

# Revisit of Flavor and CPV Data with Heavy $t'$ and $b'$

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2nd Workshop on Beyond 3 Generation Standard Model  
--- New Fermions at the Crossroads of Tevatron and LHC

# Large $\sin 2\Phi_{B_s}$ ?

SM predicts  $\sin 2\Phi_{B_s}$  (also denoted  $-\sin 2\beta_s$ )  $\sim -0.04$ .

Latest constrained fit by HFAG is  $2.2 \sigma$  away from SM.

[Barberio et al. arXiv:0808.1297]

4 generation (SM4) introduce  $t'$  to interfere with  $t$  in box diagrams

$$M_{12} = \frac{G_F^2 M_W^2}{12 \pi^2} m_{B_s} \left( f_{B_s} \sqrt{\hat{B}_{B_s}} \right)^2 \left( \lambda_t^2 \eta S_0(x_t) + \eta' \lambda_{t'}^2 S_0(x_{t'}) + 2 \tilde{\eta} \lambda_t \lambda_{t'} \tilde{S}_0(x_t, x_{t'}) \right)$$

SM4 makes SM triangle into quadrangle

$$V_{us}^* V_{ub} + V_{cs}^* V_{cb} + V_{ts}^* V_{tb} + V_{t's}^* V_{t'b} = 0$$

Combine

$$\Delta m_{B_s}$$

[box diagram]

$$\mathcal{B}(b \rightarrow s \ell \ell)$$

[Z penguin diagram]

to determine

$$V_{t's}^* V_{t'b}$$

# Use parametrization matrix

[Hou, Soni and Steger PLB 192 (1987)]

## 1st Row

$$V_{ud} = -c_{13} s_{14} s_{12} s_{24} e^{i\Phi_{sb} - i\Phi_d} - s_{13} c_{24} s_{14} s_{34} e^{-i\Phi_d - i\phi_3} + c_{14} c_{13} c_{12}$$

$$V_{us} = c_{13} c_{24} s_{12} - s_{13} s_{34} s_{24} e^{-i\Phi_{sb} - i\phi_3}$$

$$V_{ub} = s_{13} c_{34} e^{-i\phi_3}$$

$$V_{ub'} = c_{14} c_{13} s_{12} s_{24} e^{i\Phi_{sb}} + c_{13} c_{12} s_{14} e^{i\Phi_d} + s_{13} c_{14} c_{24} e^{-i\phi_3} s_{34}$$

## 4th Row

$$V_{t'd} = -c_{34} c_{24} s_{14} e^{-i\Phi_d}$$

$$V_{t's} = -c_{34} s_{24} e^{-i\Phi_{sb}}$$

$$V_{t'b} = -s_{34}$$

$$V_{t'b'} = c_{14} c_{34} c_{24}$$

## 2nd Row

$$V_{cd} = s_{14} s_{13} s_{12} s_{24} s_{23} e^{i\Phi_{sb} - i\Phi_d + i\phi_3} - c_{12} c_{23} s_{14} s_{24} e^{i\Phi_{sb} - i\Phi_d} - c_{13} c_{24} s_{14} s_{34} s_{23} e^{-i\Phi_d} - c_{14} c_{12} e^{i\phi_3} s_{13} s_{23} - c_{14} c_{23} s_{12}$$

$$V_{cs} = -c_{13} s_{34} s_{24} s_{23} e^{-i\Phi_{sb}} - c_{24} e^{i\phi_3} s_{13} s_{12} s_{23} + c_{12} c_{24} c_{23}$$

$$V_{cb} = c_{13} c_{34} s_{23}$$

$$V_{cb'} = -c_{14} s_{13} s_{12} s_{24} s_{23} e^{i\Phi_{sb} + i\phi_3} + c_{14} c_{12} c_{23} s_{24} e^{i\Phi_{sb}} - c_{12} s_{14} s_{13} s_{23} e^{i\Phi_d + i\phi_3} - c_{23} s_{14} s_{12} e^{i\Phi_d} + c_{14} c_{13} c_{24} s_{34} s_{23}$$

## 3rd Row

$$V_{td} = c_{23} s_{14} s_{13} s_{12} s_{24} e^{i\Phi_{sb} - i\Phi_d + i\phi_3} + c_{12} s_{14} s_{24} s_{23} e^{i\Phi_{sb} - i\Phi_d} - c_{13} c_{24} c_{23} s_{14} s_{34} e^{-i\Phi_d} + c_{14} s_{12} s_{23} - c_{14} c_{12} c_{23} e^{i\phi_3} s_{13}$$

$$V_{ts} = -c_{13} c_{23} s_{34} s_{24} e^{-i\Phi_{sb}} - c_{12} c_{24} s_{23} - c_{24} c_{23} e^{i\phi_3} s_{13} s_{12}$$

$$V_{tb} = c_{13} c_{34} c_{23}$$

$$V_{tb'} = -c_{14} c_{23} s_{13} s_{12} s_{24} e^{i\Phi_{sb} + i\phi_3} - c_{14} c_{12} s_{24} s_{23} e^{i\Phi_{sb}} - c_{12} c_{23} s_{14} s_{13} e^{i\Phi_d + i\phi_3} + s_{14} s_{12} s_{23} e^{i\Phi_d} + c_{14} c_{13} c_{24} c_{23} s_{34}$$

Hou, Nagashima and Soddu  
[PRD72(2005),PRD76(2007)]

illustrated the case  $m_t = 300$  GEV using

$$f_{B_s} \sqrt{B_{B_s}} = 295 \text{ MEV}$$

Get  $\sin 2\Phi_{B_s} \sim -0.50 - -0.70$

What's new in our recent study ?

## Trivial Change

1.  $m_{t'} = 300 \text{ GEV} \rightarrow 500 \text{ GEV}$  (unitarity bound)

[Chanowitz et al. PLB78(1978)]

2.  $f_{B_s} \sqrt{B_{B_s}} = 295 \text{ MEV} \rightarrow 266(18) \text{ MEV}$

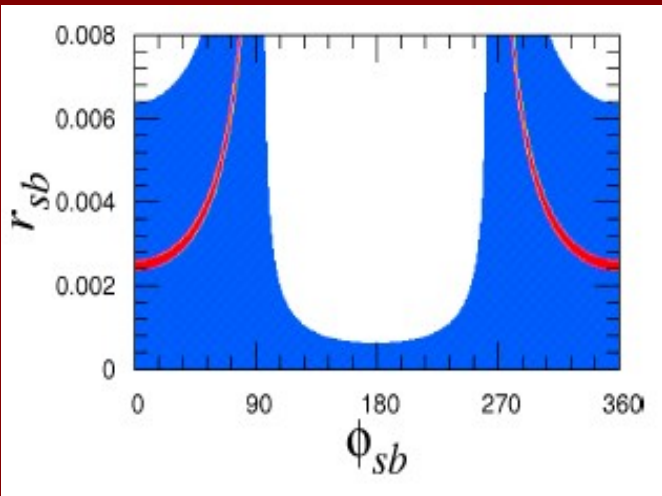
[Gamiz et al. HPQCD PRD80(2009)]

