Heavy quark mass effects on the virtual photon structure in QCD

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in collaboration with

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Plan of the talk

1. Introduction and motivation
2. Evolution equations
3. HQE on structure function
4. Parton distributions
5. Summary and outlook

1. Introduction and Motivation

Why photon structure is interesting?

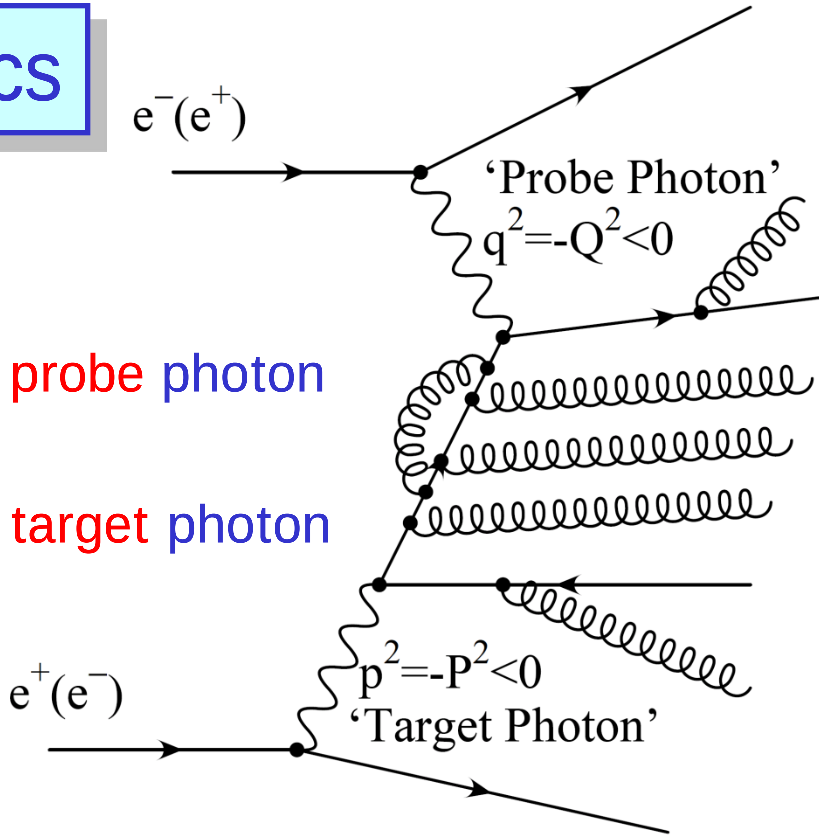
- Provides a good probe to study QCD dynamics in perturbation theory
- In the future linear collider (ILC) two-photon-process in the new kinematical region studied
- Unpolarized virtual photon structure functions studied up to NNLO in QCD for massless quarks
- Here we investigate heavy quark mass effects

Virtual-Photon Kinematics

$$x = \frac{Q^2}{2p \cdot q}$$: Bjorken variable

$$Q^2 = -q^2 > 0$$: Mass squared of probe photon

$$P^2 = -p^2 > 0$$: Mass squared of target photon



In the kinematic region:

$$\Lambda^2 \ll P^2 \ll Q^2$$

structure fns. F_2^γ and F_L^γ

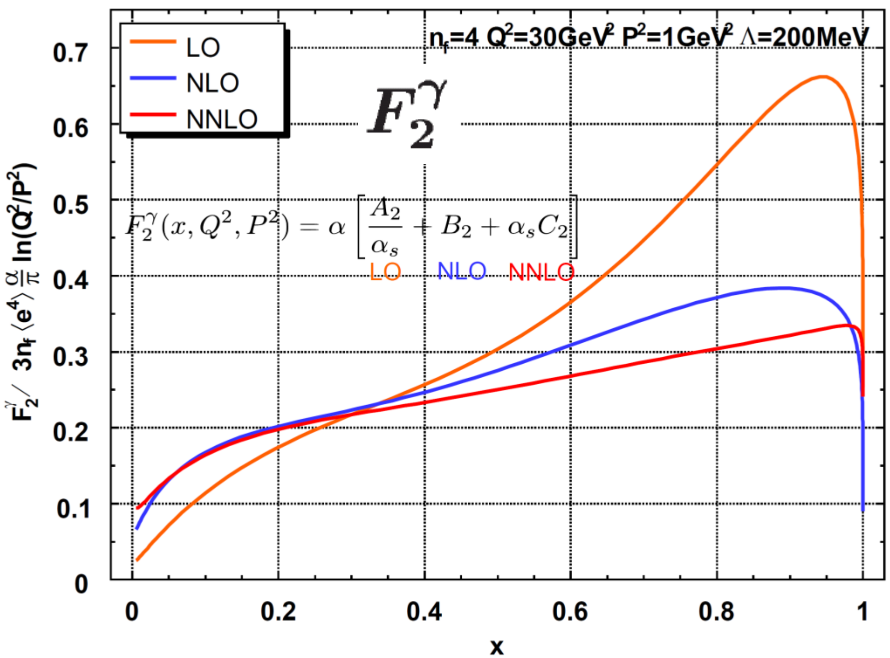
g_1^γ and g_2^γ

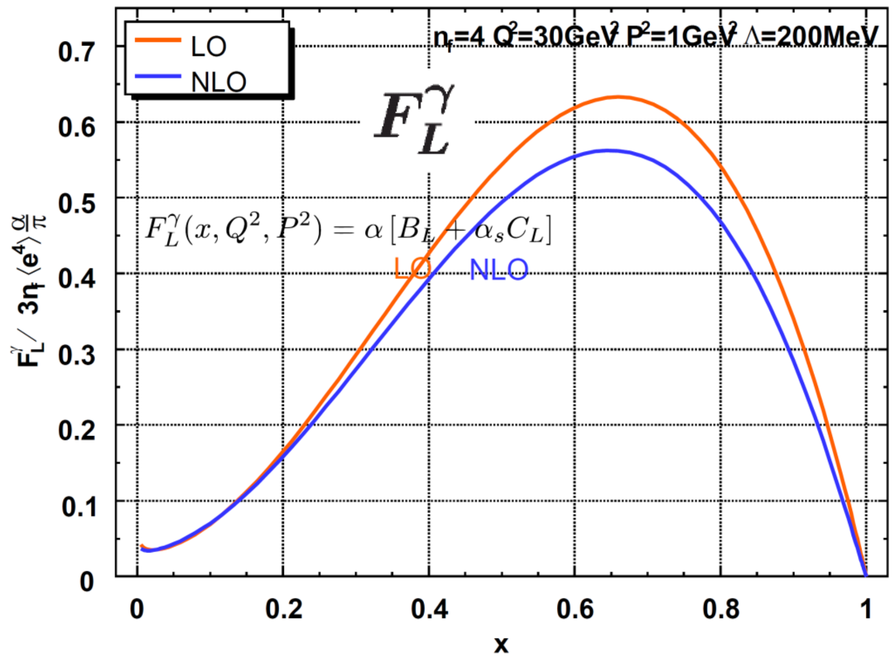
perturbatively calculable !

