Leading Effect of *CP* Violation with Four Generations

A Study of Jarlskog Invariants

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Outline

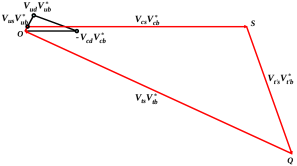
Outline

Jarlskog Invariant and *CP* Violation

In SM3, Jarlskog Invariant [Jarlskog 1985] summarizes CPV

$$J = -\frac{1}{2}\,\mathrm{Im}\,\det[S,S'] = -\frac{1}{6}\,\mathrm{Im}\,\mathrm{tr}\,[S,S']^3$$
$$J = (m_t^2 - m_u^2)(m_t^2 - m_c^2)(m_c^2 - m_u^2)(m_b^2 - m_d^2)(m_b^2 - m_s^2)(m_s^2 - m_d^2)A$$

- Mass degeneracy or trivial CKM area $\Rightarrow$ No CPV
- Falls short of n_B/n_γ by at least 10^{-10}



Hou [2009] proposed an SM4 analog

$$J_{(2,3,4)}^{sb} = (m_{t'}^2 - m_c^2)(m_{t'}^2 - m_t^2)(m_t^2 - m_c^2)(m_{b'}^2 - m_s^2)(m_{b'}^2 - m_b^2)(m_b^2 - m_s^2)A_{234}^{sb}$$

- Enhanced by $\sim 10^{15}$ $\because$ Heaviness of t' and b'

Complexity in SM4

- Only an analog?
  Hou [2009] argued $J^{sb}_{(2,3,4)}$ is leading in degeneracy limit
  $m_d \simeq m_s \simeq 0$ [Jarlskog 1987]

- SM4 is much more complex:

	SM3	SM4
unitarity	triangles	quadrangles
phases	1	3
rotation angles	3	6

- We wish to address:
  1. Is $J^{sb}_{(2,3,4)}$ really leading?
  2. Sub-leading terms? How important?

Outline

Jarlskog Invariants

- SM3: only 1 condition

$$J = \frac{1}{6}\mathrm{Im\,tr}\,[S, S']^3 = -\mathrm{Im\,tr}\,(S^2 S' S S'^2) = v(1,2,3)v'(1,2,3)A = 0$$

where $S = MM^\dagger$ is up-type squared mass matrix,
$v(\alpha,\beta,\gamma) = (m_\alpha^2 - m_\beta^2)(m_\beta^2 - m_\gamma^2)(m_\gamma^2 - m_\alpha^2)$,
primed symbols for down-type.

- Jarlskog [1987] extended to SM4 with 4 invariants, each proportional to a 3-cycle:

$$J(\alpha,\beta,\gamma) = v(\alpha,\beta,\gamma)\,\mathrm{Im\,tr}\,(P_\alpha S' P_\beta S' P_\gamma S')$$

where P_α is projection operator $SP_\alpha = m_\alpha^2 P_\alpha$.

- And proposed:
All four invariants vanish $\iff$ *CP* conservation

Features of Jarlskog Invariants

- Basis-independent
- All invariants sum up to $\mathrm{Im}\,\mathrm{tr}\,[S, S']^3$

$$\mathrm{Im}\,\mathrm{tr}\,[S, S']^3 = \sum_{\alpha,\beta,\gamma} J(\alpha,\beta,\gamma) = \sum_{\alpha,\beta,\gamma} v(\alpha,\beta,\gamma)\,\mathrm{Im}\,\mathrm{tr}\,(P_\alpha S' P_\beta S' P_\gamma S')$$
$$= 6\,[J(2,3,4) + J(1,3,4) + J(1,2,4) + J(1,2,3)] \quad \text{(in SM4)}$$

[J's indices are symmeric.]

