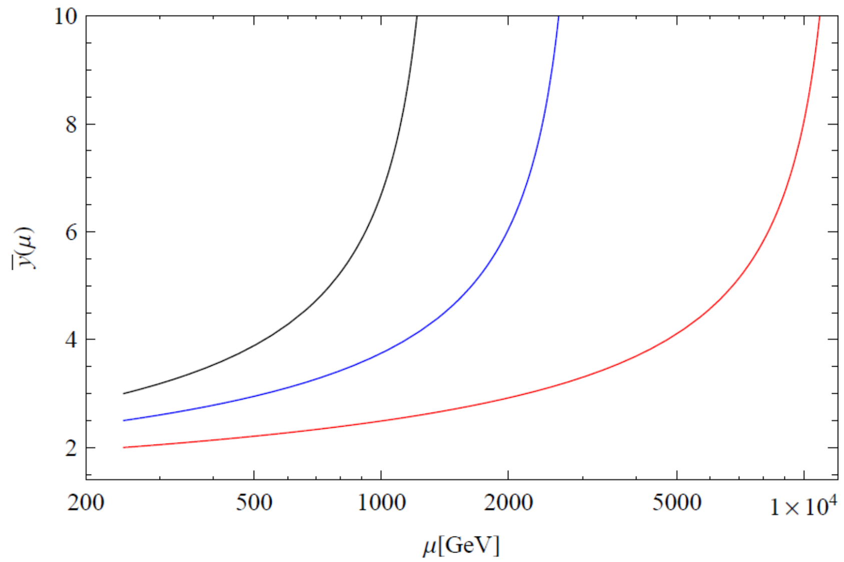
Cutoff scale of the Effective theory

one-loop RG eq.

$$\mu\frac{\partial}{\partial\mu}\bar{\lambda}_1 = \frac{1}{8\pi^2}[(N_f^2+4)\bar{\lambda}_1^2 + 4N_f\bar{\lambda}_1\bar{\lambda}_2 + 3\bar{\lambda}_2^2 + 2N_c y^2\bar{\lambda}_1],$$

$$\mu\frac{\partial}{\partial\mu}\bar{\lambda}_2 = \frac{1}{8\pi^2}(6\bar{\lambda}_1\bar{\lambda}_2 + 2N_f\bar{\lambda}_2^2 + 2N_c y^2\bar{\lambda}_2 - 2N_c\bar{y}^4),$$

$$\mu\frac{\partial}{\partial\mu}\bar{y} = \frac{1}{16\pi^2}(N_f+N_c)\bar{y}^3$$



mass bound --> y >2.1 at μ=246 GeV

blow up around 1〜10 TeV

estimation of cutoff scale Λ

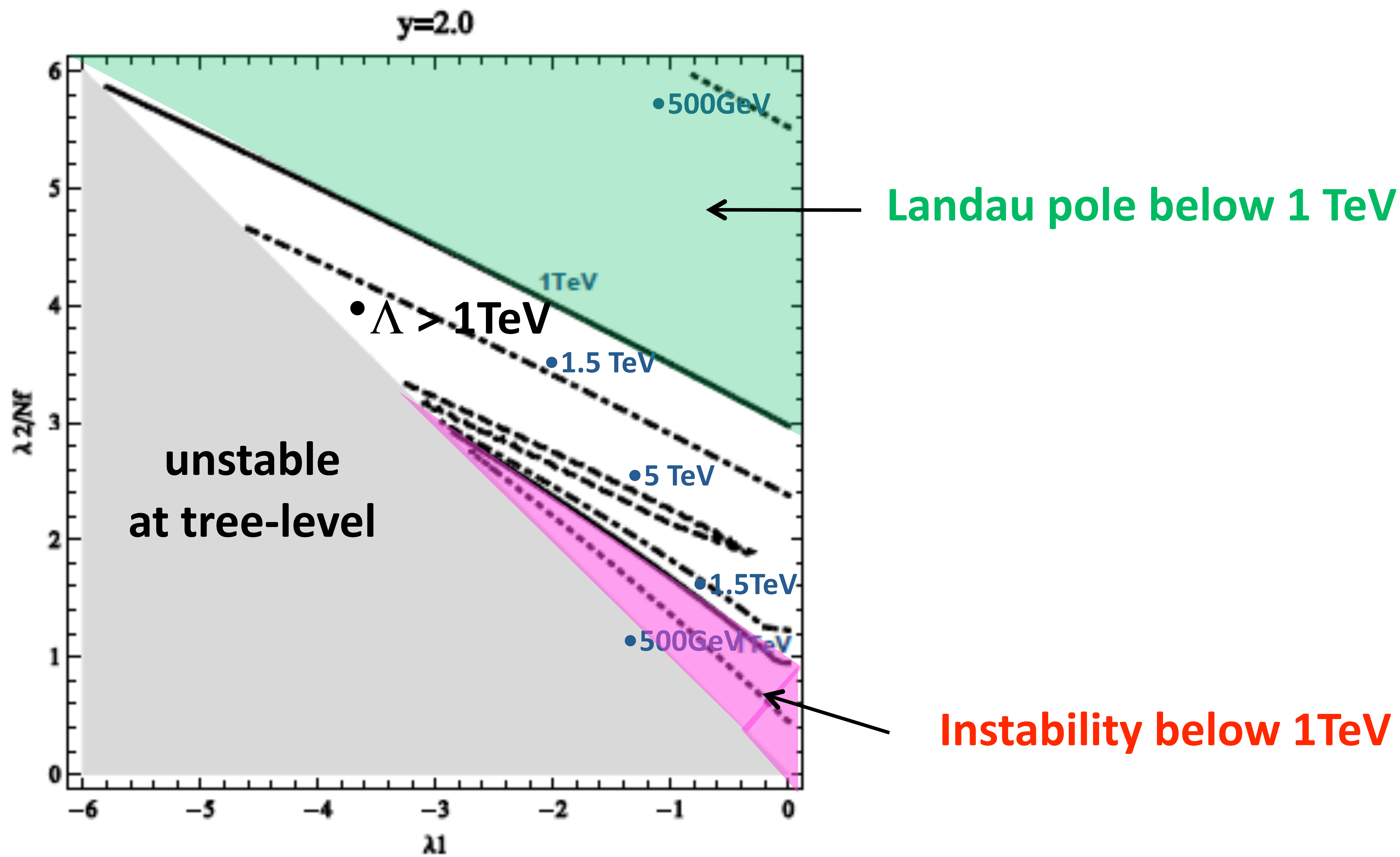
Starting from (λ₁, λ₂, y) at μ=246 GeV,
search the scale Λ at which the vacuum instability or Landau pole occurs :

$$\bar{\lambda}_1(\Lambda) + \bar{\lambda}_2(\Lambda)/N_f = 0, \quad \bar{\lambda}_2(\Lambda) = 0$$

$$\bar{y}(\Lambda)^2 = \frac{16\pi^2}{N_c}, \quad \bar{\lambda}_1(\Lambda) + \bar{\lambda}_2(\Lambda)/N_f = \frac{16\pi^2}{N_f^2}, \quad \bar{\lambda}_2(\Lambda) = \frac{16\pi^2}{N_f}$$