e^+e^-collision

& QCD interactions

NNLO QCD analysis of virtual photon structure





K. Sasaki, T. Ueda and TU, Phys. Rev.D75 (2007) 114009

- NNLO QCD analysis performed with 3-loop splitting fns. and 2-loop coefficient fns. for massless quarks ; PDFs also studied
  Eur.Phys.J.C62(2009)467
- Target mass effects studied to NNLO in QCD

Y. Kitadono, K. Sasaki, T. Ueda and TU, Phys. Rev.D77 (2008)054019

The heavy quark effects in DIS processes

Any difference? Nucleon vs. Photon

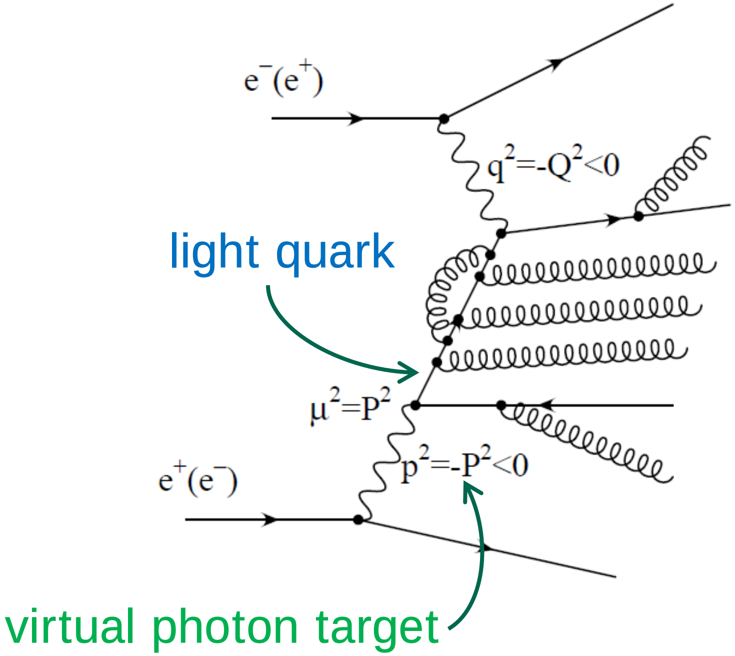
- In the case of nucleon target the heavy quarks are treated as radiatively generated from the gluon and light quarks
- For the virtual photon target, the heavy quarks are treated in the same way as light quarks

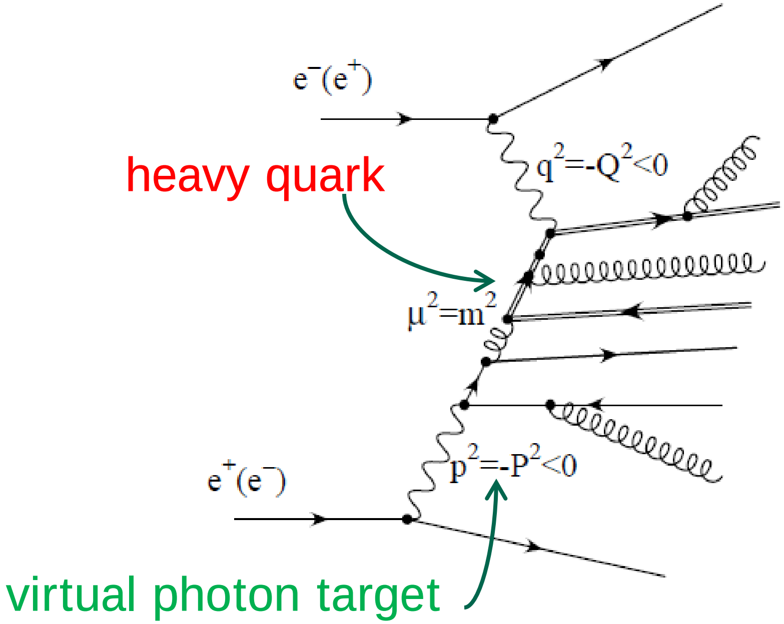
  radiatively generated from the photon target
- We study heavy quark mass effects in the framework of mass-independent renormalization group method

- Heavy quark mass effects for the real photon case are studied in the literature

  M. Gluck, E. Reya & A. Vogt; F. Cornet, P. Jankowski & M. Krawczyk, ...

- Here we treat the heavy quarks on the same footing as the light quarks except for the initial condition : evolution starts at $\mu^2=P^2$ for light quarks, at $\mu^2=m^2$ for heavy quarks





2. Evolution equations

n_f : number of flavours

Let us consider the case: $n_f - 1$ massless quarks and 1 heavy quark

$q_i^\gamma \quad i = 1, \cdots, n_f - 1 \qquad q_{n_f}^\gamma \equiv q_H^\gamma$

The evolution equation:

$$\frac{d}{d \ln Q^2} \vec{q}^{\,\gamma}(x, Q^2, P^2) = \int_x^1 \frac{dy}{y} \vec{q}^{\,\gamma}(y, Q^2, P^2) \hat{P}\left(\frac{x}{y}, Q^2\right) + \vec{k}(x, Q^2)$$

$$\vec{q}^{\,\gamma} = (q_{LS}^\gamma,\ q_H^\gamma,\ G^\gamma,\ q_{LNS}^\gamma)$$

$$q_{LS}^\gamma = \sum_{i=1}^{n_f-1} q_i^\gamma$$ light singlet quark

$$q_{LNS}^\gamma = \sum_{i=1}^{n_f-1} e_i^2 q_{LNS}^i$$ light non-singlet quark

$$q_{LNS}^i = q_i^\gamma - \frac{1}{n_f - 1} q_{LS}^\gamma$$

Splitting function

$$\hat{P} \equiv \begin{pmatrix} P_{qq}^S & P_{HL} & P_{GL} & 0 \\ P_{LH} & P_{HH} & P_{GH} & 0 \\ P_{LG} & P_{HG} & P_{GG} & 0 \\ 0 & 0 & 0 & P_{qq}^{NS} \end{pmatrix}$$

Photon-parton splitting function

$$\vec{k} = (k_{LS},\ k_H,\ k_G,\ k_{LNS})$$

Splitting functions

$$
\begin{aligned}
P^S_{qq} &= \tilde{P}_{qq} + \frac{n_f - 1}{n_f}\tilde{P}^S_{qq}, \quad P_{LH} = \frac{n_f - 1}{n_f}\tilde{P}^S_{qq}, \quad P_{LG} = (n_f - 1)\tilde{P}_{qG}, \\
P^S_{HL} &= \frac{1}{n_f}\tilde{P}^S_{qq}, \quad P_{HH} = \tilde{P}_{qq} + \frac{1}{n_f}\tilde{P}^S_{qq}, \quad P_{HG} = \tilde{P}_{qG}, \\
P^S_{GL} &= \tilde{P}_{Gq}, \quad P_{GH} = \tilde{P}_{Gq}, \quad P_{GG} = \tilde{P}_{GG}, \quad P^{NS}_{qq} = \tilde{P}_{qq}\ .
\end{aligned}
$$

