VIRTUAL CORRECTIONS TO MASSLESS QCD PROCESSES WITH D-DIMENSIONAL UNITARITY

ACHILLEAS LAZOPOULOS
RADCOR 2009
ETH ZURICH

Introduction

Introduction

- Massless quarks and gluons the necessary ingredients for any physical process.
- Four jets - landmark NLO calculation.
- $b\bar{b}b\bar{b}$ production with massless bottoms.

Introduction: the formula

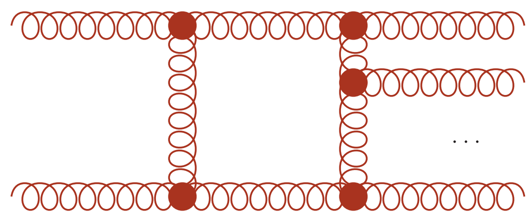
$$\sigma_{nlo} = \int_n d\sigma_{tree} + \int_n d\sigma_{virt} + \int_{n+1} d\sigma_{real}$$

WITHIN D-DIMENSIONAL UNITARITY

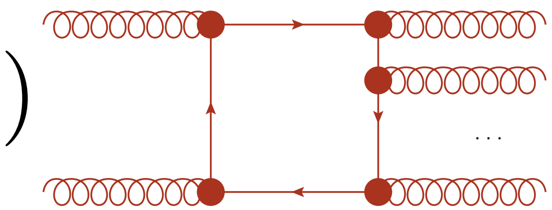
Giele, Kunszt, Melnikov [0801.2237]
Ellis, Giele, Kunszt [0708.2398]
Ellis, Giele, Kunszt, Melnikov [0806.3467]
Zanderighi [...], Winter[...], Schulze[...]
AL[0812.2998]

Introduction: Color decomposition

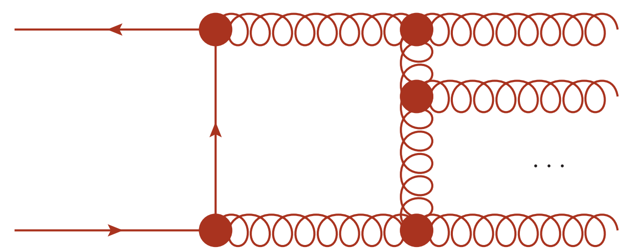
$$A^{NLO}_{\text{gluons}} = \sum_{\sigma} CF_1(\sigma)$$



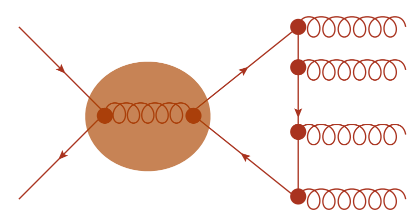
$$+ \frac{n_f}{N_c} \sum_{\sigma'} CF_2(\sigma')$$



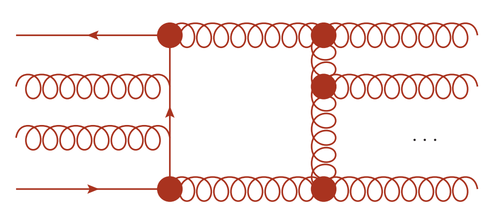
$$A^{NLO}_{q\bar{q}+Ng} = \sum_{\sigma} CF_1(\sigma)$$

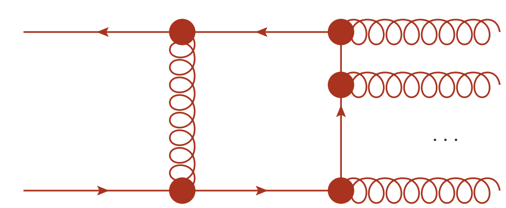


$$+ \frac{n_f}{N_c} \sum_{\sigma'} CF_2(\sigma')$$



$$+ \sum_{\sigma''} CF_3(\sigma'')$$

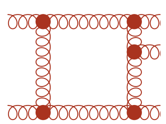




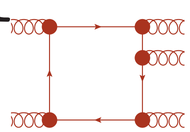
✓ No general formula for more than one fermion pair

Introduction: D-dimensional Unitarity

$$A^{D_s} = A_0 + D_s A_1 \qquad A^{FDH} = 2A^{(6)} - A^{(8)}$$



$$A^{D_s} = 2^{D_s/2-1} A_0 \qquad A^{FDH} = 8A^{(6)} = 16A^{(8)}$$



"Integrand level reduction ... partial fractioning of the amplitude over the standard base of master integrals ... OPP system"

Introduction: the OPP system in DDU

Finding $A^{(D_s)}$

$$\bar{e}_{I_5,s} = \mathcal{P}|^{I_5}_{l_s} = e_{I,0}$$

$$\bar{d}_{I_4,s} = \mathcal{P}|^{I_4}_{l_s} - \sum_{J_5/I_4} \frac{\bar{e}_{J_5/I_4}}{D_{k_1}(l_s)} = \sum_r d_{I_4,r} f_r(l_s)$$

$$\bar{c}_{I_3,s} = \mathcal{P}|^{I_3}_{l_s} - \sum_{J_5/I_3} \frac{\bar{e}_{J_5/I_3}}{D_{k_1}(l_s) D_{k_2}(l_s)} - \sum_{J_4/I_3} \frac{\bar{d}_{J_4/I_3}(l_s)}{D_{k1}(l_s)} = \sum_r c_{I_3,r} g_r(l_s)$$

$$\bar{b}_{I_2,s} = \mathcal{P}|^{I_2}_{l_s} - \sum_{J_5/I_2} \frac{\bar{e}_{J_5/I_2}}{D_{k_1}(l_s) D_{k_2}(l_s) D_{k_3}(l_s)} - \sum_{J_4/I_2} \frac{\bar{d}_{J_4/I_2}(l_s)}{D_{k_1}(l_s) D_{k_2}(l_s)} - \sum_{J_3/I_2} \frac{\bar{c}_{J_3/I_2}(l_s)}{D_{k_1}(l_s)} = \sum_r b_{I_2,r} h_r(l_s)$$

b) Solve for those

a) Fix loop momentum to evaluate numerically those

Introduction: the OPP system in DDU

Finding $A^{(D_s)}$

$$\bar{e}_{I_5,s} = \mathcal{P}|^{I_5}_{l_s} = e_{I,0} \qquad l_0$$

$$\bar{d}_{I_4,s} = \mathcal{P}|^{I_4}_{l_s} - \sum_{J_5/I_4} \frac{\bar{e}_{J_5/I_4}}{D_{k_1}(l_s)} = \sum_r d_{I_4,r} f_r(l_s) \qquad l_0, l_1 \; l_2, l_3, l_4$$

$$\bar{c}_{I_3,s} = \mathcal{P}|^{I_3}_{l_s} - \sum_{J_5/I_3} \frac{\bar{e}_{J_5/I_3}}{D_{k_1}(l_s)D_{k_2}(l_s)} - \sum_{J_4/I_3} \frac{\bar{d}_{J_4/I_3}(l_s)}{D_{k1}(l_s)} = \sum_r c_{I_3,r} g_r(l_s) \qquad l_{0\ldots6} \quad l_7, l_8, l_9$$

$$\bar{b}_{I_2,s} = \mathcal{P}|^{I_2}_{l_s} - \sum_{J_5/I_2} \frac{\bar{e}_{J_5/I_2}}{D_{k_1}(l_s)D_{k_2}(l_s)D_{k_3}(l_s)} - \sum_{J_4/I_2} \frac{\bar{d}_{J_4/I_2}(l_s)}{D_{k_1}(l_s)D_{k_2}(l_s)} - \sum_{J_3/I_2} \frac{\bar{c}_{J_3/I_2}(l_s)}{D_{k_1}(l_s)} = \sum_r b_{I_2,r} h_r(l_s)$$

$l_{0\ldots8} \quad l_9$

Cuts in D_s

Note: The system is linear $2A^{(6)} - A^{(8)} \rightarrow 2\mathcal{P}^{(6)}|^I_l - \mathcal{P}^{(8)}|^I_l$

Introduction: the OPP system in DDU

Finding $A^{(D_s)}$

$$\bar{e}_{I_5,s} = \mathcal{P}|_{l_s}^{I_5} = e_{I,0} \qquad l_0$$

$$\bar{d}_{I_4,s} = \mathcal{P}|_{l_s}^{I_4} - \sum_{J_5/I_4} \frac{\bar{e}_{J_5/I_4}}{D_{k_1}(l_s)} = \sum_r d_{I_4,r} f_r(l_s) \qquad l_0, l_1 \; l_2, l_3, l_4$$

$$\bar{c}_{I_3,s} = \mathcal{P}|_{l_s}^{I_3} - \sum_{J_5/I_3} \frac{\bar{e}_{J_5/I_3}}{D_{k_1}(l_s)D_{k_2}(l_s)} - \sum_{J_4/I_3} \frac{\bar{d}_{J_4/I_3}(l_s)}{D_{k1}(l_s)} = \sum_r c_{I_3,r} g_r(l_s) \qquad l_{0\ldots6} \quad l_7, l_8, l_9$$

$$\bar{b}_{I_2,s} = \mathcal{P}|_{l_s}^{I_2} - \sum_{J_5/I_2} \frac{\bar{e}_{J_5/I_2}}{D_{k_1}(l_s)D_{k_2}(l_s)D_{k_3}(l_s)} - \sum_{J_4/I_2} \frac{\bar{d}_{J_4/I_2}(l_s)}{D_{k_1}(l_s)D_{k_2}(l_s)} - \sum_{J_3/I_2} \frac{\bar{c}_{J_3/I_2}(l_s)}{D_{k_1}(l_s)} = \sum_r b_{I_2,r} h_r(l_s)$$