- Close analog between SM3 and SM4
- When 3 vanish, the other must also vanish [Jarlskog 1987]
  $\Rightarrow$ # conditions = # phases in V in SM4
- All are of same mass dimension
  $\Rightarrow$ Easy to see mass enhancement effect

Other Approaches

- Gronau, Kfir, and Loewy [1986] (GKL) proposed:

  $$CP \text{ conserved} \iff \operatorname{Im}\operatorname{tr}\left(P_1(S')SP_2(S')S\cdots P_k(S')S\right)=0, \quad 3\le k\le n$$

  Introducing 5 more invariants:

  $$\begin{aligned}&\operatorname{Im}\operatorname{tr}\left(S^2S'SS'^2\right)=\operatorname{Im}\operatorname{tr}\left(S^2S'SS'^3\right)=\operatorname{Im}\operatorname{tr}\left(S^2S'^2SS'^3\right)\\&=\operatorname{Im}\operatorname{tr}\left(S'SS'^2SS'^3\right)=\operatorname{Im}\operatorname{tr}\left(S^3S'SS'^2\right)=\operatorname{Im}\operatorname{tr}\left(S^3S'SS'^3\right)=0\end{aligned}$$

- Debate on if *really* necessary and sufficient for *CP* conservation:
  [Jarlskog 1987; Gronau & Loewy 1989; Jarlskog 1989]

  [In SM4, both sides agreed with that Jarlskog and GKL conditions are equivalent provided there is no vanishing element in mixing matrix.]

- What *really* matters: How do these quantities appear in processes?
  $\Rightarrow$ Difficult to answer. Algebraic study may help!

Invariants and CKM Triangle Areas

- SM3: If *the* CKM triangle area vanishes, *CP* is conserved.
- SM4: unitarity ⇒ quadrangles.

$$A^{u_1 u_2}_{d_1 d_2} \equiv \mathrm{Im}\left[(V_{u_1 d_1} V^*_{u_1 d_2})^* V_{u_2 d_1} V^*_{u_2 d_2}\right], \quad \forall\, u_1 \neq u_2, d_1 \neq d_2$$

  9 independent triangle areas in SM4 (by unitarity) [Botella & Chau 1986]
  If *all* 9 areas vanish, *CP* is conserved

- CKM triangle areas are:
  – close related to processes √
  – NOT basis-independent *X*
- Our approach: Express Jarlskog invariants in CKM triangle areas

  N.B. del Aguila & Aguilar-Saavedra [1996]

  expressed GKL invariants in (many!) CKM areas.

Outline

Decomposing Process Guideline

Start from down-type Jarlskog invariants

$$J'(a,b,c) = v'(a,b,c) \sum_{i,j,k=1}^{4} m_i^2 m_j^2 m_k^2 \,\mathrm{Im}\left(V_{ia}V_{ib}^* V_{jb} V_{jc}^* V_{kc} V_{ka}^*\right)$$

1. Collect terms with the same mass factor.
2. Factor out real numbers. Left elements form areas.

   [For those mass factors with three all different flavors, all such terms need to be considered at once to factor out real numbers.]

3. Use unitarity $V_{1a}V_{1b}^* = -V_{2a}V_{2b}^* - V_{3a}V_{3b}^* - V_{4a}V_{4b}^* + \delta_{ab}$ to eliminate index "1" in $A_{d_1 d_2}^{u_1 u_2}$.

   $\Rightarrow$ Only 9 areas appear

   $\Rightarrow$ Asymmetry between indices 1 and 2, 3, 4

   [One could choose to eliminate any other indices and obtain similar results.]

4. Keep working and you'll arrive at the result!

Jarlskog Invariants in CKM Triangle Areas

Now depart from generality ⇒ introduce quark mass hierarchy

$$\begin{aligned}
J'(2,3,4) &= -v'(2,3,4)\ \Lambda_{234} \\
J'(1,3,4) &= \ \ v'(1,3,4)\left[\Lambda_{234} - v(1,3,4)A^{34}_{34} - v(1,2,3)A^{23}_{34} - v(1,4,2)A^{42}_{34}\right] \\
J'(1,2,3) &= \ \ v'(1,2,3)\left[\Lambda_{234} - v(1,3,4)A^{34}_{23} - v(1,2,3)A^{23}_{23} - v(1,4,2)A^{42}_{23}\right] \\
J'(1,4,2) &= \ \ v'(1,4,2)\left[\Lambda_{234} - v(1,3,4)A^{34}_{42} - v(1,2,3)A^{23}_{42} - v(1,4,2)A^{42}_{42}\right]
\end{aligned}$$