Photon-parton splitting functions

$$
k_{LS} = \sum_{i=1}^{n_f-1} \tilde{P}_{i\gamma}, \quad k_H = \tilde{P}_{H\gamma}, \quad k_G = \tilde{P}_{G\gamma}, \quad k_{LNS} = \sum_{i=1}^{n_f-1} e_i^2 \left(\tilde{P}_{i\gamma} - \frac{1}{n_f - 1}\sum_{j=1}^{n_f-1} \tilde{P}_{j\gamma} \right)
$$

Mass-independent renormalization group eq.

$$
\left[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} + \gamma_m(g)m\frac{\partial}{\partial m} - \gamma_n(g,\alpha)\right]_{ij} C_n^j\left(\frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}, \bar{g}(\mu^2), \alpha\right) = 0
$$

Coefficient functions

The heavy quark mass effects in parton picture and OPE

$$F_2^\gamma(x,Q^2,P^2)=\vec{q}^{\,\gamma}(y,Q^2,P^2,m^2)\otimes\vec{C}\left(\frac{x}{y},\frac{\overline{m}^2}{Q^2},\bar{g}(Q^2)\right)$$

Photon structure function PDF Coefficient function

mass dependence

Parton interpretation of twist-2 operators $\vec{O}_n$

No mass dependence

$$\int_0^1 dx x^{n-1}\vec{q}^{\,\gamma}(x,Q^2,P^2,m^2)$$

$$=\vec{A}_n\left(1,\frac{\overline{m}^2(P^2)}{P^2},\bar{g}(P^2)\right)T\exp\left[\int_{\bar{g}(Q^2)}^{\bar{g}(P^2)}dg\frac{\gamma_n(g,\alpha)}{\beta(g)}\right]$$

where

$$\langle\gamma(P^2)|\vec{O}_n(\mu^2)|\gamma(P^2)\rangle=\vec{A}_n\left(\frac{P^2}{\mu^2},\frac{\overline{m}^2(\mu^2)}{\mu^2},\bar{g}(\mu^2)\right)$$

Perturbatively calculable !

mass dependence

3. Heavy quark effects on structure function

- We compute the deviation arising from heavy quark mass effects (HQE) on the photon matrix elements of twist-2 quark & gluon operators and corresponding coefficient functions to NLO QCD

$$M_2^\gamma(n,m^2) = M_2^\gamma(n,m^2=0) + \Delta M_2^\gamma(n,m^2)$$

$$\Delta M_2^\gamma(n,Q^2,P^2,m^2) = \int_0^1 dx x^{n-2} \Delta F_2^\gamma(x,Q^2,P^2,m^2)$$

$$= \frac{\alpha}{4\pi}\frac{1}{2\beta_0}\left[\sum_{i=\pm,NS}\Delta\mathcal{A}_i^n\left[1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_i^n}\right]\right.$$

1-loop anomalous dim.

$$d_i^n = \lambda_i^n/2\beta_0 \quad (i=\pm,NS)$$

$$\left.+\sum_{i=\pm,NS}\Delta\mathcal{B}_i^n\left[1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_i^n+1}\right]+\Delta\mathcal{C}^n\right]+\mathcal{O}(\alpha_s)$$

Master formula

$$M_2^\gamma(n,Q^2,P^2) = \int_0^1 dx F_2^\gamma(x,Q^2,P^2) \qquad r \equiv \frac{\alpha_s(Q^2)}{\alpha_s(P^2)}$$

$$= \frac{\alpha}{4\pi}\frac{1}{2\beta_0}\left[\frac{4\pi}{\alpha_s(Q^2)}\sum_i \mathcal{L}_i^n(1-r^{d_i^n+1}) + \sum_i \mathcal{A}_i^n(1-r^{d_i^n}) + \mathcal{B}_i^n(1-r^{d_i^n+1}) + \mathcal{C}^n\right] + \mathcal{O}(\alpha_s)$$

$$F_2^\gamma(x,Q^2,P^2) = F_2^\gamma(x,Q^2,P^2)|_{\mathrm{massless}} + \Delta F_2^\gamma(x,Q^2,P^2)$$

$$\Delta F_2^\gamma = \langle e^2\rangle_L \Delta q_{LS}^\gamma + e_H^2 \Delta q_H^\gamma$$

Heavy quark mass effect

$$\Delta M_2^\gamma(n)/\frac{\alpha}{8\pi\beta_0} = 2\beta_0(\langle e^2\rangle - e_H^2)\Delta\tilde{A}_H^n(1-r^{d_\psi^n})$$
$$-2\beta_0\langle e^2\rangle\frac{\gamma_{\psi\psi}^{(0),n}-\lambda_-^n}{\lambda_+^n-\lambda_-^n}\Delta\tilde{A}_H^n(1-r^{d_+^n})$$
$$-2\beta_0\langle e^2\rangle\frac{\gamma_{\psi\psi}^{(0),n}-\lambda_+^n}{\lambda_-^n-\lambda_+^n}\Delta\tilde{A}_H^n(1-r^{d_-^n})$$
$$+2\beta_0 e_H^2\Delta\tilde{A}_H^n$$

$$\Delta\tilde{A}_H^n = 6e_H^2\Delta\tilde{A}_{nG}^\psi/n_f$$

In the massive quark limit

$$\Lambda_{\rm QCD}^2 \ll P^2 \ll m^2 \ll Q^2 \qquad \langle e^2\rangle_{n_f} = \sum_{i=1}^{n_f} e_i^2/n_f$$

We obtain

e_H :Heavy quark charge

$$\Delta\mathcal{A}_{NS}^n = -12\beta_0 e_H^2 (e_H^2 - \langle e^2\rangle_{n_f})(\Delta\tilde{A}_{nG}^{\psi}/n_f)$$

$$\Delta\mathcal{A}_{\pm}^n = -12\beta_0 e_H^2 \langle e^2\rangle_{n_f}(\Delta\tilde{A}_{nG}^{\psi}/n_f)\frac{\gamma_{\psi\psi}^{0,n} - \lambda_{\mp}^n}{\lambda_{\pm}^n - \lambda_{\mp}^n}$$

$$\Delta\mathcal{B}_{NS}^n = 0\ , \quad \Delta\mathcal{B}_{\pm}^n = 0\ , \quad \Delta\mathcal{C}^n = 12\beta_0 e_H^2(\Delta\tilde{A}_{nG}^{\psi}/n_f)$$

where

$$\Delta\tilde{A}_{nG}^{\psi}/n_f = 2\left[-\frac{n^2+n+2}{n(n+1)(n+2)}\ln\frac{m^2}{P^2} + \frac{1}{n} - \frac{1}{n^2} + \frac{4}{(n+1)^2} - \frac{4}{(n+2)^2} - \frac{n^2+n+2}{n(n+1)(n+2)}\sum_{j=1}^{n}\frac{1}{j}\right]$$