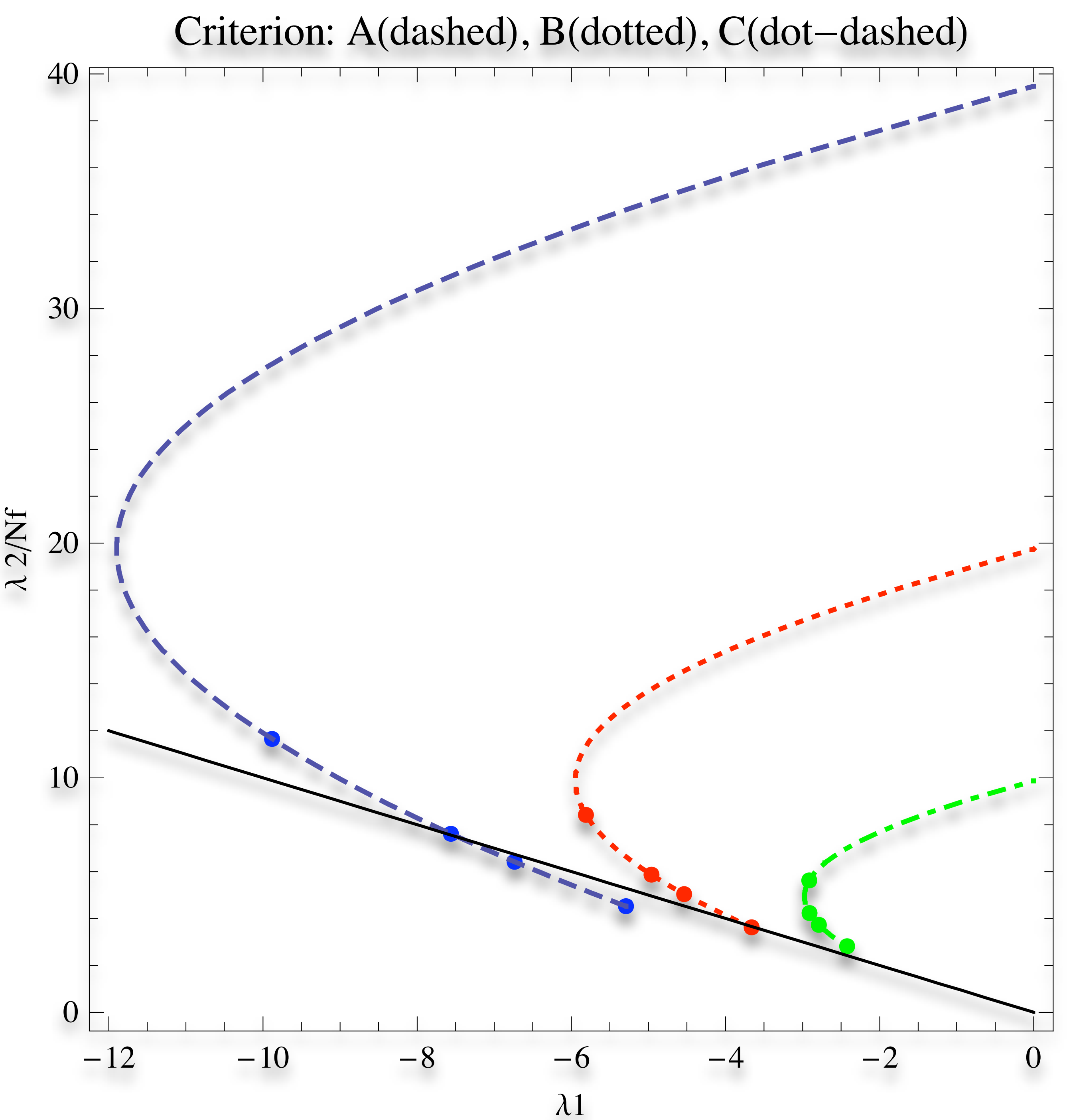
(pertubativity bound)

- contour plot of cutoff Λ

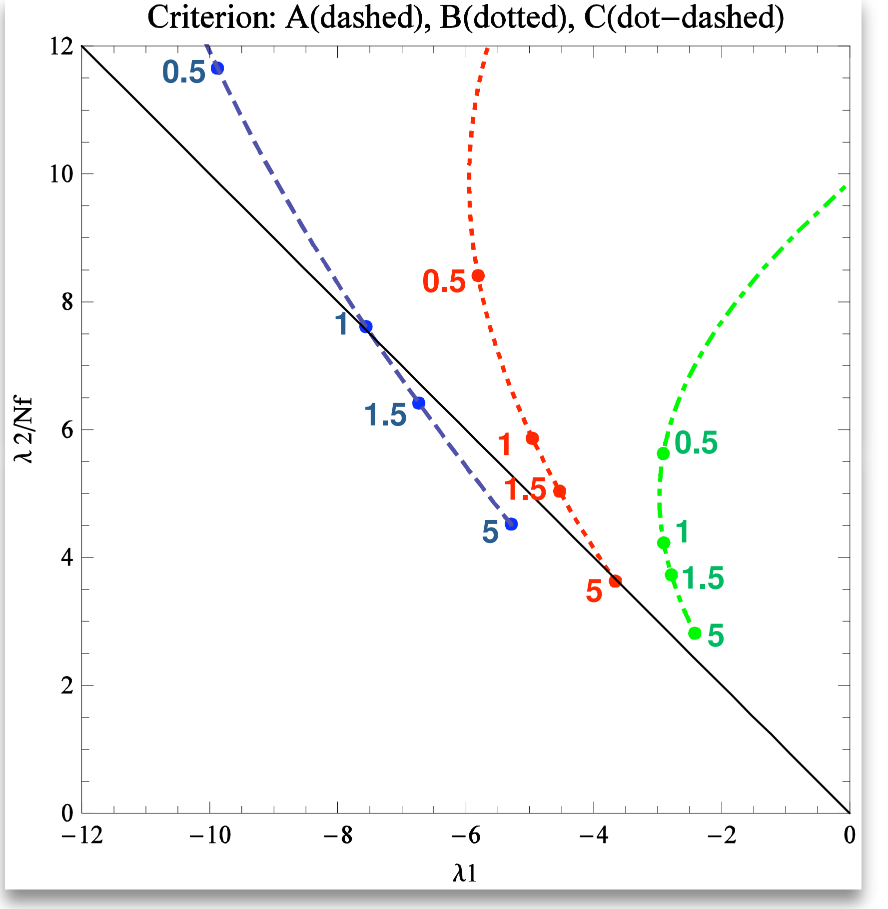


◆ To ensure the renormalizability, Λ >> m

- largest mass scale in the interesting region: $m_\xi \sim 500$ GeV
- exclude the region where Λ < 1TeV



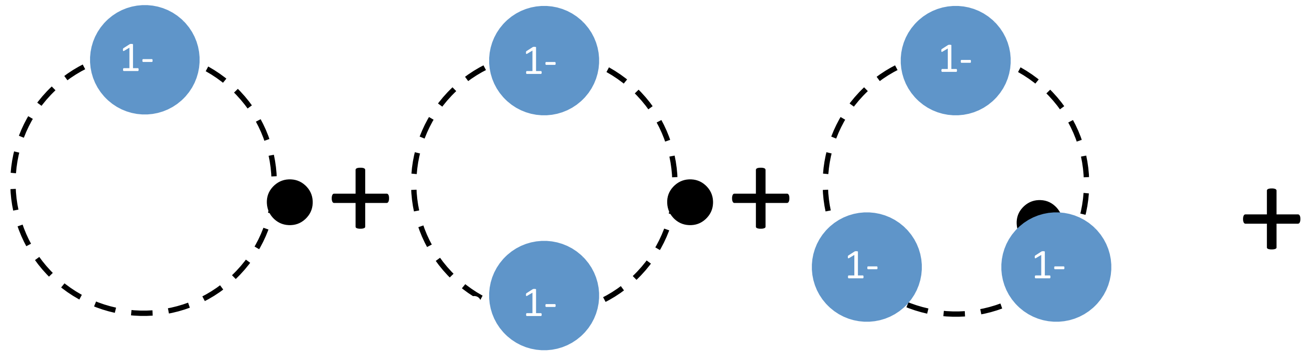
Criterion: A(dashed), B(dotted), C(dot−dashed)
λ 2/Nf
λ1
40
30
20
10
0
−12
−10
−8
−6
−4
−2
0

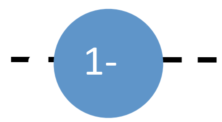


Criterion: A(dashed), B(dotted), C(dot−dashed)
λ 2/Nf
λ1
0.5
1
1.5
5
0.5
1
1.5
5
0.5
1
1.5
5

the Effective theory at finite temperature

one-loop + ring diagram contributions

$V_{\text{ring}}(\phi, T) =$ 

At high T , 

$\sim \lambda_i T^2$

$$
\begin{aligned}
V(\phi, T) =& \frac{1}{2}(m_\Phi^2 - c)\phi^2 + \frac{1}{8}\left(\lambda_1 + \frac{\lambda_2}{N_f}\right)\phi^4 \\
&+ \sum_{i=h,\xi,\eta,\pi,q'} n_i \frac{\mathcal{M}_i^4(\phi,T)}{64\pi^2}\left[\ln\frac{\mathcal{M}_i^2(\phi,T)}{\mu^2} - \frac{3}{2}\right] + \frac{1}{2}A\phi^2 \\
&+ \sum_{i=h,\xi,\eta,\pi,q'} n_i \frac{T^4}{2\pi^2}\int_0^\infty dx\, x^2 \ln\left[1 \mp \exp\left(-\sqrt{x^2 + \mathcal{M}_i^2(\phi,T)/T^2}\right)\right]
\end{aligned}
$$

$$
\begin{cases}
\mathcal{M}_i^2(\phi,T) = m_i^2(\phi) + [(N_f^2+1)\lambda_1 + 2N_f\lambda_2 + y^2]\dfrac{T^2}{12} \\
\mathcal{M}_{q'}^2(\phi,T) = m_{q'}^2(\phi)
\end{cases}
$$

the Effective theory at finite temperature (cont'd)

high temperature expansion

$$\Delta V \sim \Delta E T \phi^3 \qquad \Delta E \sim (\lambda_2/N_f)^{3/2} \sim m_\xi^3$$

