$pp \to H + 2$ jets: Compact analytic results

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based on work done in collaboration with
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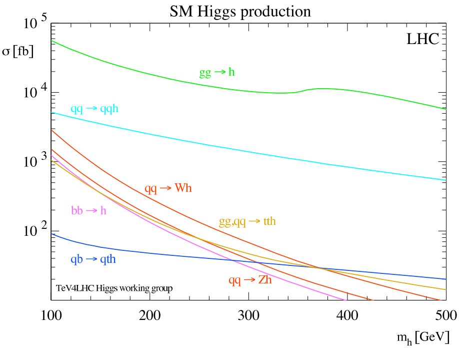
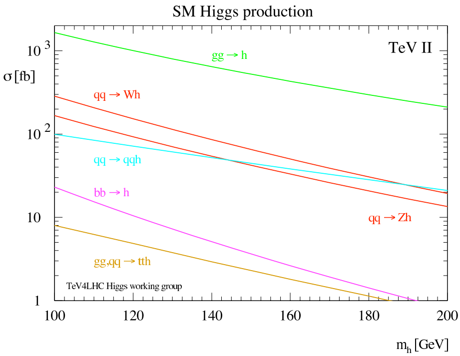
and
Simon Badger, John Campbell, Keith Ellis 0910.4481 .

Radcor,
26-30 October 2009, Ascona

Outline

Higgs Production at Hadron Colliders

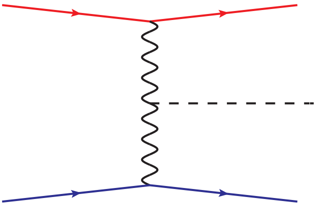




Figures: Particle Data Group

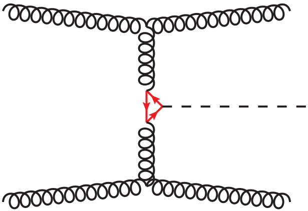
At hadron colliders the Higgs production is dominated by gluon fusion (at the LHC PDFs further enhance this dominance). At the LHC the second most important production mechanism is $qq \to qqH$. This is not the case at the Tevatron ($p\overline{p}$).

Vector Boson Fusion



VBF

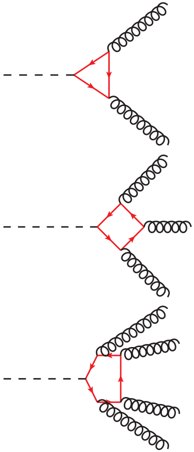
- VBF is an important Higgs discovery channel and is crucial for the measurement of the couplings between the Higgs and the weak vector bosons (W, Z).
- The next-to-leading order QCD corrections are quite small $\sim 5 - 10\%$ (Han, Valencia, Willenbrock, Figy, Oleari, Zeppenfeld, Berger, Campbell)
  Full EW+QCD corrections (Ciccolini, Denner, Dittmaier)



Gluon Fusion + Jets

- The VBF process has a characteristic distribution, one very forward jet and one very backward jet, allowing cuts to reduce major background ...
- ...which is gluon fusion plus additional jet activity. (Del Duca, Kilgore, Oleari, Schmidt, Zeppenfeld, Dawson, Kauffman)
- By excluding jets from the central region, one can maximise VBF signal over gluon fusion background. (Figy, Oleari, Zeppenfeld)
- Semi-numeric calculation at NLO (Campbell,Eliis,Giele,Zangerighi)

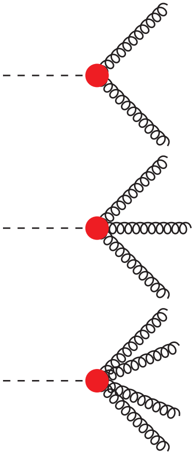
The Higgs plus Gluon Coupling



The Effective Interaction

Integrating out the top-quark loop introduces a five dimensional effective operator.
(Wilczek; Shifman, Vainshtein, Zakharov)

The Higgs plus Gluon Coupling in the Large-m_t Limit



The Effective Interaction

Integrating out the top-quark loop introduces a five dimensional effective operator.

(Wilczek; Shifman, Vainshtein, Zakharov)

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} C H \; \mathrm{tr}(G^{\mu\nu} G_{\mu\nu})$$

To leading order in α_s

$$C = \frac{\alpha_s}{6\pi v} \qquad v = 246 \text{ eV}$$

The approximation is valid over a wide range of Higgs masses (Kramer,Laenen,Spira) and is a good approximation with increased number of jets provided $p_T < m_t$. (Del Duca,Kilgore,Oleari,Schmidt,Zeppenfeld)

Motivation: A Tree level Higgs Amplitude

Looking at some simple Higgs + gluon tree level amplitudes there are hints of underlying substructure

$$A^{(0)}(H,1^-,2^+,3^-,4^+) \quad = \quad \frac{\langle 13\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} + \frac{[24]^4}{[12][23][34][41]}$$

We see that the "MHV-structure" is similar to that of QCD, the Higgs MHV amplitude looks like MHV +$\overline{MHV}$. Since MHV and $\overline{MHV}$ amplitudes are conjugates of each other, perhaps a simplification would occur if we considered complex rather than real scalars?

$\phi, \phi^\dagger$ Splitting

$H = \phi + \phi^\dagger$

The idea is to split the real scalar field H into two complex scalars such that the Higgs is given by the sum of the two scalars (Dixon, Glover, Khoze).We want the gluon field strength to also be in suitable form so we introduce the self-dual and anti-self dual field strengths

$$G^{\mu\nu}_{SD} = \frac{1}{2}(G^{\mu\nu} + {}^*G^{\mu\nu}) \quad G^{\mu\nu}_{ASD} = \frac{1}{2}(G^{\mu\nu} - {}^*G^{\mu\nu}) \quad {}^*G^{\mu\nu} = \frac{i}{2}\epsilon^{\mu\nu\rho\sigma}G_{\rho\sigma}$$

Introducing $\phi = 1/2(H + iA)$ leads to the following breakdown of the effective Lagrangian

$$\mathcal{L}^{\text{eff}}_{H,A} = \frac{C}{2}\left(H \ \text{tr}(G^{\mu\nu}G_{\mu\nu}) + A \ \text{tr}(G^{\mu\nu\,*}G_{\mu\nu})\right)$$

$$\mathcal{L}^{\text{eff}}_{H,A} = \frac{C}{2}\left(\phi \ \text{tr}(G^{\mu\nu}_{SD}G^{SD}_{\mu\nu}) + \phi^\dagger \ \text{tr}(G^{\mu\nu}_{ASD}G^{ASD}_{\mu\nu})\right)$$

Due to self-duality ϕ and $\phi^\dagger$ amplitudes are more compact than H amplitudes.

$\phi, \phi^\dagger$ Tree level Amplitudes and Parity

Parity of ϕ and $\phi^\dagger$ amplitudes (Dixon, Glover, Khoze)

Although Higgs amplitudes are made from the sum of ϕ and $\phi^\dagger$ amplitudes, in principle one need only calculate ϕ amplitudes since

$$A_n^{(m)}(\phi^\dagger, g_1^{\lambda_1}, \ldots, g_n^{\lambda_n}) = \left(A_n^{(m)}(\phi, g_1^{-\lambda_1}, \ldots, g_n^{-\lambda_n})\right)^*.$$

Simple ϕ Amplitudes (Badger, Dixon, Glover, Khoze)

- MHV ϕ + parton amplitudes have the same structure as QCD MHV amplitude.
- In addition there are new amplitudes such as the ϕ-all minus amplitude, which also has a simple structure.

