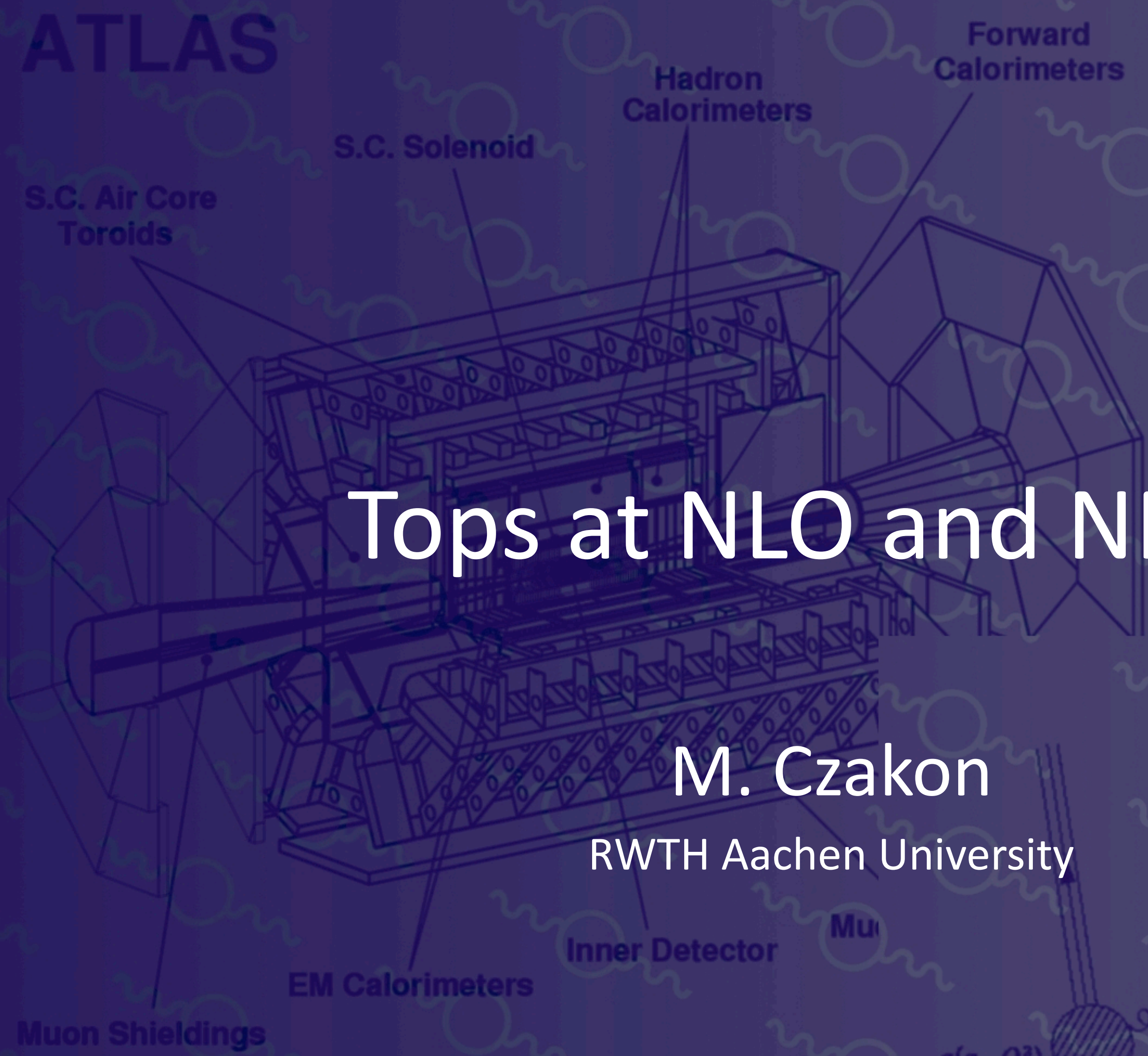
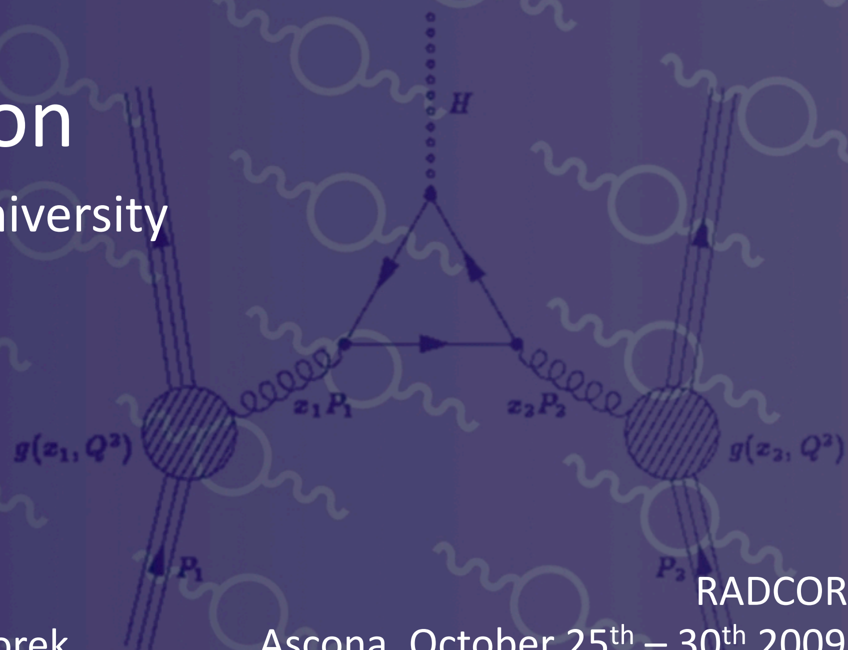