$$l_{0\ldots8} \quad l_9$$

Cuts in D_s ↘

Note: when $l_s \in 4D \quad \rightarrow \quad A^{(6)} = A^{(8)} = A^{(4)}$

Cuts in 4D ↙

NOT FERMION LOOP

Introduction: the OPP system in DDU

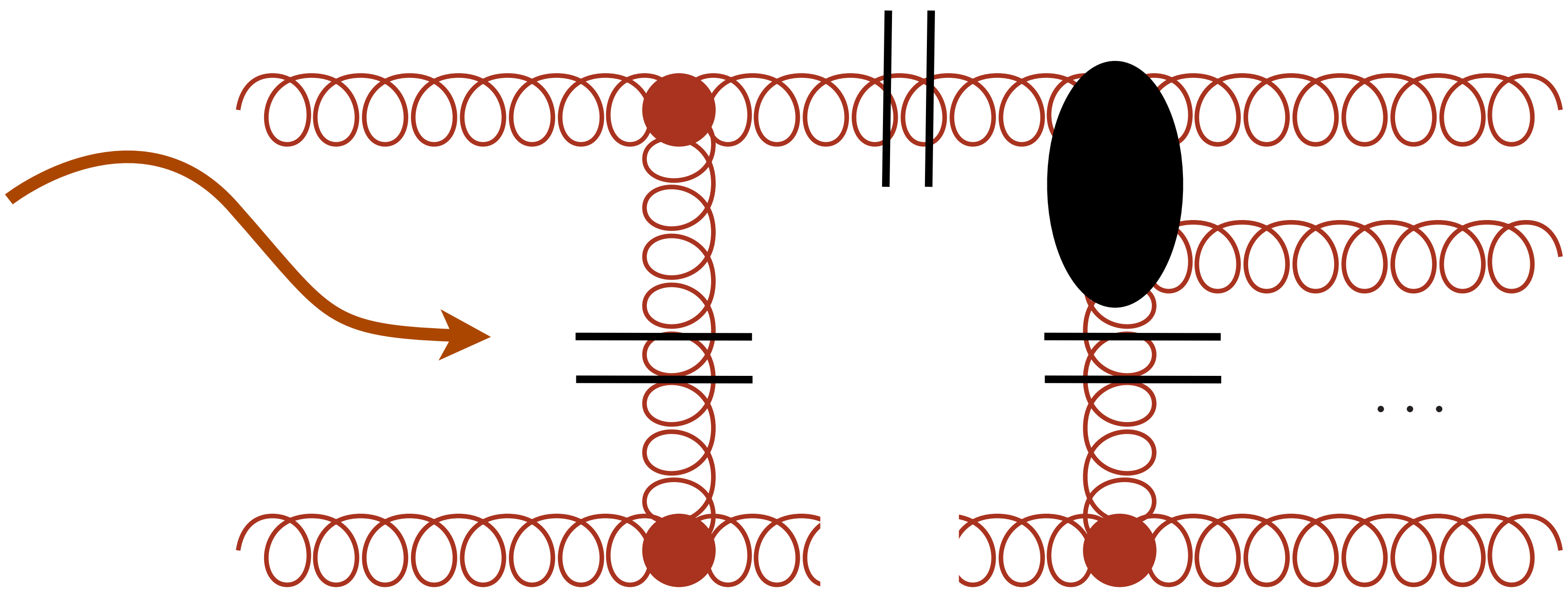
Finding $A^{(D_s)}$

$= \mathcal{P}|_{l_s}^{I_5} =$

$= \mathcal{P}|_{l_s}^{I_4} -$

$= \mathcal{P}|_{l_s}^{I_3} -$

$= \mathcal{P}|_{l_s}^{I_2} -$



$$\sum_h \epsilon_h^\mu \epsilon_h^\nu$$

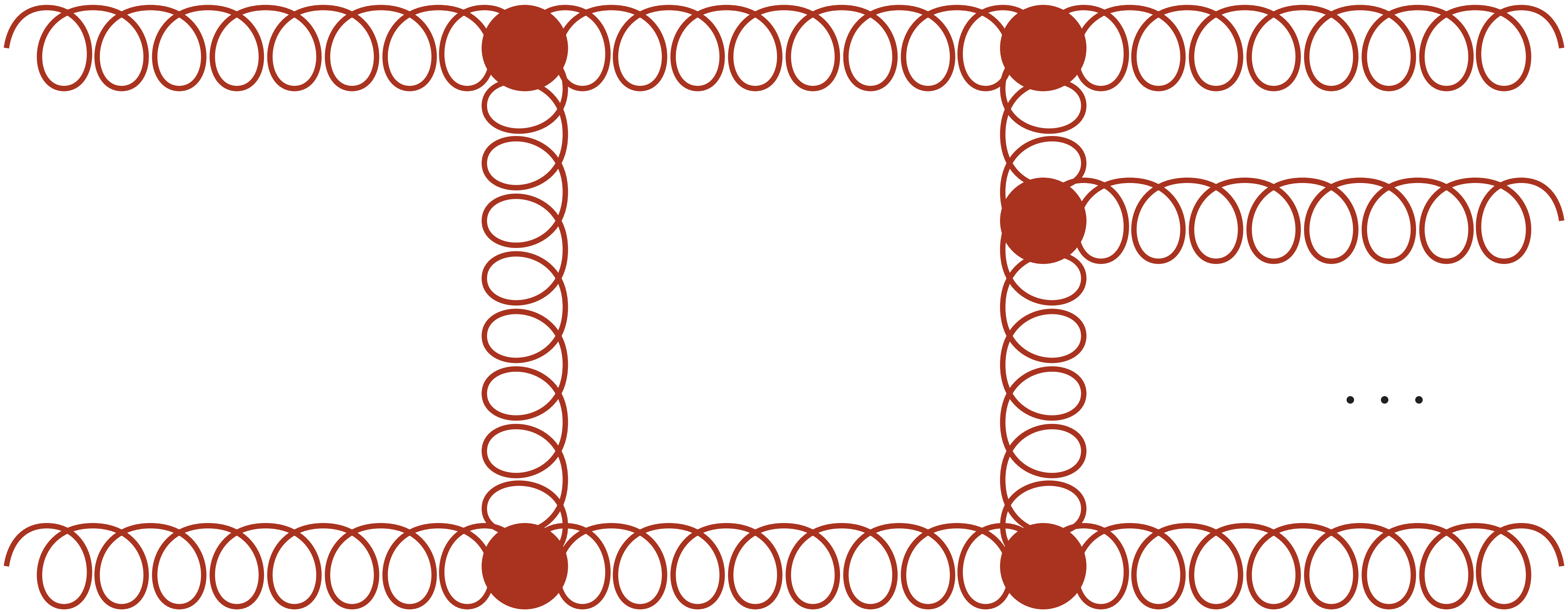
$l_s \in 5D$

$$\mathcal{P}|_{l_s}^{I_x} = 2\mathcal{P}^{(6)}|_{l_s}^{I_x} - \mathcal{P}^{(8)}|_{l_s}^{I_x} = \mathcal{P}^{(6)}|_{l_s}^{I_x} - \mathcal{P}^{(8^*)}|_{l_s}^{I_x}$$