More Complicated ϕ Tree Amplitudes

The four-point ϕ-NMHV amplitude

We will also require the following NMHV amplitude which has been derived using BCFW recursion relations (Badger, Glover, Mastrolia, CW)

$$A_n^{(0)}(\phi, 1^+, 2^-, 3^-, 4^-) =$$
$$\frac{m_\phi^4 \langle 24\rangle^4}{s_{124}\langle 12\rangle\langle 14\rangle\langle 2|p_\phi|3]\langle 4|p_\phi|3]} - \frac{\langle 4|p_\phi|1]^3}{s_{123}\langle 4|p_\phi|3][12][23]} + \frac{\langle 2|p_\phi|1]^3}{s_{134}\langle 2|p_\phi|3][14][34]}.$$

and is more compact than the previous known result (Dixon, Glover, Khoze).

The $\phi q\overline{q} - NMHV$ amplitude

The $\phi q\overline{q} - NMHV$ amplitude has a more complicated structure,

$$A_n^{(0)}(\phi, 1_q^+, 2^-, 3^-, 4_{\overline{q}}^-) =$$
$$-\frac{m_\phi^4 \langle 24\rangle^3}{s_{124}\langle 14\rangle\langle 2|p_\phi|3]\langle 4|p_\phi|3]} + \frac{\langle 4|p_\phi|1]^2}{\langle 4|p_\phi|3][12][23]} - \frac{\langle 2|p_\phi|1]^2\langle 2|p_\phi|4]}{s_{134}\langle 2|p_\phi|3][14][34]}$$

Which has been derived with BCFW recursion relations (Badger, Glover, Mastrolia, CW)

ϕ plus four parton amplitudes at one-loop

The helicity amplitudes for $\phi + 4g$ have been calculated,

H amplitude	ϕ amplitude	$\phi^\dagger$ amplitude
$\mathcal{A}(H,+,+,+,+)$	$\mathcal{A}(\phi,+,+,+,+)$ (Berger,Del Duca, Dixon)	$\mathcal{A}(\phi^\dagger,+,+,+,+)$ (Badger,Glover)
$\mathcal{A}(H,-,+,+,+)$	$\mathcal{A}(\phi,-,+,+,+)$ (Berger,Del Duca, Dixon)	$\mathcal{A}(\phi^\dagger,-,+,+,+)$ (Badger,Glover,Mastrolia,CW)
$\mathcal{A}(H,-,-,+,+)$	$\mathcal{A}(\phi,-,-,+,+)$ (Badger,Glover,Risager)	$\mathcal{A}(\phi^\dagger,-,-,+,+)$ (Badger,Glover,Risager)
$\mathcal{A}(H,-,+,-,+)$	$\mathcal{A}(\phi,-,+,-,+)$ (Glover,Mastrolia,CW)	$\mathcal{A}(\phi^\dagger,-,+,-,+)$ (Glover,Mastrolia,CW)

Whilst those with a quark pair and two gluons have also been calculated
$(Q = 1_{\bar{q}}^-, q = 2_q^+)$

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$\mathcal{A}(H,Q,q,-,^-)$	$\mathcal{A}(\phi,Q,q,-,-)$ (Badger,Campbell,Ellis,CW)	$\mathcal{A}(\phi^\dagger,Q,q,-,-)$ (Berger,Del Duca, Dixon)
$\mathcal{A}(H,Q,q,+,-)$	$\mathcal{A}(\phi,Q,q,+,-)$ (Dixon, Sofiantaos)	$\mathcal{A}(\phi^\dagger,Q,q,+,-)$ (Dixon, Sofiantaos)
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The $H(\phi)q\bar{q}Q\bar{Q}$ amplitudes have also been calculated, (Ellis, Giele, Zanderighi; Dixon, Sofiantaos)

Those marked in *red* are the most complicated *NMHV* helicity amplitudes and are the main topic of this talk.

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Rational $\phi^\dagger$ amplitudes

$\phi^\dagger \overline{q}^- q^+ g^- g^-$ amplitudes (Berger,Del Duca, Dixon)

These amplitudes are zero at tree-level and hence are finite at one-loop,

$$-iA_4^L(\phi^\dagger, 1_{\bar{q}}^-, 2_q^+, 3_g^-, 4_g^-) = 2i\, A_4^{(0)}(\phi, 1_{\bar{q}}^-, 2_q^+, 3_g^-, 4_g^-) + \frac{1}{2}\left[\frac{\langle 3|(1+4)|2]}{[1\,4]\,[3\,4]} + \frac{[2\,1]\,\langle 1\,4\rangle\,[2\,4]}{[1\,4]\,[3\,4]\,[2\,3]}\right] - \frac{1}{3}\frac{[2\,4]\,\langle 3\,4\rangle\,[2\,3]}{[1\,2]\,[3\,4]^2}\,.$$

$$-iA_4^R(\phi^\dagger, 1_{\bar{q}}^-, 2_q^+, 3_g^-, 4_g^-) = -\frac{1}{2}\left[\frac{\langle 3|(1+4)|2]}{[1\,4]\,[3\,4]} + \frac{[2\,1]\,\langle 1\,4\rangle\,[2\,4]}{[1\,4]\,[3\,4]\,[2\,3]}\right],$$

$$-iA_4^{\rm f}(\phi^\dagger, 1_{\bar{q}}^-, 2_q^+, 3_g^-, 4_g^-) = \frac{1}{3}\frac{[2\,4]\,\langle 3\,4\rangle\,[2\,3]}{[1\,2]\,[3\,4]^2}\,,$$

A useful relationship

We note that the various pieces satisfy the relation,

$$A_4^L(\phi^\dagger, 1_{\bar{q}}, 2_q, 3, 4) + A_4^R(\phi^\dagger, 1_{\bar{q}}, 2_q, 3, 4) + A_4^{\rm f}(\phi^\dagger, 1_{\bar{q}}, 2_q, 3, 4) + 2A_4^{(0)}(\phi, 1_{\bar{q}}, 2_q, 3, 4) = 0$$

Which will help us later....

The One-loop Basis

We can recast the general form of the n-point (massless) one-loop integral in the following form

$$A_n^{(1)} = \sum_i C_{4;i}(4)\,\mathcal{I}_{4;i} + \sum_i C_{3;i}(4)\,\mathcal{I}_{3;i} + \sum_i C_{2;i}(4)\,\mathcal{I}_{2;i} + R$$

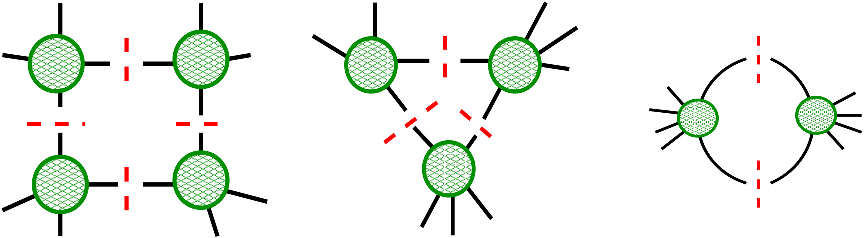
Here $\mathcal{I}_{j;i}$ represents a j-point scalar basis integral - the boxes, triangles and bubbles - with coefficient $C_{j;i}$ evaluated in 4-dimensions.

R is a finite piece that is entirely rational.

Higher point functions can always be reduced to this basis at the cost of changing R (Bern, Dixon, Dunbar,Kosower).

This breakdown led to a more generalised approach to unitarity involving up to quadruple cuts to isolate the coefficients (Britto, Cachazo, Feng).

Cut constructible pieces; methods



Box coefficients

Box coefficients are determined using the four cut method of Britto, Cachazo and Feng. Four cuts result in box coefficients determined by algebraic operation.

