Upper and lower Higgs boson mass bounds from a chirally invariant lattice Higgs-Yukawa model

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Motivation

- Higgs-Sector of SM is a trivial field theory.
  - If cutoff Λ sent to $\infty \Rightarrow$ non-interacting theory, since

$$\lambda_r = \frac{\lambda_0}{1 + \frac{3}{32\pi^2}\lambda_0 \log(\Lambda/\mu)} \leq \frac{32\pi^2}{3\log(\Lambda/\mu)}$$

  - Cutoff Λ must remain finite. (Otherwise no interaction.)
  - Consider SM as effective theory valid up to energy scale Λ.
- Due to triviality and $m_H^2 \propto \lambda_r v_r^2$ ($v_r = 246\,\mathrm{GeV}$)
  $\rightarrow$ Cutoff dependent upper Higgs boson mass bound $m_H^{up}(\Lambda)$.
- From additional arguments (vacuum stability considerations):
  $\rightarrow$ Cutoff dependent lower Higgs boson mass bound $m_H^{low}(\Lambda)$.

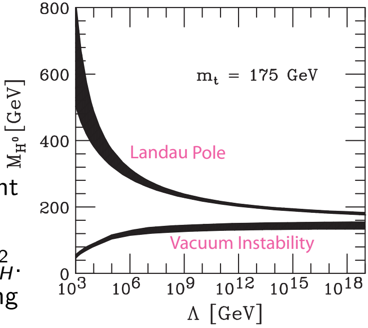
Main Motivation

Once m_H is known experimentally, Λ-dependent bounds $m_H^{up}(\Lambda)$, $m_H^{low}(\Lambda)$ allow to determine the energy scale Λ, where new physics must set in.

Aim of this investigation

- Available perturbative results:
  - Upper bound: Triviality
  - Lower bound: Vacuum instability
- Non-perturbative effects become important for ...
  - upper bound $m_H^{up}(\Lambda)$ due to $\lambda_r \propto m_H^2$.
  - heavy fourth generation due to strong Yukawa coupling.

Hagiwara *et al.*:



Aim of this investigation

Establish non-perturbative evaluation method for $m_H^{up}(\Lambda)$, $m_H^{low}(\Lambda)$ in lattice Higgs-Yukawa model.

Considered coupling structure

- Higgs-Fermion coupling in SM:
  - φ complex scalar doublet and $\tilde{\varphi} = i\tau_2\varphi$.
  - y_t, y_b, ...: Yukawa coupling constants.

$$L_Y = y_b \cdot (\bar{t}, \bar{b})_L \varphi b_R + y_t \cdot (\bar{t}, \bar{b})_L \tilde{\varphi} t_R + h.c. + \dots$$

- Higgs-Higgs self-interaction in SM:
  - λ: Quartic coupling constant

$$L_\varphi = \lambda(\varphi^\dagger\varphi)^2$$

- Higgs-dynamics dominated by ...
  - coupling to heaviest fermions (top, bottom, 4th generation).
  - quartic self-coupling (if $\lambda \gg 1$).
- In this study: Pure Higgs-fermion sector of SM:
  - All gauge fields neglected.

Coupling structure on the lattice

- Introduce finite, discrete space-time lattice with $V = L_s^3 \times L_t$ sites.
- Lattice model, obeying global $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ symmetry:

$$S = \sum_{x,\mu} \frac{1}{2} \nabla^f_\mu \varphi^\dagger_x \nabla^f_\mu \varphi_x + \sum_x \frac{1}{2} m_0^2 \varphi^\dagger_x \varphi_x + \sum_x \lambda \left(\varphi^\dagger_x \varphi_x\right)^2$$
$$+ \sum_{x,y} \bar{t}_x \mathcal{D}^{(ov)}_{x,y} t_y + \sum_{x,y} \bar{b}_x \mathcal{D}^{(ov)}_{x,y} b_y + \sum_x y_b \cdot (\bar{t}_{L,x}, \bar{b}_{L,x}) \varphi_x b_{R,x} + y_t \cdot (\bar{t}_{L,x}, \bar{b}_{L,x}) \tilde{\varphi}_x t_{R,x} + h.c.$$

Some lattice details...

- Need chiral invariance on lattice (to define $t_L, t_R, \ldots$)
  $\Rightarrow$ Ginsparg-Wilson fermions satisfying GW-relation (here: overlap op. $\mathcal{D}^{(ov)}$)

$$\gamma_5 \mathcal{D}^{(ov)} + \mathcal{D}^{(ov)} \hat{\gamma}_5 = 0 \quad \text{with} \quad \hat{\gamma}_5 = \gamma_5 \left(1 - a\mathcal{D}^{(ov)}\right)$$

- Use modified projectors $\hat{P}_\pm = \frac{1}{2}(1 \pm \hat{\gamma}_5)$ to define $t_L, \ldots$:

Continuum

$$\begin{pmatrix} t \\ b \end{pmatrix}_{L,R} = P_\mp \begin{pmatrix} t \\ b \end{pmatrix}$$
$$(\bar{t}, \bar{b})_{L,R} = (\bar{t}, \bar{b}) P_\pm$$