Tops at NLO and NNLO

M. Czakon

RWTH Aachen University



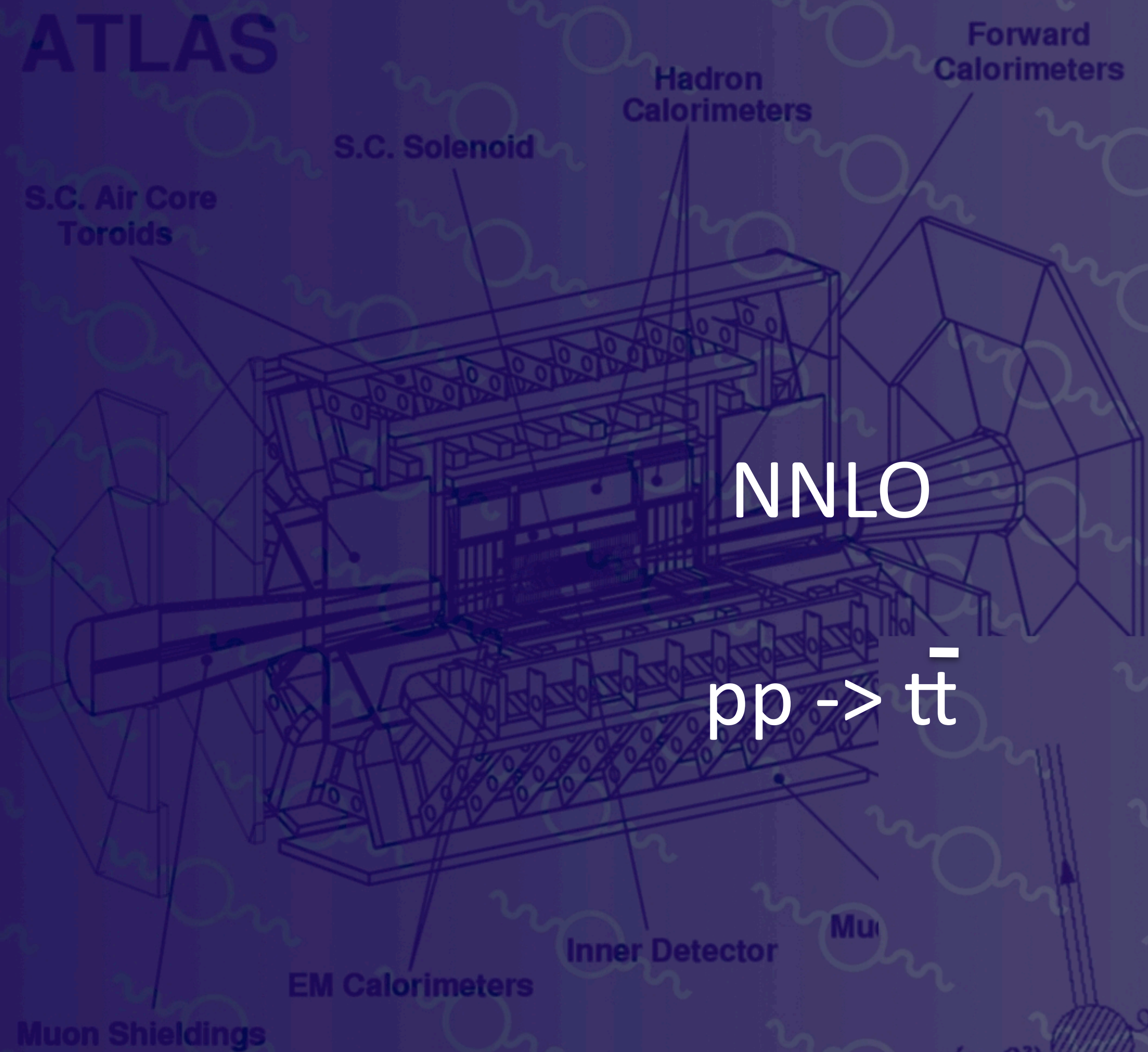
Collaborators (NNLO): A. Mitov, G. Sterman
M. Beneke, P. Falgari, C. Schwinn