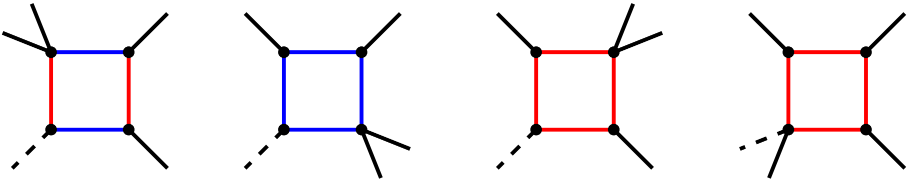
Triangle Coefficients

Triangle coefficients are calculated using the Laurent expansion method of Forde. See Badger's talk for more details.

Bubble coefficients

Bubble coefficients are calculated analytically using Mastrolia's Stoke's theorem method and numerically with Forde's method. See Mastrolia's talk for more details.

The Cut-constructible Pieces: Boxes

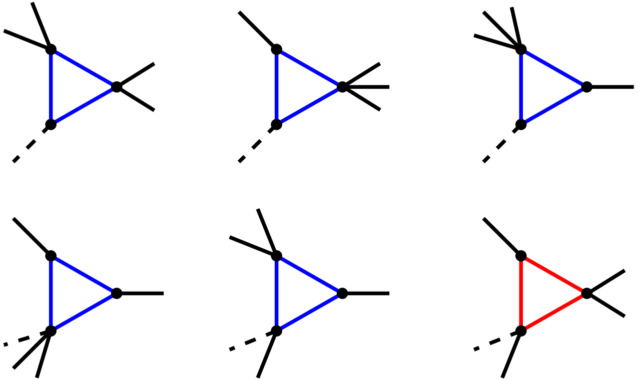


The box topologies that appear in ϕ + parton amplitudes.

One-mass (a) and two-mass easy boxes (b) contribute to MHV ($\overline{q}qgg, gggg$) and all-minus ($gggg$) amplitudes.

One-mass (a) and two-mass hard boxes (c) and (d) contribute to NMHV amplitudes ($\overline{q}qgg, gggg$).

The Cut-constructible Pieces: Triangles



The triangle topologies that appear in ϕ+parton amplitudes. One-mass (a) and two-mass (b) – (e) triangles are pure poles and contribute to all helicities. The finite three-mass triangle (f) only contributes to NMHV amplitudes.

Eliminating Triangles

The Structure of Boxes

A generic box has the structure

$$\mathcal{I}_{4;i} \propto \sum_{s_{i,j}} \frac{(-1)^a}{\epsilon^2} \left(\frac{\mu^2}{-s_{i,j}} \right)^{\epsilon} + Li_2 + \log^2 + \mathcal{O}(\epsilon)$$

i.e. it contains pieces which have ϵ^2 poles in kinematic invariants and then finite pieces which consist of dilogs and log squared terms.

The Structure of One- and Two-mass Triangles

One- and two-mass triangles have the following form

$$\mathcal{I}_{3;1m}(s_{i,j}) \propto \frac{1}{\epsilon^2} \left(\frac{\mu^2}{-s_{i,j}} \right)^{\epsilon}$$

$$\mathcal{I}_{3;2m}(s,t) \propto \frac{\mathcal{I}_{3;1m}(s) - \mathcal{I}_{3;1m}(t)}{s-t}$$

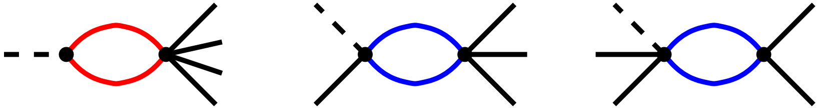
Infrared Safety

The infrared poles must satisfy the following relation.

$$A^{(1)}_{4;\epsilon^{-2}} = -A^{(0)}_4 \frac{1}{\epsilon^2} \left(\sum_{i=1}^{4} \left(\frac{\mu_R^2}{-s_{i(i+1)}} \right)^{\epsilon} \right)$$

The only role (one- and two-mass) triangles play is ensuring this relation.

The Cut-Constructible Pieces: Bubbles



Each double cut can produce box and triangle contributions, as well as logarithms.

The bubble topologies which contribute to ϕ + parton bubbles with a single gluon on either side of the cut vanish since they correspond to logs of vanishing scales.

In all helicity cases, (a) produces only box and triangle contributions and no $\log(s_{1234})$.
In all helicity cases some s_{ij} and s_{ijk} logs appear

The Structure of Bubbles

Checking coefficients

The bubble integral produces a $1/\epsilon$ pole

$$\mathcal{I}_2(s) \propto \frac{1}{(1-2\epsilon)\epsilon}\left(\frac{\mu^2}{-s}\right)^{\epsilon}$$

However, there is no $1/\epsilon$ pole in the amplitude (gluons) which implies a relation between the coefficients

$$\sum_{k=1}^{4}(C_{2;\phi k} + C_{2;\phi kk+1}) = 0.$$

Functions of Logs

We use the following functions to express our result.

$$L_i(s,t) = \frac{\log(s/t)}{(s-t)^i}$$

These functions develop unphysical singularities as $s \to t$ (for $i > 1$).

Completion Terms

To L_3 and L_2 we subtract $\frac{1}{2(s-t)^{(i-1)}}\left(\frac{1}{s}+\frac{1}{t}\right)$ to ensure correct behaviour as $s \to t$.

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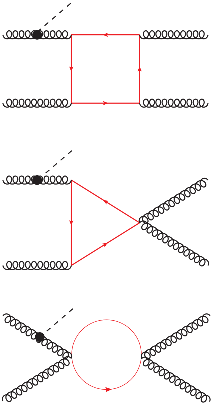
Rational Pieces: gluon amplitudes



Evaluation by Feynman Diagrams

Previous calculations have shown that

- the homogeneous piece is a simple function of tree amplitudes for ϕ and vanishes for H amplitudes.
- the rational piece of Higgs $+$ gluon amplitudes is directly proportional to the number of flavours N_f.

Of the 739 Feynman diagrams, only 136 have a fermion loop and the worst contribution is a second rank box.

Colour Ordering: gluon amplitudes

Tree-level: pure gluon amplitudes

Tree level amplitude of gluons can always be reduced to a single colour trace.

$$\mathcal{A}_n^{(0)}(\{k_i,\lambda_i,a_i\}) = ig^{n-2} \sum_{\sigma\in S_n/Z_n} \mathrm{tr}(T^{a_{\sigma(1)}}\cdots T^{a_{\sigma(n)}})\, A_n^{(0)}(\sigma(1^{\lambda_1},..,n^{\lambda_n})).$$

Here S_n/Z_n represents the group of non-cyclic permutations of n symbols. The kinematic piece which is left over is called a primitive amplitude.

One-loop; pure gluon amplitudes

At one-loop the situation is similar, now single and double traces occur,

$$\mathcal{A}_n^{(1)}(\{k_i,\lambda_i,a_i\}) = ig^{n} \sum_{c=1}^{[n/2]+1} \sum_{\sigma\in S_n/S_{n;c}} G_{n;c}(\sigma) A_{n;c}^{(1)}(\sigma(1^{\lambda_1},\ldots,n^{\lambda_n}))$$

$$G_{n;1}(1) = N_c\, \mathrm{tr}(T^{a_1}\cdots T^{a_n})$$

$$G_{n;c}(1) = \mathrm{tr}(T^{a_1}\cdots T^{a_{c-1}})\,\mathrm{tr}(T^{a_c}\cdots T^{a_n})\ ,\ c>2.$$

$A_{n;1}^{(1)}$ is called the primitive amplitude and all others can be obtained from it.