Lattice

$$\begin{pmatrix} t \\ b \end{pmatrix}_{L,R} = \hat{P}_\mp \begin{pmatrix} t \\ b \end{pmatrix}$$
$$(\bar{t}, \bar{b})_{L,R} = (\bar{t}, \bar{b}) P_\pm$$

Implemented simulation algorithm

- PHMC-algorithm implemented.
  - Allows to evaluate model numerically by lattice Monte-Carlo calculations.
  - Allows to access odd and even N_f (# of degen. fermion generations).
- Determinant $\det(\mathcal{M})$ of fermionic matrix $\mathcal{M}$:
  - $\det(\mathcal{M}) \in \mathbb{R} \Leftrightarrow y_t = y_b$, *i.e.* mass degenerate quarks
  - Even if $y_t = y_b$, negative EW possible $\Rightarrow$ Check sign of $\det(\mathcal{M})$
- Algorithmic improvements:
  - Preconditioning:
    Achieved: Condition number of $\mathcal{M}\mathcal{M}^\dagger$ reduced by factor $O(10-100)$
  - Fourier acceleration:
    Achieved: Auto-correlation time of $\langle\varphi\rangle$ reduced by $O(10-100)$.
  - Exact and fast reweighting:
    Achieved: Exactness and $O(10)$ faster than standard Gauss reweighting
  - Furthermore: Multiple time-scale integration, higher order integrators, ...

Phase diagram in large N_f-limit

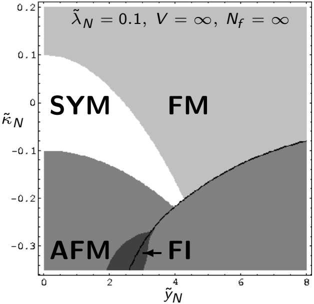
- Analytical calculation based on Constraint Effective Potential (CEP)
- Use lattice parameters $\kappa, \hat{\lambda}, \hat{y}_{t,b}$ related to $m_0, \lambda, y_{t,b}$ through

$$\lambda = \frac{\hat{\lambda}}{4\kappa^2}, \quad m_0^2 = \frac{1 - 2N_f\hat{\lambda} - 8\kappa}{\kappa}, \quad y_{t,b} = \frac{\hat{y}_{t,b}}{\sqrt{2\kappa}}$$

- Consider limit $N_f \to \infty$, while scaling $\kappa, \hat{\lambda}, \hat{y}_{t,b}$ according to:

$$\hat{y}_{t,b} = \frac{\tilde{y}_N}{\sqrt{N_f}}, \quad \hat{\lambda} = \frac{\tilde{\lambda}_N}{N_f}, \quad \varphi = N_f^{1/2}\tilde{\varphi}, \quad \kappa = \tilde{\kappa}_N, \qquad \tilde{y}_N, \tilde{\lambda}_N, \tilde{\kappa}_N, \tilde{\varphi} = \text{const}$$

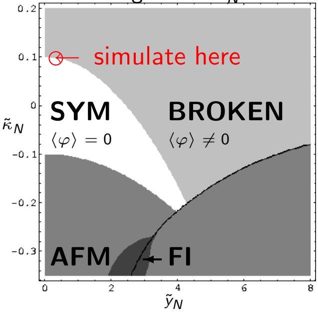
- Order parameters:
  $\langle\varphi\rangle$ and $\langle\varphi\rangle_s \equiv \langle \frac{1}{V}\sum_x (-1)^{\sum_\mu x_\mu}\varphi_x\rangle, \qquad V = L_s^3 \times L_t$
- Four phases:
  - SYM: $\langle\varphi\rangle = 0, \ \langle\varphi\rangle_s = 0$
  - FM: $\langle\varphi\rangle \neq 0, \ \langle\varphi\rangle_s = 0$
  - AFM: $\langle\varphi\rangle = 0, \ \langle\varphi\rangle_s \neq 0$
  - FI: $\langle\varphi\rangle \neq 0, \ \langle\varphi\rangle_s \neq 0$



Strategy for mass bound determination

- Idea: For given cutoff $\Lambda = a^{-1}$ find min. and max. Higgs masses in HY-model consistent with phenomenology.
- Considered phenomenology:
  - SSB: $\langle\varphi\rangle/(a\sqrt{Z_G}) \equiv v_r = 246\,\text{GeV}$
    $\rightarrow$ Fixes cutoff $\Lambda = a^{-1}$.
  - Top quark mass: $m_t/a = 175\,\text{GeV}$
    $\rightarrow$ Fixes Yukawa coupling constant y_t.
  - Bottom quark mass: $m_b/a = 4.2\,\text{GeV}$
    $\rightarrow$ Fixes Yukawa coupling constant y_b.
- 4 param. - 3 cond. = 1 freedom
  $\rightarrow \lambda$ undetermined.
- From tree-level: $m_H^2 \propto \lambda v^2$
  $\Rightarrow$ Smallest m_H at small λ. (Weak coupling.)
  Largest m_H at $\lambda \rightarrow \infty$. (Strong coupling.)