## Nontrivial New Analysis

1.  $D - D$  mixing constraint

2. Electroweak precision global fit

$$V_{t's}^* V_{t'b} \equiv r_{sb} e^{i\phi_{sb}}$$

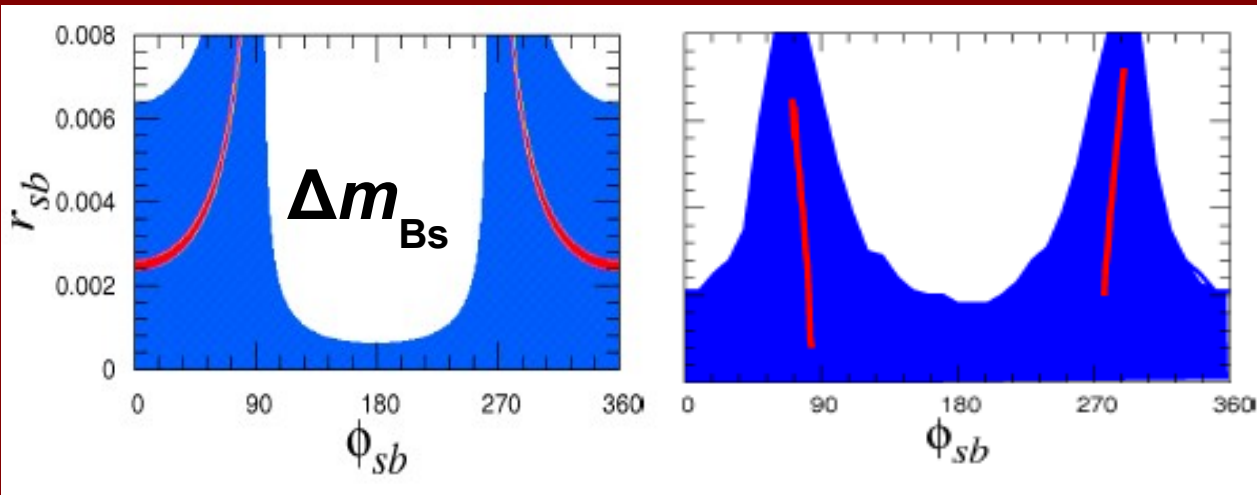


$$(\Delta m_{B_s})^{\text{exp}} = (17.77 \pm 0.12) \text{ ps}^{-1}$$

$$f_{B_s} \sqrt{\hat{B}_{B_s}} = 266(18) \text{ MEV}$$

$$M_{12} = \frac{G_F^2 M_W^2}{12 \pi^2} m_{B_s} \left( f_{B_s} \sqrt{\hat{B}_{B_s}} \right)^2 \left( \lambda_t^2 \eta S_0(x_t) + \eta' \lambda_{t'}^2 S_0(x_{t'}) + 2 \tilde{\eta} \lambda_t \lambda_{t'} \tilde{S}_0(x_t, x_{t'}) \right)$$

$$V_{t's}^* V_{t'b} \equiv r_{sb} e^{i\phi_{sb}}$$

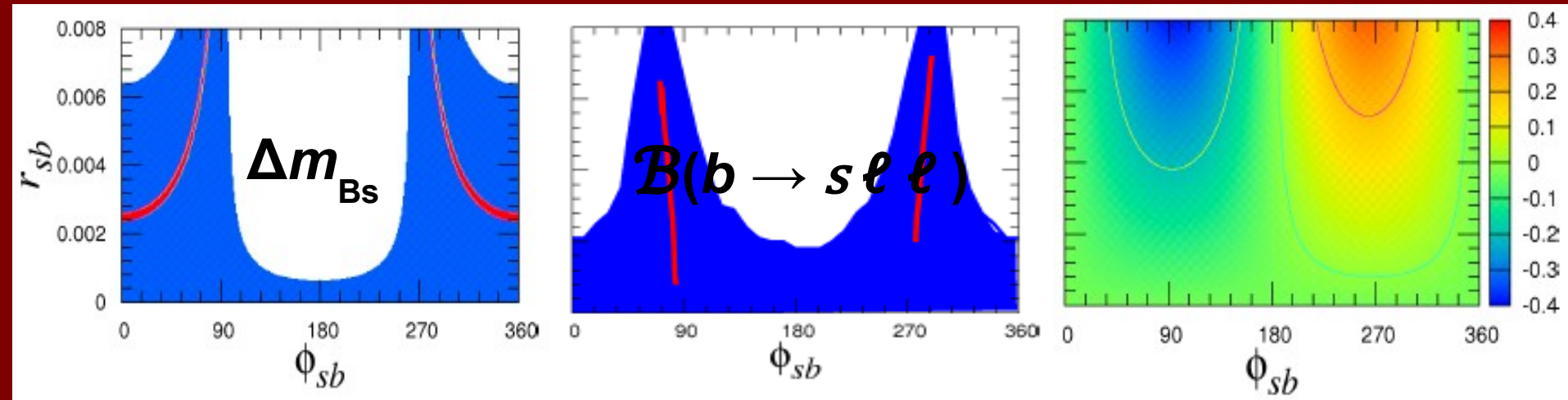


Calculation using [Bobeth *et al.* NPB(2000)]

$$\mathcal{B}^{\text{exp}}(b \rightarrow s \ell \ell) = (4.5 \pm 1.0) \times 10^{-6}$$



$$V_{t's}^* V_{t'b} \equiv r_{sb} e^{i\phi_{sb}}$$



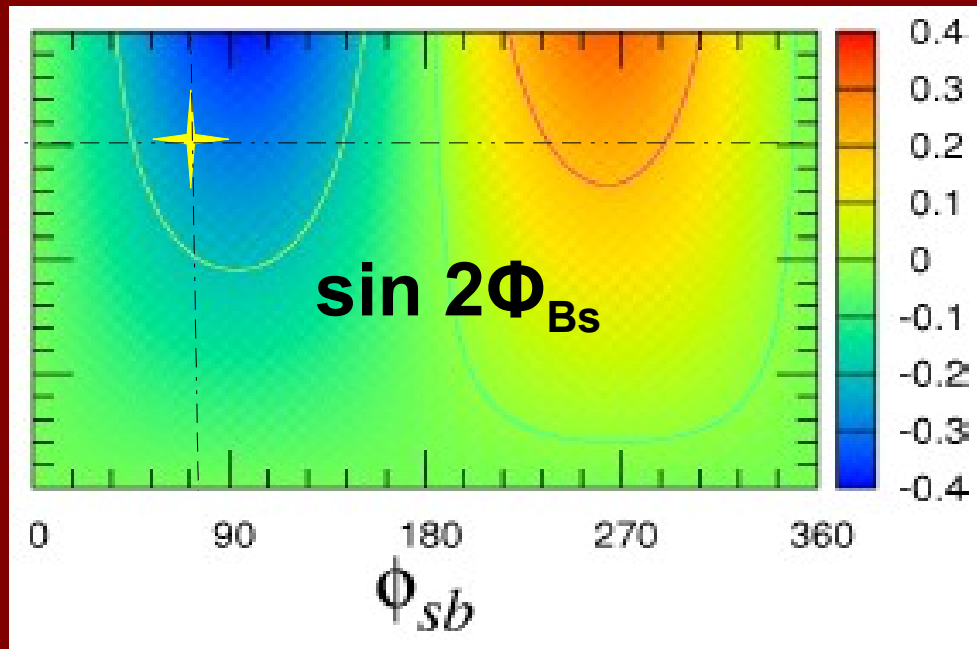
$$\sin 2\Phi_{B_s} = \text{Im}(M_{12}) / |M_{12}|$$

$$M_{12} = \frac{G_F^2 M_W^2}{12 \pi^2} m_{B_s} \left( f_{B_s} \sqrt{\hat{B}_{B_s}} \right)^2 \left( \lambda_t^2 \eta S_0(x_t) + \eta' \lambda_{t'}^2 S_0(x_{t'}) + 2 \tilde{\eta} \lambda_t \lambda_{t'} \tilde{S}_0(x_t, x_{t'}) \right)$$