Breakdown of Results

The results are presented in the following way

$$A_4^{(1)}(\phi, 1^+, 2^-, 3^-, 4^-) = ic_\Gamma(C_4(\phi, 1^+, 2^-, 3^-, 4^-) + R_4(\phi, 1^+, 2^-, 3^-, 4^-)).$$

That is, we have chosen to separate the rational and cut-constructible pieces. We further choose to spilt the cut-constructible pieces into divergent and finite pieces,

$$C_4(\phi, 1^+, 2^-, 3^-, 4^-) = V_4(\phi, 1^+, 2^-, 3^-, 4^-) + F_4(\phi, 1^+, 2^-, 3^-, 4^-).$$

Here V_4 is determined by IR safety,

$$V_4(\phi, 1^+, 2^-, 3^-, 4^-) = -A^{(0)}(\phi, 1^+, 2^-, 3^-, 4^-)\frac{1}{\epsilon^2}\left(\sum_{i=1}^{4}\left(\frac{\mu_R^2}{-s_{i(i+1)}}\right)^\epsilon\right).$$

We express our results in terms of $W_i \propto Ls_{-1} + 2 \times \widetilde{\mathrm{Ls}}_{-1}^{2\mathrm{m}h}$, (one and two-mass hard boxes) and three mass triangle functions I_3^{3m} and completed L_i functions.

$\phi g^+ g^- g^- g^-$ Cut-constructible

The most complicated helicity amplitude is the ϕ-NMHV configuration. (Badger,Glover,Mastrolia,CW)
$F_4(\phi, 1^+, 2^-, 3^-, 4^-) =$

$$\Bigg\{ -\frac{s_{234}^3}{4\langle 1|p_\phi|2]\langle 1|p_\phi|4][23][34]} W^{(1)} - \left(\frac{\langle 2|p_\phi|1]^3}{2s_{134}\langle 2|p_\phi|3][34][41]} + \frac{\langle 34\rangle^3 m_\phi^4}{2s_{134}\langle 1|p_\phi|2]\langle 3|p_\phi|2]\langle 41\rangle} \right) W^{(2)}$$

$$+ \frac{1}{4s_{124}} \left(\frac{\langle 3|p_\phi|1]^4}{\langle 3|p_\phi|2]\langle 3|p_\phi|4][21][41]} + \frac{\langle 24\rangle^4 m_\phi^4}{\langle 12\rangle\langle 14\rangle\langle 2|p_\phi|3]\langle 4|p_\phi|3]} \right) W^{(3)}$$

$$+ \left(\sum_{\gamma=\gamma_\pm(p_\phi, p_1+p_2)} \frac{2m_\phi^4 \langle K_1^\flat 2\rangle^3 \langle 34\rangle^3}{\gamma(\gamma+m_\phi^2)\langle K_1^\flat 1\rangle\langle K_1^\flat 3\rangle\langle K_1^\flat 4\rangle\langle 12\rangle} \right) \mathrm{F}_3^{3m}(m_\phi^2, s_{12}, s_{34})$$

$$+ \left(1 - \frac{N_f}{4N_c}\right) \left(-\frac{\langle 3p_\phi 1]^2}{s_{124}[24]^2} \mathrm{F}_{4F}^{1m}(s_{12}, s_{14}; s_{124}) + \frac{4\langle 24\rangle\langle 3|p_\phi|1]^2}{s_{124}[42]} \hat{L}_1(s_{124}, s_{12}) - \frac{4\langle 23\rangle\langle 4|p_\phi|1]^2}{s_{123}[32]} \hat{L}_1(s_{123}, s_{12}) \right)$$

$$+ \left(1 - \frac{N_f}{N_c} + \frac{N_s}{N_c}\right) \Bigg(\frac{[12][41]\langle 3|p_\phi|2]\langle 3|p_\phi|4]}{2s_{124}[24]^4} \mathrm{F}_{4F}^{1m}(s_{12}, s_{14}; s_{124}) + \frac{2s_{124}\langle 24\rangle\langle 34\rangle^2[41]^2}{3[42]} \hat{L}_3(s_{124}, s_{12})$$

$$+ \frac{\langle 34\rangle[41]\left(3s_{124}\langle 34\rangle[41] + \langle 24\rangle\langle 3|p_\phi|1][42]\right)}{3[42]^2} \hat{L}_2(s_{124}, s_{12}) + \left(\frac{2s_{124}\langle 34\rangle^2[41]^2}{\langle 24\rangle[42]^3} - \frac{\langle 24\rangle\langle 3|p_\phi|1]^2}{3s_{124}[42]} \right) \hat{L}_1(s_{124}, s_{12})$$

$$+ \frac{\langle 3|p_\phi|1](4s_{124}\langle 34\rangle[41] + \langle 3|p_\phi|1](2s_{14}+s_{24})}{s_{124}\langle 24\rangle[42]^3} \hat{L}_0(s_{124}, s_{12}) - \frac{2s_{123}\langle 23\rangle\langle 34\rangle^2[31]^2}{3[32]} \hat{L}_3(s_{123}, s_{12})$$

$$+ \frac{\langle 23\rangle\langle 34\rangle[31]\langle 4|p_\phi|1]}{3[32]} \hat{L}_2(s_{123}, s_{12}) + \frac{\langle 23\rangle\langle 4|p_\phi|1]^2}{3s_{123}[32]} \hat{L}_1(s_{123}, s_{12}) \Bigg) \Bigg\} + \Big\{ (2 \leftrightarrow 4) \Big\}$$

$\phi g^+ g^- g^- g^-$ Rational terms

The rational pieces are extremely compact,

$$\begin{aligned} R_4(\phi, 1^+, 2^-, 3^-, 4^-) \quad = \quad & 2A_4^{(0)}(\phi, 1^+, 2^-, 3^-, 4^-) \\ & + \left\{ \left(1 - \frac{N_f}{N_c} + \frac{N_s}{N_c}\right) \frac{1}{2} \left(\frac{\langle 23\rangle\langle 34\rangle\langle 4|p_\phi|1][31]}{3 s_{123}\langle 12\rangle[21][32]} - \frac{\langle 3|p_\phi|1]^2}{s_{124}[42]^2} \right.\right. \\ & + \frac{\langle 24\rangle\langle 34\rangle\langle 3|p_\phi|1][41]}{3 s_{124} s_{12}[42]} - \frac{[12]^2\langle 23\rangle^2}{s_{14}[42]^2} \\ & \left.\left. - \frac{\langle 24\rangle(s_{23}s_{24} + s_{23}s_{34} + s_{24}s_{34})}{3\langle 12\rangle\langle 14\rangle[23][34][42]} \right) \right\} + \Big\{ (2 \leftrightarrow 4) \Big\} \end{aligned}$$

Colour Ordering: quark plus gluon amplitudes

Tree level: quark plus gluon amplitudes

For $q\bar{q}gg$ amplitudes at tree level the situation is similar to that of the pure gluon amplitudes, now, however, there are free colour indices associated with the quarks

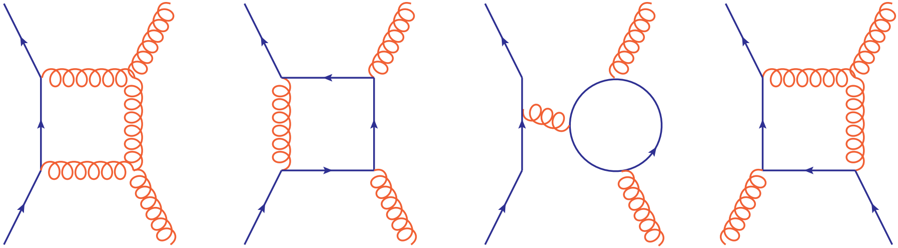
$$A_4^{(0)}(1_{\bar{q}}, 2_q, 3, 4) = g^2 \sum_{\sigma \in S_2} \left(T^{a_{\sigma(3)}} T^{a_{\sigma(4)}}\right)_{i_2}^{\bar{i}_1} A_4^{(0)}(1_{\bar{q}}, 2_q, \sigma(3), \sigma(4))$$