extra Higgs ξ induce cubic term for $m_\xi >> m_h$

$$\Rightarrow \quad \frac{\phi_c}{T_c} \sim \frac{(\lambda_2/N_f)^{3/2}}{(\lambda_1 + \lambda_2/N_f)} \sim \frac{m_\xi^3}{m_h^2}$$

strongly 1st. order for $m_\xi >> m_h$

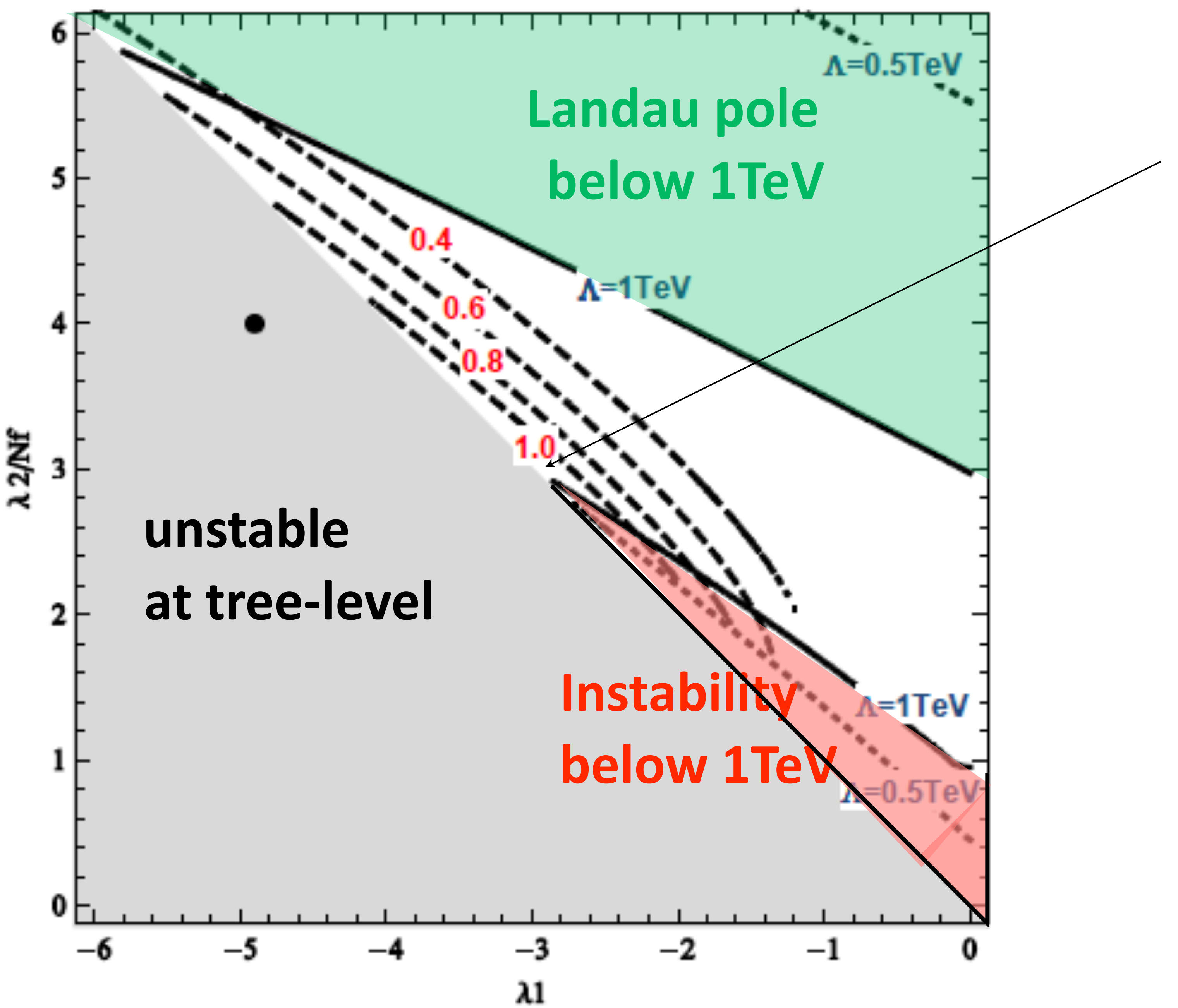
cf. Carena, Megevand, Quiros, Wagner (2004)

- *valid for λi , y << 1*
- *cannot rely on high-T expansion for strongly coupled 4th family*

Numerical results (1)

