Constraint on 4th Generation Leptons from Muon g - 2

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Phys. Rev. D 79, 2009

Outline

- **Introduction**
  - **4^{th} generation and muon g-2**

- **Effects of neutral lepton *N***

  SM4
  - **W^+W^-N loop contribution**
  - **H^+H^-N loop contribution**

  $$\begin{pmatrix} N \\ E \end{pmatrix}_L \quad \begin{matrix} N_R \\ E_R \end{matrix}$$

- **Effects of charged lepton *E***
  - **EEh^0, EEH^0, EEA^0 loop contribution**
  - **EEZ' loop contribution**

- **Comparison with MSSM**

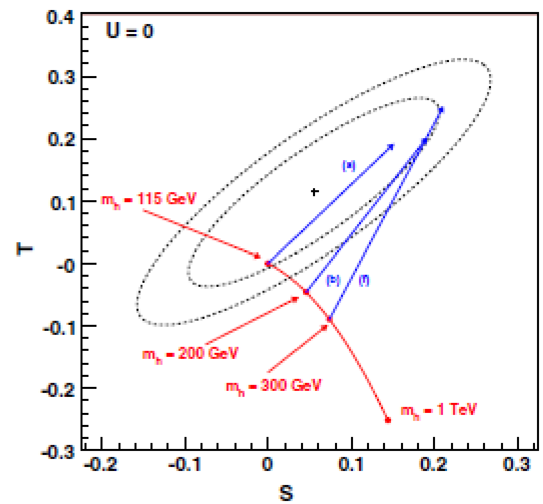
- **Summary**

Is a fourth generation allowed ?

- At least 3 generations for CP violation
- Direct measurement of invisible Z width:
  - N(v) = 2.92 ± 0.05
  - Does not exclude the existence of heavy neutrino ($m_N > m_Z/2$)
- 4^{th} generation is not excluded by EW precision data

G. D. Kribs et al. Phys. Rev. D76, 2007

parameter set	$m_{t'}$	$m_{b'}$	m_H	$\Delta S_{\rm tot}$	$\Delta T_{\rm tot}$
(a)	310	260	115	0.15	0.19
(b)	320	260	200	0.19	0.20
(c)	400	325	300	0.21	0.25



- SM source of CP violation is too small to account for the asymmetry between matter and anti-matter
  - An extra generation of quarks may resolve this problem

W.-S. Hou Chinese Journal of Phys. 2009

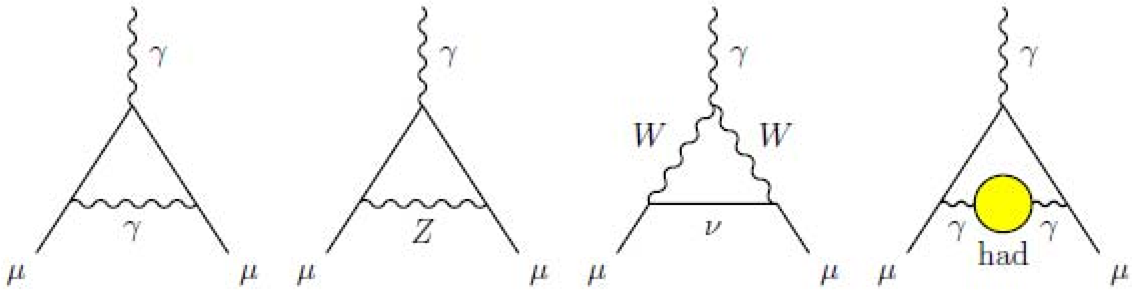
Muon g-2

- Experimental value :

E821 Collab, Phys. Rev. D73, 2006

$$a_\mu^{\exp} = \frac{g-2}{2} = 116\,592\,080\,(63)\cdot 10^{-11}$$

- SM3 prediction :



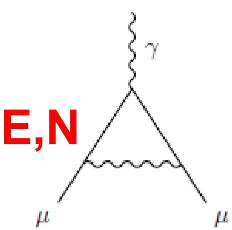
$$a_\mu^{\mathrm{SM}} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{EW}} + a_\mu^{\mathrm{Had}} = 116\,591\,788\,(2)(46)(35)\cdot 10^{-11}$$

Difference between experiment and theory:

$$\Delta a_\mu = a_\mu^{\exp} - a_\mu^{\mathrm{SM}} = 292\,(86)\cdot 10^{-11}$$ **(3.4 σ deviation)**

PDG 2009

from E,N

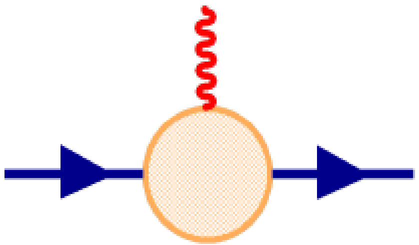


?

Muon g-2 and lepton flavor violating decay (LFV)

- Diagram for a_μ and $\mathcal{B}(\mu \to e\gamma)$ are similar, giving similar structures,

$$\varepsilon_\lambda q_\nu \sigma^{\lambda\nu}\left(C_L L + C_R R\right)$$



- Very stringent bounds on LFV decays,

$$\mathcal{B}(\mu \to e\gamma) < 1.2\times10^{-11}$$

$$\mathcal{B}(\tau \to e\gamma) < 1.1\times10^{-7}$$

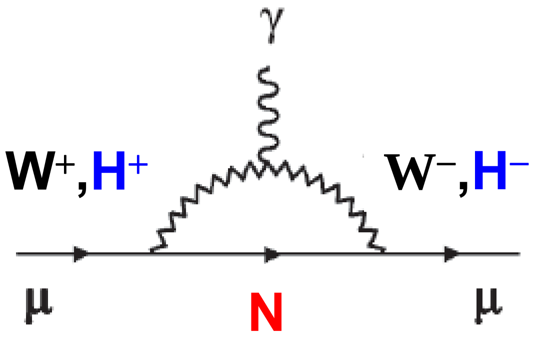
$$\mathcal{B}(\tau \to \mu\gamma) < 4.5\times10^{-8}$$

PDG 2009

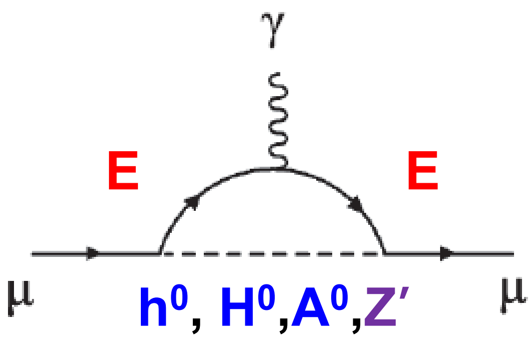
at 90% C.L.