$$\begin{aligned}
\Lambda_{234} = \ & (m_2^2 - m_1^2)(m_3^2 - m_1^2)\Big[\ (m_4^2 - m_2^2)\left(|V_{22}|^2 A^{23}_{34} + |V_{23}|^2 A^{23}_{42} + |V_{24}|^2 A^{23}_{23}\right) \\
& \qquad - (m_4^2 - m_3^2)\left(|V_{32}|^2 A^{23}_{34} + |V_{33}|^2 A^{23}_{42} + |V_{34}|^2 A^{23}_{23}\right)\Big] \\
& + (m_3^2 - m_1^2)(m_4^2 - m_1^2)\Big[\ (m_2^2 - m_3^2)\left(|V_{32}|^2 A^{34}_{34} + |V_{33}|^2 A^{34}_{42} + |V_{34}|^2 A^{34}_{23}\right) \\
& \qquad - (m_2^2 - m_4^2)\left(|V_{42}|^2 A^{34}_{34} + |V_{43}|^2 A^{34}_{42} + |V_{44}|^2 A^{34}_{23}\right)\Big] \\
& + (m_4^2 - m_1^2)(m_2^2 - m_1^2)\Big[\ (m_3^2 - m_4^2)\left(|V_{42}|^2 A^{42}_{34} + |V_{43}|^2 A^{42}_{42} + |V_{44}|^2 A^{42}_{23}\right) \\
& \qquad - (m_3^2 - m_2^2)\left(|V_{22}|^2 A^{42}_{34} + |V_{23}|^2 A^{42}_{42} + |V_{24}|^2 A^{42}_{23}\right)\Big]
\end{aligned}$$

- Down-type quark mass hierarchy $m_{b'}^2 \gg m_b^2 \gg m_s^2 \gg m_d^2$

$$
\begin{aligned}
J'(2,3,4) &= -v'(2,3,4)\,\Lambda_{234} \\
J'(1,3,4) &= \;\; v'(1,3,4)\left[\Lambda_{234} - v(1,3,4)A^{34}_{34} - v(1,2,3)A^{23}_{34} - v(1,4,2)A^{42}_{34}\right] \\
J'(1,2,3) &= \;\; v'(1,2,3)\left[\Lambda_{234} - v(1,3,4)A^{34}_{23} - v(1,2,3)A^{23}_{23} - v(1,4,2)A^{42}_{23}\right] \\
J'(1,4,2) &= \;\; v'(1,4,2)\left[\Lambda_{234} - v(1,3,4)A^{34}_{42} - v(1,2,3)A^{23}_{42} - v(1,4,2)A^{42}_{42}\right]
\end{aligned}
$$

$$
\begin{aligned}
\Lambda_{234} = \;& (m_2^2 - m_1^2)(m_3^2 - m_1^2)\Big[\;\; (m_4^2 - m_2^2)\left(|V_{22}|^2 A^{23}_{34} + |V_{23}|^2 A^{23}_{42} + |V_{24}|^2 A^{23}_{23}\right) \\
& \qquad - (m_4^2 - m_3^2)\left(|V_{32}|^2 A^{23}_{34} + |V_{33}|^2 A^{23}_{42} + |V_{34}|^2 A^{23}_{23}\right)\Big] \\
& + (m_3^2 - m_1^2)(m_4^2 - m_1^2)\Big[\;\; (m_2^2 - m_3^2)\left(|V_{32}|^2 A^{34}_{34} + |V_{33}|^2 A^{34}_{42} + |V_{34}|^2 A^{34}_{23}\right) \\
& \qquad - (m_2^2 - m_4^2)\left(|V_{42}|^2 A^{34}_{34} + |V_{43}|^2 A^{34}_{42} + |V_{44}|^2 A^{34}_{23}\right)\Big] \\
& + (m_4^2 - m_1^2)(m_2^2 - m_1^2)\Big[\;\; (m_3^2 - m_4^2)\left(|V_{42}|^2 A^{42}_{34} + |V_{43}|^2 A^{42}_{42} + |V_{44}|^2 A^{42}_{23}\right) \\
& \qquad - (m_3^2 - m_2^2)\left(|V_{22}|^2 A^{42}_{34} + |V_{23}|^2 A^{42}_{42} + |V_{24}|^2 A^{42}_{23}\right)\Big]
\end{aligned}
$$