Phase diagram at $\tilde{\lambda}_N = 0.1$



Choice of bare model parameters

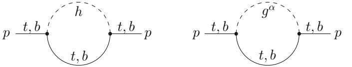
- Degenerate Yukawa coupling constants: (Otherwise $\det(\mathcal{M}) \in \mathbb{C}$)
  $\rightarrow$ Fix by tree-level relation: $y_{t,b} = \frac{m_{t,b}}{v_r} \Rightarrow y_{t,b} = \frac{175\ GeV}{246\ GeV} \approx 0.711$
- Quartic coupling constant:
  $\rightarrow$ Direct lattice calculation confirms: Min. m_H at $\lambda = 0 \quad \Rightarrow \lambda = 0$
- Accessible energy scales Λ:
  - Cutoff effects: Demand $\hat{m} < 0.5, \quad \hat{m} = m_H, m_t, m_b$
  - Finite volume effects: Demand $\hat{m} \cdot L_{s,t} > 2$ on largest lattices

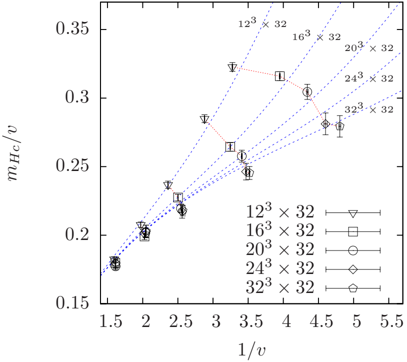
  $\Rightarrow \Lambda = 350 - 1100\ GeV$ accessible on 32^4-lattice,
  if $m_H(1100\ GeV) \approx 70\ GeV$
- Hopping parameter:
  $\rightarrow$ Vary κ such that $350\ GeV < \Lambda < 1100\ GeV$

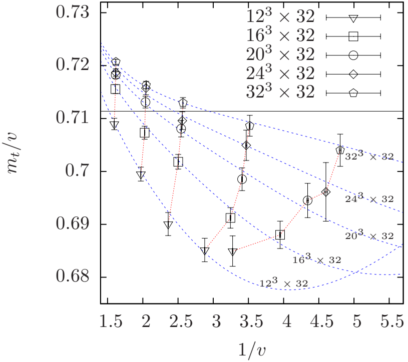
κ	L_s	L_t	N_f	λ	y_t	y_b/y_t	$1/v$	Λ
0.12301	10,12,14,16,18,20,24,32	32	1	0	0.71138	1	≈ 4.8	$\approx$ 1160 GeV
0.12303	10,12,14,16,18,20,24,32	32	1	0	0.71138	1	≈ 3.5	$\approx$ 850 GeV
0.12306	10,12,14,16,18,20,24,32	32	1	0	0.71138	1	≈ 2.6	$\approx$ 630 GeV
0.12309	10,12,14,16,18,20,24,32	32	1	0	0.71138	1	≈ 2.0	$\approx$ 500 GeV
0.12313	10,12,14,16,18,20,24,32	32	1	0	0.71138	1	≈ 1.6	$\approx$ 400 GeV

Finite volume Lattice results

- m_H: Blue lines: CEP-results at 1-loop order:
- m_t : Blue lines: LPT-results:
  - Calculate 1-loop diagrams for top propagator in lattice-PT:

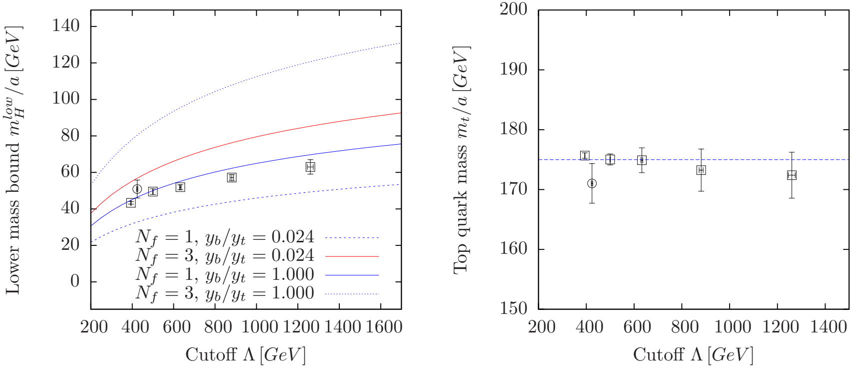






Lower Higgs boson mass bound in SM3

- m_H: Colored lines: CEP-results for $V = \infty$, different physical setups
  Red curve closest to situation in SM3
- Circular symbols: Series of lattice runs in non-degenerate case
  i.e. $m_b = 4.2\ GeV \Rightarrow y_b/y_t = 0.024$.
  Caution: Unknown systematic uncertainties due to $\det(M) \in \mathbb{C}$, if $y_t \neq y_b$