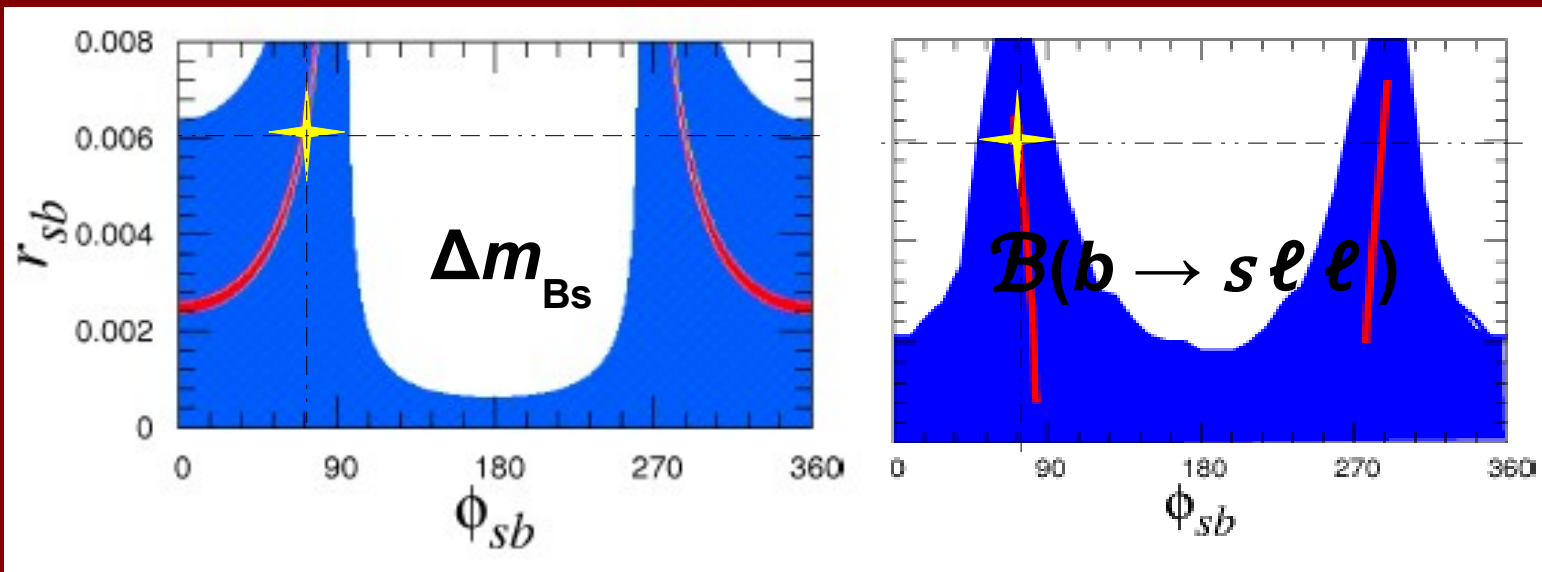
$$r_{sb} = 0.006,$$

$$\varphi_{sb} = 75^\circ,$$

$$\sin 2\Phi_{Bs} = -0.33$$



SMALLER  
 $\sin 2\Phi_{Bs}$   
 STILL  
 POSSIBLE



$$V_{ud} \quad V_{us} \quad V_{ub} \quad V_{ub'}$$

$$V_{cd} \quad V_{cs} \quad V_{cb} \quad V_{cb'}$$

$$V_{td} \quad V_{ts} \quad V_{tb} \quad V_{tb'}$$

$$V_{t'd} \quad V_{t's} \quad V_{t'b} \quad V_{t'b'}$$

$$V_{t's}^* V_{t'b} = 0.006 e^{i75^\circ}$$

$$V_{t's} = ?$$

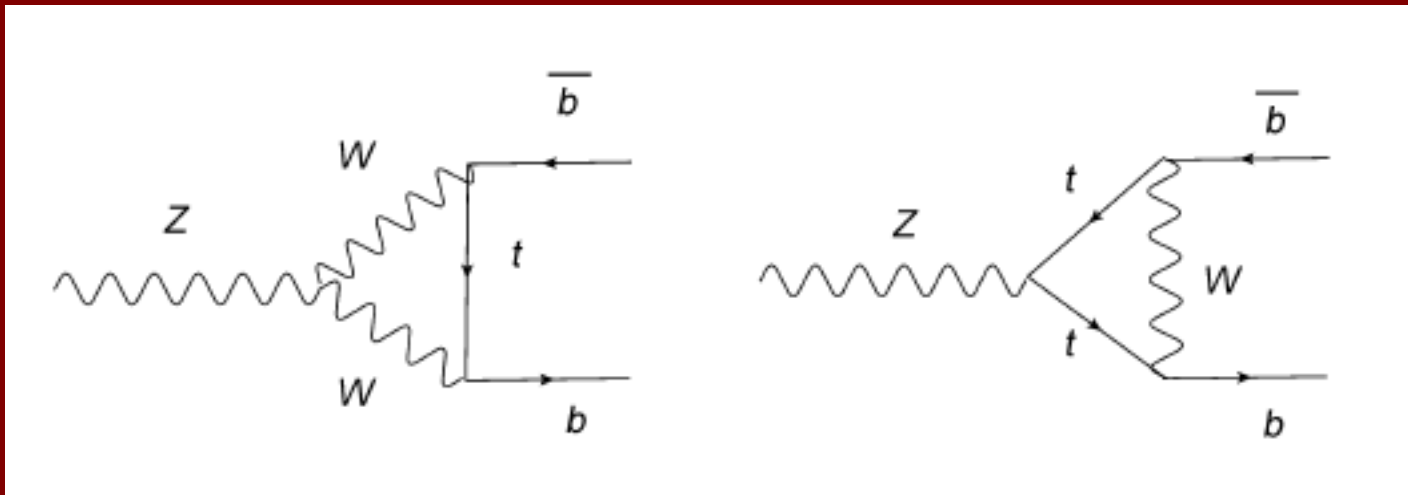
$$V_{t'b} = ?$$

# Upper Bound on $|V_{t'b}|$

$$R_b \equiv \Gamma(Z \rightarrow b\bar{b}) / \Gamma(Z \rightarrow \text{hadrons})$$

$$|V_{t'b}| < 0.13 \quad (m_{t'} = 500 \text{ GEV})$$

[Yanir, JHEP06(2002)]



What is Lower Bound on  $|V_{t'b}|$  ?

# Constraint from

$$\mathcal{B}^{\text{exp}}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (1.73_{-0.33}^{+1.15}) \times 10^{-10}$$

[Artamonov *et al.* PRL **101**(2008)]

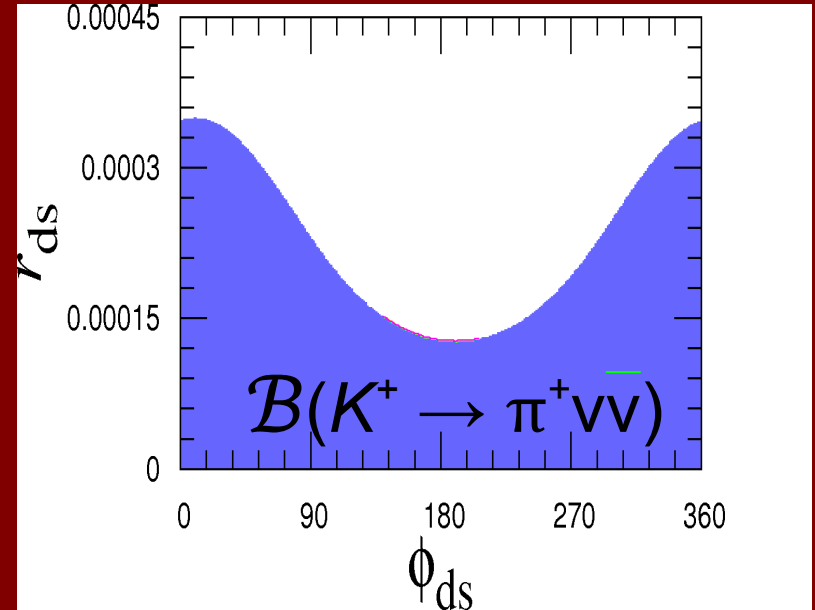
$$V_{ud} \quad V_{us} \quad V_{ub} \quad V_{ub'}$$

$$V_{cd} \quad V_{cs} \quad V_{cb} \quad V_{cb'}$$

$$V_{td} \quad V_{ts} \quad V_{tb} \quad V_{tb'}$$

$$V_{t'd} \quad V_{t's} \quad V_{t'b} \quad V_{t'b'}$$

$$V_{t'd}^* \quad V_{t's} = r_{ds} e^{i\phi_{ds}}$$



## UPPER BOUND

$$\kappa_+ |V_{us}|^{-5} |\lambda_c^{ds} |V_{us}|^4 P_c + \lambda_t^{ds} \eta_t X_0(x_t) + \lambda_{t'}^{ds} \eta_{t'} X_0(x_{t'})|^2 < 3.6 \times 10^{-10} \text{ (90\% CL)},$$