One-loop: quark plus gluon amplitudes

At one-loop new color structures arise

$$\begin{aligned} \mathcal{A}_4^{(1)}(1_{\bar{q}}, 2_q, 3, 4) &= g^4 \Bigg[N_c \sum_{\sigma \in S_2} \left(T^{a_{\sigma(3)}} T^{a_{\sigma(4)}}\right)_{i_2}^{\bar{i}_1} A_{4;1}(1_{\bar{q}}, 2_q, \sigma(3), \sigma(4)) \\ &\quad + \delta^{a_3 a_4} \delta_{i_2}^{\bar{i}_1} A_{4;3}(1_{\bar{q}}, 2_q; 3, 4) \Bigg] . \end{aligned}$$

Colour structures of one-loop quark + gluon amplitudes



These primitives can be split up into more primitive quantities,

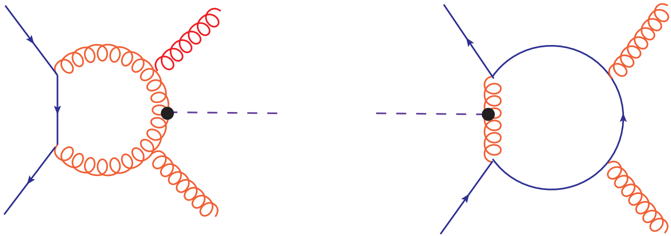
$$A_{4;1}(1_{\bar{q}}, 2_q, 3, 4) = A_4^L(1_{\bar{q}}, 2_q, 3, 4) - \frac{1}{N_c^2} A_4^R(1_{\bar{q}}, 2_q, 3, 4) + \frac{n_f}{N_c} A_4^{\mathrm{f}}(1_{\bar{q}}, 2_q, 3, 4),$$

and

$$\begin{aligned} A_{4;3}(1_{\bar{q}}, 2_q; 3, 4) &= A_4^L(1_{\bar{q}}, 2_q, 3, 4) + A_4^R(1_{\bar{q}}, 2_q, 3, 4) + A_4^L(1_{\bar{q}}, 3, 2_q, 4) \\ &+ A_4^L(1_{\bar{q}}, 2_q, 4, 3) + A_4^R(1_{\bar{q}}, 2_q, 4, 3) + A_4^L(1_{\bar{q}}, 4, 2_q, 3). \end{aligned}$$

Importantly the addition of the Higgs-gluon effective interaction does not change the colour structure, so one-loop $H+$ parton amplitudes have exactly the same colour breakdown as the pure QCD amplitudes.

Rational Pieces: quark and gluon amplitudes



The rational pieces of the $\phi\overline{q}qgg - NMHV$ amplitude do not have such a simple breakdown as the ϕ + glue amplitudes, however the following relationship

$$\mathcal{R}\Big\{A_4^L(\phi,1_{\bar{q}},2_q,3,4)+A_4^R(\phi,1_{\bar{q}},2_q,3,4)+A_4^{\mathrm{f}}(\phi,1_{\bar{q}},2_q,3,4)\Big\}$$
$$+2A_4^{(0)}(\phi^\dagger,1_{\bar{q}},2_q,3,4)=0$$

can be used to simplify the calculation since the R terms are simpler than the L terms.

Left($\phi, \overline{q}^- q^+ g^- g^-$) Cut-constructible

$$-iA_4^L(\phi, 1_{\bar q}^-, 2_q^+, 3_g^-, 4_g^-) = -iA_4^{(0)}\Bigg(-\frac{1}{\epsilon^2}\left[\sum_{i=2}^{4}\left(\frac{\mu^2}{-s_{i(i+1)}}\right)^{\epsilon}\right] + \frac{13}{6\epsilon}\left(\frac{\mu^2}{-s_{12}}\right)^{\epsilon} + \frac{119}{18} - \frac{\delta_R}{6}\Bigg)$$

$$-\frac{s_{134}^2}{[1\,4]\,[3\,4]\,\langle 2|p_\phi|3]}\left[\mathrm{Ls}_{-1}(s_{14}, s_{34}; s_{134}) + \widetilde{\mathrm{Ls}}_{-1}^{2\mathrm{m}h}(s_{12}, s_{134}; s_{34}, m_\phi^2)\right]$$

$$+\frac{\langle 1|p_\phi|2]^2}{\langle 1|p_\phi|4]\,[2\,3]\,[3\,4]}\left[\mathrm{Ls}_{-1}(s_{34}, s_{23}; s_{234}) + \widetilde{\mathrm{Ls}}_{-1}^{2\mathrm{m}h}(s_{12}, s_{234}; s_{34}, m_\phi^2)\right]$$

$$+\left[\frac{m_\phi^4\langle 1\,4\rangle^2\,\langle 2\,4\rangle}{\langle 1\,2\rangle\,\langle 2|p_\phi|3]\langle 4|p_\phi|3]s_{124}} - \frac{\langle 3|p_\phi|2]^3}{[1\,2]\,[2\,4]\,\langle 3|p_\phi|4]s_{124}}\right]\mathrm{Ls}_{-1}(s_{12}, s_{14}; s_{124})$$

$$+\left[\frac{[2\,3]^2\langle 4|p_\phi|1]^3}{[1\,2]\,[1\,3]^3\langle 4|p_\phi|3]s_{123}} - \frac{m_\phi^4\langle 1\,3\rangle^3}{\langle 1\,2\rangle\,\langle 1|p_\phi|4]\langle 3|p_\phi|4]s_{123}}\right]\mathrm{Ls}_{-1}(s_{12}, s_{23}; s_{123})$$

$$+\left[\frac{\langle 4|p_\phi|2]^3}{[1\,2]\,[2\,3]\,\langle 4|p_\phi|3]s_{123}} - \frac{m_\phi^4\langle 1\,3\rangle^3}{\langle 1\,2\rangle\,\langle 1|p_\phi|4]\langle 3|p_\phi|4]s_{123}}\right]\left[\widetilde{\mathrm{Ls}}_{-1}^{2\mathrm{m}h}(s_{34}, s_{123}; s_{12}, m_\phi^2) + \widetilde{\mathrm{Ls}}_{-1}^{2\mathrm{m}h}(s_{14}, s_{123}; s_{23}, m_\phi^2)\right]$$

$$+\left[\frac{m_\phi^4\langle 1\,4\rangle^2\,\langle 2\,4\rangle}{\langle 1\,2\rangle\,\langle 2|p_\phi|3]\langle 4|p_\phi|3]s_{124}} - \frac{\langle 3|p_\phi|2]^2\langle 3|p_\phi|1]}{[1\,2]\,[1\,4]\,\langle 3|p_\phi|4]s_{124}}\right]\left[\widetilde{\mathrm{Ls}}_{-1}^{2\mathrm{m}h}(s_{34}, s_{124}; s_{12}, m_\phi^2) + \widetilde{\mathrm{Ls}}_{-1}^{2\mathrm{m}h}(s_{23}, s_{124}; s_{14}, m_\phi^2)\right]$$

$$-\sum_{\gamma=\gamma_\pm}\frac{m_\phi^4\langle 3\,4\rangle^3\langle 1K_1^\flat\rangle^2}{\gamma(\gamma-m_\phi^2)\langle 1\,2\rangle\langle 3K_1^\flat\rangle\langle 4K_1^\flat\rangle}\mathrm{I}_3^{3\mathrm{m}}(s_{12}, s_{34}, m_\phi^2) - \sum_{\gamma=\gamma_\pm}\frac{m_\phi^4\langle 1\,4\rangle^2\langle 3K_1^\flat\rangle^2}{2\gamma(\gamma-m_\phi^2)\langle 1K_1^\flat\rangle\langle 2K_1^\flat\rangle}\mathrm{I}_3^{3\mathrm{m}}(s_{23}, s_{14}, m_\phi^2)$$

$$+\frac{2\langle 1\,3\rangle^2\,\langle 3\,4\rangle\,\langle 4|p_\phi|3]\,[1\,2]}{3}\widehat{\mathrm{L}}_3(s_{123}, s_{12}) - \frac{\langle 3\,4\rangle\,\langle 3\,1\rangle\,(\langle 4|p_\phi|2]\,[1\,3] - 3\langle 4|p_\phi|1]\,[2\,3])}{6\,[3\,1]}\widehat{\mathrm{L}}_2(s_{123}, s_{12}) + \ldots$$