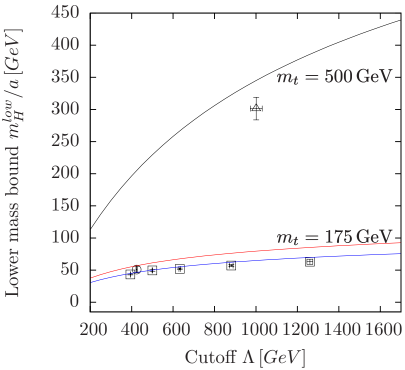


Fourth generation: Some preliminary results

- Assume mass-degenerate fourth quark doublet at 500 GeV

  Triangular symbols: m_H at $m_{t'} = m_{b'} = 500\,\text{GeV}$, $N_f = 1$ on $16^3 \times 32$ lattice

- Colored lines show CEP-results for $V = \infty$:
  - Black: $m_{t'} = m_{b'} = 500\,\text{GeV}$, $N_f = 1$
  - Blue: $m_t = m_b = 175\,\text{GeV}$, $N_f = 1$
  - Red: $m_t = 175\,\text{GeV}$, $m_b = 4.2\,\text{GeV}$, $N_f = 3$

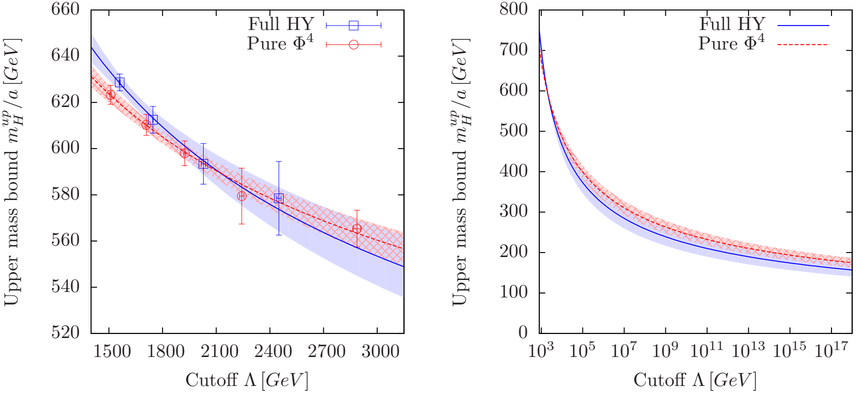


Upper Higgs boson mass bound in SM3

- Explicitly check: Largest m_H at $\lambda = \infty$
- Colored curves:

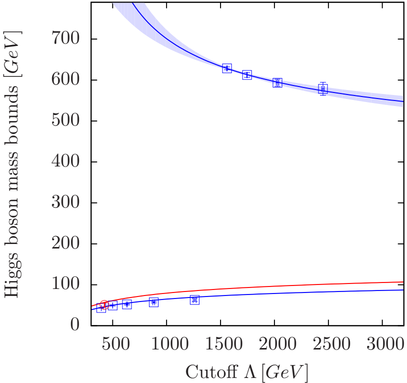
  Fits (A_m, B_m free fit parameter) with expected cutoff-dependence

$$\frac{m_{Hp}}{a} = A_m \cdot \left[\log(\Lambda^2/\mu^2) + B_m\right]^{-1/2}$$



Summary and Outlook

- Upper and lower Higgs boson mass bounds established on lattice.
  - $m_H^{low}(\Lambda = 1\,\text{TeV}) = 80\,\text{GeV}, \quad m_H^{up}(\Lambda = 1.5\,\text{TeV}) = 630\,\text{GeV}$
- Next: Push system to strong Yukawa coupling regime.
- Decay properties of Higgs boson also accessible via Lüscher's method. (Not demonstrated here.)

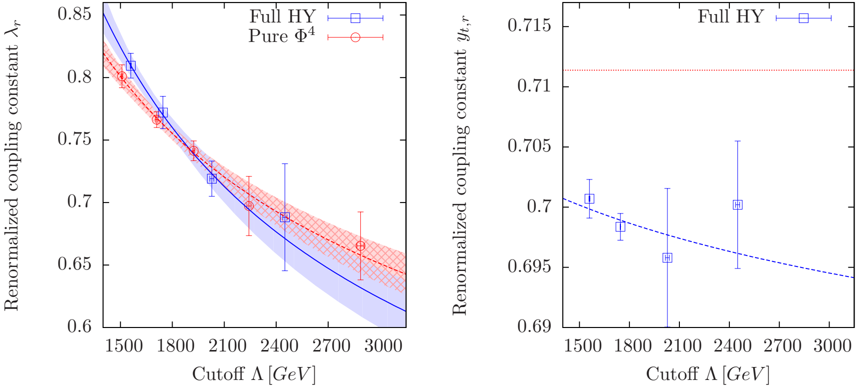


Running of renormalized coupling constants

- Fit observed running of coupling constants with expected form

$$\lambda_r = A_\lambda \left[\log(\Lambda^2/\mu^2) + B_\lambda\right]^{-1} \quad \text{and} \quad y_r = A_y \left[\log(\Lambda^2/\mu^2) + B_y\right]^{-1/2}$$

- Recall: In this notation no factor $1/4!$ in front of λ-term!