# Constraint from $D$ - $D$ mixing

$V_{ud}$	$V_{us}$	$V_{ub}$	$V_{ub'}$	} UPPER BOUND
$V_{cd}$	$V_{cs}$	$V_{cb}$	$V_{cb'}$	
$V_{td}$				
$V_{t'd}$				

$|V_{ub'}^* V_{cb'}| < 0.0021$   
for  $m_{b'} = 480$  GEV

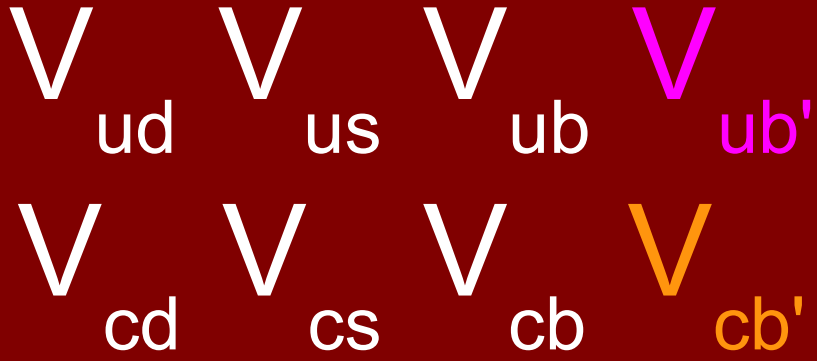
$$M_{12}^{D^0} \propto \lambda_s^2 + 2\lambda_s\lambda_b S(x_s, x_b) + \lambda_b^2 S_0(x_b) + LD$$

$$+ 2\lambda_s\lambda_{b'} S(x_s, x_{b'}) + 2\lambda_b\lambda_{b'} S(x_b, x_{b'}) + LD$$

$$+ \lambda_{b'}^2 S_0(x_{b'}), \quad (\lambda_{b'} = V_{ub'}^* V_{cb'})$$

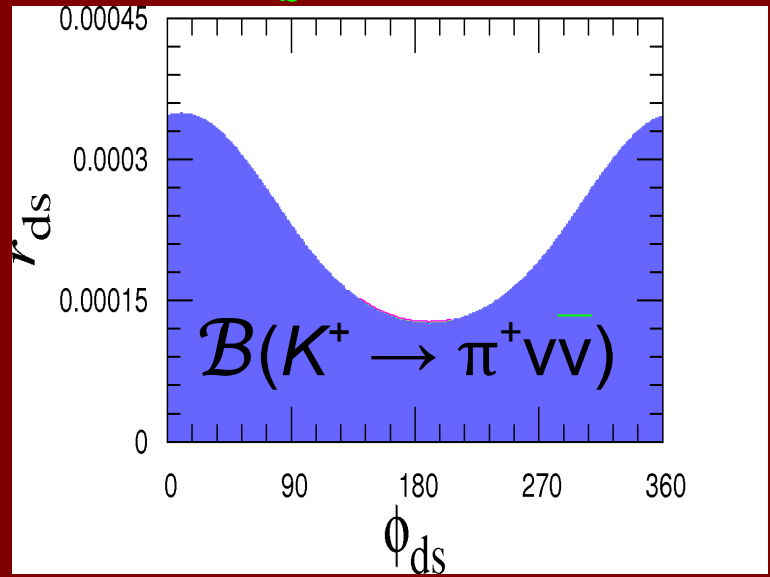
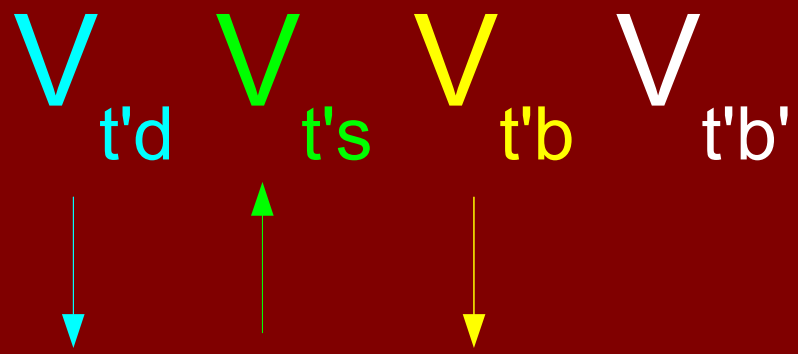
Neglect all terms, except  $\lambda_{b'}^2 S_0(x_{b'})$ ,  
 equate this term with  $x_D^{\text{exp}} = 9.1 \times 10^{-3}$   
 accommodate  $LD$  by a factor of 3  
 [Golowich *et al.* PRD76(2007),  
 Bobrowski, Lenz *et al.* PRD79(2009)]

$$V_{ub'} = c_{13}c_{12}s_{14}e^{i\Phi_d} + c_{14}c_{13}s_{12}s_{24}e^{i\Phi_{sb}} + s_{13}s_{34}c_{14}c_{24}e^{-\phi_3}$$



?  $|V_{ub'}^* V_{cb'}| < 0.0021$   
 for  $m_{b'} = 480 \text{ GEV}$

Fix  $V_{t's}^* V_{t'b} = 0.006 e^{i75^\circ}$



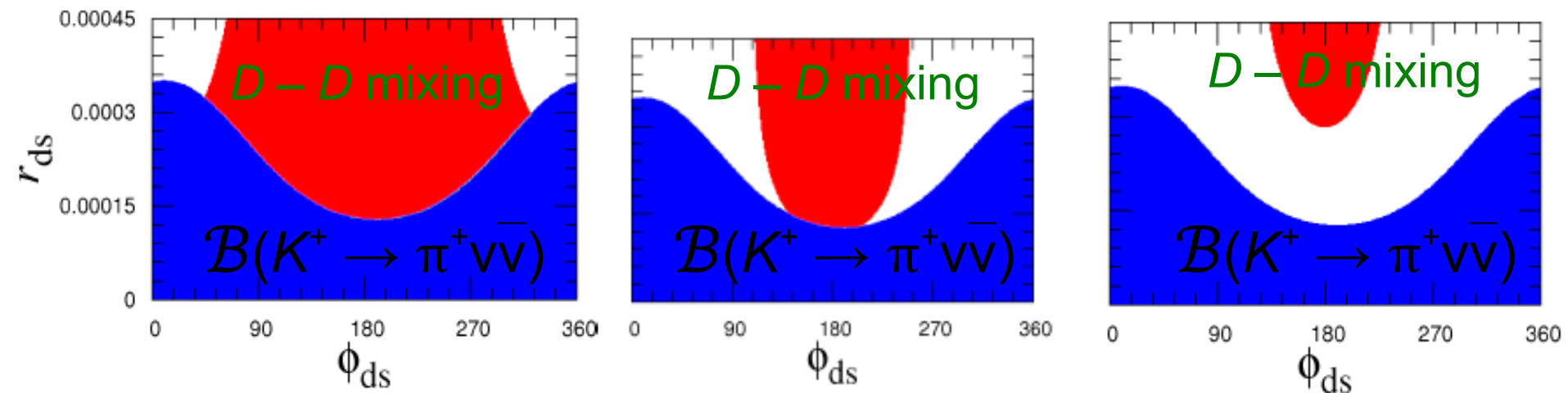
$$V_{t'd} = -c_{34}c_{24}s_{14}e^{-i\Phi_d}$$

$$V_{t's} = -c_{34}s_{24}e^{-i\Phi_{sb}}$$

$$V_{t'b} = -s_{34}$$

$$\kappa_+ |V_{us}|^{-5} |\lambda_c^{ds} |V_{us}|^4 P_c + \lambda_t^{ds} \eta_t X_0(x_t) + \lambda_{t'}^{ds} \eta_{t'} X_0(x_{t'})|^2 < 3.6 \times 10^{-10} \text{ (90\% CL)},$$