Diagrammatic point of view

- Two types of contributions: ***N*** and ***E*** in loop



Boson-Boson-*N*



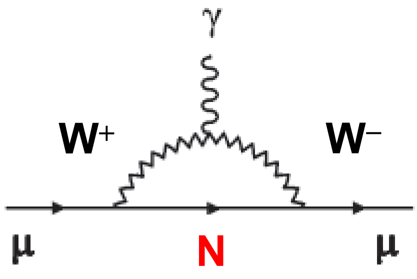
***E*-*E*-Boson**

Effects of neutral lepton *N*

W^+W^-N loop contribution **SM4**

- From weak Lagrangian

$$\mathcal{L} = -\frac{g}{2\sqrt{2}}\left(\bar{\mu}\gamma^\alpha\left(1-\gamma^5\right)V_{N\mu}N\right)W_\alpha + h.c.$$



we find

$$a_\mu\left(W^+W^-N\right) = \frac{G_F m_\mu^2}{4\sqrt{2}\pi^2}\left|V_{N\mu}\right|^2 F(x),$$

where $V_{N\mu}$ is the lepton mixing matrix element, and $x = m_N^2 / M_W^2$

$$F(x) = \int_0^1 du \frac{u(1-u)(2-u)x + 2u^2(u+1)}{(1-u)x+u} = \frac{3x^3 \log x}{(x-1)^4} + \frac{4x^3 - 45x^2 + 33x - 10}{6(x-1)^3}$$

- Replace ***N*** by ν_μ,

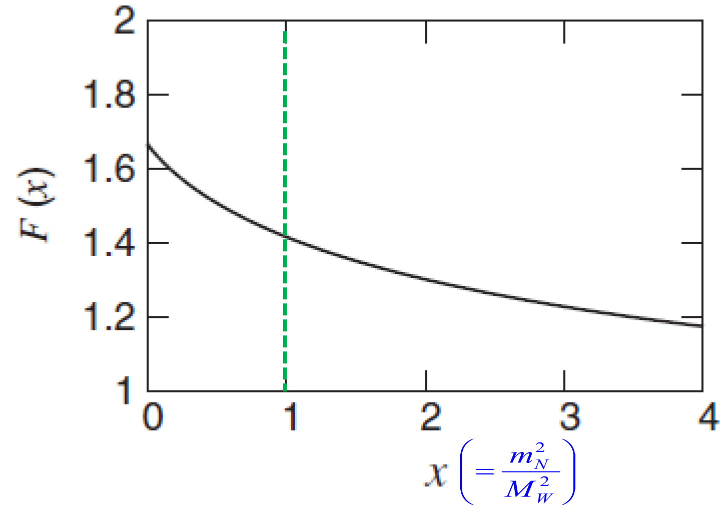
$$V_{N\mu} \to V_{\nu_\mu \mu} \cong 1 \;,\; m_N \to m_{\nu_\mu} = 0 \quad \Rightarrow \quad F(x=0) = \frac{5}{3}$$

recover SM contribution,

$$a_\mu^{SM}\left(W^+W^-\nu_\mu\right) = \frac{5G_F m_\mu^2}{12\sqrt{2}\pi^2} \qquad \left(\frac{G_F m_\mu^2}{4\sqrt{2}\pi^2} \approx 233\times10^{-11} \sim \Delta a_\mu\right)$$

- $F(x)$, an Inami-Lim loopfunction, is well-behaved and bounded.
- $F(1) = 17/12$: not singular

  Incorrectly rendered in **W. Huo and T. F. Feng arXiv:0301153** and **K. R. Lynch arXiv:0108081**
- Since $F(x) \simeq 1.4$, $|V_{N\mu}|$ needs to be $\gtrsim 0.7$ to reach within 2σ of $\triangle a_\mu$



- On the other hand,

$$\mathcal{B}_{\mu\to e\gamma}\left(W^\pm W^\pm N\right) = \frac{3\alpha\left|V^*_{Ne}V_{N\mu}\right|^2}{8\pi}F^2(x)$$

$$\approx \underbrace{1.2\times10^{-11}}_{\text{upper bound}}\left(\frac{|V_{N\mu}|}{0.7}\right)^2\left(\frac{|V_{Ne}|}{10^{-4}}\right)^2\left(\frac{F(x)}{1.6}\right)^2$$

If $|V_{N\mu}| \gtrsim 0.7$ ⇨ $|V_{Ne}| \lesssim 10^{-4}$ ⇨ **Unrealistic!**

$\triangle a_\mu$ unlikely from W^+W^-N

because of $\mu\to e\gamma$

Two Higgs Doublet Model (2HDM)

BSM4

- One Higgs doublet → Two Higgs doublets

- Neutral components of the Higgs boson doublets Φ_1 and Φ_2 acquire vevs v_1 and v_2 and ratio is $\tan\beta = v_2/v_1$.

- Electroweak breaking leaves:

  1 CP-odd scalar A^0
  2 Charged Higgs bosons $H^{\pm}$
  2 CP-even Higgs scalars h^0, H^0

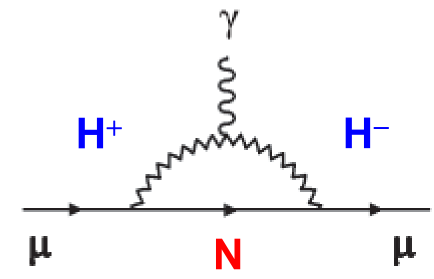
- **Type I** : only Φ_2 couples and gives masses to leptons

- **Type II** : Φ_1 and Φ_2 couple and give masses to charged and neutral leptons respectively. This model occurs for **MSSM**.