Primitives

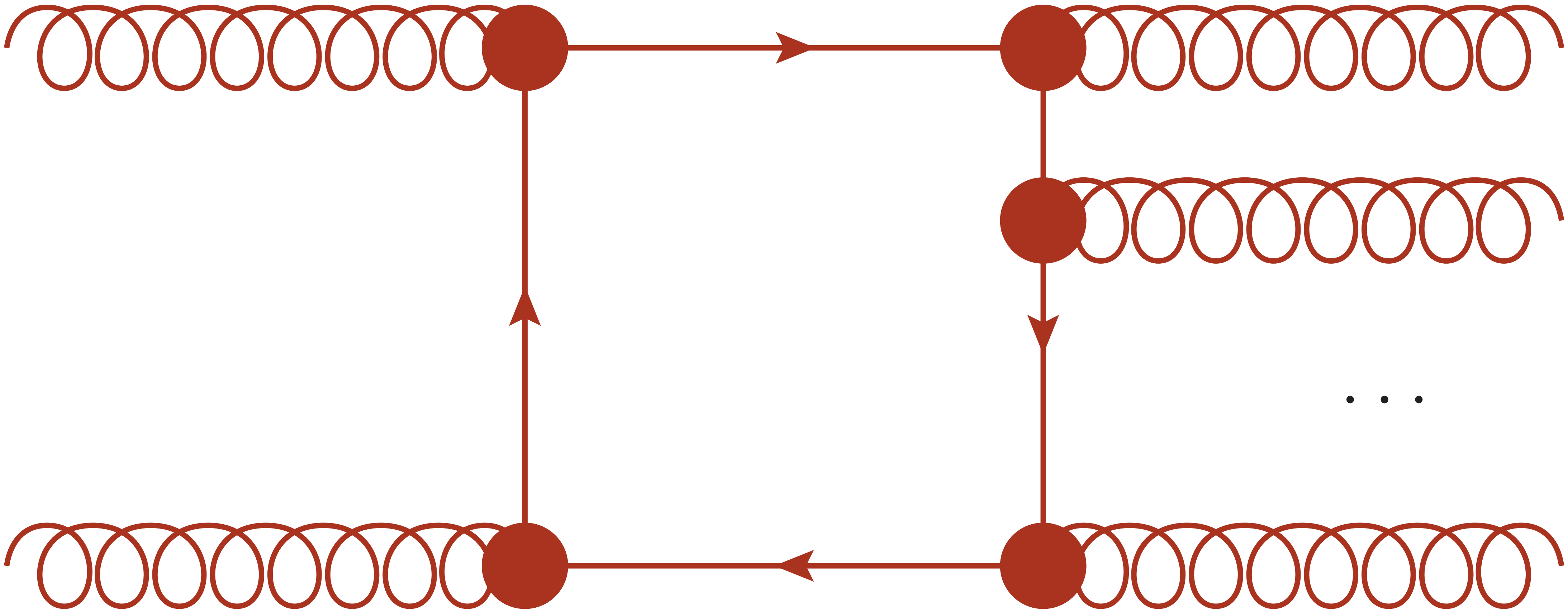
Gluons

Primitives: purely gluonic



- ✓ The most dangerous numerically
- ✓ The fastest for fixed N

Primitives: gluonic - fermion loop

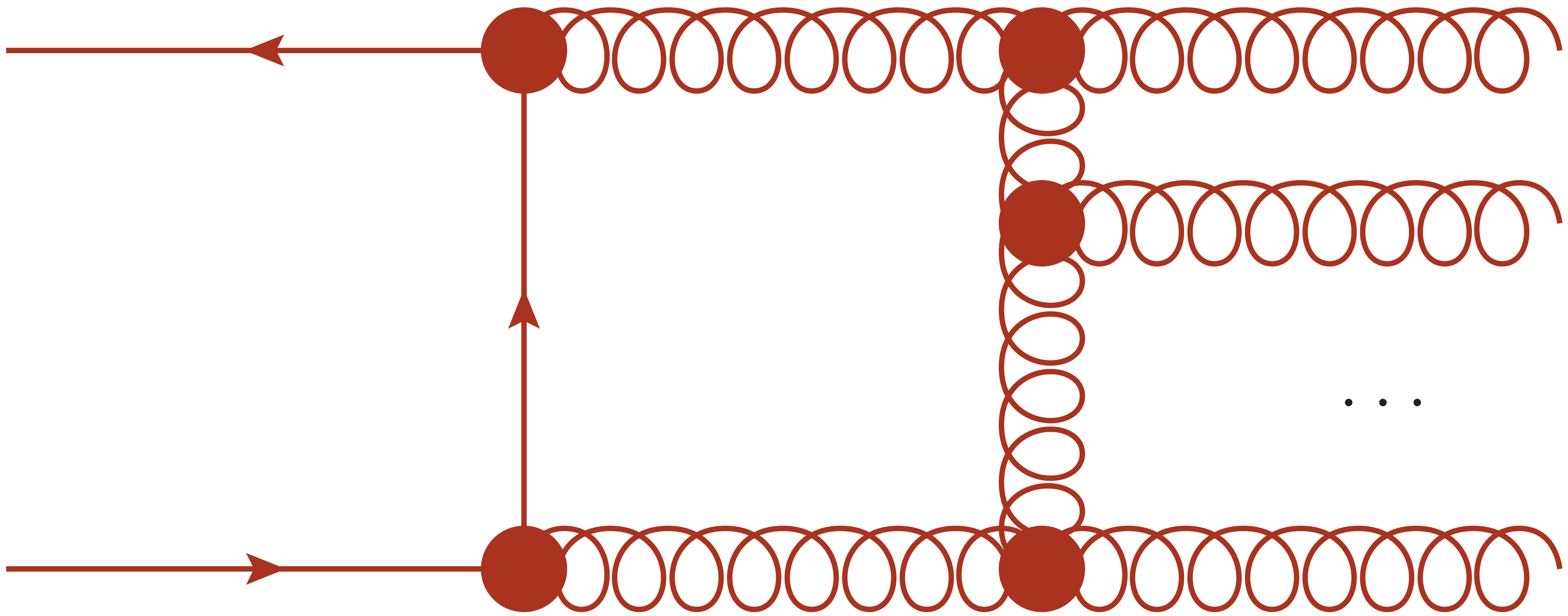


✓Much more stable

✓Propotional to n_f/N_c

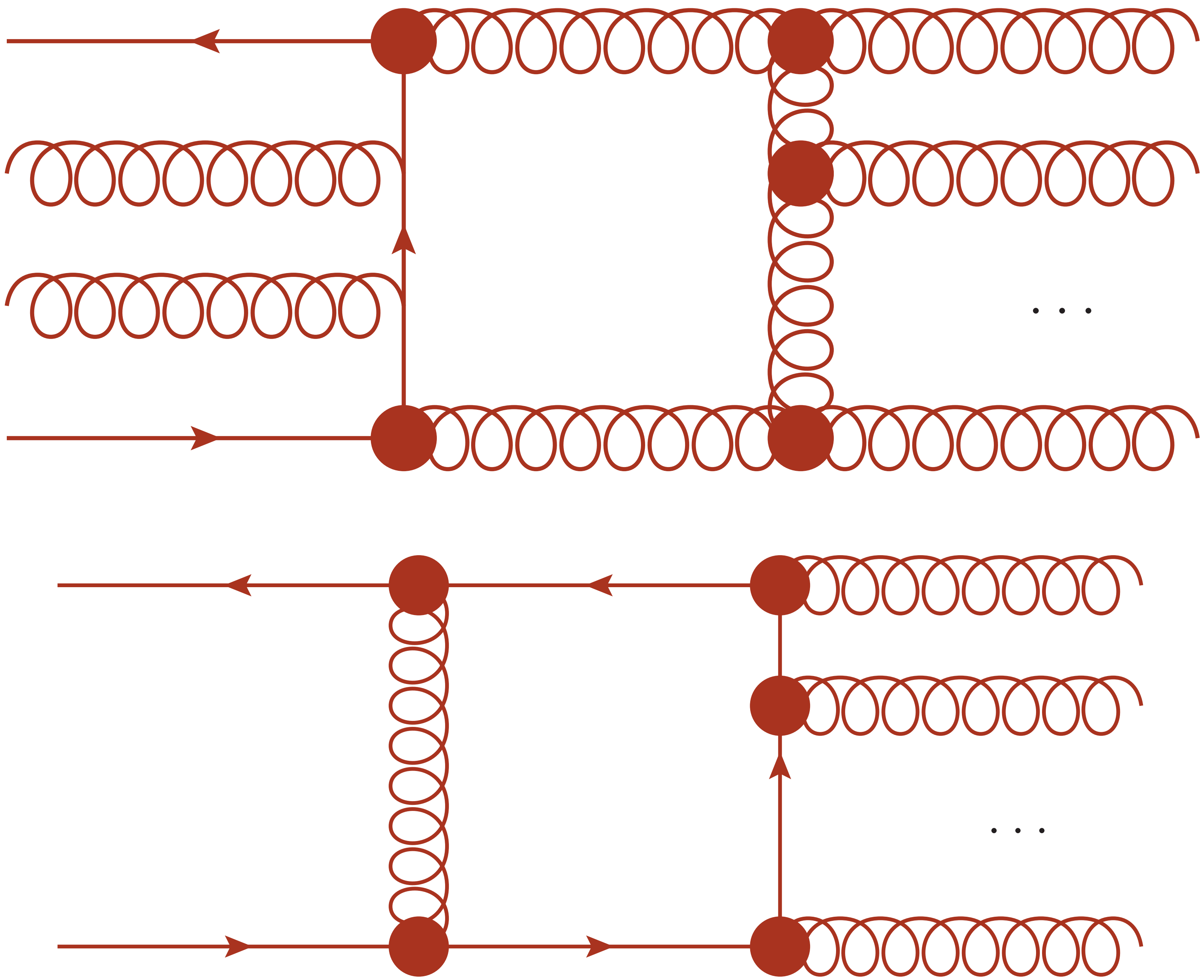
Fermions

Primitives: single fermion line



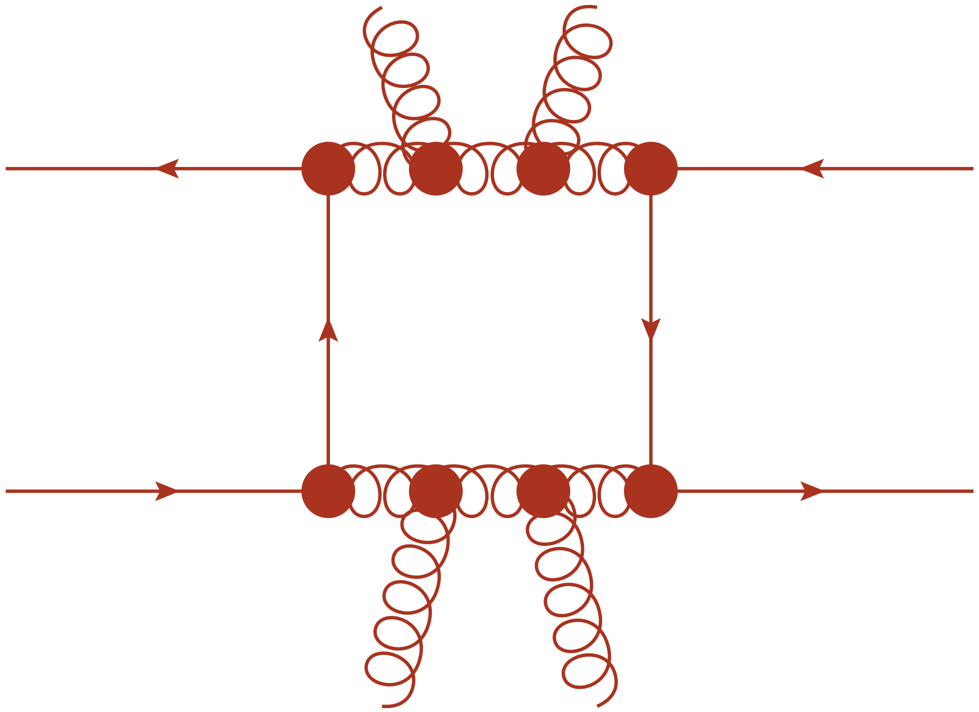
✓ Leading color

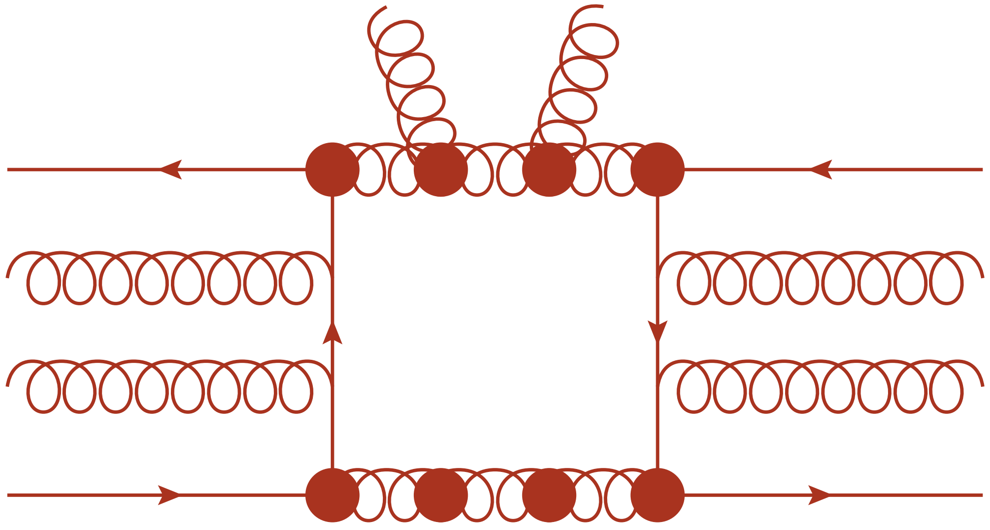
Primitives: single fermion line



✓ $1/N_c$ suppressed

Primitives: two fermion lines

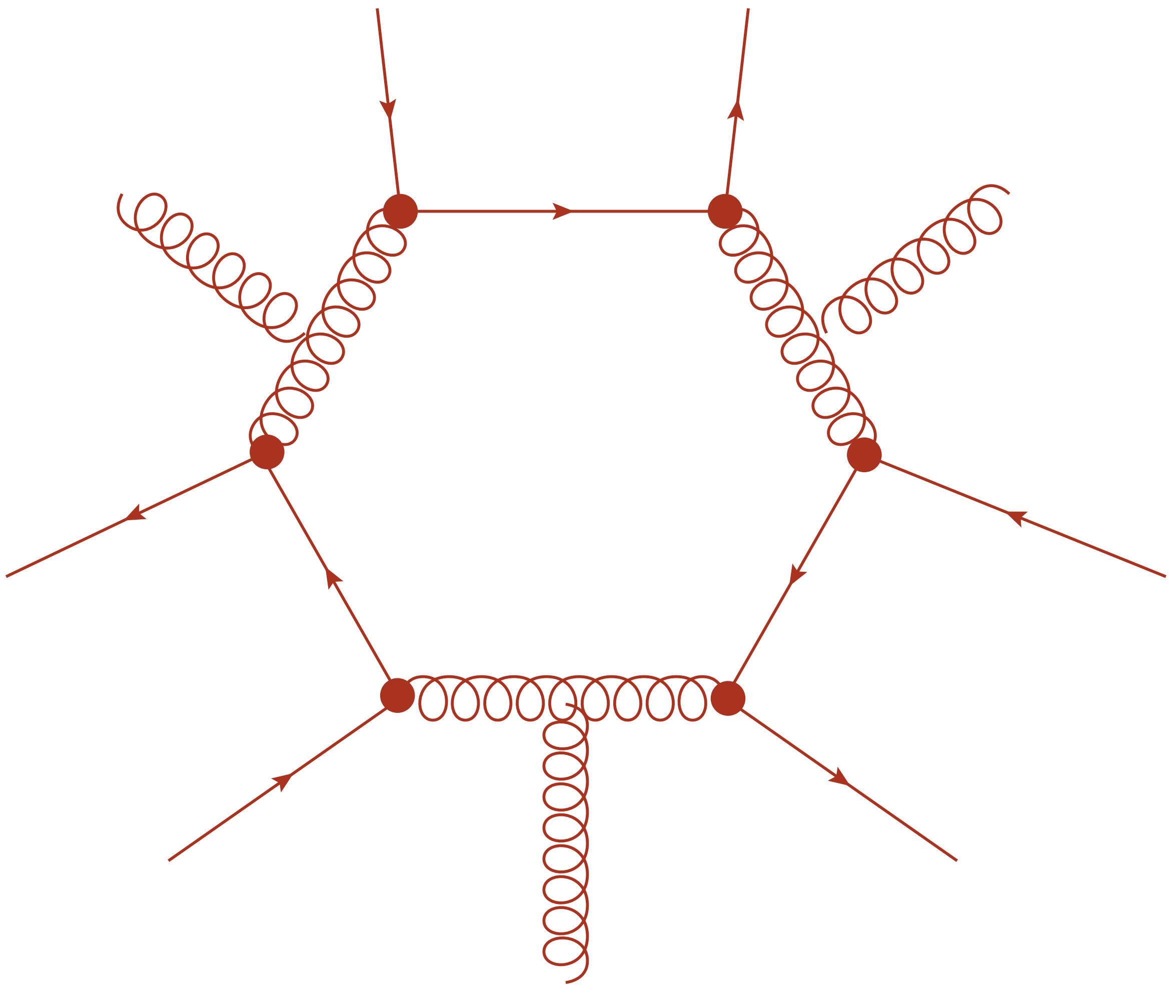


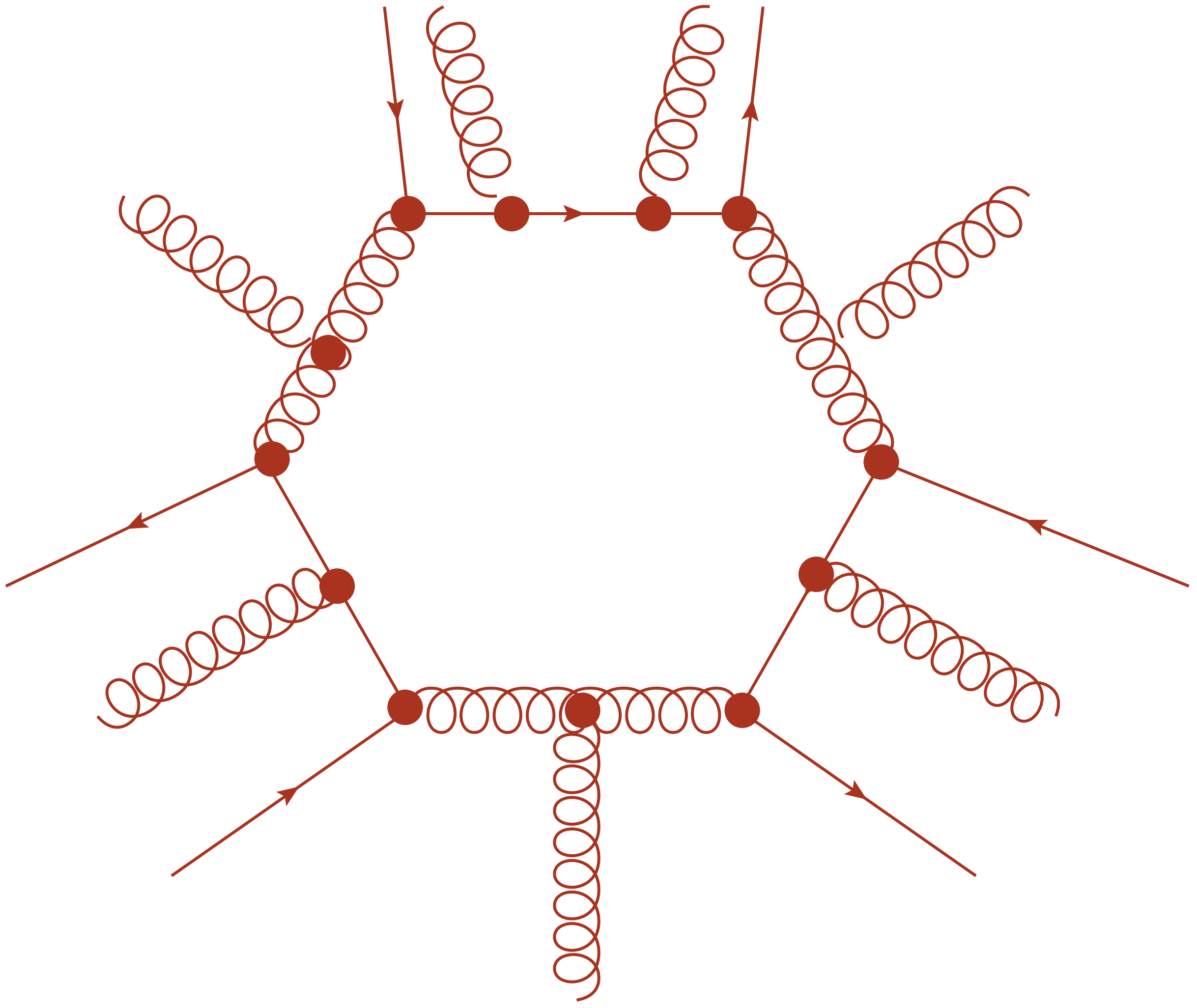


✓ Leading color
✓ Fermion line direction

✓ $1/N_c$ suppressed

Primitives: three fermion lines





✓ Leading Color
✓ Fermion line direction

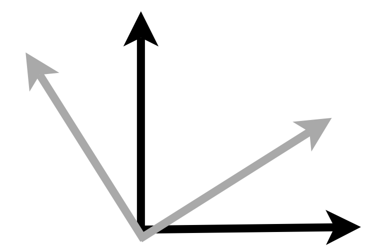
✓ $1/N_c$ suppressed

Primitives

"All necessary ingredients for a virtual amplitude with massless QCD partons are in place, tested, ready for production mode".

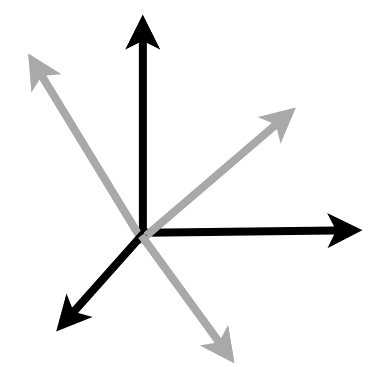