- Down-type quark mass hierarchy $m_{b'}^2 \gg m_b^2 \gg m_s^2 \gg m_d^2$
- Up-type quark mass hierarchy $m_{t'}^2 > m_t^2 \gg m_c^2 \gg m_u^2$

$$J'(2,3,4) = -v'(2,3,4)\,\Lambda_{234}$$
$$J'(1,3,4) = \;\; v'(1,3,4)\left[\Lambda_{234} - v(1,3,4)A_{34}^{34} - v(1,2,3)A_{34}^{23} - v(1,4,2)A_{34}^{42}\right]$$

$$\begin{aligned}
\Lambda_{234} = \;& (m_2^2 - m_1^2)(m_3^2 - m_1^2)\Big[\; (m_4^2 - m_2^2)\left(|V_{22}|^2 A_{34}^{23} + |V_{23}|^2 A_{42}^{23} + |V_{24}|^2 A_{23}^{23}\right) \\
&\qquad - (m_4^2 - m_3^2)\left(|V_{32}|^2 A_{34}^{23} + |V_{33}|^2 A_{42}^{23} + |V_{34}|^2 A_{23}^{23}\right)\Big] \\
&+ (m_3^2 - m_1^2)(m_4^2 - m_1^2)\Big[\; (m_2^2 - m_3^2)\left(|V_{32}|^2 A_{34}^{34} + |V_{33}|^2 A_{42}^{34} + |V_{34}|^2 A_{23}^{34}\right) \\
&\qquad - (m_2^2 - m_4^2)\left(|V_{42}|^2 A_{34}^{34} + |V_{43}|^2 A_{42}^{34} + |V_{44}|^2 A_{23}^{34}\right)\Big] \\
&+ (m_4^2 - m_1^2)(m_2^2 - m_1^2)\Big[\; (m_3^2 - m_4^2)\left(|V_{42}|^2 A_{34}^{42} + |V_{43}|^2 A_{42}^{42} + |V_{44}|^2 A_{23}^{42}\right) \\
&\qquad - (m_3^2 - m_2^2)\left(|V_{22}|^2 A_{34}^{42} + |V_{23}|^2 A_{42}^{42} + |V_{24}|^2 A_{23}^{42}\right)\Big]
\end{aligned}$$

- Down-type quark mass hierarchy $m_{b'}^2 \gg m_b^2 \gg m_s^2 \gg m_d^2$
- Up-type quark mass hierarchy $m_{t'}^2 > m_t^2 \gg m_c^2 \gg m_u^2$

$$J'(2,3,4) = -v'(2,3,4)\ \Lambda_{234}$$
$$J'(1,3,4) = \quad v'(1,3,4)\left[\Lambda_{234} - v(1,3,4)A_{34}^{34} \qquad\qquad \right]$$

$$\Lambda_{234} =$$

$$+ (m_3^2 - m_1^2)(m_4^2 - m_1^2)\Big[\quad (m_2^2 - m_3^2)\left(|V_{32}|^2 A_{34}^{34} + |V_{33}|^2 A_{42}^{34} + |V_{34}|^2 A_{23}^{34}\right)$$
$$- (m_2^2 - m_4^2)\left(|V_{42}|^2 A_{34}^{34} + |V_{43}|^2 A_{42}^{34} + |V_{44}|^2 A_{23}^{34}\right)\Big]$$