Left($\phi, \overline{q}^- q^+ g^- g^-$) Cut-constructible + Rat

$$+\ldots$$

$$+\frac{\langle 1\,3\rangle\left(16\langle 4|p_\phi|2]^2[1\,3]^2-3\langle 4|p_\phi|2]\langle 4|p_\phi|3]\,[2\,1]\,[3\,1]+6\langle 4|p_\phi|1]^2[2\,3]^2\right)}{6s_{123}[3\,1]^2\,[3\,2]}\widehat{\mathrm{L}}_1(s_{123},s_{12})$$

$$-\frac{2s_{124}\langle 3\,4\rangle^2\,\langle 1\,4\rangle\,[4\,2]}{3}\widehat{\mathrm{L}}_3(s_{124},s_{12})+\langle 3\,4\rangle\,\langle 1\,4\rangle\,\frac{2\langle 3|p_\phi|2]\,[1\,4]-3\langle 3|p_\phi|4]\,[1\,2]}{6\,[4\,1]}\widehat{\mathrm{L}}_2(s_{124},s_{12})$$

$$+\frac{\langle 3|p_\phi|2](9\,s_{124}\,\langle 3\,4\rangle\,[2\,1]+22\,\langle 3|p_\phi|2]\,\langle 4\,2\rangle\,[1\,2])}{6\,s_{124}\,[4\,1]\,[2\,1]}\widehat{\mathrm{L}}_1(s_{124},s_{12})-\frac{\langle 1\,4\rangle\,\langle 1\,3\rangle\,\langle 4|p_\phi|1]\,[1\,2]}{2\,[3\,1]}\widehat{\mathrm{L}}_2(s_{123},s_{23})$$

$$-\langle 1\,3\rangle\,\langle 4|p_\phi|1]\frac{3\,\langle 4|p_\phi|2]\,[1\,3]+2\,\langle 4|p_\phi|1]\,[2\,3]}{2s_{123}[1\,3]^2}\widehat{\mathrm{L}}_1(s_{123},s_{23})$$

$$+\frac{s_{234}\,\langle 1\,4\rangle\,\langle 3\,4\rangle\,[4\,2]}{2\,[4\,3]}\widehat{\mathrm{L}}_2(s_{234},s_{23})-3\frac{\langle 3\,4\rangle\,\langle 1|p_\phi|2]}{2\,[4\,3]}\widehat{\mathrm{L}}_1(s_{234},s_{23})$$

$$R^L(\phi,1_{\overline{q}}^-,2_q^+,3_g^-,4_g^-)=+\frac{\langle 2\,3\rangle\,\langle 4|p_\phi|2]^2\left(3\,\langle 1\,2\rangle\,[2\,1]-2\,\langle 2\,3\rangle\,[3\,2]\right)+2\,\langle 1\,3\rangle^2\,\langle 2\,4\rangle\,\langle 4|p_\phi|1]\,[2\,1]\,[3\,2]}{12s_{123}\,\langle 1\,2\rangle\,\langle 2\,3\rangle\,[2\,1]\,[3\,1]\,[3\,2]}$$

$$-\frac{\langle 3\,4\rangle\,\langle 3|p_\phi|2]\left(2\,\langle 2\,4\rangle\,[4\,2]-\langle 1\,2\rangle\,[2\,1]\right)}{12s_{124}\,\langle 1\,2\rangle\,[2\,1]\,[4\,1]}+\frac{5\,\langle 3\,4\rangle^2}{12\,\langle 2\,3\rangle\,[3\,1]}-\frac{5\,\langle 3\,4\rangle\,\langle 4|p_\phi|2]}{6\,\langle 2\,3\rangle\,[3\,1]\,[3\,2]}+\frac{\langle 4|p_\phi|2]^2}{6\,\langle 1\,2\rangle\,[2\,1]\,[3\,1]\,[3\,2]}$$

$$-\frac{\langle 1\,3\rangle\,\langle 1\,4\rangle\,\langle 2\,4\rangle\,[2\,1]}{3\,\langle 1\,2\rangle\,\langle 2\,3\rangle\,[3\,1]\,[3\,2]}-\frac{\langle 1\,3\rangle\,\langle 3\,4\rangle}{12\,\langle 1\,2\rangle\,[4\,1]}-\frac{\langle 3\,4\rangle^2\,[4\,2]}{6\,\langle 1\,2\rangle\,[2\,1]\,[4\,1]}-\frac{\langle 1\,3\rangle\,\langle 2\,4\rangle\,\langle 4|(1+3)|4]}{4\,\langle 1\,2\rangle\,\langle 2\,3\rangle\,[3\,1]\,[4\,3]}$$

$$-\frac{\langle 1\,3\rangle\,\langle 4|(1+3)|4]}{3\,\langle 1\,2\rangle\,[4\,1]\,[4\,3]}-\frac{5\,\langle 1\,4\rangle^2\,[4\,1]}{12\,\langle 1\,2\rangle\,[3\,1]\,[4\,3]}+\frac{\langle 1\,4\rangle^2\,[4\,2]}{6\,\langle 1\,2\rangle\,[3\,2]\,[4\,3]}\,.$$

Right($\phi, \overline{q}^- q^+ g^- g^-$) Cut-constructible

$$-iA_4^R(\phi, 1_{\bar q}^-, 2_q^+, 3_g^-, 4_g^-) = -iA_4^{(0)}(\phi, 1_{\bar q}^-, 2_q^+, 3_g^-, 4_g^-)\left(-\frac{1}{\epsilon^2}\left(\frac{\mu^2}{-s_{12}}\right)^{\epsilon} - \frac{3}{2\epsilon}\left(\frac{\mu^2}{-s_{12}}\right)^{\epsilon} - \frac{7}{2} - \frac{\delta_R}{2}\right)$$

$$+ \frac{[1\,2]^2\langle 4|p_\phi|3]^2}{[1\,3]^3[2\,3]s_{123}}\,\mathrm{Ls}_{-1}(s_{12}, s_{23}; s_{123}) + \frac{\langle 3|p_\phi|2]^2}{[1\,4]\,[2\,4]\,s_{124}}\,\mathrm{Ls}_{-1}(s_{14}, s_{12}; s_{124})$$

$$- \frac{\langle 1|p_\phi|2]^2}{[2\,3][3\,4]\langle 1|p_\phi|4]}\,\widetilde{\mathrm{Ls}}_{-1}^{2\mathrm{m}h}(s_{14}, s_{234}; s_{23}, m_\phi^2) - \frac{s_{134}^2}{[1\,4][3\,4]\langle 2|p_\phi|3]}\,\widetilde{\mathrm{Ls}}_{-1}^{2\mathrm{m}h}(s_{23}, s_{134}; s_{14}, m_\phi^2)$$

$$- \sum_{\gamma=\gamma_\pm}\frac{m_\phi^4\langle 1\,4\rangle^2\langle 3K_1^\flat\rangle^2}{2\gamma(\gamma - m_\phi^2)\langle 1K_1^\flat\rangle\langle 2K_1^\flat\rangle}\mathrm{I}_3^{3\mathrm{m}}(s_{23}, s_{14}, m_\phi^2) - \frac{1}{2}\frac{\langle 1\,4\rangle^2[1\,2]^2\langle 3|p_\phi|4]^2}{[1\,4]\,[2\,4]\,s_{124}}\,\widehat{\mathrm{L}}_2(s_{124}, s_{12})$$