- **Type III** : Φ_1 and Φ_2 couple and yield masses to both charged and neutral leptons. This model possess flavor changing neutral couplings (**FCNC**) effect.

H^+H^-N loop contribution in 2HDM II

- In 2HDM II, the relevant Lagrangian is



$$\mathcal{L}_Y = \frac{g}{2\sqrt{2}M_W} V_{N\mu} \bar{\mu} \left[\left(\cot\beta\, m_N + \tan\beta\, m_\mu \right) + \left(\cot\beta\, m_N - \tan\beta\, m_\mu \right) \gamma^5 \right] N H^- + h.c.$$

we get

$$a_\mu^{\text{2HDM-II}}\left(H^+H^-N\right) = -\frac{G_F m_\mu^2}{4\sqrt{2}\pi^2} \left|V_{N\mu}\right|^2 \left[f_{H^+}(x) + g_{H^+}(x)\cot^2\beta + x_\mu q_{H^+}(x)\tan^2\beta \right],$$

$$\begin{cases} f_{H^+}(x) = \int_0^1 du \frac{2u(1-u)x}{(1-u)x+u} = -\frac{2x^2 \log x}{(x-1)^3} + \frac{x(x+1)}{(x-1)^2} \\ g_{H^+}(x) = \int_0^1 du \frac{u^2(1-u)x}{(1-u)x+u} = -\frac{x^3 \log x}{(x-1)^4} + \frac{x(2x^2+5x-1)}{6(x-1)^3} \\ q_{H^+}(x) = \int_0^1 du \frac{u^2(1-u)}{(1-u)x+u} = -\frac{x^2 \log x}{(x-1)^4} + \frac{2x^2+5x-1}{6(x-1)^3} \end{cases}$$

$$x = \frac{m_N^2}{M_{H^+}^2}, \quad x_\mu = \frac{m_\mu^2}{M_{H^+}^2} << 1$$

- Replace N by ν_μ

$$f_{H^+}(x=0)=0\,,\ g_{H^+}(0)=0\,,\ q_{H^+}(0)=\frac{1}{6}$$

we get

$$a_\mu^{\rm 2HDM\text{-}II}(H^+H^-\nu_\mu)=-\frac{G_F m_\mu^2}{4\sqrt{2}\pi^2}\cdot\frac{x_\mu\tan^2\beta}{6}$$

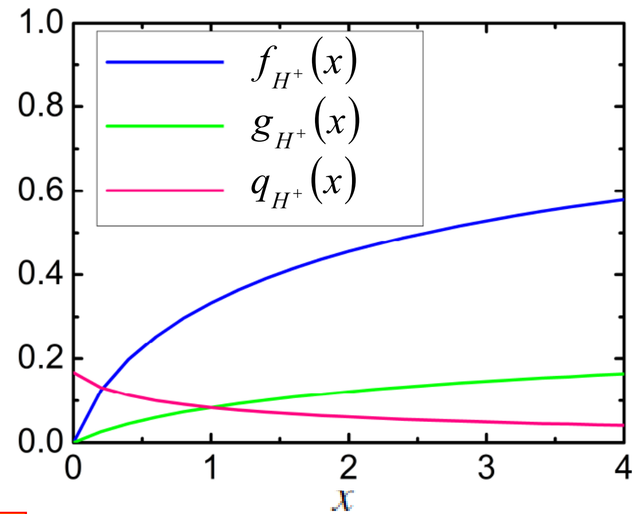
A. Dedes and H.E. Haber JHEP 2001

- $$a_\mu^{\rm 2HDM\text{-}II}(H^+H^-N)=-\frac{G_F m_\mu^2}{4\sqrt{2}\pi^2}|V_{N\mu}|^2\Big[\underbrace{f_{H^+}(x)}_{\rm A}+\underbrace{g_{H^+}(x)\cot^2\beta}_{\rm B}+\underbrace{x_\mu q_{H^+}(x)\tan^2\beta}_{\rm C}\Big],$$

A: For $1\le x\le 10$, so $0.4\le f_{H^+}(x)\le 0.8$. Suffer from $|V_{N\mu}|^2$ suppression.

B: Finite contribution for large cotβ and x, such that $|V_{N\mu}\cot\beta|^2\sim\mathcal{O}(1)$. But *negative*.
[Large cotβ gives $|g_{t\bar{t}H^0(h^0)}|$ >> 1, which become nonperturbative]

C: Suppressed by $x_\mu\left(=m_\mu^2/M_H^2\right)<<1$. Compare to **B**

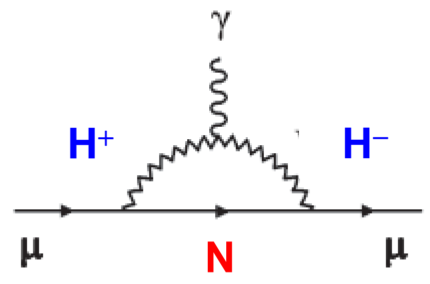


Can not give rise to $\triangle a_\mu$

H^+H^-N loop contribution in 2HDM I

- In 2HDM I, the relevant Lagrangian is

$$\mathcal{L}_Y = \frac{g\cot\beta}{2\sqrt{2}M_W} V_{N\mu}\bar{\mu}\left[\left(m_N - m_\mu\right) + \left(m_N + m_\mu\right)\gamma^5\right] N H^- + h.c.$$