Further Expansion

- Return to physicsl labels,

$$
\begin{aligned}
J'(2,3,4) &\simeq -(m_{b'}^2 - m_s^2)(m_{b'}^2 - m_b^2)(m_b^2 - m_s^2)\Lambda_{234}, \\
J'(1,3,4) &\simeq \quad (m_{b'}^2 - m_d^2)(m_{b'}^2 - m_b^2)(m_b^2 - m_d^2) \\
&\qquad \times \left[\Lambda_{234} - (m_{t'}^2 - m_u^2)(m_{t'}^2 - m_t^2)(m_t^2 - m_u^2) A_{bb'}^{tt'}\right],
\end{aligned}
$$

$$
\begin{aligned}
\Lambda_{234} \simeq (m_t^2 - m_u^2)(m_{t'}^2 - m_u^2) \\
\times \Big[\quad (m_t^2 - m_c^2) \left(-|V_{ts}|^2 A_{bb'}^{tt'} + |V_{tb}|^2 A_{sb'}^{tt'} - |V_{tb'}|^2 A_{sb}^{tt'}\right) \\
+ (m_{t'}^2 - m_c^2) \left(|V_{t's}|^2 A_{bb'}^{tt'} - |V_{t'b}|^2 A_{sb'}^{tt'} + |V_{t'b'}|^2 A_{sb}^{tt'}\right) \Big].
\end{aligned}
$$

- Still quite a few terms $\Rightarrow$ Expand in strength of $|V_{ij}|^2$

Further Expansion

- Return to physicsl labels,

$$J'(2,3,4) \simeq -(m_{b'}^2 - m_s^2)(m_{b'}^2 - m_b^2)(m_b^2 - m_s^2)\Lambda_{234},$$
$$J'(1,3,4) \simeq (m_{b'}^2 - m_d^2)(m_{b'}^2 - m_b^2)(m_b^2 - m_d^2) \times \left[\Lambda_{234} - (m_{t'}^2 - m_u^2)(m_{t'}^2 - m_t^2)(m_t^2 - m_u^2)A_{bb'}^{tt'}\right],$$

$$\Lambda_{234} \simeq (m_t^2 - m_u^2)(m_{t'}^2 - m_u^2) \times \Big[(m_t^2 - m_c^2)\left(-|V_{ts}|^2 A_{bb'}^{tt'} + |V_{tb}|^2 A_{sb'}^{tt'} - |V_{tb'}|^2 A_{sb}^{tt'}\right) + (m_{t'}^2 - m_c^2)\left(|V_{t's}|^2 A_{bb'}^{tt'} - |V_{t'b}|^2 A_{sb'}^{tt'} + |V_{t'b'}|^2 A_{sb}^{tt'}\right)\Big].$$

- Still quite a few terms ⇒ Expand in strength of $|V_{ij}|^2$
- Cabibbo angle should be the largest rotation angle
  $|V_{ts}|^2, |V_{t's}|^2, |V_{tb'}|^2, |V_{t'b}|^2 \ll 1$ [Talk by A. Arhrib, H. Lacker, C.-Y. Ma]

Leading Effects in Jarlskog Invariants

- After both mass and angle expansion,

$$J'(2,3,4) \simeq -(m_{t'}^2 - m_u^2)(m_t^2 - m_u^2)(m_{b'}^2 - m_s^2)(m_{b'}^2 - m_b^2)(m_b^2 - m_s^2) \times \left[(m_t^2 - m_c^2)\left|V_{tb}\right|^2 A_{sb'}^{tt'} + (m_{t'}^2 - m_c^2)\left|V_{t'b'}\right|^2 A_{sb}^{tt'}\right],$$

$$J'(1,3,4) \simeq (m_t^2 - m_u^2)(m_{t'}^2 - m_u^2)(m_{b'}^2 - m_d^2)(m_{b'}^2 - m_b^2)(m_b^2 - m_d^2) \times \left[(m_t^2 - m_c^2)\left|V_{tb}\right|^2 A_{sb'}^{tt'} + (m_{t'}^2 - m_c^2)\left|V_{t'b'}\right|^2 A_{sb}^{tt'} - (m_{t'}^2 - m_t^2) A_{bb'}^{tt'}\right].$$