# Lower Bound on $|V_{t'b}|$



$$|V_{t'b}| = 0.065$$

$$|V_{t'b}| = 0.060$$

$$|V_{t'b}| = 0.058$$

$$|V_{t'b}| > 0.06$$

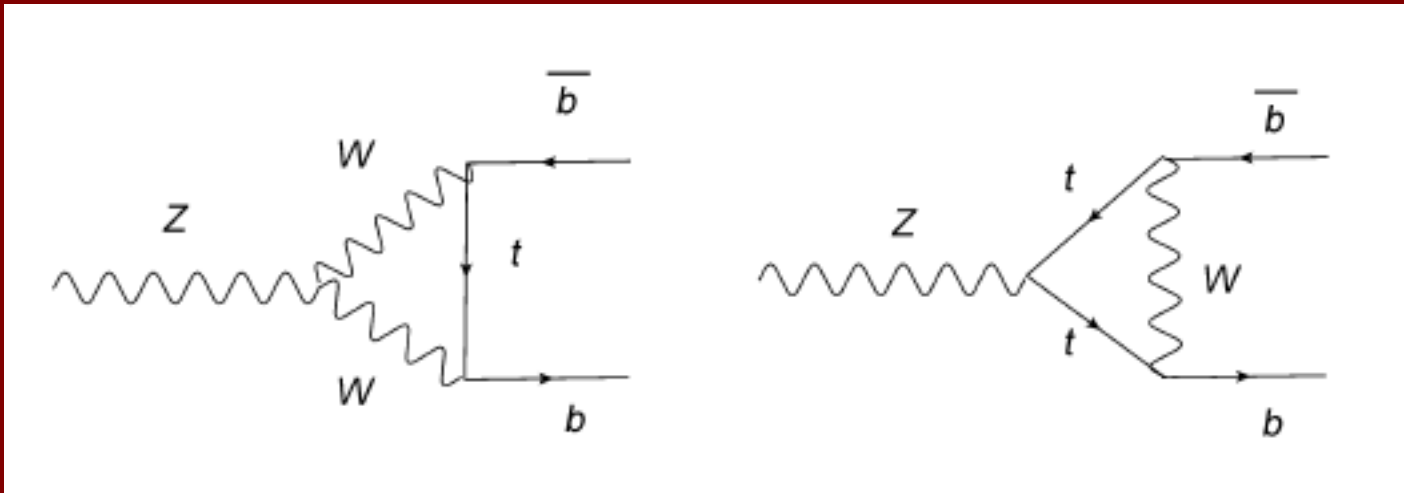


# Electroweak precision global fit

[Chanowitz, PRD79(2009)]

$$S_4 = \frac{N_C}{6\pi} \left( 1 - \frac{1}{3} \ln \frac{x_1}{x_2} \right)$$

$$T_4 = \frac{1}{8\pi x_W(1-x_W)} \left\{ 3 \left[ F_{t'b'} + s_{34}^2 (F_{t'b} + F_{tb'} - F_{tb} - F_{t'b'}) \right] + F_{l_4\nu_4} \right\} \\ + \frac{3}{8\pi x_W(1-x_W)} \left[ |V_{t's}|^2 F_{t's} + |V_{cb'}|^2 F_{cb'} \right].$$



# Electroweak precision global fit

[Maksymyk *et al.* PRD50(1994)]

$$M_W^2 = (M_W^{\text{SM}})^2 \left[ 1 - \frac{\alpha \Delta S}{2(c_w^2 - s_w^2)} + \frac{c_w^2 \alpha \Delta T}{(c_w^2 - s_w^2)} \right],$$

$$\Gamma_W = \Gamma_W^{\text{SM}} \left[ 1 - \frac{\alpha \Delta S}{2(c_w^2 - s_w^2)} + \frac{c_w^2 \alpha \Delta T}{(c_w^2 - s_w^2)} \right],$$

$$\sin^2 \theta_{\text{eff}}^{\text{lept}} = \sin^2 \theta_{\text{eff}}^{\text{lept SM}} \left[ 1 + \frac{\alpha \Delta S}{4s_w^2 (c_w^2 - s_w^2)} - \frac{c_w^2 \alpha \Delta T}{(c_w^2 - s_w^2)} \right],$$

$$\Gamma(Z \rightarrow \nu \bar{\nu}) = \Gamma^{\text{SM}}(Z \rightarrow \nu \bar{\nu}) [1 + \alpha \Delta T].$$

# Electroweak precision global fit

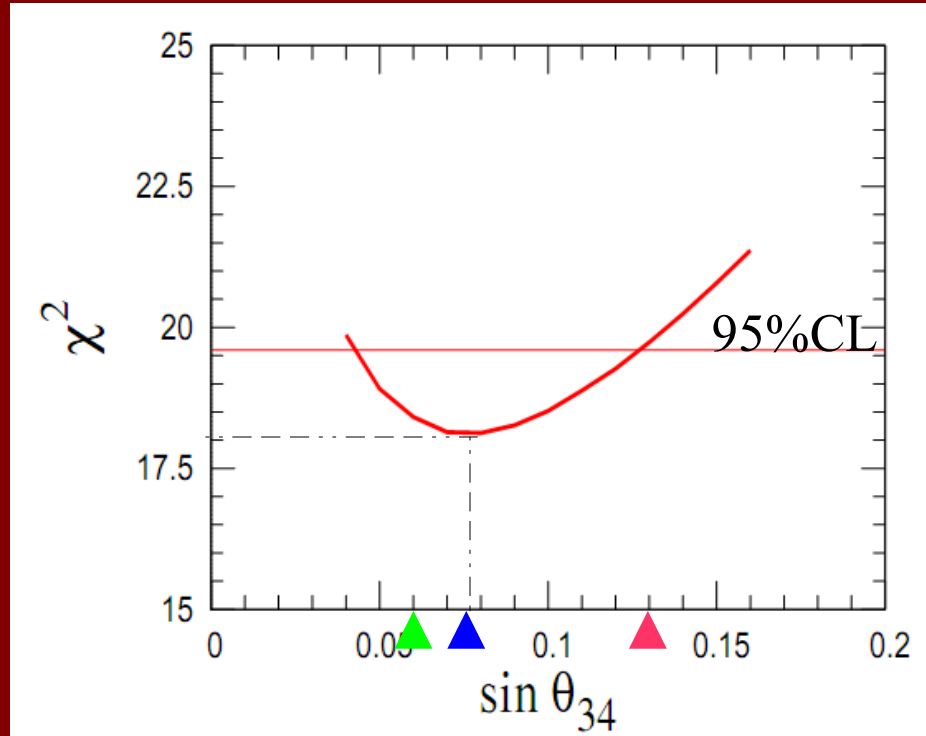
	Measurement with Total error	SM4 Chanowitz' Fit	SM4 Chanowitz's Fit (reproduce)
$\Delta\alpha_{\text{had}}^{(5)}(m_Z^2)$	$0.02758 \pm 0.00035$	0.02732	0.02732
$m_Z$ [GeV]	$91.1875 \pm 0.0021$	—	91.1875
$\Gamma_Z$ [GeV]	$2.4952 \pm 0.0023$	2.4988	2.4983
$\sigma_{\text{had}}^0$ [nb]	$41.540 \pm 0.037$	41.487	41.489
$R_\ell^0$	$20.767 \pm 0.025$	20.726	20.716
$A_{\text{FB}}^{0,\ell}$	$0.01714 \pm 0.00095$	0.01592	0.0160
$\mathcal{A}_\ell(P_\tau)$	$0.1465 \pm 0.0032$	0.1457	0.1459
$\mathcal{A}_\ell(\text{SLD})$	$0.1513 \pm 0.0021$	0.1457	0.1459
$R_b^0$	$0.21629 \pm 0.00066$	0.21547	0.21549
$R_c^0$	$0.1721 \pm 0.0030$	0.1723	0.1723
$A_{\text{FB}}^{0,b}$	$0.0992 \pm 0.0016$	0.1021	0.1022
$A_{\text{FB}}^{0,c}$	$0.0707 \pm 0.0035$	0.0729	0.0730
$\mathcal{A}_b$	$0.923 \pm 0.020$	0.934	0.934
$\mathcal{A}_c$	$0.670 \pm 0.027$	0.667	0.667
$\sin^2 \theta_{\text{eff}}^{\text{lept}}(Q_{\text{FB}}^{\text{had}})$	$0.2324 \pm 0.0012$	0.23169	0.2317
$m_t$ [GeV]	$172.6 \pm 1.4$	172.3	172.3
$m_W$ [GeV]	$80.398 \pm 0.025$	80.413	80.410
$\alpha_s(m_Z)$		0.1186	0.1186
$M_H$		810	810
		$s_{34} = 0.15$	$s_{34} = 0.15$
		$\Delta T = 0.48$	$\Delta T = 0.49$
		$\Delta S = 0.15$	$\Delta S = 0.15$
		$\chi^2 = 20.9$	$\chi^2 = 21.4$

Use  $m_H, m_t, \Delta\alpha_5, \alpha_s, m_Z$   
to fit 17 observables

12 degrees of freedom  
to each four-family fit.