Numerical analysis

Y. Kitadono, K. Sasaki, T. Ueda and TU,
Prog. Theor. Phys. 121 (2009)054019

We now evaluate

Effective photon structure function

$$F_{\rm eff}^{\gamma}(x,Q^2,P^2) \;=\; F_2^{\gamma}(x,Q^2,P^2)+\frac{3}{2}F_L^{\gamma}(x,Q^2,P^2)$$

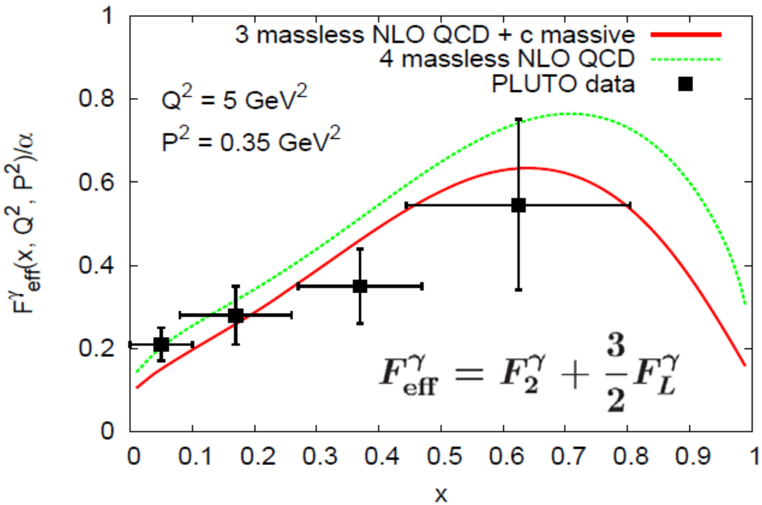
and compare the exisisting exp. data from PLUTO & L3

Heavy quark mass inputs

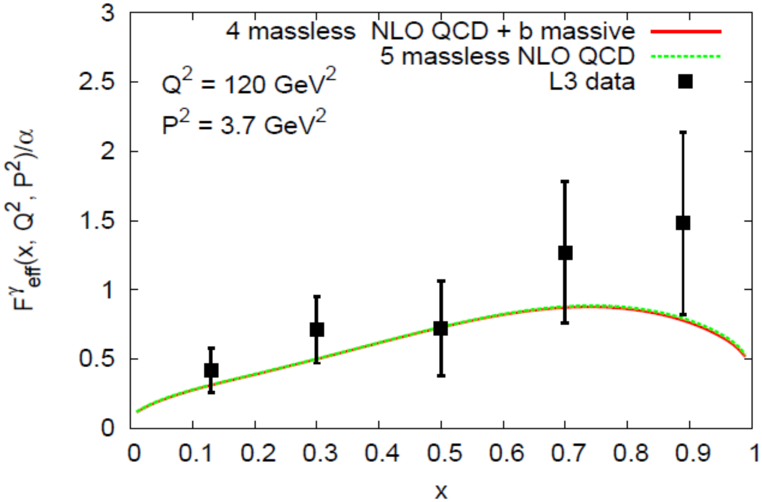
$$m_c = 1.3\mathrm{GeV} \quad (\mathrm{for\ PLUTO})$$

$$m_b = 4.2\mathrm{GeV} \quad (\mathrm{for\ L3})$$

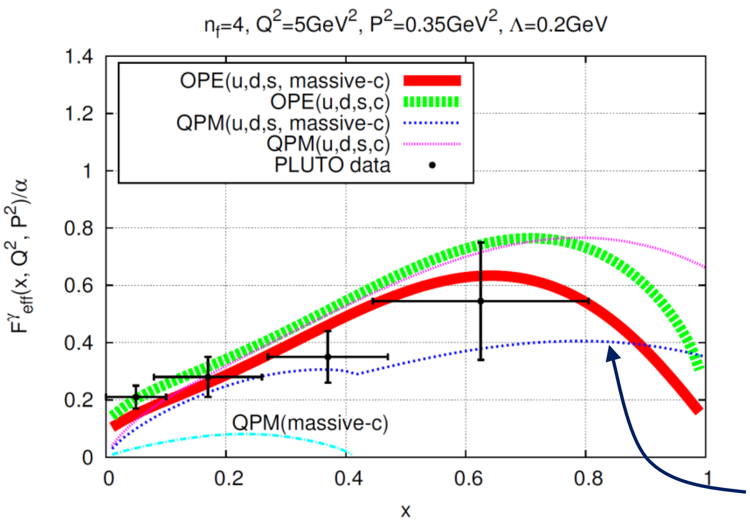
Heavy flavor effects vs. experimental data for $F_{\rm eff}^{\gamma}$



QCD prediction vs. PLUTO data



QCD prediction vs. L3 data



In the massive quark limit

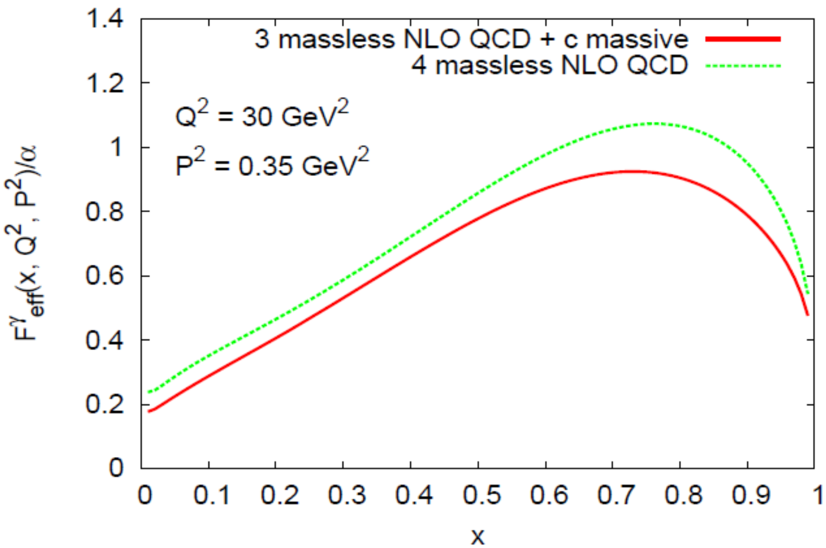
$$\Lambda_{\rm QCD}^2 \ll P^2 \ll m^2 \ll Q^2$$