- Contour plot for various $\phi c/Tc$ on $\lambda 1$-$\lambda 2/Nf$ plane

y=2 ($m_{q'}$=246 GeV) and m_η=0

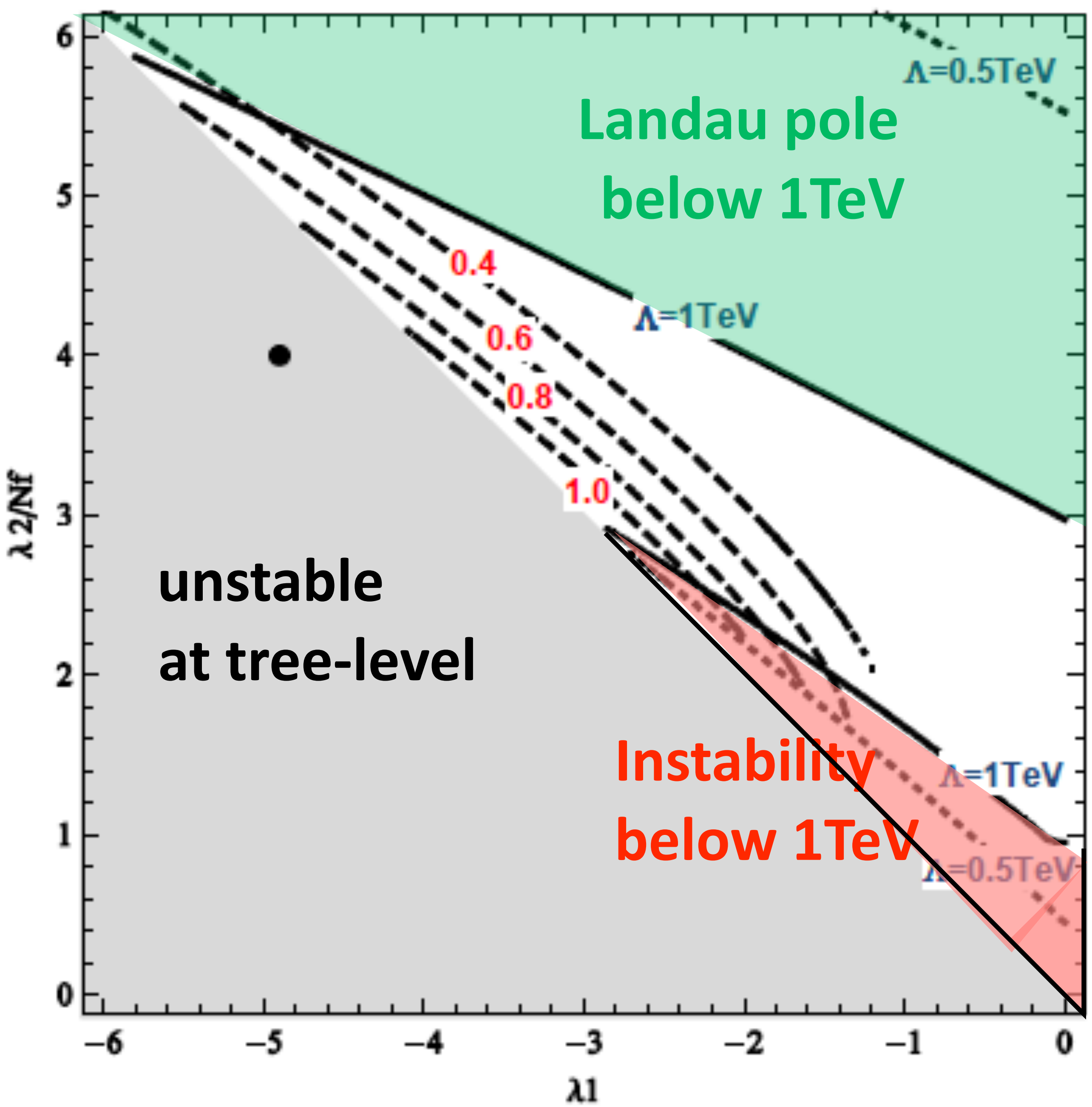


$$\frac{\phi_c}{T_c} \gtrsim 1$$

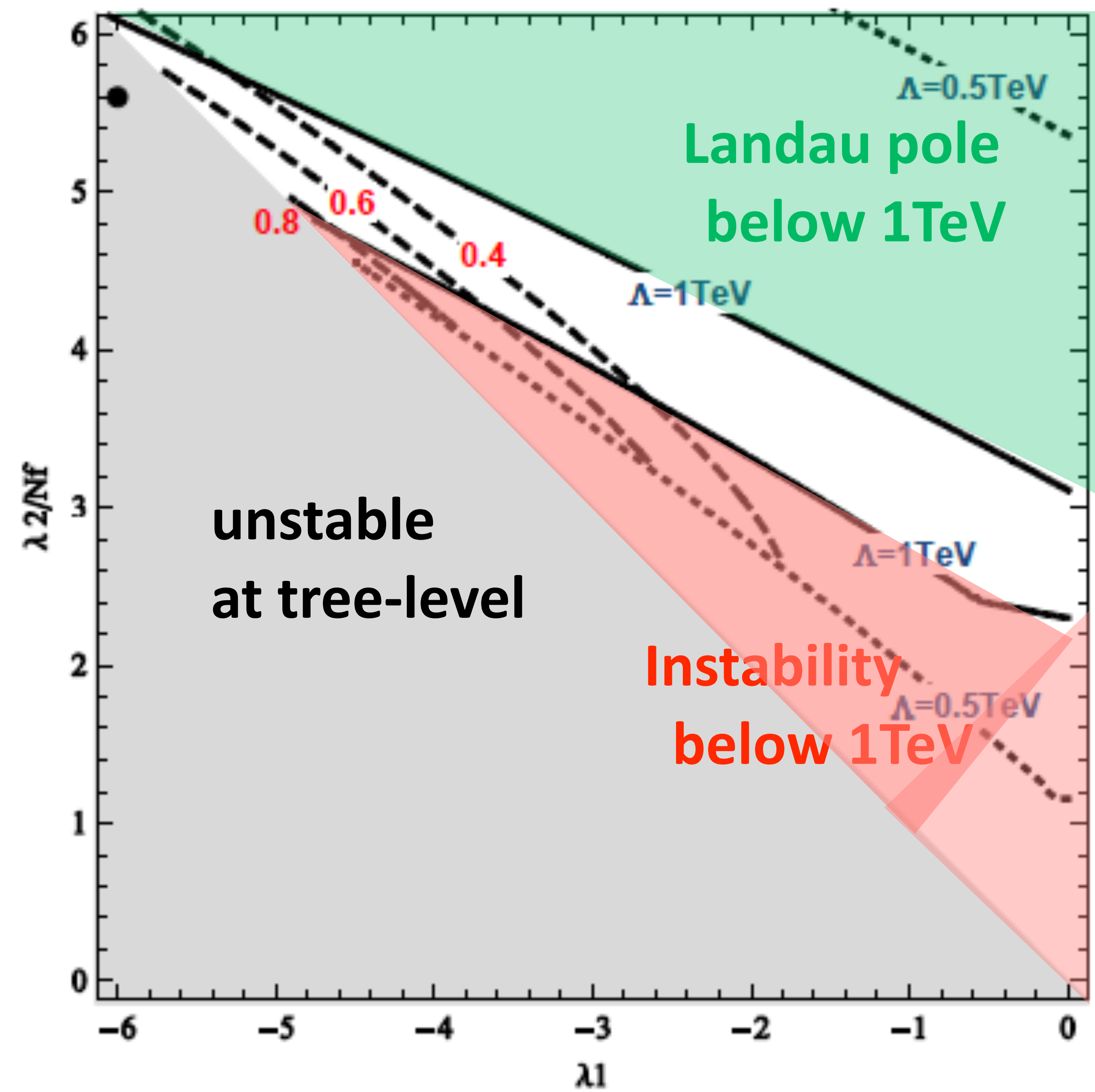
Numerical results (1)

◆ Contour plot for various ϕ_c/T_c on λ_1-λ_2/N_f plane

y=2 ($m_{q'}$=246 GeV) and m_η=0



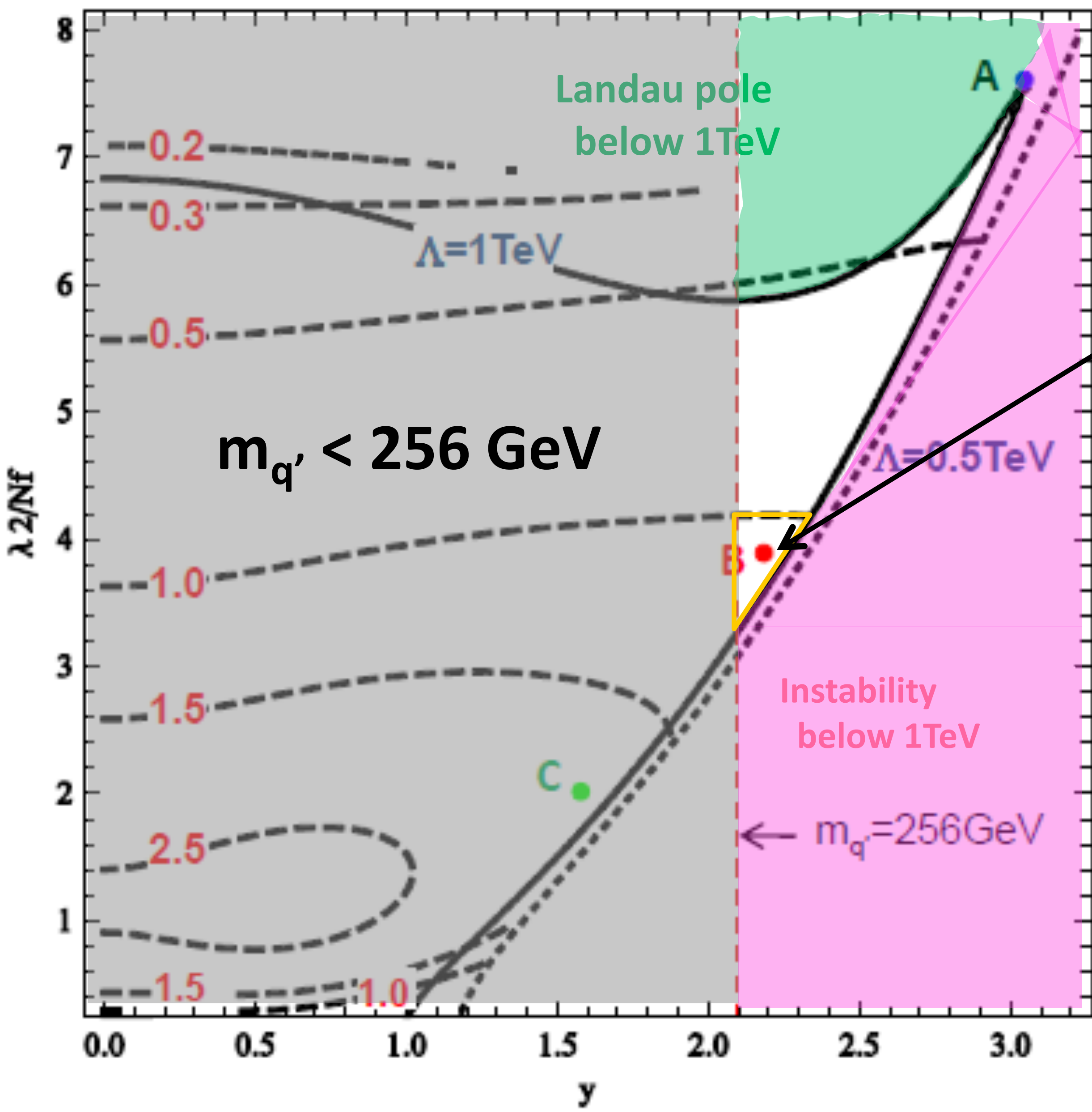
y=2.5 ($m_{q'}$=310 GeV) and m_η=0



Numerical results (2)