# Electroweak precision global fit Using ZFITTER6.4.2

- ▲ Upper bound from  $Z \rightarrow bb$
- ▲ Best global fit
- ▲ Lower bound from  $D$ - $D$  mixing



Fix

$$V_{t's}^* V_{t'b} = 0.006 e^{i75^\circ}$$

$$V_{t'b} = -s_{34}$$

$V_{ud}$	$V_{us}$	$V_{ub}$	$V_{ub'}$
$V_{cd}$	$V_{cs}$	$V_{cb}$	$V_{cb'}$
$V_{td}$	$V_{ts}$	$V_{tb}$	$V_{tb'}$
$V_{t'd}$	$-0.06e^{-i75^\circ}$	$-0.10$	$V_{t'b'}$

?

$$V_{t's}^* V_{t'b} = 0.006 e^{i75^\circ}$$

From  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$  to  $\sin 2\Phi_D$

Current Bound

$$\mathcal{B}^{\text{exp}}(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 6.7 \times 10^{-8}$$

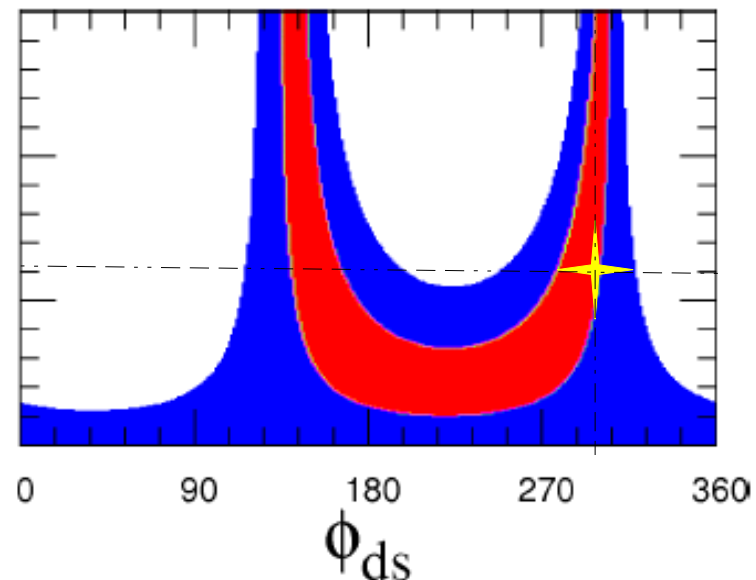
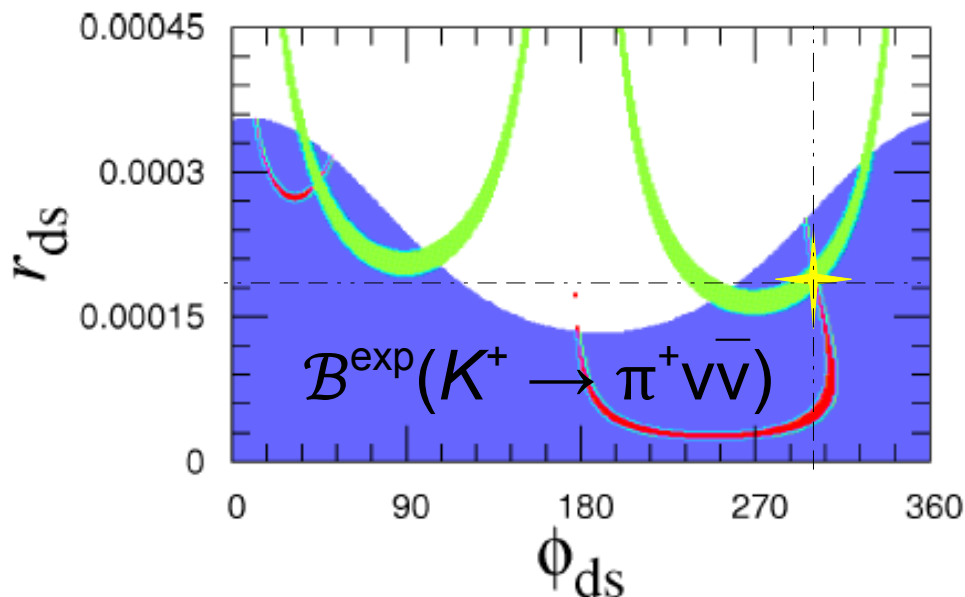
[E391a Collab. PRL100 (2008)]

KOTO(E14) will have a three-year run beginning in 2011.

$$\text{Suppose } \mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) \sim 1.00 \times 10^{-9}$$

There will be  $O(100)$  events observed.

$$V_{t'd}^* V_{t's} \equiv r_{ds} e^{i\phi_{ds}}$$



$$(\varepsilon_K)^{\text{exp}} = (2.229 \pm 0.012) \times 10^{-3}$$

[PDG PLB(2008)]

$$(\sin 2\Phi_{B_d})^{\text{exp}} = 0.672 \pm 0.023$$

[HFAG arXiv:0704.3575(2008)]

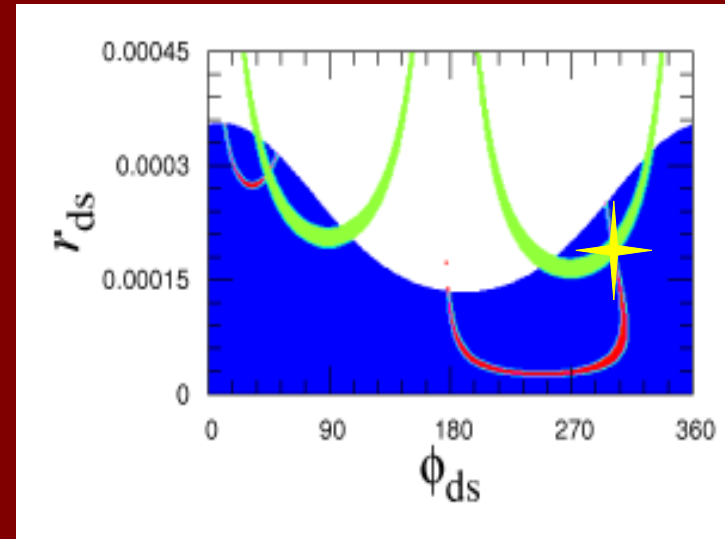
$$\mathcal{B}(K_L \rightarrow \pi^0 V\bar{V}) \sim (1.00 \pm 0.14) \times 10^{-9}$$

$$(\sin 2\Phi_{B_d})^{\text{exp}} = 0.672 \pm 0.008$$

# From $\mathcal{B} (K_L \rightarrow \pi^0 \nu \bar{\nu})$ to $\sin 2\Phi_D$

$V_{ud}$	$V_{us}$	$V_{ub}$	$V_{ub'}$
$V_{cd}$	$V_{cs}$	$V_{cb}$	$V_{cb'}$
$V_{td}$	$V_{ts}$	$V_{tb}$	$V_{tb'}$
$V_{t'd}$	$-0.06e^{-i75^\circ}$	$-0.10$	$V_{t'b'}$

$$V_{t'd}^* V_{t's} \equiv r_{ds} e^{i\phi_{ds}}$$



$$V_{t'd} = -0.003167 e^{-i18^\circ}$$



From  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$  to  $\sin 2\Phi_D$

$$(\lambda_{b'}^2 = V_{ub'}^* V_{cb'})$$

$$M_{12}^{D^0} = \frac{G_F^2 M_W^2 m_D}{12\pi^2} f_D^2 B_D \eta(m_c, M_W) \times (\lambda_{b'}^2 + R_{LD}) S_0(x_{b'}),$$