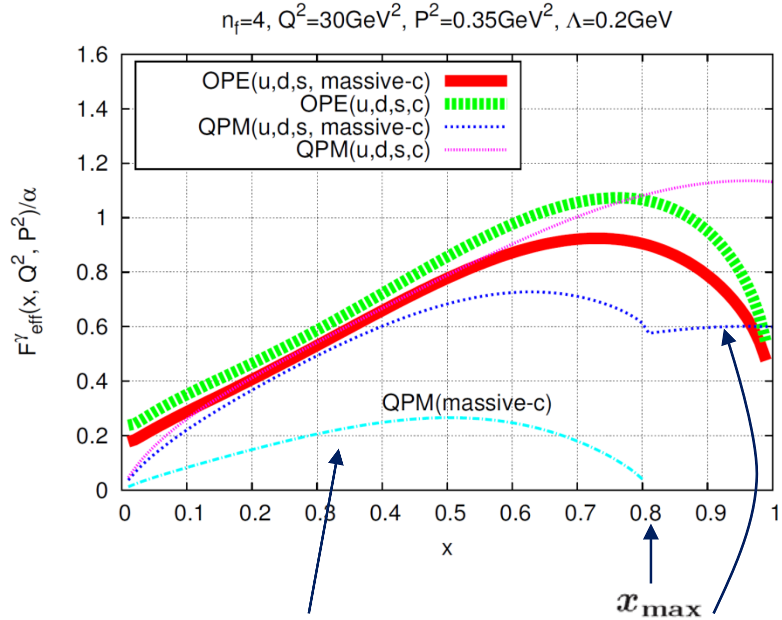
Heavy quark mass threshold effects illustrated by PM

PM Box

$$x_{\max} = \frac{1}{1+\frac{4m^2}{Q^2}}$$

Theoretical prediction for F_{eff} with flavor effects





$$Q^2 = 30\mathrm{GeV}^2 \quad P^2 = 0.35\mathrm{GeV}^2$$

For which massive quark limit satisfied

$$\Lambda^2_{\mathrm{QCD}} \ll P^2 \ll m^2 \ll Q^2$$

parton model box contribution with heavy charm threshold effects

The general case for which P^2 and m^2 are of same order is now under investigation

4. Parton distributions

$q_i^\gamma \qquad (i = 1, \cdots, n_f - 1)$ light quarks (massless)

$q_H^\gamma \qquad (q_{n_f}^\gamma \equiv q_H^\gamma)$ heavy quark (massive)

decomposition

$$\vec{q}^\gamma = (q_{LS}^\gamma,\ q_H^\gamma,\ G^\gamma,\ q_{LNS}^\gamma)$$

$$q_{LS}^\gamma = \sum_{i=1}^{n_f-1} q_i^\gamma$$ light singlet quark

$$q_{LNS}^\gamma = \sum_{i=1}^{n_f-1} e_i^2 q_{LNS}^i$$ light non-singlet quark

$$\left(q_{LNS}^i = q_i^\gamma - \frac{1}{n_f - 1} q_L^\gamma\right)$$

G^γ gluon

one-loop anomalous dim. matrix

$$\widehat{\gamma}_n^{(0)} = \left(\begin{array}{ccc|c} \gamma_{\psi\psi}^{(0),n} & 0 & \gamma_{G\psi}^{(0),n} & 0 \\ 0 & \gamma_{\psi\psi}^{(0),n} & \gamma_{G\psi}^{(0),n} & 0 \\ \frac{n_f-1}{n_f}\gamma_{\psi G}^{(0),n} & \frac{1}{n_f}\gamma_{\psi G}^{(0),n} & \gamma_{GG}^{(0),n} & 0 \\ \hline 0 & 0 & 0 & \gamma_{\psi\psi}^{(0),n} \end{array}\right)$$

diagonalization

$$\widehat{\gamma}_n^{(0)} = \sum_{i=\psi,+,-,LNS} \lambda_i^n P_i^n$$

$$\lambda_\psi^n = \gamma_{\psi\psi}^{(0),n}$$

$$\lambda_\pm^n = \frac{1}{2}\left\{\gamma_{\psi\psi}^{(0),n} + \gamma_{GG}^{(0),n} \pm \sqrt{(\gamma_{\psi\psi}^{(0),n} - \gamma_{GG}^{(0),n})^2 + 4\gamma_{\psi G}^{(0),n}\gamma_{G\psi}^{(0),n}}\right\}$$

$$\lambda_{LNS}^n = \gamma_{\psi\psi}^{(0),n}$$

eigenvalues

projection operators

$$P_\psi^n = \left(\begin{array}{ccc|c} \frac{1}{n_f} & -\frac{1}{n_f} & 0 & 0 \\ -\frac{n_f-1}{n_f} & \frac{n_f-1}{n_f} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right) \qquad P_{LNS}^n = \left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

$$P_\pm^n = \frac{1}{\lambda_\pm^n - \lambda_\mp^n}\left(\begin{array}{ccc|c} \frac{n_f-1}{n_f}(\gamma_{\psi\psi}^{0,n} - \lambda_\mp^n) & \frac{1}{n_f}(\gamma_{\psi\psi}^{0,n} - \lambda_\mp^n) & \gamma_{G\psi}^{0,n} & 0 \\ \frac{n_f-1}{n_f}(\gamma_{\psi\psi}^{0,n} - \lambda_\mp^n) & \frac{1}{n_f}(\gamma_{\psi\psi}^{0,n} - \lambda_\mp^n) & \gamma_{G\psi}^{0,n} & 0 \\ \frac{n_f-1}{n_f}\gamma_{\psi G}^{0,n} & \frac{1}{n_f}\gamma_{\psi G}^{0,n} & \gamma_{GG}^{0,n} - \lambda_\mp^n & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right)$$