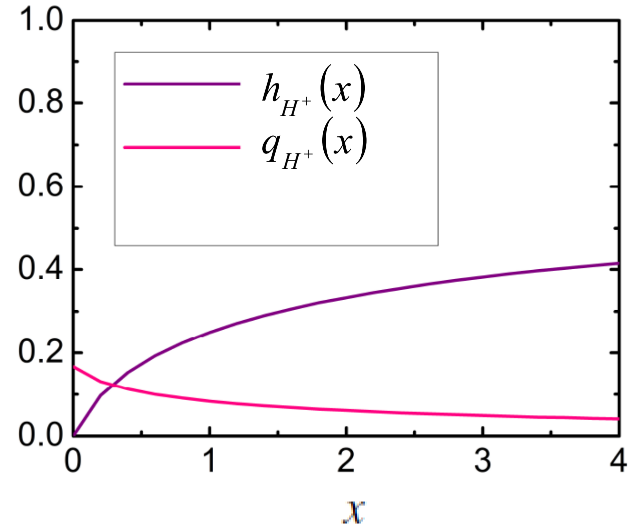


we get

$$a_\mu^{\text{2HDM-I}}\left(H^+H^-N\right) = \frac{G_F m_\mu^2}{4\sqrt{2}\pi^2}\left|V_{N\mu}\right|^2 \cot^2\beta\left[h_{H^+}(x) - x_\mu q_{H^+}(x)\right],$$

$$h_{H^+}(x) = \int_0^1 du \frac{u(1-u)(2-u)x}{(1-u)x+u} = -\frac{x^2\,(x-2)\log x}{(x-1)^4} + \frac{x\left(4x^2-5x-5\right)}{6(x-1)^3}$$

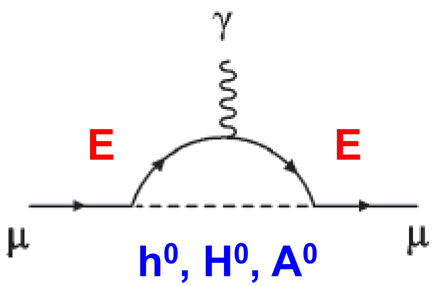
$$q_{H^+}(x) = \int_0^1 du \frac{u^2(1-u)}{(1-u)x+u} = -\frac{x^2\log x}{(x-1)^4} + \frac{2x^2+5x-1}{6(x-1)^3}$$



By same argument, insufficient for $\triangle a_\mu$

Effects of charged lepton *E*

EEh^0,EEH^0,EEA^0 loop contribution in 2HDN III



- In 2HDM III, tanβ is no longer a physical parameter. The relevant Lagrangian is

$$-\mathcal{L}_Y = \frac{\zeta_{\mu E}}{\sqrt{2}} \bar{\mu} \left(-\sin\alpha\, H^0 + \cos\alpha\, h^0 + i\gamma^5 A^0\right) E + h.c.$$

where $\zeta_{\mu E}$ is FCNC Yukawa coupling. α is mixing angle in CP-even Higgs.

- Extend Cheng-Sher ansatz to charged lepton

$$\zeta_{ij} \sim \Delta_{ij} \sqrt{m_i m_j} / \upsilon, \quad \Delta_{ij} \sim \mathcal{O}(1)$$

- $$a_\mu^{\text{2HDM-III}} = \frac{G_F m_\mu^2}{4\sqrt{2}\pi^2} \left(C_{H^0} + C_{h^0} + C_{A^0}\right)$$

$$\begin{cases} C_{H^0} = \sin^2\alpha \int_0^1 du \dfrac{u^2\left[m_E^2 + (1-u) m_\mu m_E\right]}{u m_E^2 + (1-u) M_{H^0}^2} > 0 \\ C_{h^0} = \cos^2\alpha \int_0^1 du \dfrac{u^2\left[m_E^2 + (1-u) m_\mu m_E\right]}{u m_E^2 + (1-u) M_{h^0}^2} > 0 \\ C_{A^0} = -\int_0^1 du \dfrac{u^2\left[m_E^2 - (1-u) m_\mu m_E\right]}{u m_E^2 + (1-u) M_{A^0}^2} < 0 \end{cases}$$

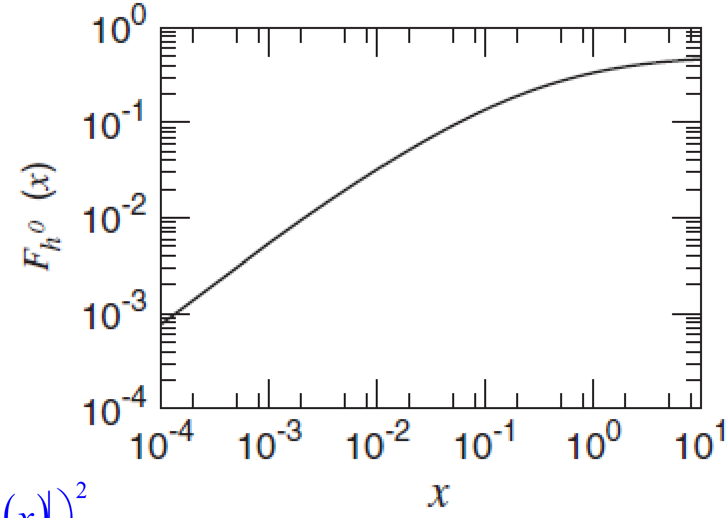
Because $C_{A^0} < 0$, we assume $M_{A^0}, M_{H^0} >> M_{h^0}$ ⇒ C_{A^0}, C_{H^0} can be ignored

- Then we get

$$a_\mu^{\text{2HDM-III}} \sim 233\times 10^{-11} F_{h^0}(x), \qquad x = \frac{m_E^2}{M_{h^0}^2}$$

$$F_{h^0}(x) = \int_0^1 du \frac{u^2 x}{ux + (1-u)} = \frac{x\log x}{(x-1)^3} + \frac{x(x-3)}{2(x-1)^2}$$

If we assume $a_\mu^{\text{2HDM-III}} \approx \Delta a_\mu$, we have $F_{h^0}(x) = \mathcal{O}(1)$