Scattering phases

- Lüscher [Nucl. Phys. B354, Nucl. Phys. B364]:
  Finite size analysis in finite space-time volume L^4 relates finite volume center-of-mass energies E_{2G} of 2-Goldstone states ($\leftrightarrow$ decay to WW states due to Goldstone equivalence theorem) to infinite volume scattering phase shifts $\delta_l(k)$ according to:

$$
\begin{aligned}
E_{2G} &= \sqrt{(2m_G)^2 + (2k)^2} \\
\delta_0(k) &= -\omega(q) \text{ modulo } \pi, \\
\tan(-\omega(q)) &= \frac{q\pi^{3/2}}{\mathcal{Z}(1,q^2)} \\
q &= \frac{kL}{2\pi} \\
\mathcal{Z}(s,q^2) &= \frac{1}{\sqrt{4\pi}} \sum_{\vec{n}\in\mathbb{Z}^3} \left(\vec{n}^2 - q^2\right)^{-s} \quad \text{zeta function}
\end{aligned}
$$

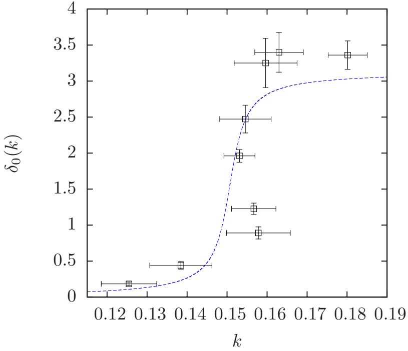
and $\mathcal{Z}(1,q^2)$ given as analytical continuation:

$$
\sqrt{4\pi}\mathcal{Z}(1,q^2) = (2\pi)^3 \int_0^1 \mathrm{d}t \left[(4\pi t)^{-\frac{3}{2}}(e^{tq^2}-1) + (4\pi t)^{-\frac{3}{2}} e^{tq^2} \sum_{\vec{n}\neq\vec{0}} e^{-\frac{\pi^2}{t}\vec{n}^2}\right] + \sum_{\vec{n}\in\mathbb{Z}^3} \frac{e^{q^2-\vec{n}^2}}{\vec{n}^2 - q^2} - 2\pi^{\frac{3}{2}}
$$

Scattering phases

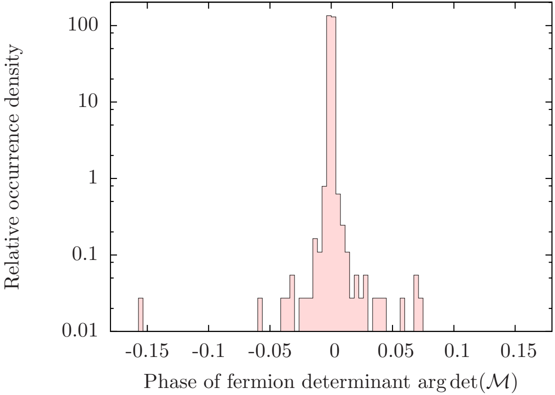
- Use Breit-Wigner function to fit x-section $\sigma(k) \approx \frac{4\pi}{k^2}\sin^2(\delta_0(k))$
- Compare with PT-result as reference. For test purpose consider small value of λ_r. Here: $\lambda_r \approx 0.12$

	Resonance analysis	Reference value
Mass	$m_{Hr} = 0.4281(31)$	$m_{Hp} = 0.4328(32)$
Decay width	$\Gamma_{Hr} = 0.0086(33)$	$\Gamma_H = 0.0076(2)$



Fermion Determinant

- In degenerate case with $y_t = y_b$: fermion determinant $\det(\mathcal{M}) \in \mathbb{R}$ however not necessarily $\det(\mathcal{M}) \geq 0$.
  $\rightarrow \det(\mathcal{M}) \geq 0$ is checked, but no neg. signs found in practice.
- In non-degenerate case $\det(\mathcal{M}) \in \mathbb{C} \rightarrow$ check phase $\arg\det(\mathcal{M})$



$y_t \neq y_b$

4^4-lattice

φ Gauss sampled

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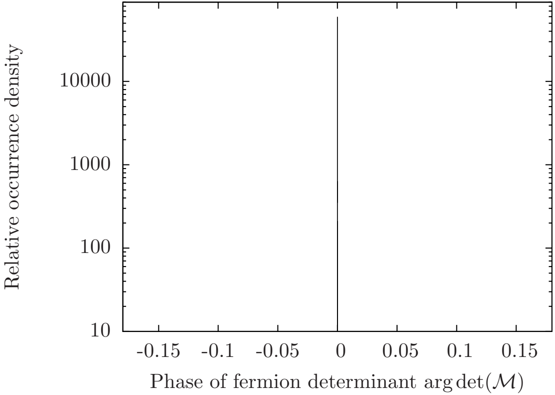

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φ from MC-sim. in broken phase