Take  $R_{LD}$  real for illustration and  $m_{b'} = 460_{-30}^{+30}$  GEV

real $R_{LD}$ scenario $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = 10^{-9}$ $\varepsilon_K = 2.229 \times 10^{-3}$	$V_{t'd} = -0.003167 e^{-i18^\circ}$
$x_D = 9.1 \times 10^{-3}$ $m_{t'} = 500$ GeV $m_{b'} = 460_{-30}^{+30}$ GeV $V_{t'b} = -0.10$ $V_{t's} = -0.06 e^{-i75^\circ}$	$\lambda_{b'}^2 = (8.017 + 2.142 i) \times 10^{-7}$ $ \lambda_{b'}^2 + R_{LD}  = (16.15_{+1.86}^{-1.59}) \times 10^{-7}$ $\sin 2\Phi_D = 0.133_{-0.014}^{+0.014}$ $\cos 2\Phi_D = \pm(0.991_{+0.002}^{-0.002})$

# Conclusion

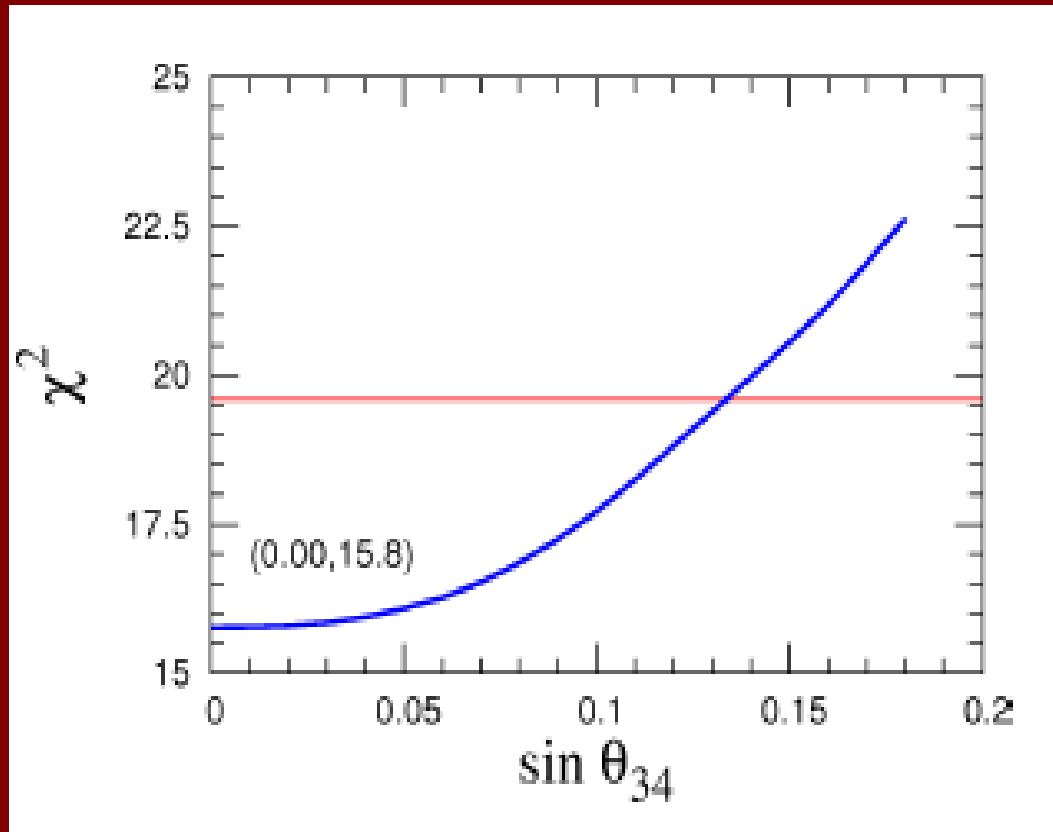
For  $f_{B_s} \sqrt{B_{B_s}} = 266(18)$  MEV,  $m_{t'} = 500$  GEV

I. For  $V_{t's}^* V_{t'b} = 0.006 e^{i75^\circ}$ ,  $|V_{t'b}| < 0.13$  on  $Z \rightarrow bb$ ,  
 $|V_{t'b}| > 0.06$  on  $D$ - $D$  mixing, best global fit at  $|V_{t'b}| = 0.08$

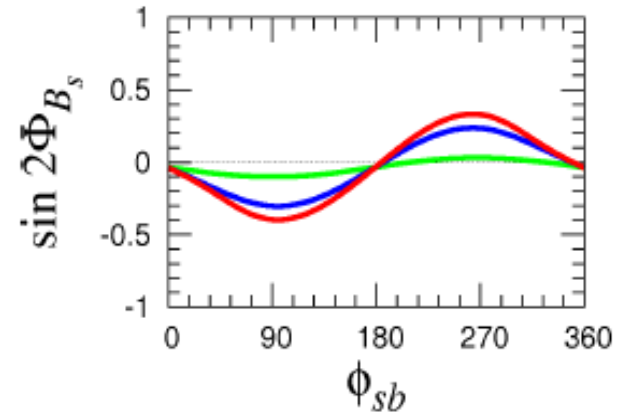
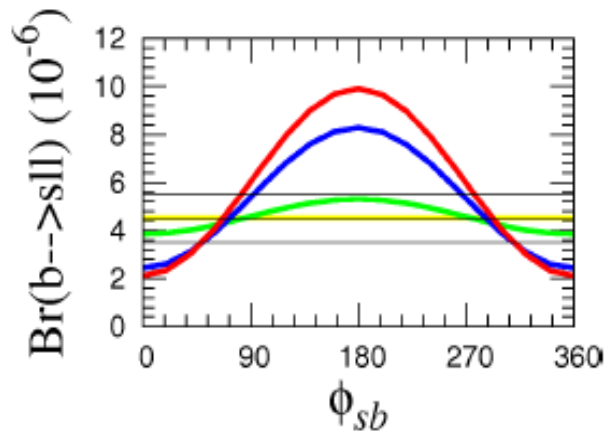
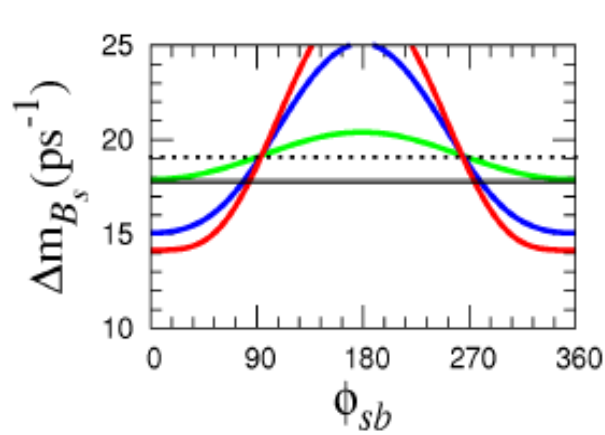
II.  $\sin 2\Phi_{B_s} \sim -0.33 \in [0.00, -0.40]$ ,  
Larger  $m_{t'}$  implies larger  $\sin 2\Phi_{B_s}$

III. Future measurement  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$   
and  $\sin 2\Phi_{B_d}$  can determine  $V_{t'd}$ .

Back Up



$$V_{t's}^* V_{t'b} \equiv r_{sb} e^{i\varphi_{sb}}$$



$$r_{sb} = 0.006,$$

$$\varphi_{sb} = 75^\circ,$$

$$\sin 2\Phi_{B_s} = -0.33$$

(central-value scenario)

facility in Frascati and  $K \rightarrow \pi \nu \bar{\nu}$  experiments like NA62 and KOTO.