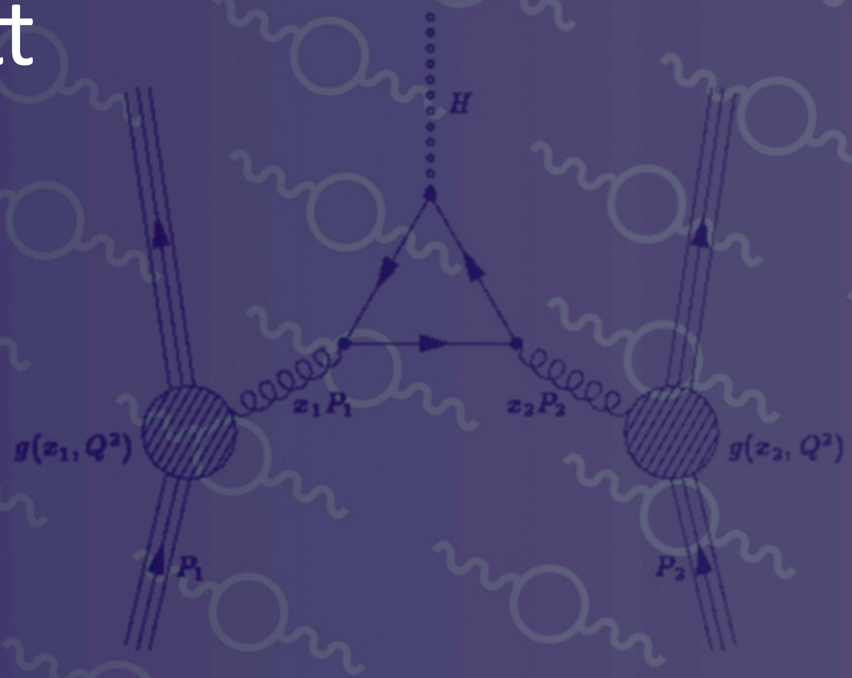
Collaborators (NLO): Bevilacqua, Papadopoulos, Pittau, Worek