- $$\mathcal{B}^{\text{2HDM-III}}(\mu \to e\gamma) = \frac{3\alpha}{2\pi}\frac{m_e}{m_\mu}\left|F_{h^0}(x)\right|^2 \sim \underset{\text{upper bound}}{1.2\times 10^{-11}} \left(\frac{\left|F_{h^0}(x)\right|}{10^{-3}}\right)^2$$

  we get $F_{h^0}(x) < 10^{-3}$

➡ The $\mu \to e\gamma$ constraint rules out Cheng-Sher ansatz + 4th generation lepton in loop

- Suppose eEh^0 vanish,

$$\mathcal{B}^{\text{2HDM-III}}(\tau \to \mu\gamma) = \mathcal{B}(\tau \to \mu\bar{\nu}_\mu \nu_\tau)\frac{3\alpha}{2\pi}\frac{m_\mu}{m_\tau}\left|F_{h^0}(x)\right|^2 \sim \underset{\text{upper bound}}{4.5\times 10^{-8}} \left(\frac{\left|F_{h^0}(x)\right|}{10^{-1}}\right)^2$$

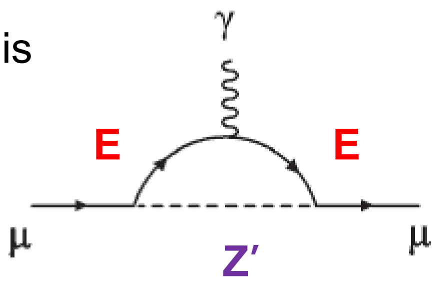
  we get $F_{h^0}(x) < 10^{-1}$

➡ The $\tau \to \mu\,\gamma$ constraint rules out Cheng-Sher ansatz + 4th generation lepton in loop

If 4th generation found, Cheng-Sher ansatz can not hold for lepton sector

EEZ′ loop contribution

- For extra $U(1)$ with FCNC coupling, the relevant Lagrangian is



$$\mathcal{L}_{Z'} = g_{Z'}\bar{e}_a\gamma^\mu\left[\varepsilon^L_{ab}L + \varepsilon^R_{ab}R\right]e_b Z'_\mu$$

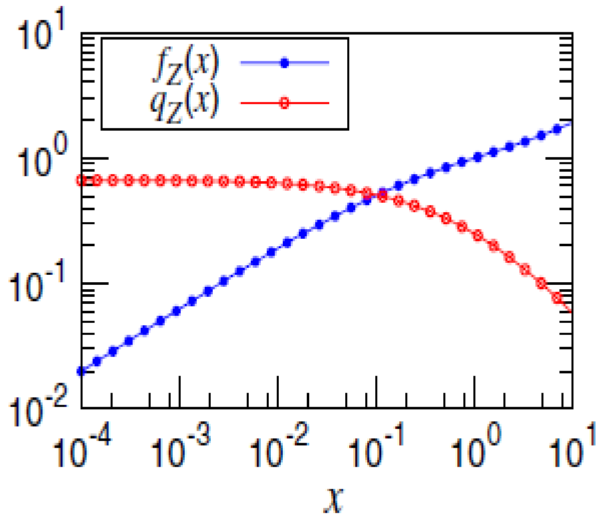
we get

$$a_\mu(EEZ') = \frac{g^2_{Z'}}{8\pi^2}\left[x^{1/2}_\mu \mathrm{Re}\left(\varepsilon^L_{\mu E}\varepsilon^R_{\mu E}\right)f_Z(x) - x_\mu\left(\left|\varepsilon^L_{\mu E}\right|^2 + \left|\varepsilon^R_{\mu E}\right|^2\right)q_Z(x)\right]$$

where $x = \frac{m_E^2}{M_{Z'}^2}$, $x_\mu = \frac{m_\mu^2}{M_{Z'}^2}$

$$f_Z(x) = \int_0^1 du\,\frac{4u(1-u)x^{1/2} + u^2x^{3/2}}{ux + (1-u)} = -\frac{3x^{3/2}\log x}{(x-1)^3} + \frac{x^{1/2}(x^2+x+4)}{2(x-1)^2}$$

$$q_Z(x) = \int_0^1 du\,\frac{u(1-u)(2-u)}{ux + (1-u)} = -\frac{x(2x-1)\log x}{(x-1)^4} + \frac{5x^2+5x-4}{6(x-1)^3}$$



can recover SM by proper substitution.

- On the other hand

$$\mathcal{B}^{Z'}(\mu \to e\gamma) = \frac{3\alpha\, g_{Z'}^4}{8\pi G_F^2 m_\mu^2 M_{Z'}^2} |\, f_Z(x)|^2 \left[\left|\varepsilon_{eE}^R \varepsilon_{\mu E}^L\right|^2 + \left|\varepsilon_{eE}^L \varepsilon_{\mu E}^R\right|^2 \right]$$

$$\mathcal{B}^{Z'}(\tau \to \mu\gamma) = \mathcal{B}(\tau \to \mu \bar{\nu}_\mu \nu_\tau) \frac{3\alpha\, g_{Z'}^4}{8\pi G_F^2 m_\tau^2 M_{Z'}^2} |\, f_Z(x)|^2 \left[\left|\varepsilon_{\mu E}^R \varepsilon_{\tau E}^L\right|^2 + \left|\varepsilon_{\mu E}^L \varepsilon_{\tau E}^R\right|^2 \right]$$

- We take $M_{Z'} = 1\,\mathrm{TeV},\ M_E = 250\,\mathrm{GeV},\ g_{Z'} = 0.105$ (P. Langacker *et al.* Phys.Rev. D 2000) and denote $\varepsilon_{\mu E}^L = \varepsilon_{\mu E}^R \equiv \varepsilon_{\mu E}\,,\ \varepsilon_{eE}^L = \varepsilon_{eE}^R \equiv \varepsilon_{eE}\,,\ \varepsilon_{\tau E}^L = \varepsilon_{\tau E}^R \equiv \varepsilon_{\tau E}$ and all ε are real; then

$a_\mu(EEZ') \sim 620 \times 10^{-11} \varepsilon_{\mu E}^2$ —— If we assume $\varepsilon_{\mu E} = \mathcal{O}(1)$ ——→ $a_\mu(EEZ') \approx \Delta a_\mu$