Deviation due to the heavy quark mass effects

light singlet quark

$$\Delta q^{\gamma}_{LS}(n)/\frac{\alpha}{8\pi\beta_0} = 2\beta_0\frac{n_f-1}{n_f}\Delta\tilde{A}^n_H(1-r^{d^n_\psi})$$

$$-\frac{n_f-1}{n_f}2\beta_0\frac{\gamma^{(0),n}_{\psi\psi}-\lambda^n_-}{\lambda^n_+-\lambda^n_-}\Delta\tilde{A}^n_H(1-r^{d^n_+})$$

$$-\frac{n_f-1}{n_f}2\beta_0\frac{\gamma^{(0),n}_{\psi\psi}-\lambda^n_+}{\lambda^n_--\lambda^n_+}\Delta\tilde{A}^n_H(1-r^{d^n_-})$$

heavy quark

$$\Delta q^{\gamma}_{H}(n)/\frac{\alpha}{8\pi\beta_0} = -2\beta_0\frac{n_f-1}{n_f}\Delta\tilde{A}^n_H(1-r^{d^n_\psi})$$

$$-\frac{1}{n_f}2\beta_0\frac{\gamma^{(0),n}_{\psi\psi}-\lambda^n_-}{\lambda^n_+-\lambda^n_-}\Delta\tilde{A}^n_H(1-r^{d^n_+})$$

$$-\frac{1}{n_f}2\beta_0\frac{\gamma^{(0),n}_{\psi\psi}-\lambda^n_+}{\lambda^n_--\lambda^n_+}\Delta\tilde{A}^n_H(1-r^{d^n_-})$$

$$+2\beta_0\Delta\tilde{A}^n_H$$

$$\Delta\tilde{A}^n_H = 6e^2_H\Delta\tilde{A}^\psi_{nG}/n_f$$

gluon

$$\Delta G^{\gamma}(n)/\frac{\alpha}{8\pi\beta_0} = 2\beta_0 \Delta\tilde{A}_H^n \left\{ \frac{\gamma_{G\psi}^{0,n}}{\lambda_+^n - \lambda_-^n}(1 - r^{d_+^n}) + \frac{\gamma_{G\psi}^{0,n}}{\lambda_-^n - \lambda_+^n}(1 - r^{d_-^n}) \right\}$$

$$\Delta q_{LNS}^{\gamma}(n)/\frac{\alpha}{8\pi\beta_0} = 0$$

no correction !

light non-singlet quark

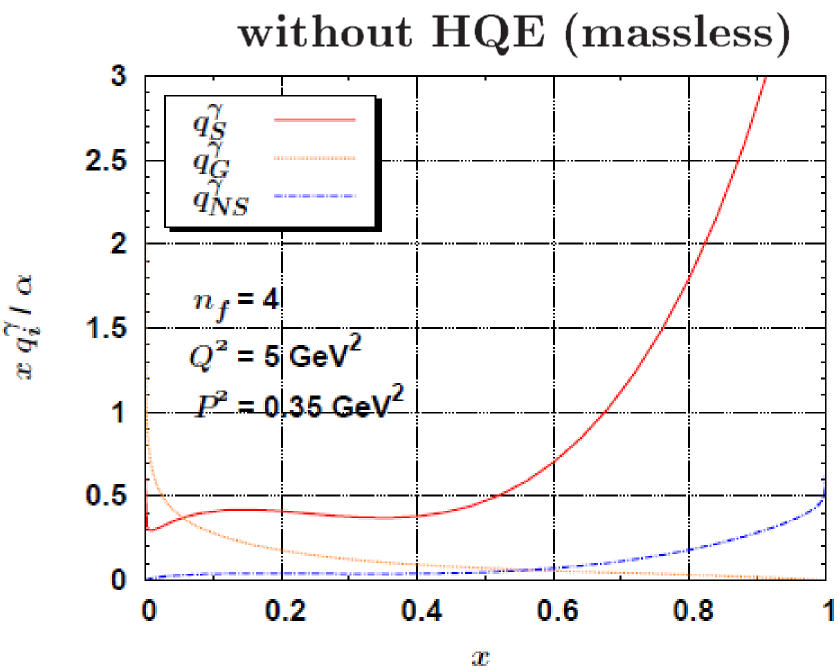
$$\Delta\tilde{A}_H^n = 6e_H^2 \Delta\tilde{A}_{nG}^{\psi}/n_f$$

In the massive quark limit: $\Lambda_{\rm QCD}^2 \ll P^2 \ll m^2 \ll Q^2$

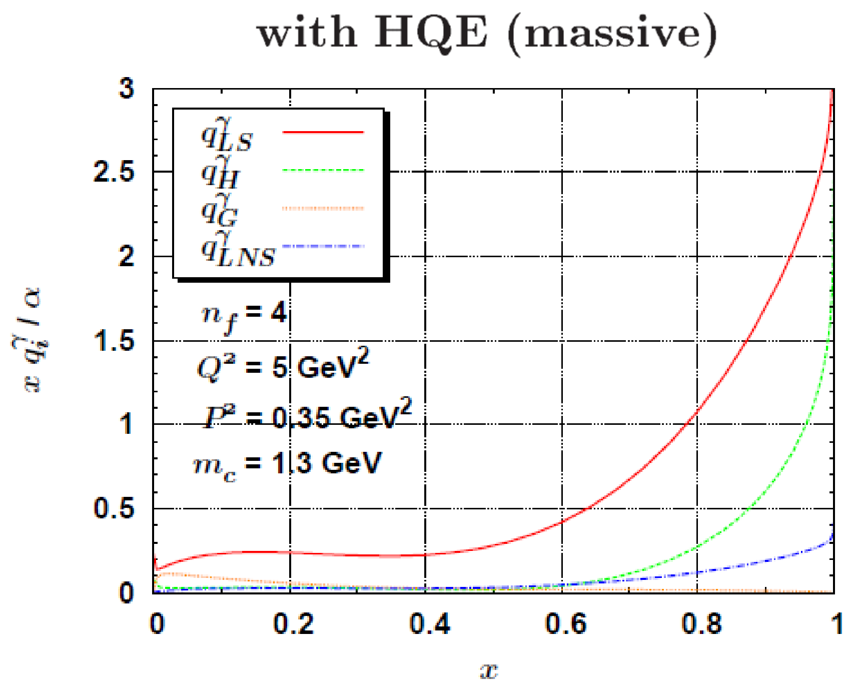
$$\begin{aligned} \Delta\tilde{A}_{nG}^{\psi}/n_f &= 2\left[-\frac{n^2+n+2}{n(n+1)(n+2)} \ln\frac{m^2}{P^2} + \frac{1}{n} - \frac{1}{n^2} \right. \\ &\left. + \frac{4}{(n+1)^2} - \frac{4}{(n+2)^2} - \frac{n^2+n+2}{n(n+1)(n+2)} \sum_{j=1}^{n} \frac{1}{j} \right] \end{aligned}$$

$$\Delta B_{\psi}^n = 0, \quad \Delta B_G^n = 0, \quad \Delta B_{\gamma}^n = 2\Delta B_G^n/n_f = 0.$$

Numerical analysis



PDFs for the massless quarks (MSbar)



PDFs for the massive c quark (MSbar)