Numerical Stability

Numerical Stability: accidental instabilities



TRIPLE CUT

$$l^\mu = V^\mu + a_1 n_1^\mu + a_2 n_2^\mu + a_5 n_5^\mu$$



DOUBLE CUT

$$l^\mu = V^\mu + a_1 n_1^\mu + a_2 n_2^\mu + a_3 n_3^\mu + a_5 n_5^\mu$$

Diagnosed by redundant OPP equation
Solved by picking another set of n-vectors

$$\bar{e}_{I_5,s} = \mathcal{P}|_{l_s}^{I_5} = e_{I,0}$$

$$\bar{d}_{I_4,s} = \mathcal{P}|_{l_s}^{I_4} - \sum_{J_5/I_4} \frac{\bar{e}_{J_5/I_4}}{D_{k_1}(l_s)} = \sum_r d_{I_4,r} f_r(l_s)$$

$$\bar{c}_{I_3,s} = \mathcal{P}|_{l_s}^{I_3} - \sum_{J_5/I_3} \frac{\bar{e}_{J_5/I_3}}{D_{k_1}(l_s) D_{k_2}(l_s)} - \sum_{J_4/I_3} \frac{\bar{d}_{J_4/I_3}(l_s)}{D_{k1}(l_s)} = \sum_r c_{I_3,r} g_r(l_s)$$

$$\bar{b}_{I_2,s} = \mathcal{P}|_{l_s}^{I_2} - \sum_{J_5/I_2} \frac{\bar{e}_{J_5/I_2}}{D_{k_1}(l_s) D_{k_2}(l_s) D_{k_3}(l_s)} - \sum_{J_4/I_2} \frac{\bar{d}_{J_4/I_2}(l_s)}{D_{k_1}(l_s) D_{k_2}(l_s)} - \sum_{J_3/I_2} \frac{\bar{c}_{J_3/I_2}(l_s)}{D_{k_1}(l_s)} = \sum_r b_{I_2,r} h_r(l_s)$$