- $\mathcal{B}^{Z'}(\mu \to e\gamma) \sim \underset{\text{upper bound}}{1.2 \times 10^{-11}}\ \varepsilon_{\mu E}^2 \left(\frac{\varepsilon_{eE}}{10^{-4}}\right)^2$: $\varepsilon_{\mu E} \sim 1$ ➡ $\varepsilon_{eE} = \mathcal{O}(10^{-4})$

- $\mathcal{B}^{Z'}(\tau \to \mu\gamma) \sim \underset{\text{upper bound}}{4.5 \times 10^{-8}}\ \varepsilon_{\mu E}^2 \left(\frac{\varepsilon_{\tau E}}{10^{-2}}\right)^2$: $\varepsilon_{\mu E} \sim 1$ ➡ $\varepsilon_{\tau E} = \mathcal{O}(10^{-2})$

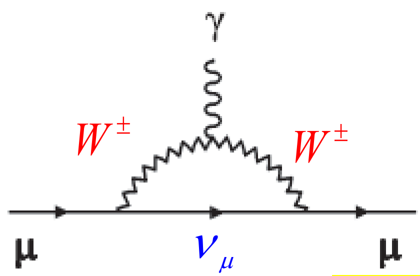
Z' unlikely the source for muon g - 2

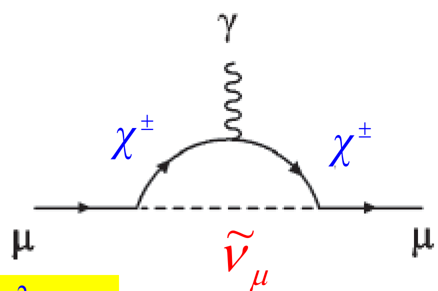
unless $\varepsilon_{\mu E} = \mathcal{O}(1)$ but $\varepsilon_{eE}\,, \varepsilon_{\tau E} << 1$

Why MSSM works?

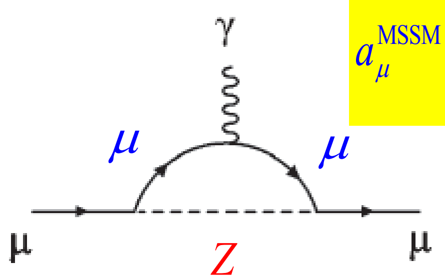
◆ **MSSM** doubles number of diagrams of **SM**

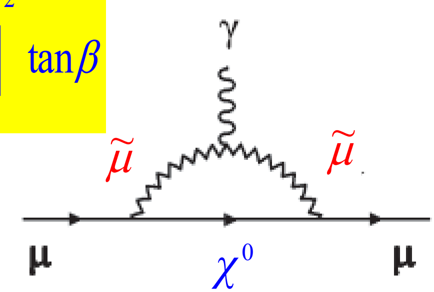
- Coupling is gauge coupling
- Can compensate $1/M_{\mathrm{SUSY}}^2$ suppression by large $\tan\beta$





$$a_\mu^{\mathrm{MSSM}} \cong 130\times10^{-11}\left(\frac{100\,\mathrm{GeV}}{M_{\mathrm{SUSY}}}\right)^2 \tan\beta$$





n.b. subtlety of mass degeneracy condition

(A) $m_{\mathrm{Higgsino}} = m_{\mathrm{Wino}} = M_{\tilde{\nu}_\mu} = M_{\mathrm{SUSY}}$ ➡ $\sum_k a_\mu\left(\chi_k^+\ \chi_k^-\ \tilde{\nu}_\mu\right) \sim \frac{g_2^2 m_\mu^2\,\mathrm{sgn}\left(\mu M_2\right)\tan\beta}{32\pi^2 M_{\mathrm{SUSY}}^2}$ **(Moroi)** **(O)**

(B) $m_{\mathrm{chargino}} = M_{\tilde{\nu}_\mu} = M_{\mathrm{SUSY}}$ ➡ $\sum_k a_\mu\left(\chi_k^+\ \chi_k^-\ \tilde{\nu}_\mu\right) \sim 837\times10^{-5} x_\mu\,,\quad x_\mu = \frac{m_\mu^2}{M_{\mathrm{SUSY}}^2} \le 10^{-7}$ **too small** **(X)**

(Czarnecki and Marciano)

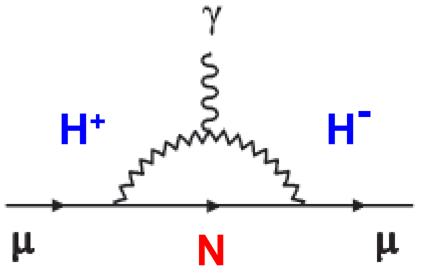
Summary

- Coupling strength is crucial factor, not the loop function.
- In SM, 2HDM-I and II, the 4th generation is irrelevant to $\triangle a_\mu$ puzzle, because of smallness of $|V_{N\mu}|$.
- If 4th generation found, Cheng-Sher ansatz can not hold in lepton sector.
- Enhancement to a_μ and suppression to $\mathcal{B}(\mu \to e\gamma)$ in MSSM both similar to SM.
- Since large $\tan\beta$ suppresses the *negative* contribution from H^+H^-N, MSSM and 4th generation can coexist.