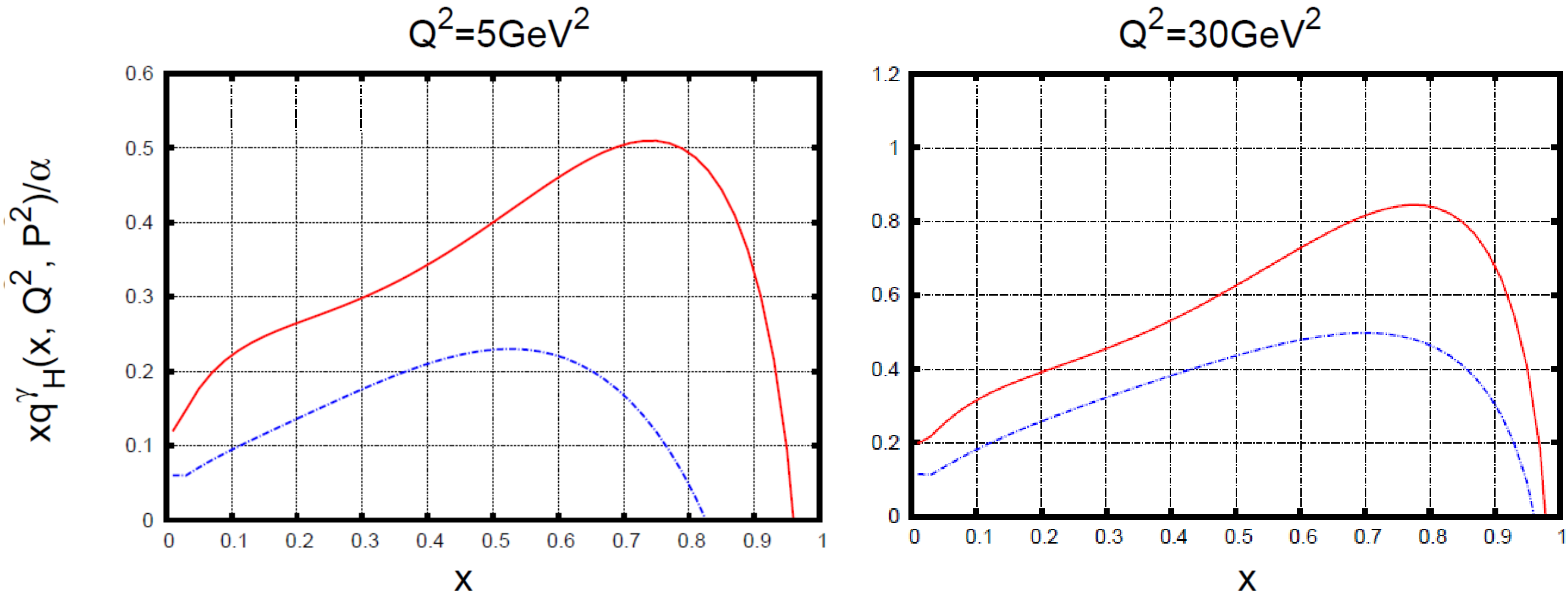
PDFs for the heavy quark (DISgamma)

$n_f=4$, $P^2=0.35\text{GeV}^2$, $\Lambda=0.2\text{GeV}$, $m_c=1.3\text{GeV}$

$$q_H^\gamma(x, Q^2, P^2)$$

without HQE (massless)

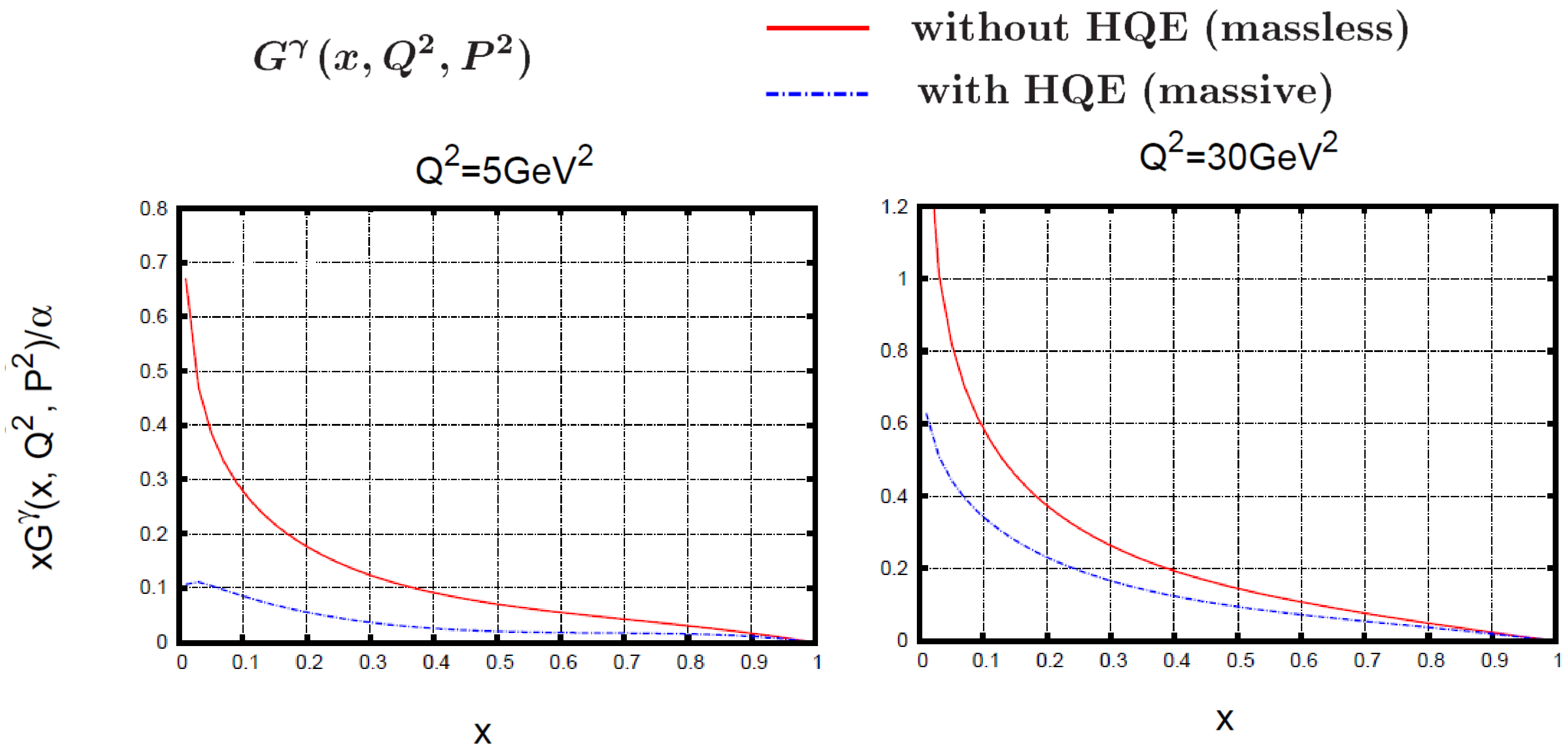
with HQE (massive)