$$\frac{\bar{d}(l_s)}{(l_s + q)^2}$$

There is a special check for this implemented

Numerical Stability: pentagon contamination

$$\bar{e}_{I_5,s} = \mathcal{P}|_{l_s}^{I_5} = e_{I,0}$$

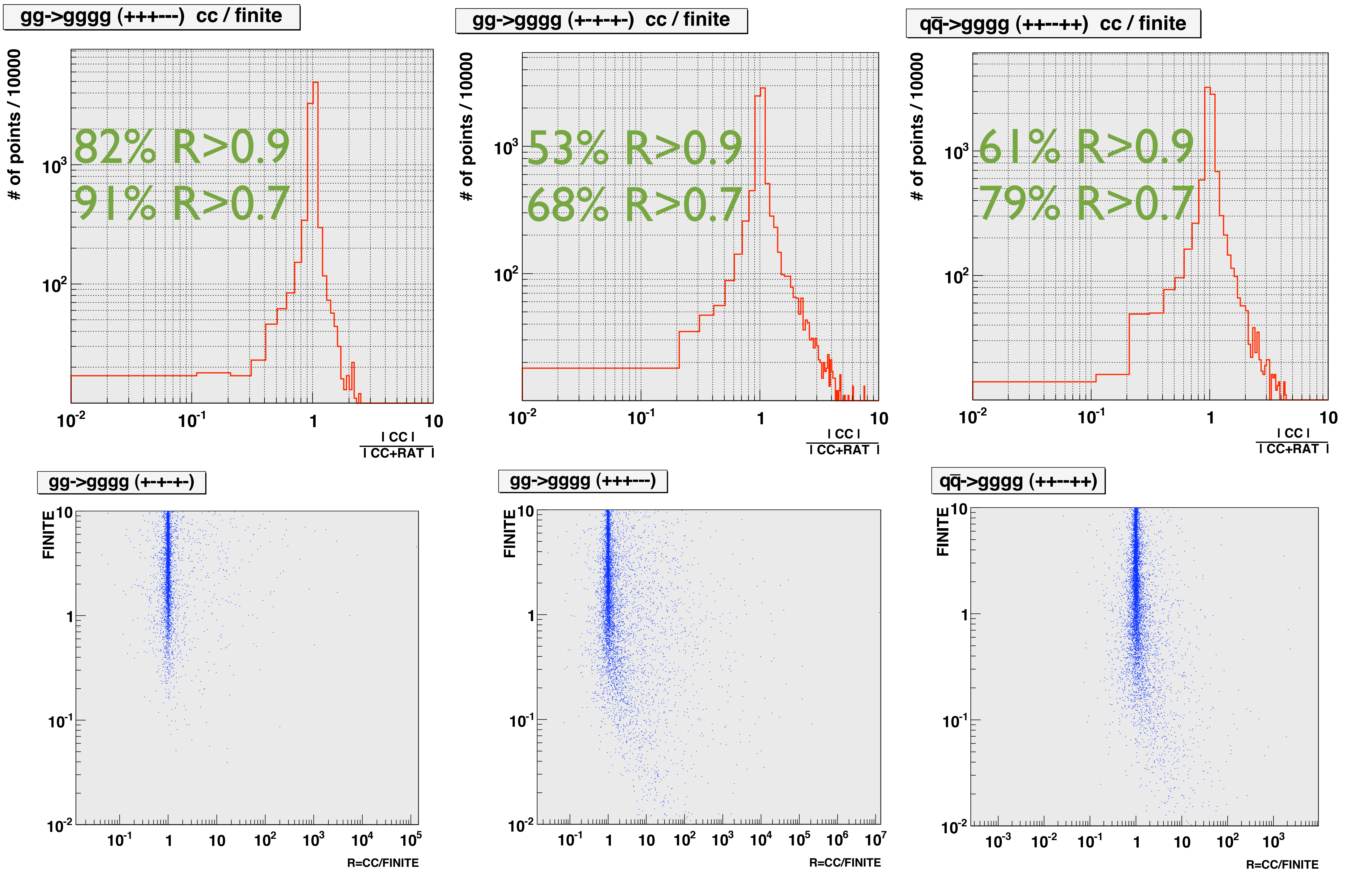
$$\bar{d}_{I_4,s} = \mathcal{P}|_{l_s}^{I_4} - \sum_{J_5/I_4} \frac{\bar{e}_{J_5/I_4}}{D_{k_1}(l_s)} = \sum_r d_{I_4,r} f_r(l_s)$$

$$\bar{c}_{I_3,s} = \mathcal{P}|_{l_s}^{I_3} - \sum_{J_5/I_3} \frac{\bar{e}_{J_5/I_3}}{D_{k_1}(l_s)D_{k_2}(l_s)} - \sum_{J_4/I_3} \frac{\bar{d}_{J_4/I_3}(l_s)}{D_{k1}(l_s)} = \sum_r c_{I_3,r} g_r(l_s)$$

$$\bar{b}_{I_2,s} = \mathcal{P}|_{l_s}^{I_2} - \sum_{J_5/I_2} \frac{\bar{e}_{J_5/I_2}}{D_{k_1}(l_s)D_{k_2}(l_s)D_{k_3}(l_s)} - \sum_{J_4/I_2} \frac{\bar{d}_{J_4/I_2}(l_s)}{D_{k_1}(l_s)D_{k_2}(l_s)} - \sum_{J_3/I_2} \frac{\bar{c}_{J_3/I_2}(l_s)}{D_{k_1}(l_s)} = \sum_r b_{I_2,r} h_r(l_s)$$

- ✓ Large cancellations in cases of small gram dets.
- ✓ Particularly for the pure gluonic case.
- ✓ Even true for the Cut Constructible part even though it can also be evaluated in 4d, i.e. without pentagons.
- ✓ Would be desirable to remove the pentagon contamination from the CC part.
- ✓ And to control its effect better in Rational.

Numerical Stability: pentagon contamination



Numerical Stability: the cure for pentagon contamination

$$\bar{d}_{I_4,s} = \bar{d}^{(1)}_{I_4,s} + \bar{d}^{(2)}_{I_4,s}$$

$$\bar{d}^{(1)}_{I_4,s} = \mathcal{P}|^{I_4}_{l_s} = \sum_r d^{(1)}_{I_4,r} f_r(l_s)$$

$$\bar{d}^{(2)}_{I_4,s} = -\sum_{J_5/I_4} \frac{\bar{e}_{J_5/I_4}}{D_{k_1}(l_s)} = \sum_r d^{(2)}_{I_4,r} f_r(l_s)$$

$$\bar{c}_{I_3,s} = \bar{c}^{(1)}_{I_3,s} + \bar{c}^{(2)}_{I_3,s}$$

$$\bar{c}^{(1)}_{I_3,s} = \mathcal{P}|^{I_3}_{l_s} - \sum_{J_4/I_3} \frac{\bar{d}^{(1)}_{J_4/I_3}(l_s)}{D_{k1}(l_s)} = \sum_r c^{(1)}_{I_3,r} g_r(l_s)$$

$$\bar{c}^{(2)}_{I_3,s} = -\sum_{J_5/I_3} \frac{\bar{e}_{J_5/I_3}}{D_{k_1}(l_s) D_{k_2}(l_s)} - \sum_{J_4/I_3} \frac{\bar{d}^{(2)}_{J_4/I_3}(l_s)}{D_{k1}(l_s)} = \sum_r c^{(2)}_{I_3,r} g_r(l_s)$$

$$\bar{b}_{I_2,s} = \bar{b}^{(1)}_{I_2,s} + \bar{b}^{(2)}_{I_2,s}$$

$$\bar{b}^{(1)}_{I_2,s} = \mathcal{P}|^{I_2}_{l_s} - \sum_{J_4/I_2} \frac{\bar{d}^{(1)}_{J_4/I_2}(l_s)}{D_{k_1}(l_s) D_{k_2}(l_s)} - \sum_{J_3/I_2} \frac{\bar{c}^{(1)}_{J_3/I_2}(l_s)}{D_{k_1}(l_s)} = \sum_r b^{(1)}_{I_2,r} h_r(l_s)$$

$$\bar{b}^{(2)}_{I_2,s} = -\sum_{J_5/I_2} \frac{\bar{e}_{J_5/I_2}}{D_{k_1}(l_s) D_{k_2}(l_s) D_{k_3}(l_s)} - \sum_{J_4/I_2} \frac{\bar{d}^{(2)}_{J_4/I_2}(l_s)}{D_{k_1}(l_s) D_{k_2}(l_s)} - \sum_{J_3/I_2} \frac{\bar{c}^{(2)}_{J_3/I_2}(l_s)}{D_{k_1}(l_s)} = \sum_r b^{(2)}_{I_2,r} h_r(l_s)$$

Numerical Stability: the cure for pentagon contamination

4D

$$\bar{d}_{I_4,s} = \bar{d}^{(1)}_{I_4,s} + \bar{d}^{(2)}_{I_4,s}$$

$$\bar{d}^{(1)}_{I_4,s} = \mathcal{P}|^{I_4}_{l_s} = \sum_r d^{(1)}_{I_4,r} f_r(l_s)$$

$$\bar{d}^{(2)}_{I_4,s} = -\sum_{J_5/I_4} \frac{\bar{e}_{J_5/I_4}}{D_{k_1}(l_s)} = \sum_r d^{(2)}_{I_4,r} f_r(l_s)$$ X

($d_0^{(2)}$ cancels against pentagon reduced to boxes)

$$\bar{c}_{I_3,s} = \bar{c}^{(1)}_{I_3,s} + \bar{c}^{(2)}_{I_3,s}$$

$$\bar{c}^{(1)}_{I_3,s} = \mathcal{P}|^{I_3}_{l_s} - \sum_{J_4/I_3} \frac{\bar{d}^{(1)}_{J_4/I_3}(l_s)}{D_{k1}(l_s)} = \sum_r c^{(1)}_{I_3,r} g_r(l_s)$$

$$\bar{c}^{(2)}_{I_3,s} = -\sum_{J_5/I_3} \frac{\bar{e}_{J_5/I_3}}{D_{k_1}(l_s) D_{k_2}(l_s)} - \sum_{J_4/I_3} \frac{\bar{d}^{(2)}_{J_4/I_3}(l_s)}{D_{k1}(l_s)} = \sum_r c^{(2)}_{I_3,r} g_r(l_s)$$ X