Back up

$H^+ H^- N$ loop contribution in 2HDM II

- In the 2HDM II, the Yukawa's Lag. of lepton sector is



$$-\mathcal{L}_Y = \eta_{ij}^{e,0} \bar{l}_{iL}^0 \Phi_1 e_{jR}^0 + \xi_{ij}^{\nu,0} \bar{l}_{iL}^0 \overline{\Phi}_2 \nu_{jR}^0 + h.c.$$

When we expand in terms of mass eigenstates, the relevant Lag. with $H^\pm$ reads

$$\mathcal{L}_Y = \frac{g}{2\sqrt{2}M_W} V_{N\mu} \bar{\mu} \left[\left(\cot\beta\, m_N + \tan\beta\, m_\mu\right) + \left(\cot\beta\, m_N - \tan\beta\, m_\mu\right)\gamma^5 \right] N H^- + h.c.$$

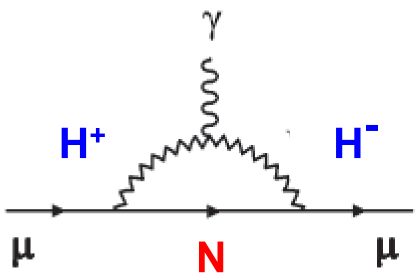
Then we can get

$$a_\mu^{\text{2HDM-II}}\left(H^+ H^- N\right) = -\frac{G_F m_\mu^2}{4\sqrt{2}\pi^2} \left|V_{N\mu}\right|^2 \left[f_{H^+}(x) + g_{H^+}(x)\cot^2\beta + x_\mu q_{H^+}(x)\tan^2\beta \right],\quad x = \frac{m_N^2}{M_{H^+}^2},\quad x_\mu = \frac{m_\mu^2}{M_{H^+}^2}$$

$$\begin{cases} f_{H^+}(x) = \int_0^1 du \frac{2u(1-u)x}{(1-u)x+u} = -\frac{2x^2 \log x}{(x-1)^3} + \frac{x(x+1)}{(x-1)^2} \\ g_{H^+}(x) = \int_0^1 du \frac{u^2(1-u)x}{(1-u)x+u} = -\frac{x^3 \log x}{(x-1)^4} + \frac{x(2x^2+5x-1)}{6(x-1)^3} \\ q_{H^+}(x) = \int_0^1 du \frac{u^2(1-u)}{(1-u)x+u} = -\frac{x^2 \log x}{(x-1)^4} + \frac{2x^2+5x-1}{6(x-1)^3} \end{cases}$$

H^+H^-N loop contribution in 2HDM I

- In the 2HDM I, the Yukawa's Lag. of lepton sector is



$$-\mathcal{L}_Y = \xi_{ij}^{e,0}\bar{l}_{iL}^0\Phi_2 e_{jR}^0 + \xi_{ij}^{\nu,0}\bar{l}_{iL}^0\overline{\Phi}_2\nu_{jR}^0 + h.c.$$

When we expand in terms of mass eigenstates, the relevant Lag. with $H^{\pm}$ reads

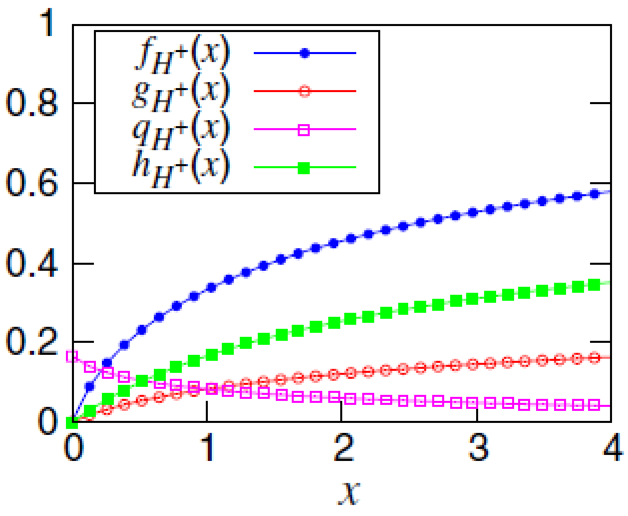
$$\mathcal{L}_Y = \frac{g\cot\beta}{2\sqrt{2}M_W}V_{N\mu}\bar{\mu}\left[\left(m_N - m_\mu\right)+\left(m_N+m_\mu\right)\gamma^5\right]NH^- + h.c.$$

Then we can get

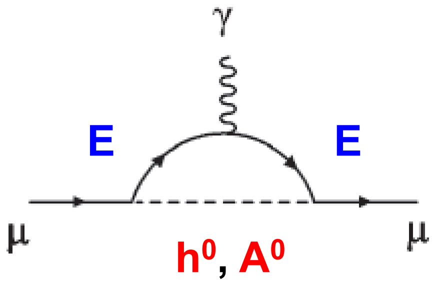
$$a_\mu^{\text{2HDM-I}}\left(H^+H^-N\right)=\frac{G_F m_\mu^2}{4\sqrt{2}\pi^2}\left|V_{N\mu}\right|^2\cot^2\beta\left[h_{H^+}(x)-x_\mu q_{H^+}(x)\right],$$

$$h_{H^+}(x)=\int_0^1 du\frac{u(1-u)(2-u)x}{(1-u)x+u} = -\frac{x^2(x-2)\log x}{(x-1)^4}+\frac{x\left(4x^2-5x-5\right)}{6(x-1)^3}$$

$$q_{H^+}(x)=\int_0^1 du\frac{u^2(1-u)}{(1-u)x+u} = -\frac{x^2\log x}{(x-1)^4}+\frac{2x^2+5x-1}{6(x-1)^3}$$



EEh^0, EEA^0 loop contribution in 2HDN III



- In the 2HDM III, the Yukawa's Lag. of lepton sector is

$$-\mathcal{L}_Y = \eta_{ij}^{\nu,0}\bar{l}_{iL}^{0}\overline{\Phi}_1\nu_{jR}^{0} + \eta_{ij}^{e,0}\bar{l}_{iL}^{0}\Phi_1 e_{jR}^{0} + \xi_{ij}^{\nu,0}\bar{l}_{iL}^{0}\overline{\Phi}_2\nu_{jR}^{0} + \xi_{ij}^{e,0}\bar{l}_{iL}^{0}\Phi_2 e_{jR}^{0} + h.c.$$

where η and ξ are 4×4 matrices and i, j are flavor indices. In this model, by redefining Φ_1, Φ_2 and η, ξ simultaneously, which still leaves the Lag. Invariant. We may assume $\langle\Phi_1^0\rangle = v/\sqrt{2}$ and $\langle\Phi_2^0\rangle = 0$, and $\tan\beta$ is not a physical parameter.