- Not exactly same as proposed $J^{sb}_{(2,3,4)}$:
  - More than one area have leading mass factor
  - m_u^2, m_d^2 involved
  - Magnitude of CKM matrix elements involved

Outline

More General (Unphysical) Cases

- 4th generation decouples from all lower generations

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} & 0 \\ V_{cd} & V_{cs} & V_{cb} & 0 \\ V_{td} & V_{ts} & V_{tb} & 0 \\ 0 & 0 & 0 & V_{t'b'} \end{pmatrix}$$

- Expected effective 3-generation case
- Any CKM areas with index "4" and Λ_{234} are 0

$$J'(2,3,4) = J'(1,3,4) = J'(1,4,2) = 0,$$
$$J'(1,2,3) = -v'(1,2,3)v(1,2,3)A_{23}^{23}$$

- Indeed SM3 CPV contribution!

More General (Unphysical) Cases

- 4th generation partly decouples from first 2 generations

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} & 0 \\ V_{cd} & V_{cs} & V_{cb} & 0 \\ V_{td} & V_{ts} & V_{tb} & V_{tb'} \\ V_{t'd} & V_{t's} & V_{t'b} & V_{t'b'} \end{pmatrix}$$

- A_{sb}^{ct}, $A_{sb}^{ct'}$ not vanishing, $|V_{tb'}|^2 A_{sb}^{ct} = |V_{t'b'}|^2 A_{sb}^{ct'}$ by unitarity
  $\Rightarrow$ only 1 CPV phase
- All Jarlskog invariants proportional to this area
- Leading effect in $J'(2,3,4)$ is,

$$-(m_{t'}^2 - m_t^2)^2(m_c^2 - m_u^2)(m_{b'}^2 - m_s^2)(m_{b'}^2 - m_b^2)(m_b^2 - m_s^2)\,|V_{tb'}|^2\,A_{sb}^{ct}$$

Proposed $J^{sb}_{(2,3,4)}$?

- If wildly assuming $A^{tt'}_{sb'} \sim -A^{tt'}_{sb}$ (may due to $|V_{t'd}|^2 \sim 0$)

$$J'(2,3,4) \simeq -(m_{t'}^2 - m_u^2)(m_t^2 - m_u^2)(m_{b'}^2 - m_s^2)(m_{b'}^2 - m_b^2)(m_b^2 - m_s^2) \times \left[(m_t^2 - m_c^2)\left|V_{tb}\right|^2 A^{tt'}_{sb'} + (m_{t'}^2 - m_c^2)\left|V_{t'b'}\right|^2 A^{tt'}_{sb}\right]$$

becomes (with $|V_{tb}|^2 \simeq |V_{t'b'}|^2 \simeq 1$)

$$J'(2,3,4) \sim -(m_{t'}^2 - m_u^2)(m_{t'}^2 - m_t^2)(m_t^2 - m_u^2) \times (m_{b'}^2 - m_s^2)(m_{b'}^2 - m_b^2)(m_b^2 - m_s^2) A^{tt'}_{sb}$$

- This is the proposed $J^{sb}_{(2,3,4)}$ (except minus sign)
- Further discussion on the assumption needed

Outline

Summary and Outlook

- Motivated by George's SM4 analog of Jarlskog invariant
- Relation b/w processes and invariants are NOT fully understood
- Algebraic study would help for more insight
- Express general form of J's in CKM triangle areas (with generality)
- Expansion with physical values:
  - Quark mass hierarchy
  - Small rotation angle assumption
- For leading effect, George's proposal is close but not complete
- But proposed $J^{sb}_{(2,3,4)}$ obtained if with a wild assumption (really wild?)
- More study on physical interpretation needed

Thanks for Your Attention!