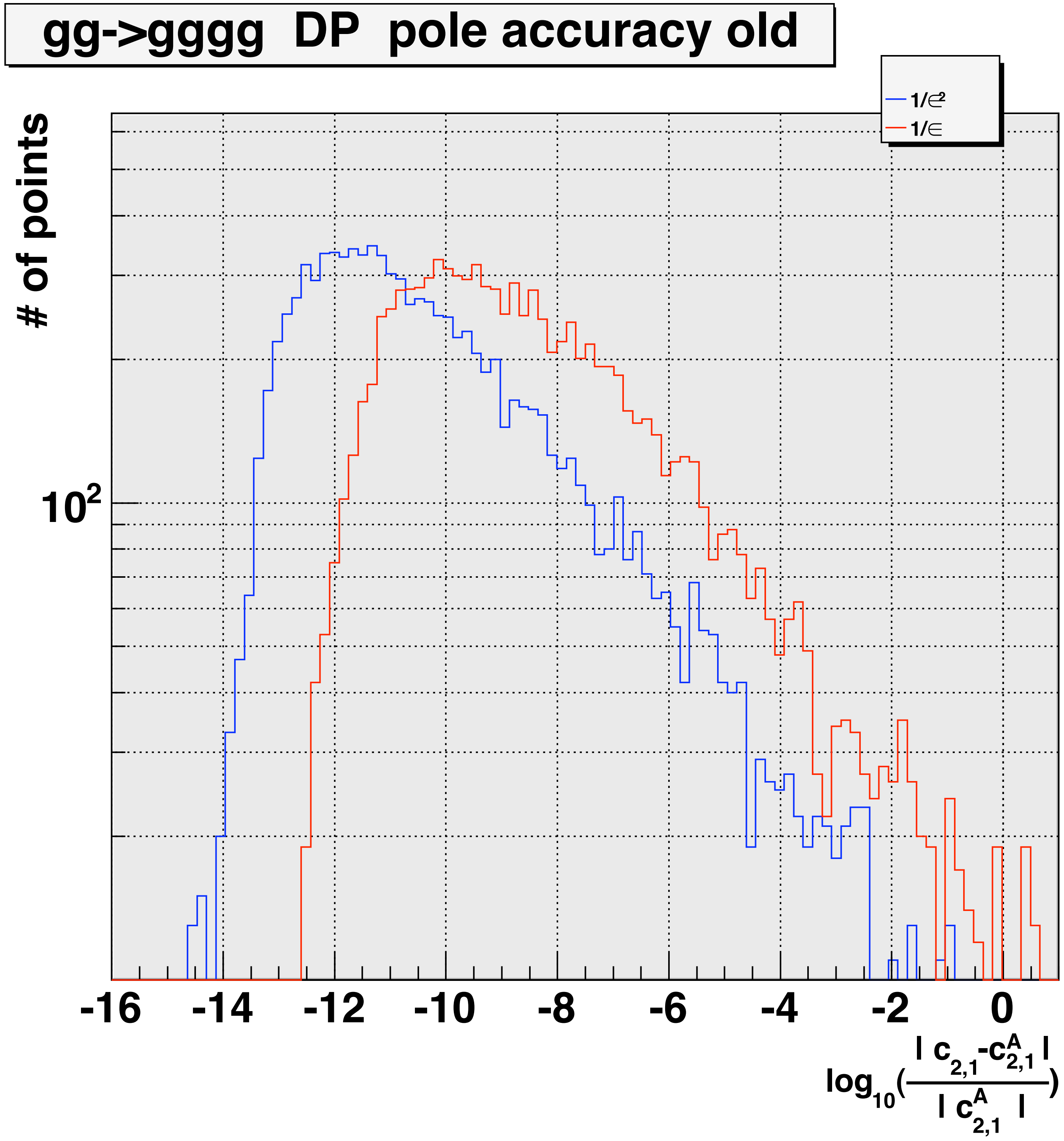
$$\bar{b}_{I_2,s} = \bar{b}^{(1)}_{I_2,s} + \bar{b}^{(2)}_{I_2,s}$$

$$\bar{b}^{(1)}_{I_2,s} = \mathcal{P}|^{I_2}_{l_s} - \sum_{J_4/I_2} \frac{\bar{d}^{(1)}_{J_4/I_2}(l_s)}{D_{k_1}(l_s) D_{k_2}(l_s)} - \sum_{J_3/I_2} \frac{\bar{c}^{(1)}_{J_3/I_2}(l_s)}{D_{k_1}(l_s)} = \sum_r b^{(1)}_{I_2,r} h_r(l_s)$$

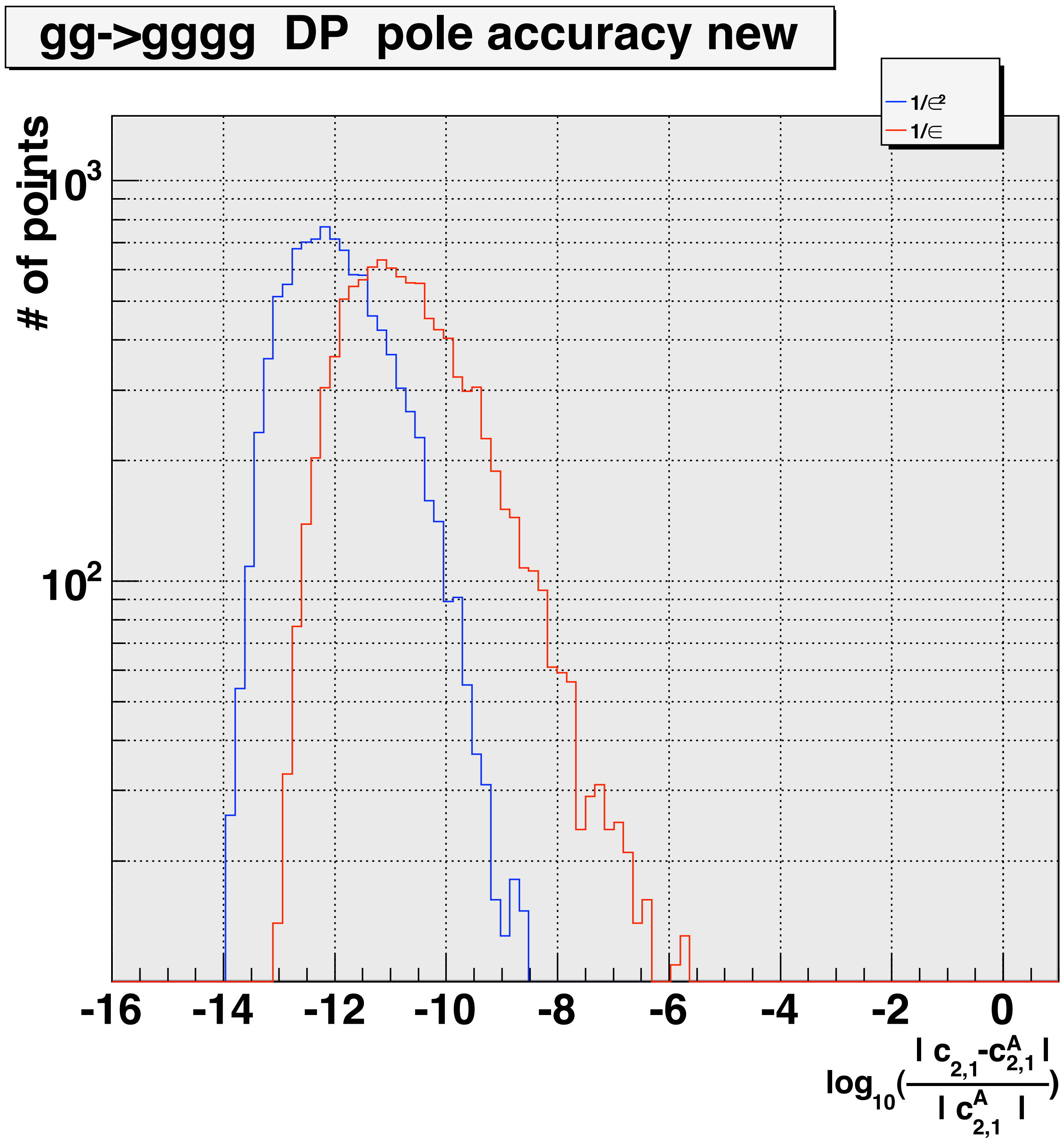
$$\bar{b}^{(2)}_{I_2,s} = -\sum_{J_5/I_2} \frac{\bar{e}_{J_5/I_2}}{D_{k_1}(l_s) D_{k_2}(l_s) D_{k_3}(l_s)} - \sum_{J_4/I_2} \frac{\bar{d}^{(2)}_{J_4/I_2}(l_s)}{D_{k_1}(l_s) D_{k_2}(l_s)} - \sum_{J_3/I_2} \frac{\bar{c}^{(2)}_{J_3/I_2}(l_s)}{D_{k_1}(l_s)} = \sum_r b^{(2)}_{I_2,r} h_r(l_s)$$ X

✓ Pentagon decoupling from 4-D!