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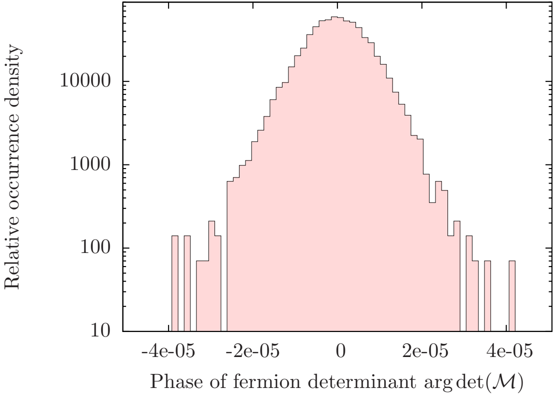

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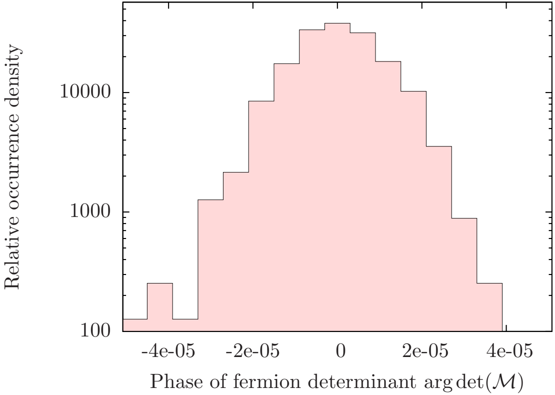


$y_t \neq y_b$

6^4-lattice

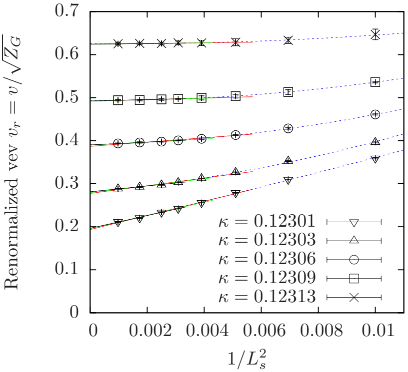
φ from MC-sim. in broken phase

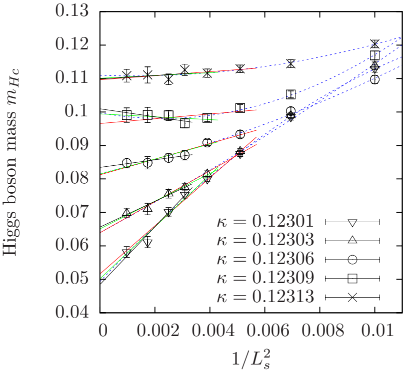
Infinite volume extrapolation

- Goldstone modes induce algebraic FSE of order $O(L_s^{-2})$, $O(L_s^{-4})$, ...
- Perform infinite volume extrapolation with fit ansatz

$$\text{Linear:} \quad f^{(l)}_{v,m}(L_s^{-2}) = A^{(l)}_{v,m} + B^{(l)}_{v,m} \cdot L_s^{-2} \quad \text{for } L \geq 14(\text{red}),\ 16(\text{green}),\ 18(\text{black})$$

$$\text{Parabolic:} \quad f^{(p)}_{v,m}(L_s^{-2}) = A^{(p)}_{v,m} + B^{(p)}_{v,m} \cdot L_s^{-2} + C^{(p)}_{v,m} \cdot L_s^{-4} \quad \text{for all } L \text{ (blue)}$$





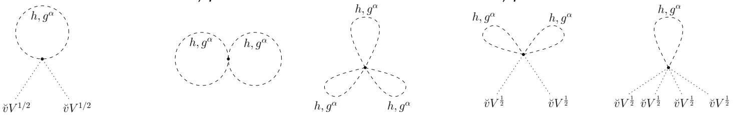
Model extension by higher order interactions

- Question: Lower bounds depend on form of Higgs self-interaction?
- Consider model extension by six-point coupling $\lambda_6|\varphi|^6$
- Evaluate extended model. Search for $m_H < m_H^{low}$.
  - Problem: Many parameters: κ, $y_{t,b}$, λ, λ_6 $\rightarrow$ Tuning problem.
  - Need analytical estimation of bare parameters targeting at $m_H < m_H^{low}$.
- CEP in extended model:
  - Include bosonic loop diagrams at order $O(\lambda, \lambda_6)$

$$\check{U}[\check{v}] = \underbrace{\check{U}_0[\check{v}]}_{\text{tree-level CEP of original HY-model}} + \lambda_6 \check{v}^6 + \lambda \sum_{\alpha=1}^{2} \sum_{\beta=0}^{\alpha} C_{4,\alpha,\beta} \cdot \check{v}^{4-2\alpha} P_H^{\alpha-\beta} P_G^{\beta}$$

$$+ \lambda_6 \sum_{\alpha=1}^{3} \sum_{\beta=0}^{\alpha} C_{6,\alpha,\beta} \cdot \check{v}^{6-2\alpha} P_H^{\alpha-\beta} P_G^{\beta}$$

with $\quad P_H = \frac{1}{V} \sum_{0 \neq p \in \mathcal{P}} \frac{1}{\hat{p}^2 + m_H^2} \quad$ and $\quad P_G = \frac{1}{V} \sum_{0 \neq p \in \mathcal{P}} \frac{1}{\hat{p}^2 + 0}$