◆ Contour plot for various ϕc/Tc on y-λ2/Nf plane

λ1+λ2/Nf ～ 0 and m_η=0



$$\phi_c/T_c \geq 1$$

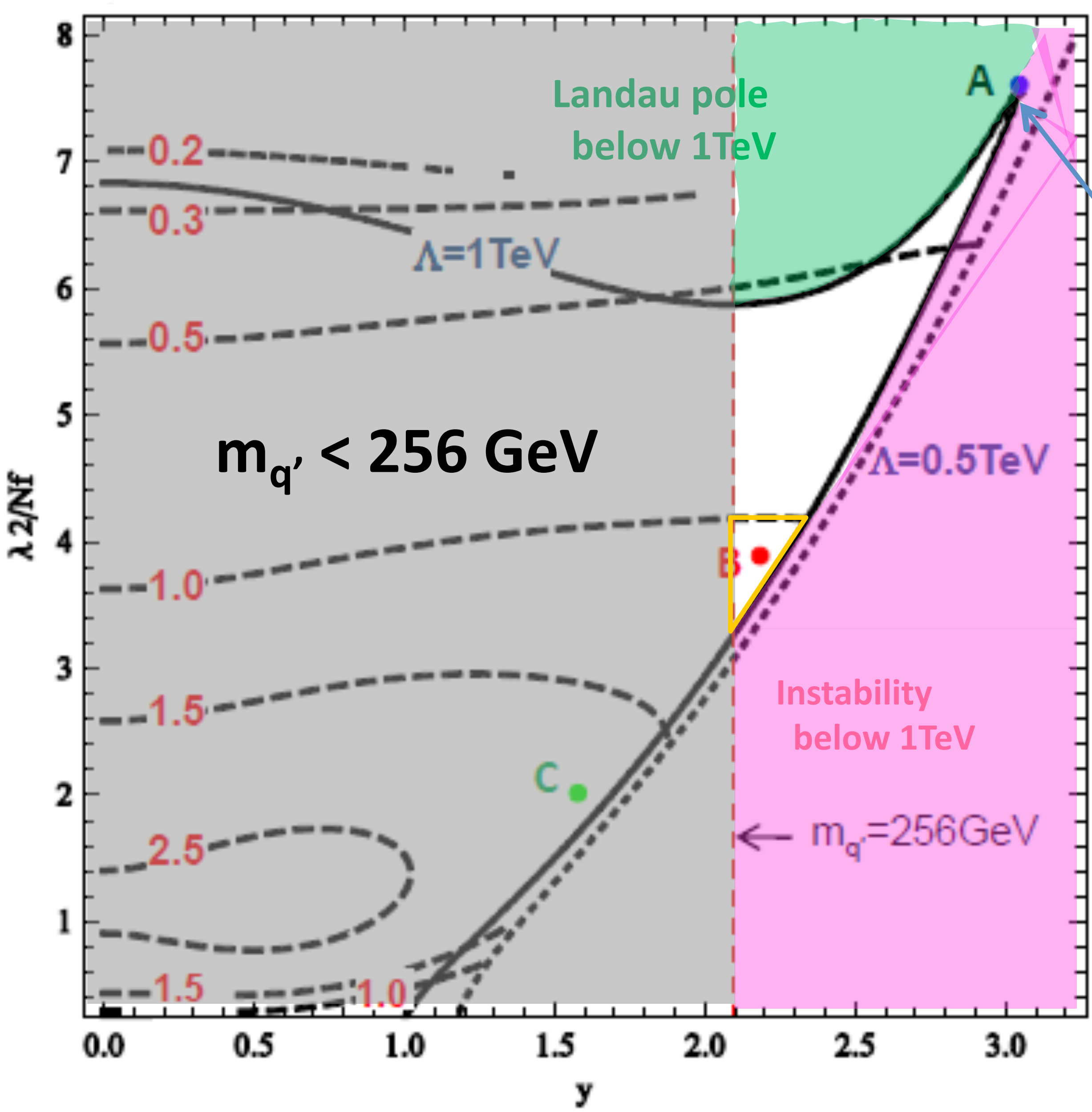
upper bound on λ2/Nf and y
⇒ upper bound on m_ξ and $m_{q'}$

256 GeV < $m_{q'}$ < 290 GeV
430GeV < m_ξ < 500 GeV
200 GeV < m_h(1-loop) < 300 GeV

Numerical results (2)

◆ Contour plot for various ϕc/Tc on y-λ2/Nf plane

λ1+λ2/Nf ～ 0 and m_η=0



compositeness condition

$$\lambda_1(\Lambda) = 0,\ \lambda_2(\Lambda) = \frac{16\pi^2}{N_f},\ y^2(\Lambda) = \frac{16\pi^2}{N_c}$$

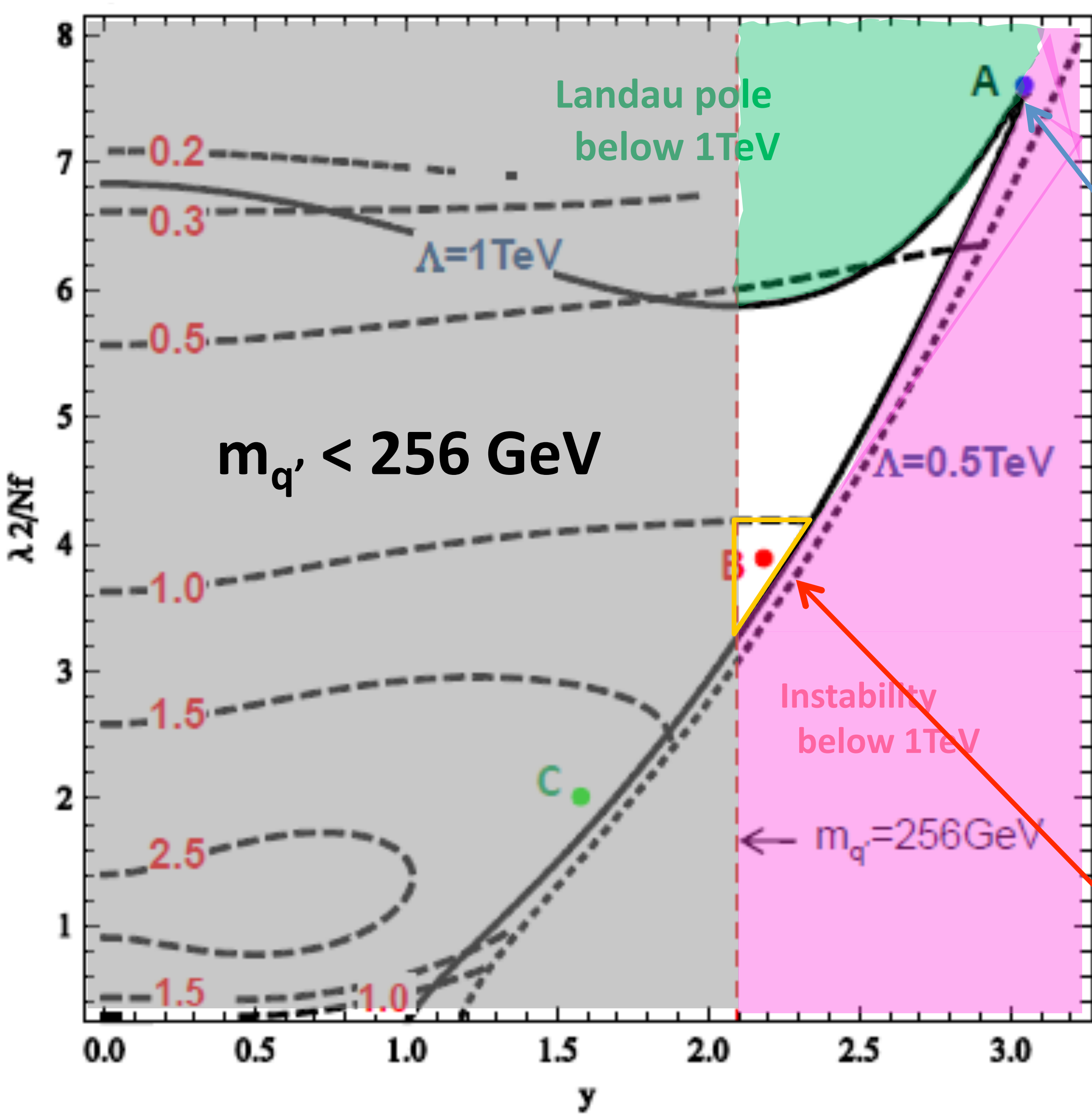
Λ = 1 TeV

⇒ ϕc/Tc ～ 0

Numerical results (2)

◆ Contour plot for various ϕc/Tc on y-λ2/Nf plane

λ1+λ2/Nf ∼ 0 and m_η=0



compositeness condition

$$\lambda_1(\Lambda) = 0,\ \lambda_2(\Lambda) = \frac{16\pi^2}{N_f},\ y^2(\Lambda) = \frac{16\pi^2}{N_c}$$