Numerical Stability: improved CC plot



Numerical Stability: improved CC plot



Numerical Stability: the cure for pentagon contamination

5D

$$\bar{d}_{I_4,s} = \bar{d}^{(1)}_{I_4,s} + \bar{d}^{(2)}_{I_4,s}$$

$$\bar{d}^{(1)}_{I_4,s} = \mathcal{P}|^{I_4}_{l_s} = \sum_r d^{(1)}_{I_4,r} f_r(l_s)$$

$$\bar{d}^{(2)}_{I_4,s} = -\sum_{J_5/I_4} \frac{\bar{e}_{J_5/I_4}}{D_{k_1}(l_s)} = \sum_r d^{(2)}_{I_4,r} f_r(l_s)$$

! can be handled with care

$$\bar{c}_{I_3,s} = \bar{c}^{(1)}_{I_3,s} + \bar{c}^{(2)}_{I_3,s}$$

$$\bar{c}^{(1)}_{I_3,s} = \mathcal{P}|^{I_3}_{l_s} - \sum_{J_4/I_3} \frac{\bar{d}^{(1)}_{J_4/I_3}(l_s)}{D_{k1}(l_s)} = \sum_r c^{(1)}_{I_3,r} g_r(l_s)$$

$$\bar{c}^{(2)}_{I_3,s} = -\sum_{J_5/I_3} \frac{\bar{e}_{J_5/I_3}}{D_{k_1}(l_s) D_{k_2}(l_s)} - \sum_{J_4/I_3} \frac{\bar{d}^{(2)}_{J_4/I_3}(l_s)}{D_{k1}(l_s)} = \sum_r c^{(2)}_{I_3,r} g_r(l_s)$$

! can be handled with care

$$\bar{b}_{I_2,s} = \bar{b}^{(1)}_{I_2,s} + \bar{b}^{(2)}_{I_2,s}$$

$$\bar{b}^{(1)}_{I_2,s} = \mathcal{P}|^{I_2}_{l_s} - \sum_{J_4/I_2} \frac{\bar{d}^{(1)}_{J_4/I_2}(l_s)}{D_{k_1}(l_s) D_{k_2}(l_s)} - \sum_{J_3/I_2} \frac{\bar{c}^{(1)}_{J_3/I_2}(l_s)}{D_{k_1}(l_s)} = \sum_r b^{(1)}_{I_2,r} h_r(l_s)$$

$$\bar{b}^{(2)}_{I_2,s} = -\sum_{J_5/I_2} \frac{\bar{e}_{J_5/I_2}}{D_{k_1}(l_s) D_{k_2}(l_s) D_{k_3}(l_s)} - \sum_{J_4/I_2} \frac{\bar{d}^{(2)}_{J_4/I_2}(l_s)}{D_{k_1}(l_s) D_{k_2}(l_s)} - \sum_{J_3/I_2} \frac{\bar{c}^{(2)}_{J_3/I_2}(l_s)}{D_{k_1}(l_s)} = \sum_r b^{(2)}_{I_2,r} h_r(l_s)$$

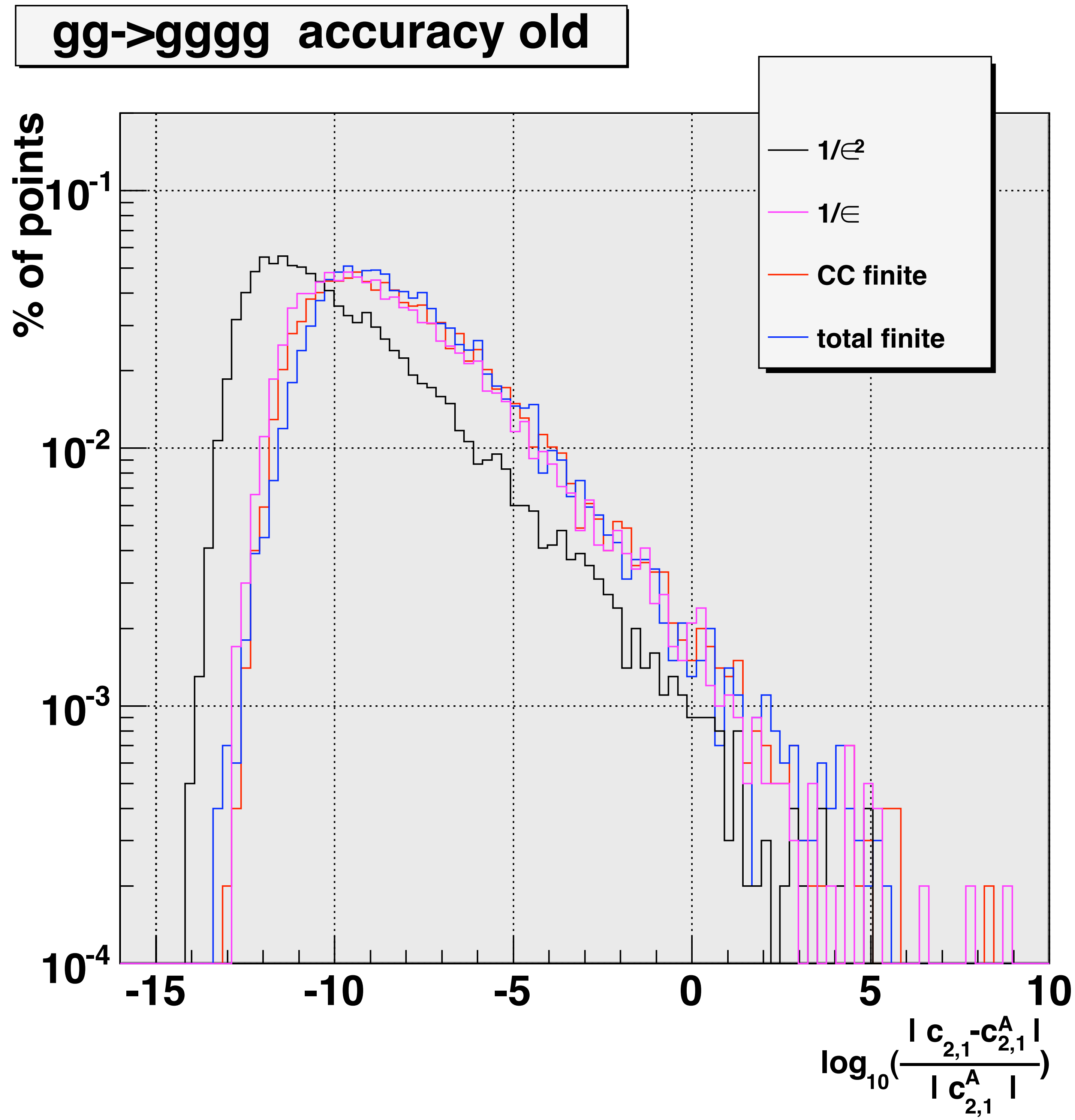
!

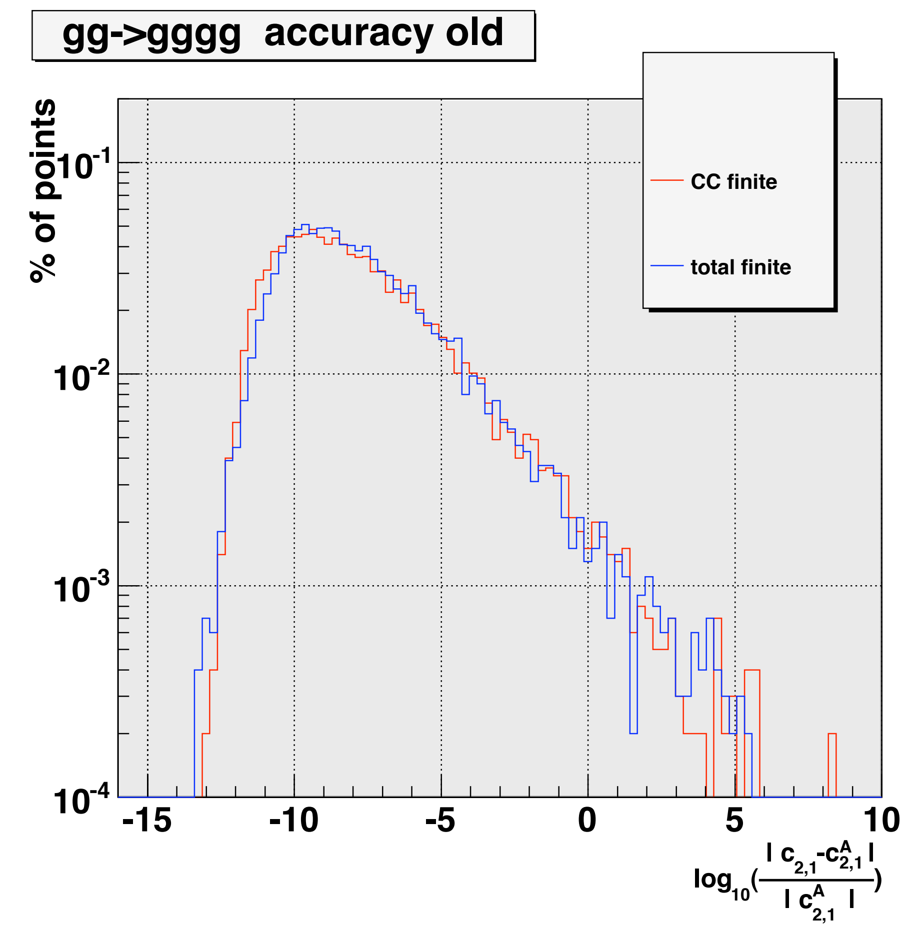
✓ Both subsystems contribute to the rational part

Numerical Stability: the cure for pentagon contamination

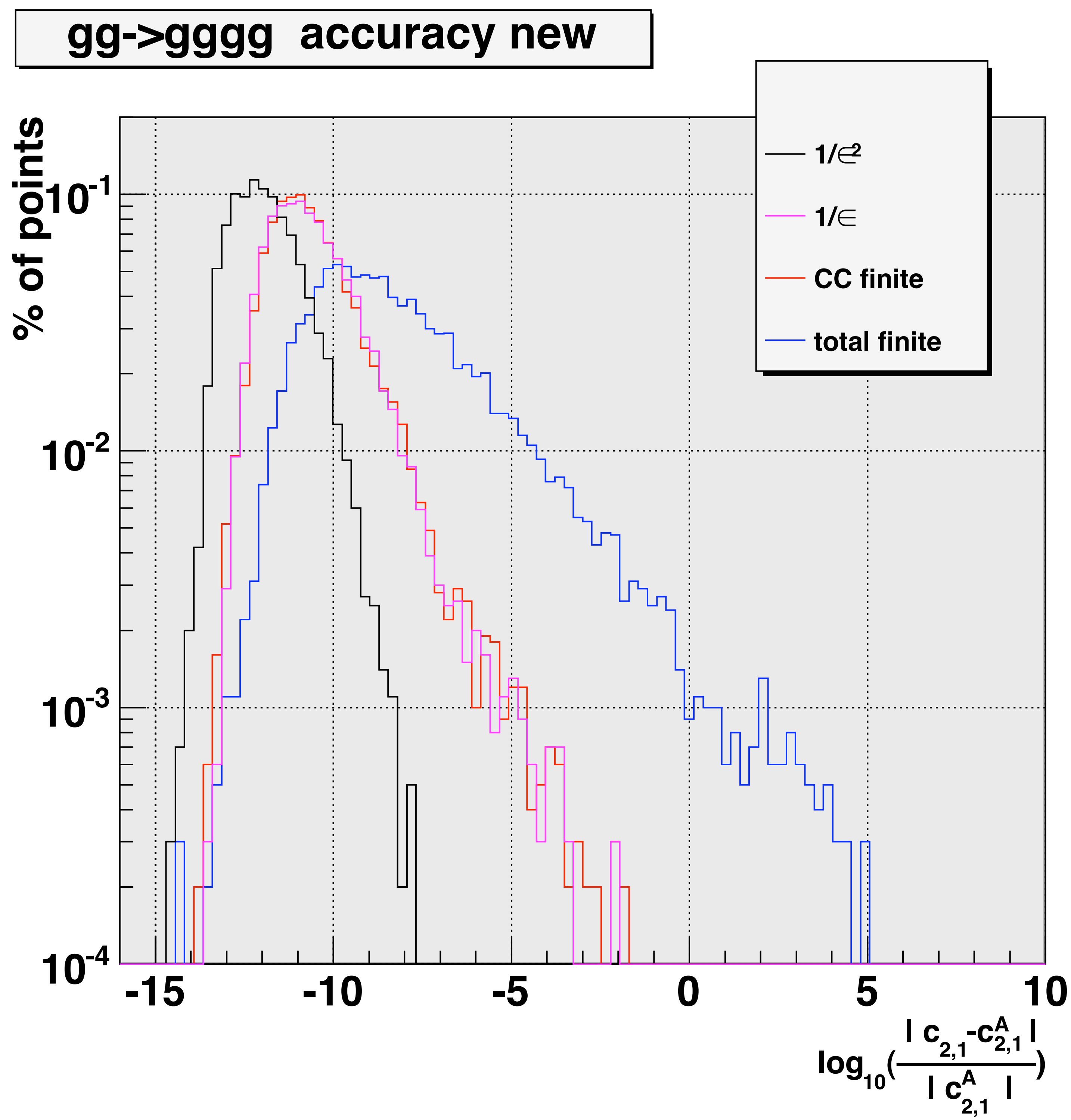
“The pentagon coefficient should always be factored out of any subtractions”