$B^0$

$$I(J^P) = \frac{1}{2}(0^-)$$

$I, J, P$  need confirmation. Quantum numbers shown are quark-model predictions.

$$\text{Mass } m_{B^0} = 5279.50 \pm 0.30 \text{ MeV}$$

$$m_{B^0} - m_{B^\pm} = 0.33 \pm 0.06 \text{ MeV}$$

$$\text{Mean life } \tau_{B^0} = (1.525 \pm 0.009) \times 10^{-12} \text{ s}$$

$$c\tau = 457.2 \text{ } \mu\text{m}$$

$$\tau_{B^+}/\tau_{B^0} = 1.071 \pm 0.009 \quad (\text{direct measurements})$$

Mode	BABAR	Belle
$J/\psi K_S^0$	$0.657 \pm 0.036 \pm 0.012$	$0.643 \pm 0.038$
$J/\psi K_L^0$	$0.694 \pm 0.061 \pm 0.031$	$0.641 \pm 0.057$

C. Amsler et al. (Particle Data Group), Physics Letters B667, 1 (2008)

$$S_{J/\psi(nS)K^0} (B^0 \rightarrow J/\psi(nS)K^0) = 0.658 \pm 0.024$$

# From $x_D$ to $\mathcal{B}$ ( $K_L \rightarrow \pi^0 \nu \bar{\nu}$ )

Short-distance dominance limit ( $R_{LD}=0$ )

and  $m_{b'} = 460^{+30}_{-30}$  GeV

$$\begin{aligned} x_D &= 9.1 \times 10^{-3} \\ m_{t'} &= 500 \text{ GeV} \\ m_{b'} &= 460^{+30}_{-30} \text{ GeV} \end{aligned}$$

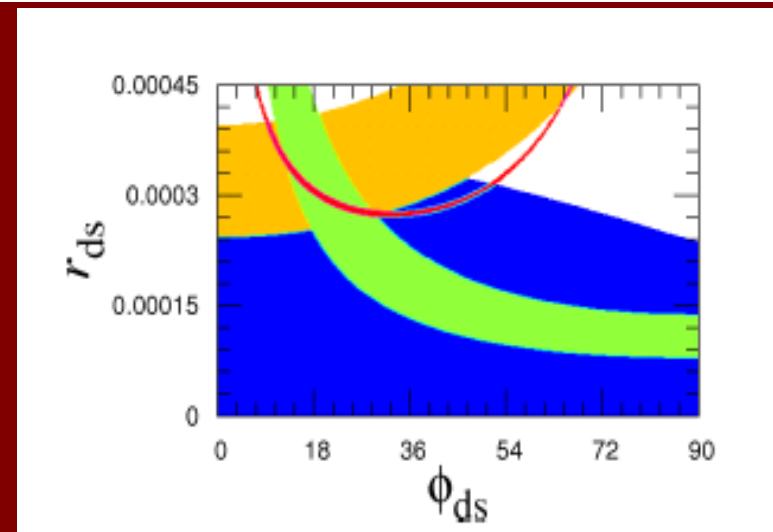
$$M_{12}^{D^0} = \frac{G_F^2 M_W^2 m_D}{12\pi^2} f_D^2 B_D \eta(m_c, M_W) \times (\lambda_{b'}^2 + R_{LD}) S_0(x_{b'}),$$

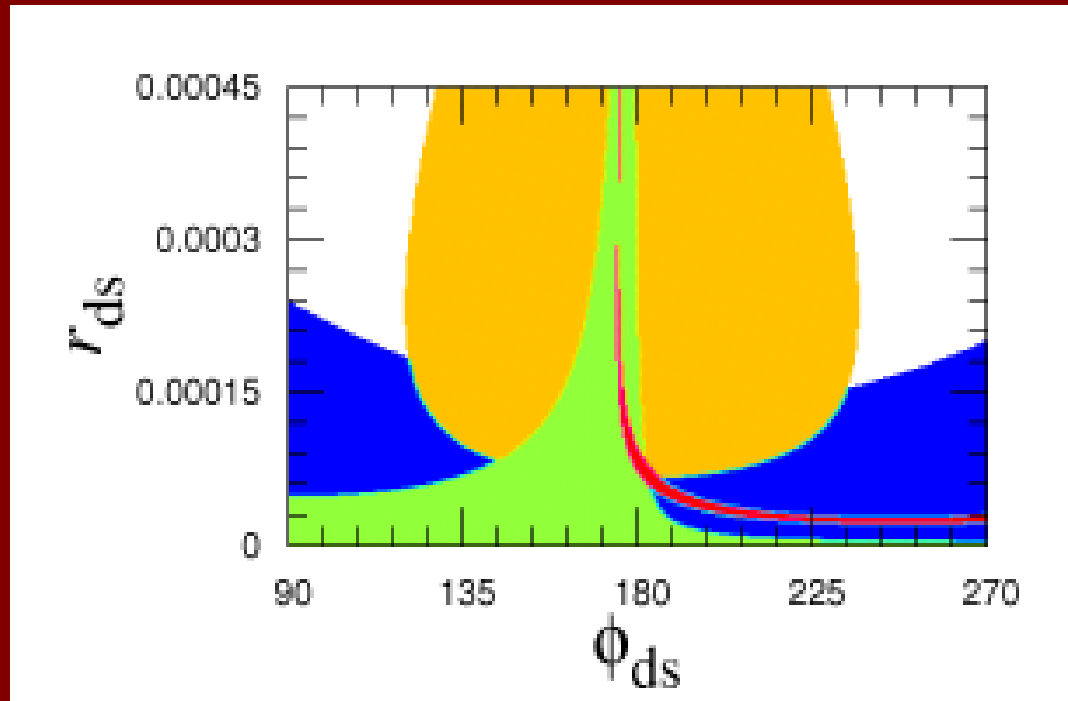
$\mathcal{B}^{\text{exp}}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$

$X_D^{\text{exp}}$

$\epsilon_K^{\text{exp}}$

$$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = (0.4 \sim 1.0) \times 10^{-9}$$





In the long-distance dominated case of  $D - \bar{D}$  mixing, *e.g.*  $|R_{LD}| > 10|\lambda_{b'}^2|$  for  $(m_{t'}, m_{b'}) = (500, 460_{-30}^{+30})$  GeV, we have  $|V_{ub'}^* V_{cb'}| < 4 \times 10^{-4}$ . Then we can see that  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 2 \times 10^{-11}$ , as shown in Fig. 7(R).



$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \kappa_+ \left| \frac{\lambda_c^{ds}}{|V_{us}|} P_c + \frac{\lambda_t^{ds}}{|V_{us}|^5} \eta_t X_0(x_t) + \frac{\lambda_{t'}^{ds}}{|V_{us}|^5} \eta_{t'} X_0(x_{t'}) \right|^2,$$

$$\kappa_+ = (4.84 \pm 0.06) \times 10^{-11} \times (0.224/|V_{us}|)^8$$

It is interesting to see what are the implications for the CPV decay  $K_L \rightarrow \pi^0 \nu \bar{\nu}$ . The formula for  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$  is analogous to Eq. (11), except [15] the change of  $\kappa_+$  to  $\kappa_L = (2.12 \pm 0.03) \times 10^{-10} \times (|V_{us}|/0.224)^8$ , and taking only the imaginary part for the various CKM products.

$$M_{12} = M_{12}^{\text{SM}} e^{i\phi_M^{\text{SM}}} + M_{12}^{\text{NP}} e^{i\phi_M^{\text{NP}}}$$

$$\sin \phi_{12}^D = \left| \frac{M_{12}^{\text{NP}}}{M_{12}} \right| \sin(\phi_M^{\text{NP}} - \phi_M^{\text{SM}})$$

$$\frac{\text{Im}(M_{12}^{\text{NP}})}{|M_{12}|} \in [-0.06, +0.10] \quad (1\sigma)$$

$$\begin{aligned} \frac{\text{Im}(M_{12}^{\text{NP}})}{|M_{12}|} &\in [-0.11, +0.08] \quad (1\sigma), \\ &\in [-0.30, +0.30] \quad (95\% \text{ CL}) \end{aligned}$$

$$\frac{\text{Im}(M_{12}^{\text{NP}})}{|M_{12}|} \in [-0.01, +0.15]$$

$$|\text{Im}(M_{12}^{\text{NP}})/M_{12}| \leq 0.30 \text{ at } 95\% \text{ CL.}$$