PDFs for the gluon (DISgamma)

n_f=4, P^2=0.35GeV^2, Λ=0.2GeV, m_c=1.3GeV

$G^\gamma(x, Q^2, P^2)$

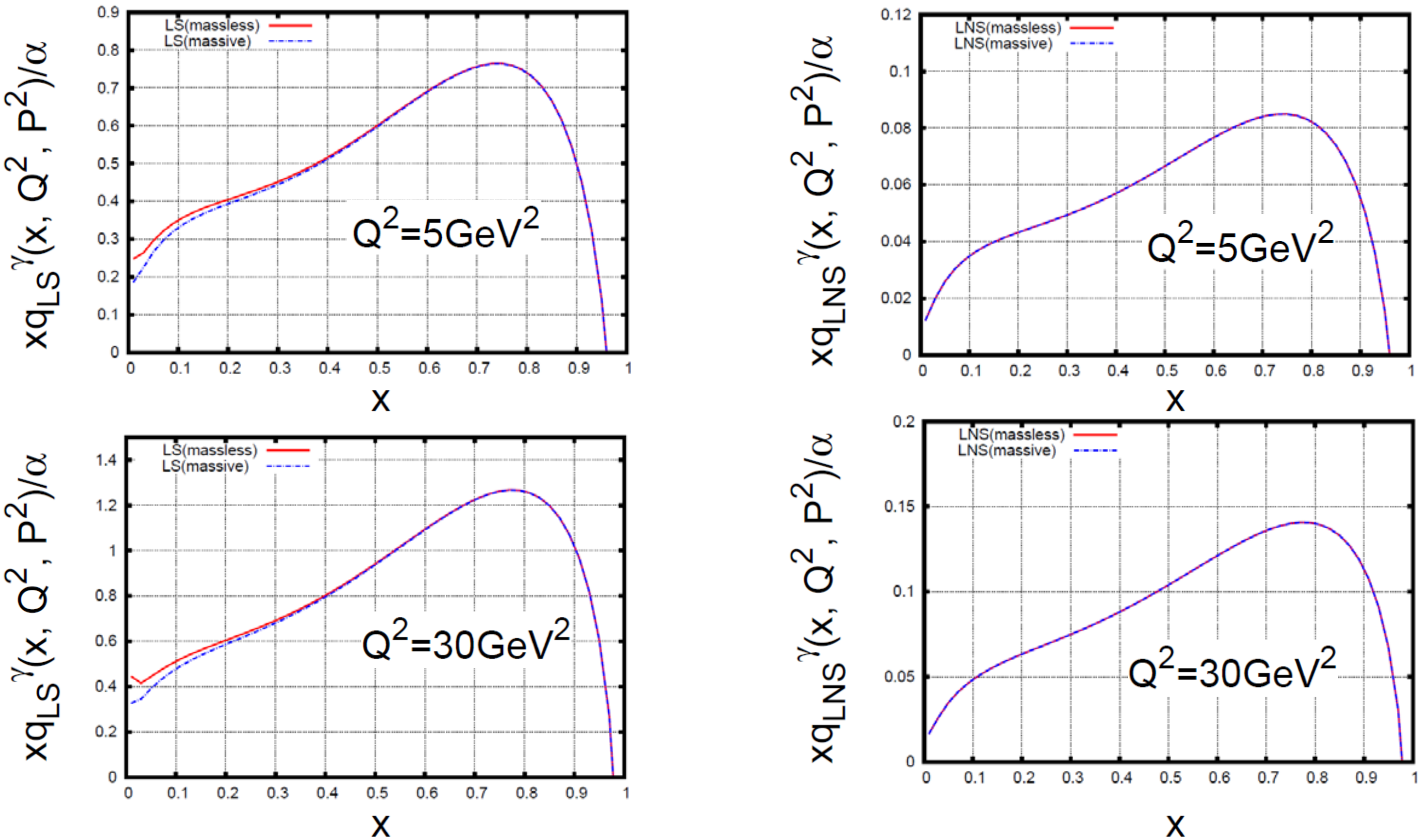


PDFs for the LS and LNS quark (DISgamma)

without HQE (massless)
with HQE (massive)

n_f=4, P^2=0.35GeV^2, Λ=0.2GeV, m_c=1.3GeV

$q^{\gamma}_{LS}(x, Q^2, P^2)$ $\qquad$ $q^{\gamma}_{LNS}(x, Q^2, P^2)$



Initial condition $t = 0$ $\quad t = \frac{2}{\beta_0} \ln \frac{\alpha_s(P^2)}{\alpha_s(Q^2)}$

$$\vec{q}^{\gamma}(n, 0) = (0,\ \hat{q}^H(n),\ 0,\ 0)$$

such that

$$q_H^{\gamma}(t = t_m) = 0, \qquad t_m = \frac{2}{\beta_0} \ln \frac{\alpha_s(P^2)}{\alpha_s(m^2)}$$

Note that $\quad Q^2 = P^2 \Longleftrightarrow t = 0$

$$Q^2 = m^2 \Longleftrightarrow t_m = 0$$

We can reproduce the deviation if

$$\hat{q}^H(n) = \frac{\alpha}{4\pi} \Delta \tilde{A}_H^n$$ holds

Note $\quad \langle \gamma(p) | O_n^H(\mu) | \gamma(p) \rangle |_{\mu^2 = P^2} = \frac{\alpha}{4\pi} \tilde{A}_H^n |_{\text{massless}} + \frac{\alpha}{4\pi} \Delta \tilde{A}_H^n$

Moment sum rule of polarized structure fn.

1 st moment of $g_1^\gamma(x,Q^2,P^2)$

$$\int_0^1 dx g_1^\gamma(x,Q^2,P^2) = -\frac{3\alpha}{\pi}\sum_{i=1}^{n_f} e_i^4 + \mathcal{O}(\alpha_s)$$

HQE for the coefficient fn.

$$\mathcal{C}^{n=1} + \Delta\mathcal{C}^{n=1} = -12\beta_0(n_f\langle e^4\rangle - e_H^4) = -12\beta_0\sum_{i=1}^{n_f-1} e_i^4$$

since $\Delta\mathcal{C}^{n=1} = 12\beta_0 e_H^4$

Heavy quark decouples from the sum rule as

$$\int_0^1 dx g_1^\gamma(x,Q^2,P^2) = -\frac{3\alpha}{\pi}\sum_{i=1}^{n_f-1} e_i^4 + \mathcal{O}(\alpha_s)$$

5. Summary and outlook

- The heavy quark mass effects investigated in the framework of parton model/operator product expansion based on the mass-independent renormalization group
- Heavy quark mass effects reside in the photon matrix element of the twist-2 quark & gluon operators and coefficient functions
- The deviation due to the heavy quark mass effects on the structure function and parton distribution functions studied

- Although we take some approximation, our theoretical prediction shows a right trend of describing the experimental data
- We should investigate the general kinematical region where P^2 and m^2 are of same order
- We have not included the kinematical threshold effects which should be taken into account in the future investigation
- General Mass Variable Flavor Number Scheme (GMVFNS) should be implemented in the present analysis which is under investigation