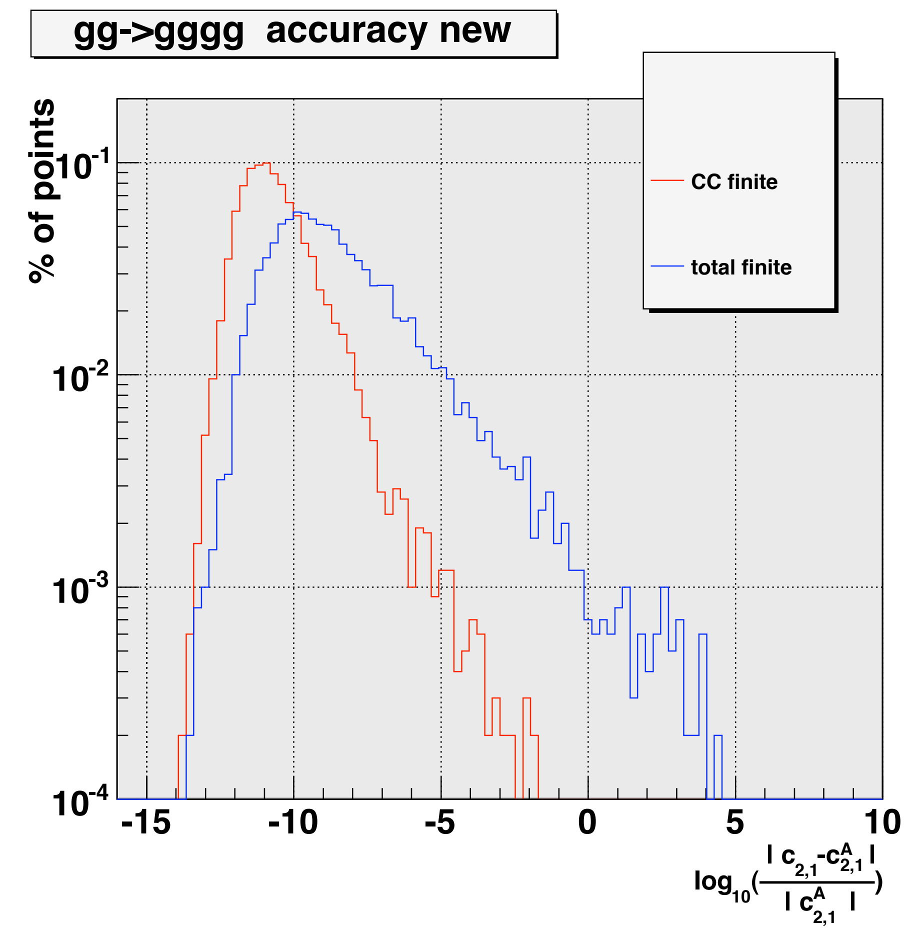
Numerical Stability: improved Finite plot





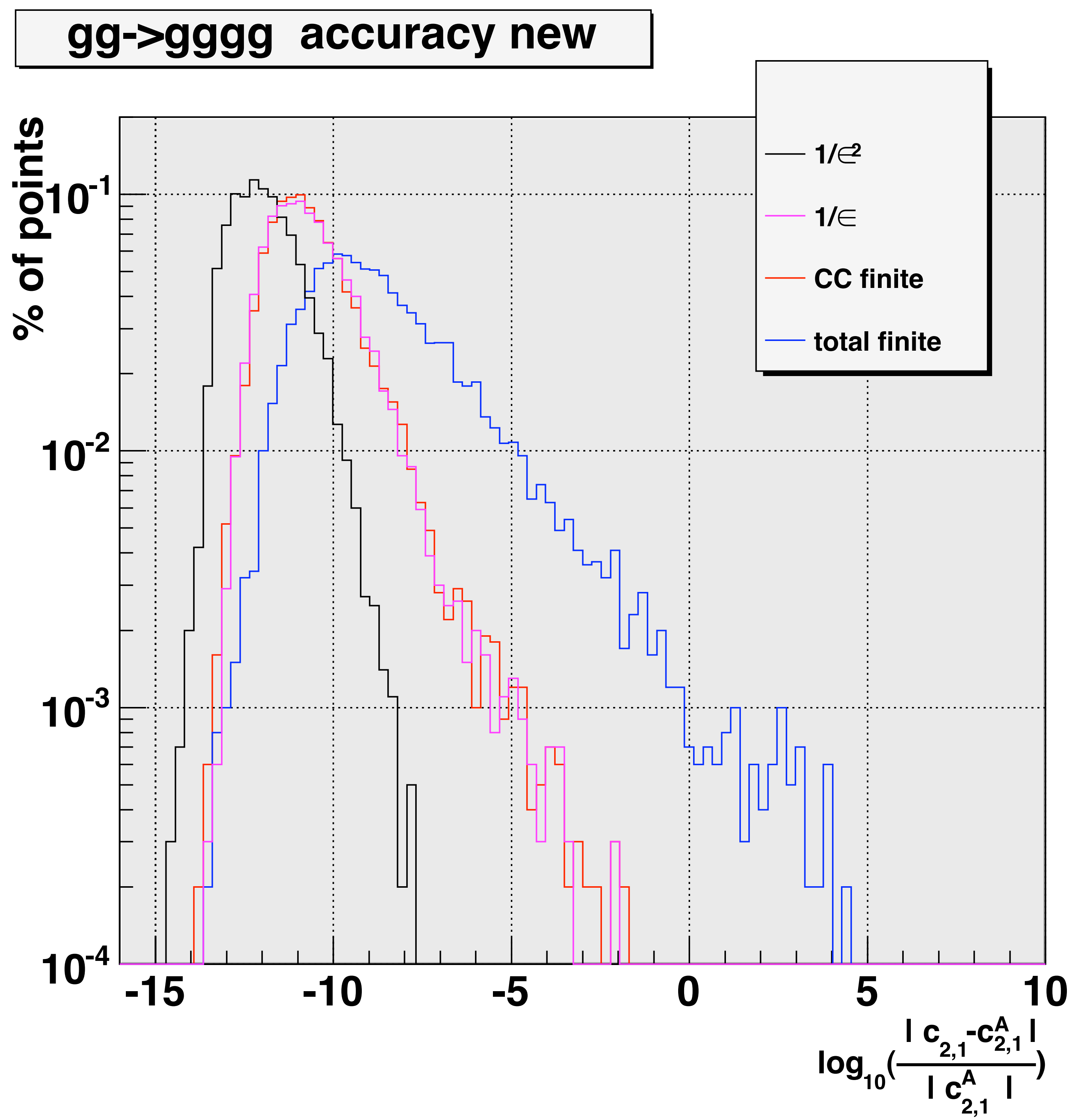
Numerical Stability: improved Finite plot





Splitting in two subsystems.

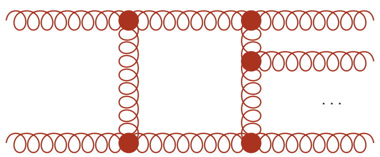
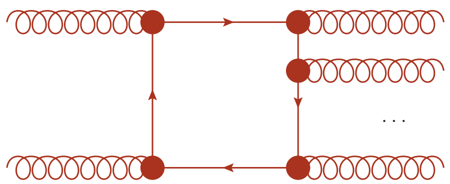
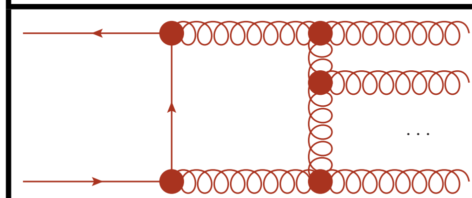
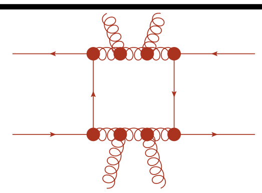
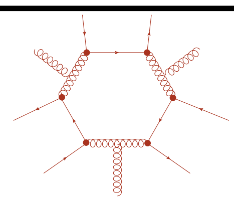
Numerical Stability: improved Finite plot



Improving the way the box coefficients are treated in subsystem 2

Performance

Performance table

	N=6	N=7	N=8
	50ms	141ms	350ms
	57ms	153ms	380ms
	60ms	155ms	373ms
	59ms	157ms	373ms
	60ms	152ms	369ms

QUAD PENALTY

x100

“Thanks to improved accuracy, quadruple precision is only called rarely, which decreases drastically the realistic cpu time per PSP”.

Summary

Summary

- We can now do all primitives necessary for massless QCD partonic processes, including four- and six- fermion subprocesses.
- Numerical instability issues due to pentagon contamination are removed from the CC part and controlled much better at the RAT part.
- Ready for production mode.