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$$+ \lambda_6 \sum_{\alpha=1}^{3} \sum_{\beta=0}^{\alpha} C_{6,\alpha,\beta} \cdot \breve{v}^{6-2\alpha} P_H^{\alpha-\beta} P_G^{\beta}$$

with $P_H = \frac{1}{V} \sum_{0 \neq p \in \mathcal{P}} \frac{1}{\hat{p}^2 + m_H^2}$ and $P_G = \frac{1}{V} \sum_{0 \neq p \in \mathcal{P}} \frac{1}{\hat{p}^2 + 0}$

$C_{4,\alpha,\beta}$

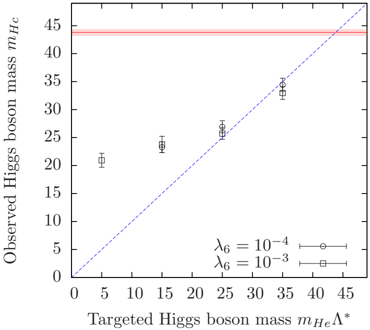
	$\beta = 0$	$\beta = 1$	$\beta = 2$
$\alpha = 1$	6	6	–
$\alpha = 2$	3	6	15

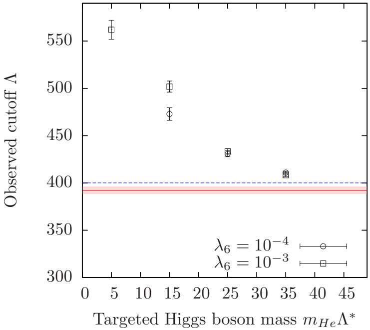
$C_{6,\alpha,\beta}$

	$\beta = 0$	$\beta = 1$	$\beta = 2$	$\beta = 3$
$\alpha = 1$	15	9	–	–
$\alpha = 2$	45	54	45	–
$\alpha = 3$	15	27	45	105

Lower bound not universal

- Use CEP to estimate bare parameters targeting at ...
  - targeted cutoff parameter $\Lambda^* = 400\,\text{GeV}$
  - targeted Higgs boson mass $m_{He} < m_H^{low}$
- Check with direct lattice calculation
  16^4-lattice, $y_t \equiv y_b$, $y_t = m_t/v_r$, different λ_6 tested, κ, λ fixed by CEP.
- Red band: $m_H^{low}(\Lambda)$ at $\Lambda = 396\ GeV$ in original HY-model.





Constraint effective potential (CEP)

CEP: $U[\underline{\tilde{\Phi}}_0, \underline{\tilde{\Phi}}_{p_s}] = -\frac{1}{V}\log\left(\int D\psi\, D\bar{\psi} \prod_{0\neq k\neq p_s} d\tilde{\Phi}_k\; e^{-S[\Phi,\psi,\bar{\psi}]} \Big|_{\tilde{\Phi}_0=\underline{\tilde{\Phi}}_0,\, \tilde{\Phi}_{p_s}=\underline{\tilde{\Phi}}_{p_s}} \right)$

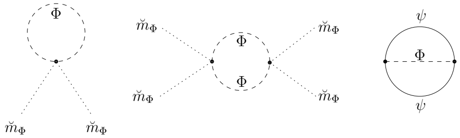
with $\tilde{\Phi}_p = \frac{1}{\sqrt{V}} \sum_x e^{-ipx} \cdot \Phi_x$ and $p_s = (\pi, \pi, \pi, \pi)$

$\Rightarrow \langle O[\tilde{\Phi}_0, \tilde{\Phi}_{p_s}] \rangle = \int d\tilde{\Phi}_0\, d\tilde{\Phi}_{p_s}\; O[\tilde{\Phi}_0, \tilde{\Phi}_{p_s}] \cdot e^{-VU[\tilde{\Phi}_0, \tilde{\Phi}_{p_s}]}$ $\leftarrow$ dominated by minima of U

- Perturbative expansion of CEP:
  - Split up action $S[\Phi, \psi, \bar{\psi}]$ around expansion point $\Phi' \equiv (\underline{\tilde{\Phi}}_0, \underline{\tilde{\Phi}}_{p_s})$

$$
\begin{aligned}
\Phi'_x &= \left(\breve{m}_\Phi + \breve{s}_\Phi e^{ip_s x} \right) \hat{\Phi} \quad \text{with} \quad \hat{\Phi} \in \mathbb{R}^4,\ |\hat{\Phi}| = 1 \\
S[\Phi, \psi, \bar{\psi}] &\equiv S_\Phi[\Phi'] + \underbrace{S_0[\Phi', \Phi, \psi, \bar{\psi}]}_{\text{Gaussian}} + \underbrace{S_I[\Phi', \Phi, \psi, \bar{\psi}]}_{\text{Interaction}} \\
S_0[\Phi', \Phi, \psi, \bar{\psi}] &= \sum_{i=1}^{N_f} \bar{\psi}^{(i)} \mathcal{M}[\Phi'] \psi^{(i)} + \frac{1}{2} \sum_{0\neq k\neq p_s} \tilde{\Phi}_k^\dagger \left[2 - 4\tilde{\lambda}_N - \sum_\mu 4\kappa \cos(k_\mu) \right] \tilde{\Phi}_k
\end{aligned}
$$