Some Algebra I

Using unitarity condition, whenever i, j, or k equals 1 in the summation, we replace those terms.

$$J'(a,b,c) = v'(a,b,c) \sum_{i,j,k=1}^{4} m_i^2 m_j^2 m_k^2 \,\mathrm{Im}\,(V_{ia}V_{ib}^*V_{jb}V_{jc}^*V_{kc}V_{ka}^*)$$
$$= v'(a,b,c) \sum_{i,j,k=2}^{4} (m_i^2 - m_1^2)(m_j^2 - m_1^2)(m_k^2 - m_1^2)$$
$$\times \,\mathrm{Im}\,(V_{ia}V_{ib}^*V_{jb}V_{jc}^*V_{kc}V_{ka}^*)$$

Some Algebra II

We can apply the similar trick to the down-type indices of V. Consider the case $a=1$ and b,c are chosen differently from $\{2,3,4\}$.

$$\begin{aligned}
\mathrm{Im}\,(V_{i1}V_{ib}^{*}V_{jb}V_{jc}^{*}V_{kc}V_{k1}^{*}) &= -\mathrm{Im}\,(V_{i2}V_{ib}^{*}V_{jb}V_{jc}^{*}V_{kc}V_{k2}^{*}) - \mathrm{Im}\,(V_{i3}V_{ib}^{*}V_{jb}V_{jc}^{*}V_{kc}V_{k3}^{*}) \\
&\quad -\mathrm{Im}\,(V_{i4}V_{ib}^{*}V_{jb}V_{jc}^{*}V_{kc}V_{k4}^{*}) + \mathrm{Im}\,(V_{ib}^{*}V_{jb}V_{jc}^{*}V_{kc})\delta_{ik} \\
&= -\mathrm{Im}\,(V_{ib}V_{ib}^{*}V_{jb}V_{jc}^{*}V_{kc}V_{kb}^{*}) - \mathrm{Im}\,(V_{ic}V_{ib}^{*}V_{jb}V_{jc}^{*}V_{kc}V_{kc}^{*}) \\
&\quad -\mathrm{Im}\,(V_{id}V_{ib}^{*}V_{jb}V_{jc}^{*}V_{kc}V_{kd}^{*}) + \mathrm{Im}\,(V_{ib}^{*}V_{jb}V_{jc}^{*}V_{kc})\delta_{ik} \\
&= -\mathrm{Im}\,(V_{id}V_{ib}^{*}V_{jb}V_{jc}^{*}V_{kc}V_{kd}^{*}) + \mathrm{Im}\,(V_{ib}^{*}V_{jb}V_{jc}^{*}V_{kc})\delta_{ik} \\
&\quad +\left|V_{ib}\right|^{2}A_{bc}^{jk} - \left|V_{kc}\right|^{2}A_{bc}^{ij},
\end{aligned}$$

where d is taken to be different from a,b,c, and the second equality follows from that we can replace $2,3,4$ by b,c,d by reordering the first three terms.

Some Algebra III

Now we obtain

$$J'(1,b,c) = v'(1,b,c)\Bigg\{ \sum_{i,j\in\{2,3,4\}} (m_i^2 - m_1^2)^2 (m_j^2 - m_1^2)\, A_{bc}^{ij} - \sum_{i,j,k\in\{2,3,4\}} (m_i^2 - m_1^2)(m_j^2 - m_1^2)(m_k^2 - m_1^2)\, \mathrm{Im}\,(V_{id}V_{ib}^* V_{jb} V_{jc}^* V_{kc} V_{kd}^*) + \sum_{i,j,k\in\{2,3,4\}} (m_i^2 - m_1^2)(m_j^2 - m_1^2)(m_k^2 - m_1^2)(|V_{ib}|^2 A_{bc}^{jk} - |V_{kc}|^2 A_{bc}^{ij}) \Bigg\},$$

The third summation would vanish since the upper indices of A are anti-symmetric so each component will cancel one another.
And we define

$$\Lambda_{dbc} = - \sum_{i,j,k\in\{2,3,4\}} (m_i^2 - m_1^2)(m_j^2 - m_1^2)(m_k^2 - m_1^2) \mathrm{Im}\,(V_{id}V_{ib}^* V_{jb} V_{jc}^* V_{kc} V_{kd}^*).$$

Then the four Jarlskog invariants in SM4 can be written as shown previously.