Λ = 1 TeV

⇒ ϕc/Tc ∼ 0

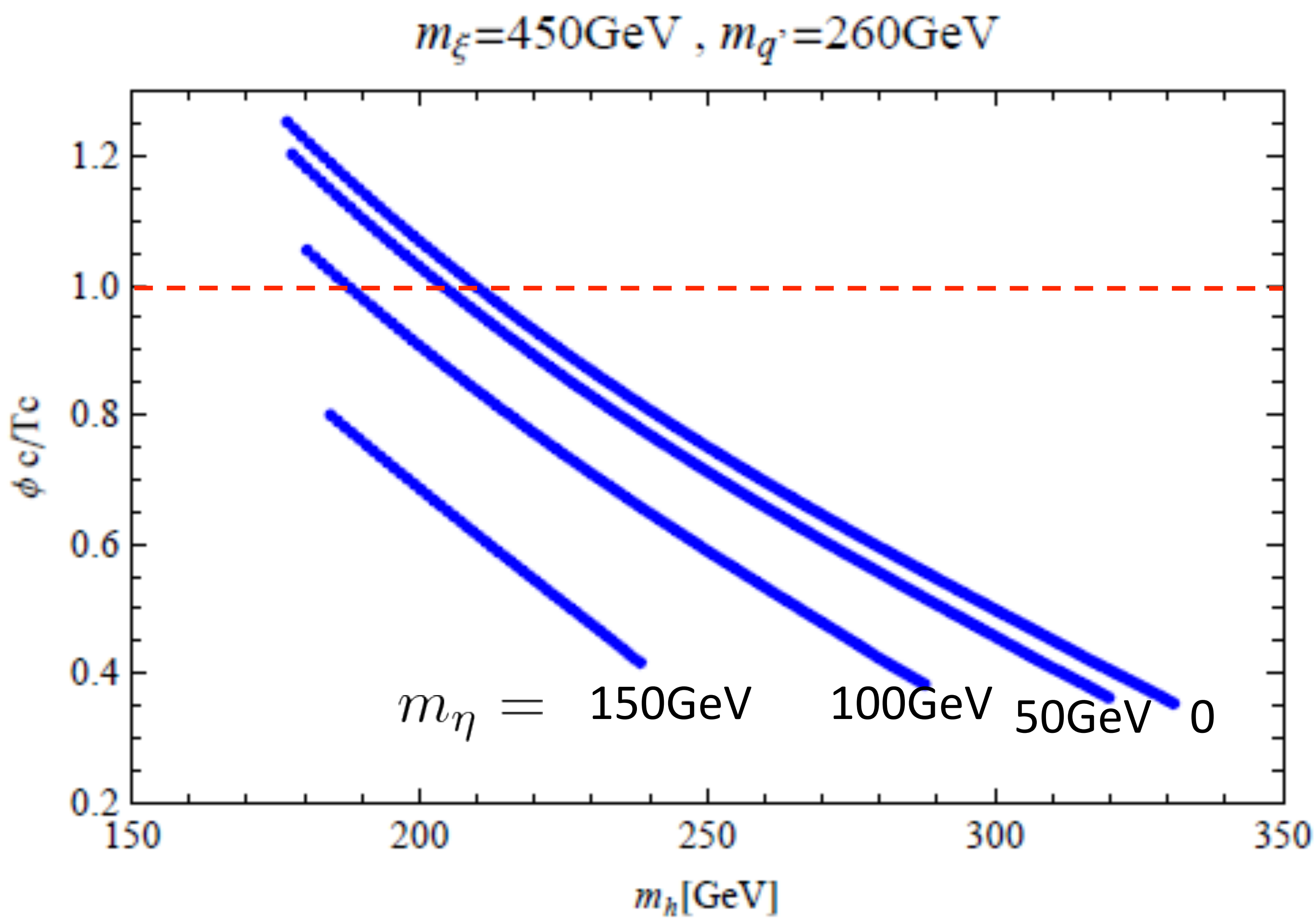
$$\lambda_1(\Lambda) = 0,\ \lambda_2(\Lambda) = \frac{8\pi^2}{N_f},\ y^2(\Lambda) = \frac{8\pi^2}{N_c}$$

Λ = 4 TeV

⇒ ϕc/Tc > 1

Numerical results (3)

◆ effect of pseudo scalar Higgs mass : m_η



For larger m_η , ϕc/Tc becomes smaller

for srongly 1st order PT, $m_\eta \lesssim$ 100 GeV

Summary & Discussion

with extremely heavy/ strongly coupled fourth family,

- EW vacuum fluctuation may be described by multiple scalar(Higgs) fields
- EWPT at finite temperature can be first order due to "fluctuation-induced first order PT" (due to the absence of IR stable fixed point)

"semi-quantitative" estimate of the strength of the 1st. order PT with an effective field theory:

$SU(2)_L \times \underline{SU(2)_R}$ Linear sigma model + t' b' + Cutoff Λ + Compositeness cond.

τ' ν' and top omitted; $SU(2)_L \times U(1)_Y$ switched off

one-loop, ring-improved finite-temperature effective pot. (perturbativity)

- a region with strongly first order PT in Yukawa theory caused by extra heavy Higgs (cubic term in high T exp.) narrow around
  $m_{q'}$ =260 GeV, m_h =180GeV, m_ξ =450GeV, m_η=100 GeV
- does not overlap with the compositeness cond.
- perturbativity --> seems hard to probe larger Yukawa coupling
- $SU(2)_R$ breaking (mass splitting, top), leptons should be included

Back Up Slides

Nambu – Jona-Lasinio model of 4th family

◆ EW symmetry breaking scale is given by

Pagels-Stokar formula :

$$v^2 = f_\pi^2 \simeq \frac{N_f N_c m_{q'}^2}{8\pi^2} \ln \frac{\Lambda^2}{m_{q'}^2} \qquad (\ \Lambda >> m_{q'}\ )$$

Nf =2 : # of 4th family quark flavor

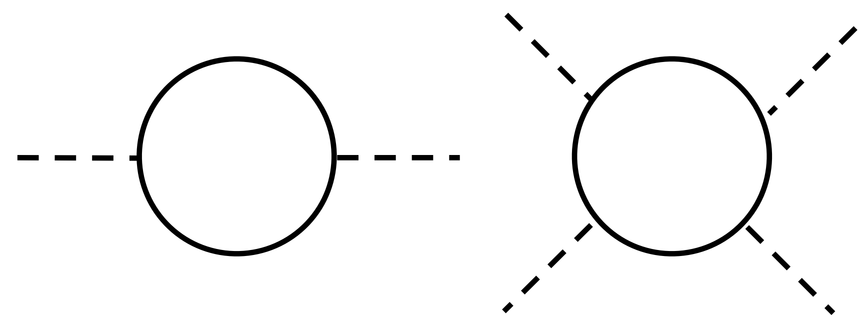
Construction of effective theory

• [Bardeen, Hill and Lindner]

$\mu = \Lambda_{4\mathrm{f}}$ $\quad \mathcal{L}_{4\mathrm{f}} = G_{q'}(\bar{q}'_{Li} q'_{Rj})(\bar{q}'_{Rj} q'_{Li})$

• auxiliary field: $\Phi_{ij} \sim \bar{q}'_{Rj} q'_{Li}$

$$\mathcal{L}'_{4\mathrm{f}} = -y_0(\bar{q}'_{Li}\Phi_{ij} q'_{Rj} + h.c.) - m^2_{\Phi 0} tr(\Phi^\dagger \Phi) \qquad G_{q'} = y_0^2/m^2_{\Phi 0}$$



$\mu \lesssim \Lambda_{4\mathrm{f}}$

$$\mathcal{L}''_{4\mathrm{f}} = -y_0(\bar{q}'_{Li}\Phi_{ij} q'_{Rj} + h.c.) + Z_\Phi tr(\partial^\mu \Phi^\dagger \partial_\mu \Phi) - m^2_{\Phi,0} tr(\Phi^\dagger \Phi)$$
$$-\frac{\lambda_{1,0}}{2}[tr(\Phi^\dagger \Phi)]^2 - \frac{\lambda_{2,0}}{2} tr(\Phi^\dagger \Phi)^2$$

• When, $\mu \to \Lambda_{4\mathrm{f}}$•, $\quad Z_\Phi \to 0 \quad \lambda_{1,0} \to 0 \quad \lambda_{2,0} \to 0$