- Example diagrams contributing to $U[\breve{m}_\Phi \hat{\Phi}, \breve{s}_\Phi \hat{\Phi}]$:



Considered observables

- On lattice: Always $\langle\varphi\rangle \equiv 0$. To study SSB:
  $\rightarrow$ Rotate each φ-configuration: $\varphi_x^{rot} = U\varphi_x$, $U \in \mathrm{SU}(2)$ such that

$$\sum_x \varphi_x^{rot} = \begin{pmatrix} 0 \\ \left|\sum_x \varphi_x\right| \end{pmatrix}. \text{ Then define } \langle\varphi^{rot}\rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}.$$

- Define Higgs-/Goldstone-modes: $\varphi_x^{rot} = \begin{pmatrix} g_x^2 + ig_x^1 \\ v + h_x - ig_x^3 \end{pmatrix}$
- Propagators: $\tilde{G}_H(p) = \langle \tilde{h}_p \tilde{h}_{-p}\rangle$, $\tilde{G}_G(p) = \frac{1}{3}\sum_{\alpha=1}^{3}\langle \tilde{g}_p^\alpha \tilde{g}_{-p}^\alpha\rangle$
- Goldstone mass and Z_G:

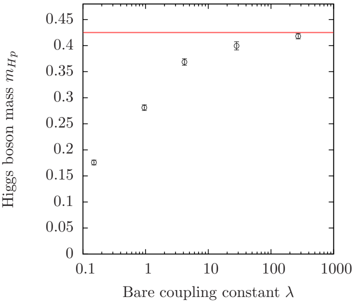
$$Z_G^{-1} = \frac{\mathrm{d}}{\mathrm{d}p_c^2}\left[\tilde{G}_G^c(p_c^2)\right]^{-1}\Big|_{p_c^2=-m_G^2} \quad \text{and} \quad \left[G_G^c(p_c^2)\right]^{-1}\Big|_{p_c^2=-m_{Gp}^2} = 0$$

- Higgs, top and bottom mass: m_{Hc}, m_t, m_b
  $\rightarrow$ From exponential decay of time correlation functions

$$C_H(\Delta t) = \sum_{\vec{x},\vec{y}}\langle h_{\Delta t,\vec{x}} h_{0,\vec{y}}\rangle \quad \text{and} \quad C_f(\Delta t) = \sum_{\vec{x},\vec{y}}\left\langle 2\,\mathrm{Re\,Tr}\left(f_{L,\Delta t,\vec{x}}\cdot \bar{f}_{R,0,\vec{y}}\right)\right\rangle, \quad f = t, b$$

Choice of bare model parameters

- At $\lambda \gg 1$:
  $C_H(\Delta t)$ contamin. by Goldstone modes
  $\rightarrow$ Instead consider m_{Hp}: [Lüscher et al.]
  $$\text{Re}\left(\left[G_H^c(p_c^2)\right]^{-1}\right)\Big|_{p_c^2=-m_{Hp}^2} = 0.$$
- m_H rises monoton. with $\lambda \rightarrow \infty$
  $\Rightarrow$ Choose $\lambda = \infty$
- Accessible energy scales:
  Require: $\hat{m} > 0.5$ and $\hat{m} \cdot L_{s,t} > 2$
  $\Rightarrow \Lambda = 1400 - 2800\ GeV$ accessible
  (if $m_H/a < 700\ GeV$)



$16^3 \times 32$-lattice, $\Lambda \approx 1500\ GeV$, $y_{t,b} = 0.71138$
Red band: $\lambda = \infty$ result

κ	L_s	L_t	N_f	λ	y_t	y_b/y_t	$1/v$	Λ
0.30039	12,16,20,24,32	32	1	∞	0.71138	1	≈ 7.7	$\approx$ 2370 GeV
0.30148	12,16,20,24,32	32	1	∞	0.71138	1	≈ 6.5	$\approx$ 1990 GeV
0.30274	12,16,20,24,32	32	1	∞	0.71138	1	≈ 5.6	$\approx$ 1730 GeV
0.30400	12,16,20,24,32	32	1	∞	0.71138	1	≈ 5.0	$\approx$ 1550 GeV
0.30570	12,16,20,24,32	32	1	∞	0	–	≈ 9.0	$\approx$ 2810 GeV
0.30680	12,16,20,24,32	32	1	∞	0	–	≈ 7.1	$\approx$ 2220 GeV
0.30780	12,16,20,24,32	32	1	∞	0	–	≈ 6.2	$\approx$ 1910 GeV
0.30890	12,16,20,24,32	32	1	∞	0	–	≈ 5.5	$\approx$ 1700 GeV
0.31040	12,16,20,24,32	32	1	∞	0	–	≈ 4.9	$\approx$ 1500 GeV