$$- 2\frac{\langle 3\,4\rangle\,\langle 3|p_\phi|2]}{[1\,4]}\,\widehat{\mathrm{L}}_1(s_{124}, s_{12}) + \frac{1}{2}\frac{\langle 3|p_\phi|2]^2}{[1\,4]\,[2\,4]\,s_{124}}\,\widehat{\mathrm{L}}_0(s_{124}, s_{12}) + \frac{1}{2}\frac{\langle 1\,4\rangle^2[2\,4]^2 s_{234}^2}{[2\,3]\,[3\,4]\,\langle 1|p_\phi|4]}\,\widehat{\mathrm{L}}_2(s_{234}, s_{23})$$

$$+ 2\frac{\langle 3\,4\rangle\,\langle 1|p_\phi|2]}{[3\,4]}\,\widehat{\mathrm{L}}_1(s_{234}, s_{23}) - \frac{1}{2}\frac{\langle 1|p_\phi|2]^2}{[2\,3]\,[3\,4]\,\langle 1|p_\phi|4]}\,\widehat{\mathrm{L}}_0(s_{234}, s_{23}) - \frac{1}{2}\frac{\left(\langle 1\,2\rangle\,[1\,2]\,\langle 4|p_\phi|1]\right)^2\,[2\,3]}{[1\,3]^3 s_{123}}\,\widehat{\mathrm{L}}_2(s_{123}, s_{23})$$

$$+ 2\frac{\langle 1\,3\rangle\,[1\,2]\,\langle 4|p_\phi|3]\langle 4|p_\phi|1]}{\langle 2\,3\rangle\,[1\,3]^2\,[2\,3]}\,\widehat{\mathrm{L}}_1(s_{123}, s_{23}) + \left[-2\frac{\langle 1\,3\rangle\,[1\,2]\,\langle 4|p_\phi|3]\langle 4|p_\phi|1]}{s_{123}[1\,3]^2\,\langle 2\,3\rangle\,[2\,3]} + \frac{1}{2}\frac{\langle 4|p_\phi|1]^2\,[2\,3]}{[1\,3]^3 s_{123}}\right]\widehat{\mathrm{L}}_0(s_{123}, s_{23})$$

$$- \frac{1}{2}\frac{\left(\langle 1\,3\rangle\,[1\,2]\,\langle 4|p_\phi|3]\right)^2}{[1\,3]\,[2\,3]\,s_{123}}\,\widehat{\mathrm{L}}_2(s_{123}, s_{12}) - \langle 3\,4\rangle\,[1\,2]\,\langle 4|p_\phi|3]\frac{(-2\,\langle 1\,3\rangle\,[1\,3] - \langle 2\,3\rangle\,[2\,3])}{\langle 2\,3\rangle\,[1\,3]^2\,[2\,3]}\,\widehat{\mathrm{L}}_1(s_{123}, s_{12})$$

$$+ [1\,2]\,\langle 4|p_\phi|3]\frac{\langle 2\,3\rangle\,\langle 4|p_\phi|2] + 2\,\langle 1\,3\rangle\,\langle 4|p_\phi|1]}{[1\,3]^2\,\langle 2\,3\rangle\,[2\,3]\,s_{123}}\,\widehat{\mathrm{L}}_0(s_{123}, s_{12})$$

Right($\phi, \overline{q}^- q^+ g^- g^-$) Rational and n_f pieces.

The rational terms for the right moving amplitude have the following form,

$$R^R(\phi, 1^-_{\overline{q}}, 2^+_q, 3^-_g, 4^-_g) = -\frac{\langle 2\,4\rangle^2\,[2\,1]^2}{2\,\langle 2\,3\rangle\,[3\,1]^3} + \frac{\langle 3|p_\phi|2]^2}{2s_{124}\,[4\,1]\,[4\,2]} + \frac{\langle 4|p_\phi|3]^2\,[2\,1]^2}{2s_{123}\,[3\,1]^3\,[3\,2]} - \frac{\langle 1\,4\rangle^2\,[2\,1]}{2\,\langle 1\,2\rangle\,[3\,1]\,[3\,2]}$$
$$+\frac{[2\,1]\left(\langle 1\,3\rangle^2\,\langle 2\,3\rangle\,\langle 4|p_\phi|3]^2\,[3\,1]^2 + \langle 1\,2\rangle^3\,\langle 4|p_\phi|1]^2\,[2\,1]\,[3\,2]\right)}{4s_{123}^2\,\langle 1\,2\rangle\,\langle 2\,3\rangle\,[3\,1]^3\,[3\,2]} - \frac{\langle 1\,3\rangle^2\,[2\,1]}{2\,\langle 1\,2\rangle\,[4\,1]\,[4\,2]}$$
$$+\frac{\langle 1\,4\rangle^2\,\langle 3|p_\phi|4]^2\,[2\,1]}{4s_{124}^2\,\langle 1\,2\rangle\,[4\,1]\,[4\,2]} + \frac{\langle 1\,3\rangle\,\langle 1\,4\rangle\,[4\,2]}{2\langle 1|p_\phi|4]\,[4\,3]} + \frac{s_{234}\,\langle 1\,4\rangle^2\,[4\,2]^2}{4\,\langle 2\,3\rangle\,\langle 1|p_\phi|4]\,[3\,2]^2\,[4\,3]} + \frac{\langle 1\,4\rangle^2\,[4\,2]^2}{2\langle 1|p_\phi|4]\,[3\,2]\,[4\,3]}\,.$$

And fermion loop,

$$-iA_4^{\mathrm{f}}(\phi, 1^-_{\overline{q}}, 2^+_q, 3^-_g, 4^-_g) = -iA_4^{(0)}(\phi, 1^-_{\overline{q}}, 2^+_q, 3^-_g, 4^-_g) \times \left[-\frac{2}{3\epsilon}\left(\frac{\mu^2}{-s_{12}}\right)^\epsilon - \frac{10}{9}\right]$$
$$-\frac{2}{3}\langle 1\,3\rangle^2\,\langle 3\,4\rangle\,[1\,2]\,\langle 4|p_\phi|3]\,\widehat{\mathrm{L}}_3(s_{123}, s_{12})$$
$$-\frac{2}{3}\langle 1\,4\rangle^2\,\langle 3\,4\rangle\,[1\,2]\,\langle 3|p_\phi|4]\,\widehat{\mathrm{L}}_3(s_{124}, s_{12}) - \frac{1}{3}\,\langle 1\,3\rangle\,\langle 3\,4\rangle\,\langle 4|p_\phi|2]\,\widehat{\mathrm{L}}_2(s_{123}, s_{12})$$
$$-\frac{1}{3}\,\langle 1\,4\rangle\,\langle 3\,4\rangle\,\langle 3|p_\phi|2]\,\widehat{\mathrm{L}}_2(s_{124}, s_{12}) - \frac{2}{3}\,\frac{\langle 1\,3\rangle\,\langle 4|p_\phi|2]^2}{\langle 1\,2\rangle\,[2\,3]\,[1\,2]}\,\widehat{\mathrm{L}}_1(s_{123}, s_{12}) + \frac{2}{3}\,\frac{\langle 1\,4\rangle\,\langle 3|p_\phi|2]^2}{\langle 1\,2\rangle\,[2\,4]\,[1\,2]}\,\widehat{\mathrm{L}}_1(s_{124}, s_{12})$$
$$+\frac{2}{3}\,\frac{\langle 1\,3\rangle\,\langle 4|p_\phi|2]^2}{\langle 1\,2\rangle\,[1\,2]\,[2\,3]\,s_{123}}\,\widehat{\mathrm{L}}_0(s_{123}, s_{12}) + \frac{2}{3}\,\frac{(s_{12}+s_{14})\langle 3|p_\phi|2]^2}{\langle 1\,2\rangle\,[1\,4]\,[2\,4]\,[1\,2]\,s_{124}}\,\widehat{\mathrm{L}}_0(s_{124}, s_{12})$$
$$+\frac{\langle 1\,3\rangle\,\langle 3\,4\rangle\,\langle 4|p_\phi|2]}{6\,\langle 1\,2\rangle\,[1\,2]\,s_{123}} + \frac{\langle 1\,4\rangle\,\langle 3\,4\rangle\,\langle 3|p_\phi|2]}{6\,\langle 1\,2\rangle\,[1\,2]\,s_{124}} - \frac{1}{3}\,\frac{\langle 1\,3\rangle\,\langle 1\,4\rangle}{\langle 1\,2\rangle\,[3\,4]}\,.$$