◆ by rescaling the field $\Phi \to \Phi/\sqrt{Z_\Phi}$

$\mu << \Lambda_{\rm 4f}$

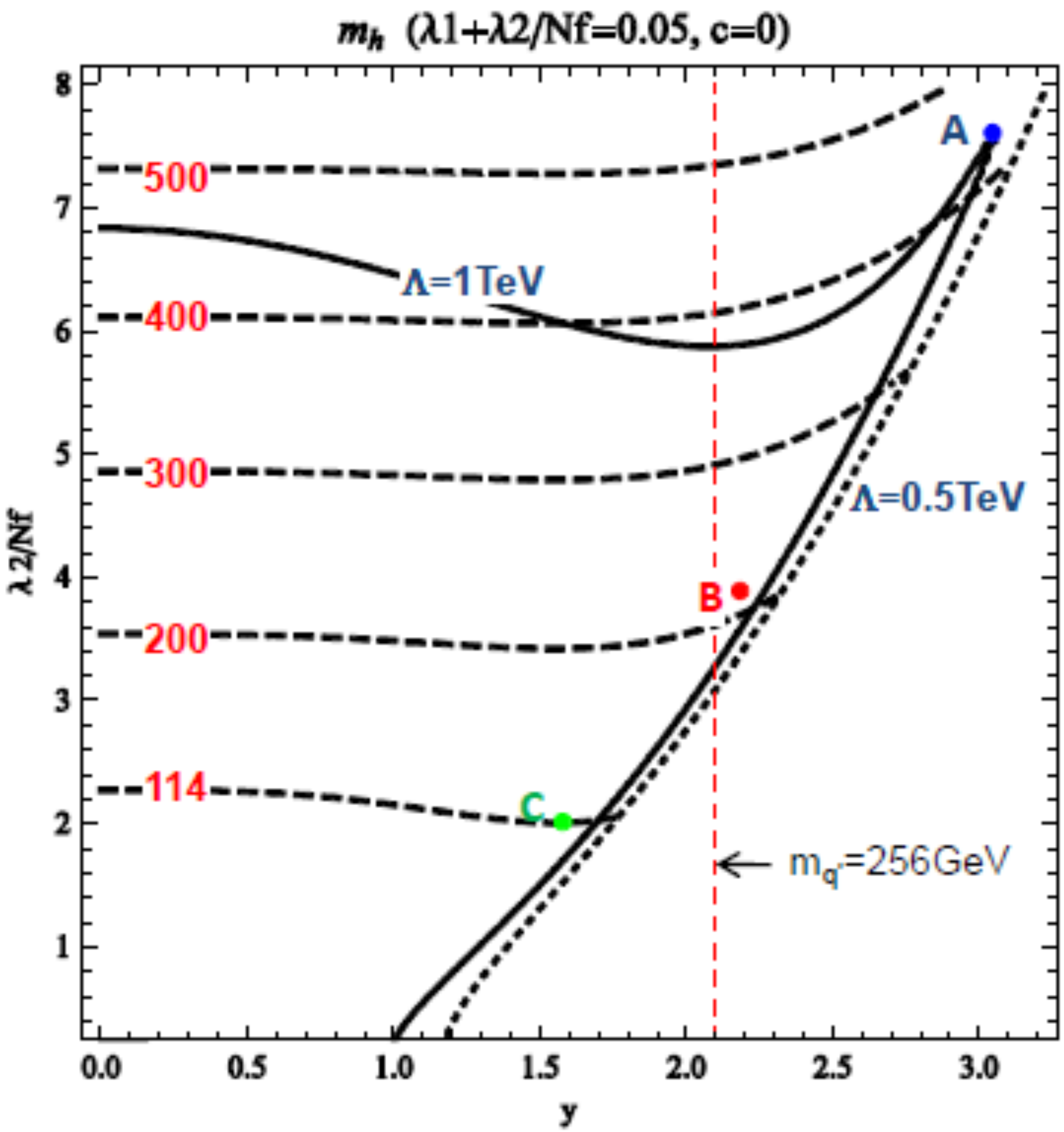
$$\mathcal{L} = \bar{q}' i\gamma^\mu \partial_\mu q' - y(\bar{q}'_L \Phi q'_R + h.c.) + \underline{tr(\partial_\mu \Phi^\dagger \partial^\mu \Phi)} - m_\Phi^2 tr(\Phi^\dagger \Phi) - \frac{\lambda_1}{2}[tr(\Phi^\dagger \Phi)]^2 - \frac{\lambda_2}{2} tr(\Phi^\dagger \Phi)^2$$

◆ compositeness condition

when $\mu \to \Lambda_{\rm 4f}$

$$\lambda_1(\mu) \to 0 \qquad \lambda_2(\mu) \to \infty \qquad y(\mu) \to \infty$$

one-loop Higgs mass on y-λ2/Nf plane (m_η=0)



the Glashow-Weinberg-Salam model

(SU(2)x U(1) sector of the standard model without SU(3) color int.)

- a chiral gauge theory with $SU(2)_L \times U(1)_Y$
- gauge symmetry breaking via Higgs mechanism
- baryon number violation due to chiral anomaly
- etc.

Weakly coupled theory,
Still, non-perturbative dynamics may be relevant

but ...

- no gauge-invariant regularization is known

  (cf. dimensional reg.)
- non-perturbative definition is missing

the Glashow-Weinberg-Salam model

(^^;

(SU(2)x U(1) sector of the standard model without SU(3) color int.)

- a chiral gauge theory with $SU(2)_L \times U(1)_Y$
- gauge symmetry breaking via Higgs mechanism
- baryon number violation due to chiral anomaly
- etc.

Weakly coupled theory,
Still, non-perturbative dynamics may be relevant

but ...

- no gauge-invariant regularization is known

  (cf. dimensional reg.)

- non-perturbative definition is missing

previous attempts to put on the lattice ...

- *Eichten-Preskill* approach (symmetry/symmetry breaking)
- Wilson-Yukawa model (*Smit, Swift, Aoki*)
- Rome (gauge-fixing) approach (*Testa et al, Golterman-Shamir*)
- domain-wall + *Eichten-Preskill* hybrid (*Creutz*)
- Mirror GW fermion approach (*Poppitz*) etc.

in our formulation ...

based on :

D. Kadoh and Y.K., JHEP 0805:095 (2008), 0802:063 (2008)

D.~Kadoh, Y.~Nakayama and Y.K., JHEP 0412, 006 (2004)

Y. Nakayama and Y.K., Nucl. Phys. B597, 519 (2001)

★ a gauge-invariant construction of GWS model on the lattice

- use of overlap Dirac operator (the Ginsparg-Wilson relation)
- cf. U(1) chiral gauge theory with exact gauge invariance *Luscher (99)*
- the first invariant / non-perturbative regularization of the model (at finite V)
- all SU(2) togological sectors with vanishing U(1) magnetic fluxes

overlap Dirac op. / the GW rel.

Neuberger(1997,98)

$$D = \frac{1}{2a}\left(1 + \gamma_5 \frac{H_{\rm w}}{\sqrt{H_{\rm w}^2}}\right)$$

$$\gamma_5 D + D\gamma_5 = 2aD\gamma_5 D$$

chiral operator

Luscher ; Hasenfratz, Niedermayer(1998)

$$\hat{\gamma}_5 \equiv \gamma_5(1-2aD) = -\frac{H_{\rm w}}{\sqrt{H_{\rm w}^2}}$$

chiral fermion

$$\hat{\gamma}_5 \psi_\pm(x) = \pm\,\psi_\pm(x)$$

$$\bar{\psi}_\pm(x)\gamma_5 = \mp\,\bar{\psi}_\pm(x)$$

Path Integral Quantization

Path Integral Measure depends on gauge fields !

$$\psi_-(x) = \sum_i v_i(x) c_i$$

$$\bar{\psi}_-(x) = \sum_i \bar{c}_i \bar{v}_i(x)$$

$$\tilde{v}_i(x) = v_j(x)\left(\tilde{Q}^{-1}\right)_{ji}$$

$$\tilde{c}_i = \tilde{Q}_{ij} c_j$$

$$\{v_i(x) \mid \hat{\gamma}_5 v_i(x) = -v_i(x)\ (i=1,\cdots,N_-)\}$$

$$\{\bar{v}_i(x) \mid \bar{v}_i(x)\gamma_5 = +\bar{v}_i(x)\ (i=1,\cdots,\bar{N}_-)\}$$

$\det \tilde{Q}$ **complex phase !**

$$Z = \int \mathcal{D}[\psi_-]\mathcal{D}[\bar{\psi}_-]\, \mathrm{e}^{-a^4\sum_x \bar{\psi}_- D\psi_-(x)}$$

$$= \int \prod_i dc_i \prod_j d\bar{c}_j\, \mathrm{e}^{-\sum_{ij}\bar{c}_j M_{ji} c_i}$$

$$= \det M_{ji} \qquad M_{ji} = a^4 \sum_x \bar{v}_j D v_i(x)$$

"overlap formula"