- In terms of physical field, η^{ν} and η^{e} can be diagonalized to be mass matrix. ξ^{ν} and ξ^{e} are trasformed to ζ^{ν} and ζ^{e}. The relevant Lag. with $\zeta_{\mu E}$

$$-\mathcal{L}_Y = \frac{\zeta_{\mu E}}{\sqrt{2}}\bar{\mu}\left(-\sin\alpha\, H^0 + \cos\alpha\, h^0 + i\gamma^5 A^0\right)E + \frac{\zeta_{\mu E}}{2}\overline{N}\left(1+\gamma^5\right)\mu H^+ + h.c.$$

- We extend Cheng and Sher ansatz for quark sector to charged lepton sector with 4th gen.

$$\zeta_{ij} \sim \Delta_{ij}\sqrt{m_i m_j}/\upsilon, \qquad \Delta_{ij} \sim \mathcal{O}(1)$$

- $$a_{\mu}^{\text{2HDM-III}} = \frac{G_F m_{\mu}^2}{4\sqrt{2}\pi^2} \times \left(C_{EEH^0} + C_{EEh^0} + C_{EEA^0}\right)$$

$$C_{EEH^0} = \sin^2\alpha \int_0^1 du\, \frac{u^2\left[m_E^2 + (1-u)m_{\mu}m_E\right]}{u m_E^2 + (1-u)M_{H^0}^2} > 0$$

$$C_{EEh^0} = \cos^2\alpha \int_0^1 du\, \frac{u^2\left[m_E^2 + (1-u)m_{\mu}m_E\right]}{u m_E^2 + (1-u)M_{h^0}^2} > 0$$

$$C_{EEA^0} = -\int_0^1 du\, \frac{u^2\left[m_E^2 - (1-u)m_{\mu}m_E\right]}{u m_E^2 + (1-u)M_{A^0}^2} < 0$$

$$\sum_k a_\mu(\chi_k^+\chi_k^-\bar{\nu}_\mu) = \frac{m_\mu}{16\pi^2}\sum_k\left\{\frac{m_\mu}{12M^2_{\tilde{\nu}_\mu}}(g_2^2|V_{k1}|^2 + y_\mu^2|U_{k2}|^2)F_1^C(x_k) - g_2 y_\mu \frac{2m_{\chi_k^\pm}}{3M^2_{\tilde{\nu}_\mu}}\,\mathrm{Re}[U_{k2}V_{k1}]F_2^C(x_k)\right\}. \tag{30}$$

where $x_k = m^2_{\chi_k^\pm}/M^2_{\tilde{\nu}_\mu}$, and we follow the notations of Ref. [11], except for a factor of 2 for $F_2^C(x)$. The biunitary transformation matrices U and V diagonalize the chargino mass matrix X:

$$X = \begin{pmatrix} M_2 & M_W\sqrt{2}\sin\beta \\ M_W\sqrt{2}\cos\beta & \mu \end{pmatrix}. \tag{31}$$

Assuming $|\mu| = |M_2| = M_{\tilde{\nu}_\mu} = M_{\rm SUSY}$ and taking the large $\tan\beta$ limit, we make a Taylor expansion of the factor $\sum_k \mathrm{Re}(U_{k2}V_{k1})m_{\chi_k^\pm}F_2^C(x_k)$ in Eq. (30) around $x_k = 1$ and express it in terms of X:

$$\sum_k \mathrm{Re}(U_{k2}V_{k1})m_{\chi_k^\pm}F_2^C(x_k) = \left(X - \frac{3}{4}XE + \frac{6}{5}XE^2 + \cdots\right)_{21}. \tag{32}$$

Since $E = X^\dagger X/M^2_{\rm SUSY} - I$ is a matrix at $O(M_W/M_{\rm SUSY})$, we can neglect high order terms. Approximating $X_{22}E_{21} = \mathrm{sgn}(\mu M_2)\sqrt{2}M_W\sin\beta$ in Eq. (32), we obtain a sizable contribution

$$\sum_k a_\mu(\chi_k^+\chi_k^-\bar{\nu}_\mu) \sim \frac{g_2^2 m_\mu^2\,\mathrm{sgn}(\mu M_2)\tan\beta}{32\pi^2 M^2_{\rm SUSY}}. \tag{33}$$

However, if one takes the degeneracy limit $m_{\chi^\pm_{1,2}} = M_{\tilde{\nu}_\mu} = M_{\rm SUSY}$, one gets $F_2^C(x_{1,2}) = 1$, the factor $\sum_k \mathrm{Re}(U_{k2}V_{k1})m_{\chi_k^\pm}F_2^C(x_k)$ becomes exactly equal to $X_{21} = \sqrt{2}M_W\cos\beta$, and one inadvertently loses the $\tan\beta$ enhancement. After some calculation, we get

$$\sum_k a_\mu(\chi_k^+\chi_k^-\bar{\nu}_\mu) \sim 837\times 10^{-5}x_\mu. \tag{34}$$

With $x_\mu = m_\mu^2/M^2_{\rm SUSY} \lesssim 10^{-7}$ typically, the chargino-$\bar{\nu}_\mu$ loop contribution would become insufficient for the $g_\mu - 2$ discrepancy.

V. SUMMARY

In this paper, we consider the existence of fourth generation leptons and discussed their impact on a_μ. In the SM, 2HDM-I and II, the fourth generation seems irrelevant to the Δa_μ puzzle because of the smallness of $|V_{N\mu}|$. However, this off-diagonal factor also protects these models from the stringent $\mu\to e\gamma$ and $\tau\to\mu\gamma$ constraints. In the 2HDM-III, applying the Cheng-Sher ansatz with a fourth generation to charged leptons, one has a strong conflict with $\mathcal{B}(\mu\to e\gamma)$ and even $\mathcal{B}(\tau\to\mu\gamma)$. Hence, if a fourth generation is found, the Cheng-Sher ansatz cannot hold for the lepton sector. This may be reasonable since the lepton mixing pattern seems different from quarks.