Sub-leading pieces

The combination of the various sub-leading pieces is interesting,

$$
\begin{aligned}
&-iA_{4;3}(\phi, 1_{\bar{q}}^-, 2_q^+, 3_g^-, 4_g^-) = -iA_4^{(0)}(\phi, 1_{\bar{q}}^-, 2_q^+, 3_g^-, 4_g^-) \times V_5(s_{12}, s_{34}, s_{13}, s_{24}) \\
&+ \frac{1}{s_{123}} \left[\frac{\langle 4|p_\phi|3]^2\,[1\,2]^2}{[1\,3]^3\,[2\,3]} + \frac{[2\,3]^2\,\langle 4|p_\phi|1]^3}{[1\,3]^3\,[1\,2]\,\langle 4|p_\phi|3]} - \frac{m_\phi^4\,\langle 1\,3\rangle^3}{\langle 1\,2\rangle\,\langle 1|p_\phi|4]\langle 3|p_\phi|4]} \right] \mathrm{Ls}_{-1}(s_{12}, s_{23}; s_{123}) \\
&+ \frac{1}{s_{124}} \left[\frac{m_\phi^4\,\langle 1\,4\rangle^2\,\langle 2\,4\rangle}{\langle 1\,2\rangle\,\langle 2|p_\phi|3]\langle 4|p_\phi|3]} - \frac{\langle 3|p_\phi|2]^2\,\langle 3|p_\phi|1]}{[1\,4]\,\langle 3|p_\phi|4]\,[1\,2]} \right] \mathrm{Ls}_{-1}(s_{12}, s_{14}; s_{124}) \\
&+ \frac{1}{s_{123}} \left[\frac{m_\phi^4\,\langle 1\,3\rangle^2}{\langle 1|p_\phi|4]\langle 2|p_\phi|4]} - \frac{\langle 4|p_\phi|2]^2}{[1\,3]\,[2\,3]} \right] \mathrm{Ls}_{-1}(s_{13}, s_{23}; s_{123}) + \frac{\langle 1|p_\phi|2]^2}{\langle 1|p_\phi|3]\,[2\,4]\,[3\,4]} \widetilde{\mathrm{Ls}}_{-1}^{2\mathrm{m}h}(s_{14}, s_{234}, s_{23}, m_\phi^2) \\
&- \frac{s_{341}^2}{[1\,3]\,[3\,4]\,\langle 2|p_\phi|4]} \left[\mathrm{Ls}_{-1}(s_{13}, s_{14}; s_{341}) + \widetilde{\mathrm{Ls}}_{-1}^{2\mathrm{m}h}(s_{23}, s_{341}, s_{14}, m_\phi^2) \right] \\
&- \frac{s_{341}^2}{[1\,4]\,[3\,4]\,\langle 2|p_\phi|3]} \left[\mathrm{Ls}_{-1}(s_{14}, s_{34}; s_{341}) + \widetilde{\mathrm{Ls}}_{-1}^{2\mathrm{m}h}(s_{12}, s_{341}, s_{34}, m_\phi^2) \right] \\
&+ \frac{\langle 1|p_\phi|2]^2}{[2\,3]\,[3\,4]\,\langle 1|p_\phi|4]} \left[\mathrm{Ls}_{-1}(s_{23}, s_{34}; s_{234}) + \widetilde{\mathrm{Ls}}_{-1}^{2\mathrm{m}h}(s_{12}, s_{234}, s_{34}, m_\phi^2) \right] - \frac{[2\,4]^2\,\langle 1|p_\phi|3]^2}{[2\,3]\,[3\,4]^3\,\langle 1|p_\phi|4]} \mathrm{Ls}_{-1}(s_{23}, s_{24}; s_{234}) \\
&+ \ldots
\end{aligned}
$$

Sub-leading pieces cont

$$
\begin{aligned}
&+\dots\\
&+\frac{1}{s_{123}}\left[-\frac{m_\phi^4\,\langle 1\,3\rangle^2\,\langle 2\,3\rangle}{\langle 1\,2\rangle\,\langle 2|p_\phi|4]\langle 3|p_\phi|4]}+\frac{\langle 4|p_\phi|2]^2}{\langle 4|p_\phi|3]}\frac{\langle 4|p_\phi|1]}{[1\,2]\,[1\,3]}\right]\widetilde{\mathrm{Ls}}_{-1}^{2\mathrm{m}h}(s_{14},s_{123},s_{23},m_\phi^2)\\
&+\frac{1}{s_{123}}\left[\frac{m_\phi^4\,\langle 1\,3\rangle^3}{\langle 1\,2\rangle\,\langle 3|p_\phi|4]\langle 1|p_\phi|4]}-\frac{\langle 4|p_\phi|2]^3}{\langle 4|p_\phi|3]\,[1\,2]\,[2\,3]}\right]\widetilde{\mathrm{Ls}}_{-1}^{2\mathrm{m}h}(s_{24},s_{123},s_{13},m_\phi^2)\\
&+\frac{1}{s_{123}}\left[-\frac{m_\phi^4\,\langle 1\,3\rangle^2}{\langle 2|p_\phi|4]\langle 1|p_\phi|4]}+\frac{\langle 4|p_\phi|2]^2}{[1\,3]\,[2\,3]}\right]\widetilde{\mathrm{Ls}}_{-1}^{2\mathrm{m}h}(s_{34},s_{123},s_{12},m_\phi^2)\\
&+\left\{3\leftrightarrow 4\right\},
\end{aligned}
$$

Since they contain only the finite pieces of boxes! It is more efficient to code up the total rather than the individual pieces from the colour breakdown. *c.f*

$$
\begin{aligned}
A_{4;3}(1_{\bar q},2_q;3,4) &= A_4^L(1_{\bar q},2_q,3,4)+A_4^R(1_{\bar q},2_q,3,4)+A_4^L(1_{\bar q},3,2_q,4)\\
&+ A_4^L(1_{\bar q},2_q,4,3)+A_4^R(1_{\bar q},2_q,4,3)+A_4^L(1_{\bar q},4,2_q,3)\,.
\end{aligned}
$$

Conclusions

- Strong need for fast and efficient evaluation of Higgs phenomenology at the LHC, best achieved by evaluation of compact formulae.
- The Higgs is produced dominantly at the LHC through gluon fusion, which in the standard model proceeds through a top quark loop. Calculations can be drastically simplified by working in an effective theory in which the mass of the top tends to infinity.
- The evaluation of Higgs helicity amplitudes can be further simplified by considering the Higgs as a real part of a complex scalar field ϕ. This field couples to the self-dual piece of the gluon field strength tensor and produces compact helicity amplitudes.

Conclusions cont

- Amplitudes can be split into two pieces called the cut-constructible and rational parts. The cut-constructible pieces are calculated from four-dimensional cuts. By applying multiple cuts one can isolate specific coefficients of integrals. Rational pieces need additional techniques.

- The helicity amplitudes for the process $pp \to H + 2j$ are now known analytically and fortran code can be downloaded from mcfm.fnal.gov