Some Algebra IV

Λ_{234} can be also expressed in terms of triangle areas. Consider

$$-(m_i^2 - m_1^2)(m_j^2 - m_1^2)(m_k^2 - m_1^2)\mathrm{Im}\,(V_{ib}V_{ic}^*V_{jc}V_{jd}^*V_{kd}V_{kb}^*) = -\mathcal{M}_1^{ijk}\omega_{bcd}^{ijk},$$

Note that ω_{bcd}^{ijk} has the following properties,

$$\begin{aligned}\sum_{i\in\{2,3,4\}} \omega_{bcd}^{ijk} &= -\omega_{bcd}^{1jk},\\ \omega_{bcd}^{ijk} &= \omega_{cdb}^{jki} = \omega_{dbc}^{kij},\\ \omega_{bcd}^{iii} &= 0,\\ \omega_{bcd}^{iji} &= |V_{ib}|^2\, A_{cd}^{ij}.\end{aligned}$$

Relations between CKM Triangle Areas

- In SM4 there are nine independent CKM triangle areas [Botella & Chau 1986] .
- But provided the magnitude of each CKM mixing matrix element is known, the degree of freedom of the CKM triangles can be further reduced to at most three.

$$\left[(|V_{u_1 n}|^2 - |V_{u_2 n}|^2) A_{lm}^{u_1 u_2} + (|V_{u_3 n}|^2 - |V_{u_4 n}|^2) A_{lm}^{u_3 u_4}\right] + \text{cyclic permu. of } (l,m,n) = 0$$

$$\left[(|V_{n d_1}|^2 - |V_{n d_2}|^2) A_{d_1 d_2}^{lm} + (|V_{n d_3}|^2 - |V_{n d_4}|^2) A_{d_3 d_4}^{lm}\right] + \text{cyclic permu. of } (l,m,n) = 0$$

The Two Different Schemes for

- To see the CPV with fourth generation, we may take the two following scheme.
  - Calculating the decay amplitudes with a proposed 4×4 mixing matrix and masses of fourth generation quarks.
  - Physics measurable must be proportional to quantity that is invariant under the choice of basis. Calculating these invariants and find hints from them.
- Examples...
- Hou found that the Jarlskog invariant [Jarlskog 1985, 1987] with fourth generation will enhance 10^{15} order of magnitude. This implies the fourth generation may be the source for baryon asymmetry of the Universe!

Mass Degeneracy Limits

- Consider only almost but not exactly degeneracy
  $\Rightarrow$ No extra freedom in mixing matrix
- d-s degeneracy limit $\Rightarrow J'(1,2,3) \simeq J'(1,4,2) \simeq 0$
  Leading effects same as using mass hierarchy expansion
- t-t' degeneracy limit:
  - None of the four down-type Jarlskog invariants vanishes
  - Leading effects should still fall down at least one order
  - Assuming $m_t/m_{t'} \sim 1-g$,

$$J'(2,3,4) \simeq -(m_{t'}^2 - m_u^2)(m_t^2 - m_u^2)(m_{b'}^2 - m_s^2)(m_{b'}^2 - m_b^2)(m_b^2 - m_s^2) \times \left[(m_{t'}^2 - m_c^2)\left(|V_{tb}|^2 A_{sb'}^{tt'} + |V_{t'b'}|^2 A_{sb}^{tt'}\right) - g m_{t'}^2 |V_{tb}|^2 A_{sb'}^{tt'}\right]$$

  - $A_{sb'}^{tt'}$ and $A_{sb}^{tt'}$ may cancel when t-t' degenerated. $|V_{tb}|^2 \simeq |V_{t'b'}|^2 \simeq 1$

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