Narayanan-Neuberger(1993)

our approach

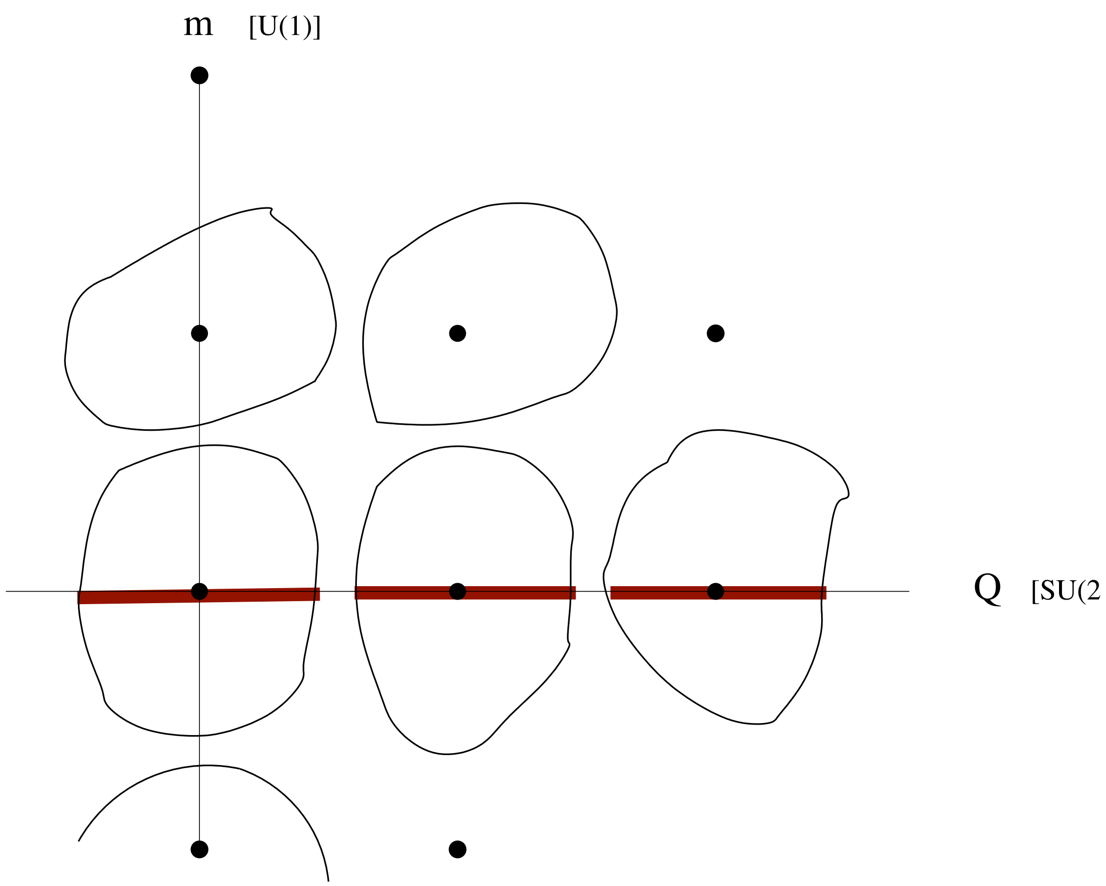
pure SU(2) theory   *cf. Nuberger(98) Bar-Campos (00)*

measure defined globally !

a pair of doublets (a,b)

$$v_j^{(a)}(x) = v_j(x)$$

$$v_j^{(b)}(x) = \left(\gamma_5 C^{-1} \otimes i\sigma_2\right)\ [v_j(x)]^*$$

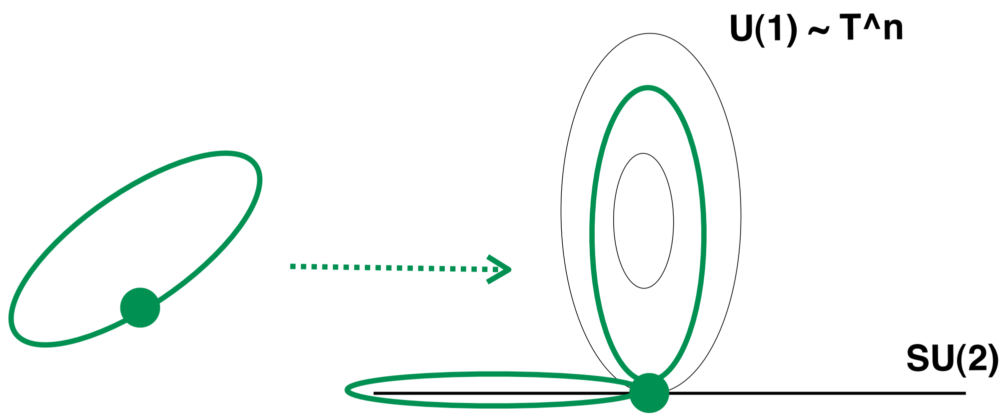


U(1) degrees of freedom

$$U_\mu(x) = \mathrm{e}^{iA_\mu^T(x)} g(x) g(x+\hat\mu)^{-1} U_{[\mathrm{w}]}(x,\mu) V_{[\mathrm{m}]}(x,\mu)$$

measure term smooth on $T^n[U(1)] \times M[SU(2)]$

proof of the global integrability condition



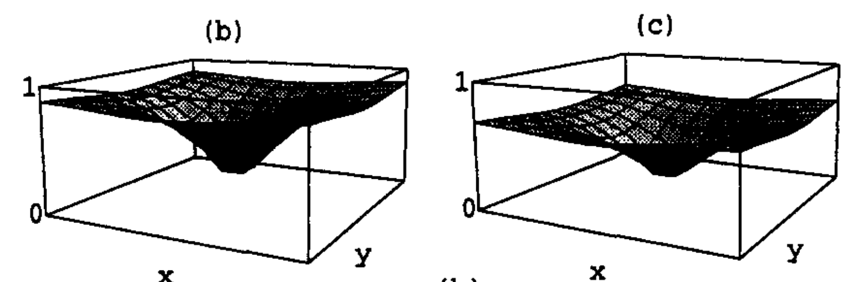
non-contractible loops

introduction of Higgs field & Yukawa-couplings

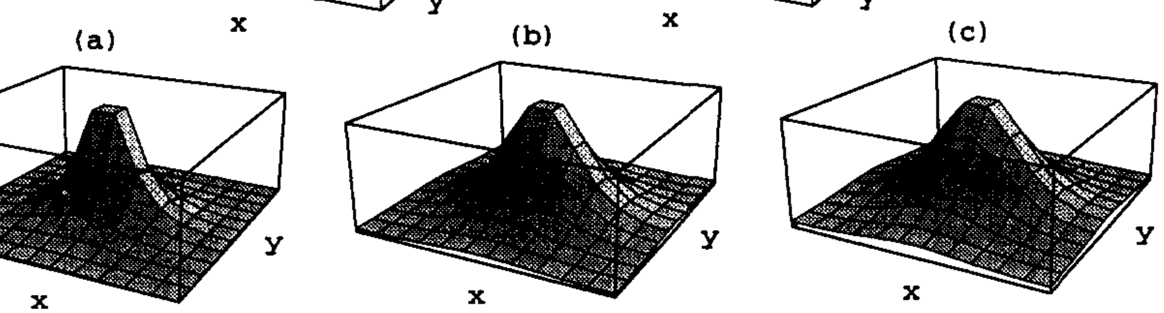
$$S_{EW} = S_G + S_F + \sum_x \{\nabla_\mu \phi^\dagger \nabla_\mu \phi + V(\phi)\}$$
$$- \sum_x \left\{ y_t\, \bar{Q}_- \tilde{\phi}\, t_+(x) + y_b\, \bar{Q}_- \phi\, b_+(x) + c.c. \right\}$$

sphaleron on the lattice

$$U_\mu^{(2)}(x), U_\mu^{(1)}(x), \phi(x)\ (x \in \mathbb{L}^3)$$



saddle point cooling
Perez- van Baal (96)



fermion fluctuation det.

cf. Moore (96)

$$\kappa_F(v, \lambda, y_t, \cdots) \equiv \prod_{q,l} \prod_{\omega_n} \det \mathcal{M} / \det \mathcal{M}_0$$

$$\mathcal{M}_t = \begin{pmatrix} (\bar{v}_k D v_j) & y_t(\bar{v}_k \tilde{\phi} u_j) \\ y_t(\bar{u}_k \tilde{\phi}^\dagger v_j) & (\bar{u}_k D u_j) \end{pmatrix}$$

- sum over matsubara freq.
- one-loop renormarizations
- dependence on the Higgs, Yukawa coupling
- comparison to other methods

cf. Bodeker et. (00)

possible applications of lattice EW theory

- a perturbative computation of the EW contributions to muon g-2 (one-loop check, beyond one-loop)
- a computation of the effect of quarks & leptons to the sphaleron rate at finite temp. (at one-loop, top quark?)
- a lattice construction of a model of dynamical EW symmetry breaking
  - "SU(2) minimal walking" technicolor model can be put on the lattice (!) *Dietrich, Sannino, Tuominen (05)*
  - cf. recent activitiy to study QCD-like theories in/close to the conformal window
    - T. Appelquist, G.T. Flemming and E.T. Neil (Yale Univ.) Phys.Rev.Lett. 100:171607,2008 (arXiv:0712.0609)
    - ...
- some other non-perturbative applications ?