RADCOR
Ascona, October 25th – 30th 2009

NNLO

pp -> $t\bar{t}$





Threshold Expansion

- Top quark pair production at NNLO is the hot topic

- Many phenomenological applications (not discussed here)

- Main progress of the last 12 months is in the threshold behaviour and virtual corrections

Hagiwara, Sumino, Yokoya`08; MC, Mitov `08 (matching coefficients)
Beneke, Falgari, Schwinn `09; MC, Mitov, Sterman `09 (NNLL soft gluon resummation)
Beneke, MC, Falgari, Mitov, Schwinn, in preparation (potential effects)

- Threshold expansion of the gg -> tt cross section at NNLO

$$\sigma_{gg}^{(0)} \left(\frac{\alpha_s}{4\pi}\right)^2 \Big(68.5471\frac{1}{\beta^2} + (496.3\log^2\beta + 321.137\log\beta - 8.62261)\frac{1}{\beta} + 4608\log^4\beta - 1894.91\log^3\beta - 912.349\log^2\beta + 2456.74\log\beta + C_{gg}^{(2)}\Big)$$

Total Cross Section Resummation

NLL a classic result by
Bonciani, Catani, Mangano, Nason `98

- Resummation in Mellin space

$$\frac{\hat{\sigma}^{N}_{ij,I}(m_t^2,\mu_f^2,\mu_r^2)}{\hat{\sigma}^{(0),N}_{ij,I}(m_t^2,\mu_f^2,\mu_r^2)} = g^{0}_{ij,I}(m_t^2,\mu_f^2,\mu_r^2)\cdot \exp\left(G^{N+1}_{ij,I}(m_t^2,\mu_f^2,\mu_r^2)\right) + o\left(N^{-1}\ln^n N\right)$$

with

$$G^{N}_{q\bar{q}/gg,I} = G^{N}_{\mathrm{DY/Higgs}} + \delta_{I,8}\, G^{N}_{Q\bar{Q}}$$

$$G^{N}_{\mathrm{DY/Higgs}} = \int_0^1 dz \frac{z^{N-1}-1}{1-z} \int_{\mu_f^2}^{4m_t^2(1-z)^2} \frac{dq^2}{q^2}\, 2A_i(\alpha_s(q^2)) + D_i(\alpha_s(4m_t^2[1-z]^2))$$

$$G^{N}_{Q\bar{Q}} = \int_0^1 dz \frac{z^{N-1}-1}{1-z} D_{Q\bar{Q}}(\alpha_s(4m_t^2[1-z]^2))$$

final state radiation in the octet state

required at the two-loop level

- The requirements stand for NNLL resummation

The Soft Function

- Factorization in Mellin space

$$J_1(N,\alpha_s(\mu^2))\ldots J_k(N,M/\mu,m/\mu,\alpha_s(\mu^2))$$
$$\times\ \mathrm{Tr}\left[\mathbf{H}^P\left(\frac{M^2}{\mu^2},\frac{m^2}{\mu^2},\alpha_s(\mu^2)\right)\mathbf{S}^P\left(\frac{N^2\mu^2}{M^2},\frac{M^2}{m^2},\alpha_s(\mu^2)\right)\right]+\mathcal{O}(1/N)$$

- The soft function satisfies a renormalization group equation

$$\overline{\mathcal{P}}\exp\left\{\int_0^1 dx\frac{x^{N-1}-1}{1-x}\,\mathbf{\Gamma}_S^\dagger\left(\beta_i\cdot\beta_j,\alpha_s\left((1-x)^2M^2\right)\right)\right\}$$
$$\times\mathbf{S}\left(1,\beta_i\cdot\beta_j,\alpha_s\left(M^2/N^2\right)\right)$$
$$\times\mathcal{P}\exp\left\{\int_0^1 dx\frac{x^{N-1}-1}{1-x}\,\mathbf{\Gamma}_S\left(\beta_i\cdot\beta_j,\alpha_s\left((1-x)^2M^2\right)\right)\right\}$$

anomalous dimension
to be expanded at threshold

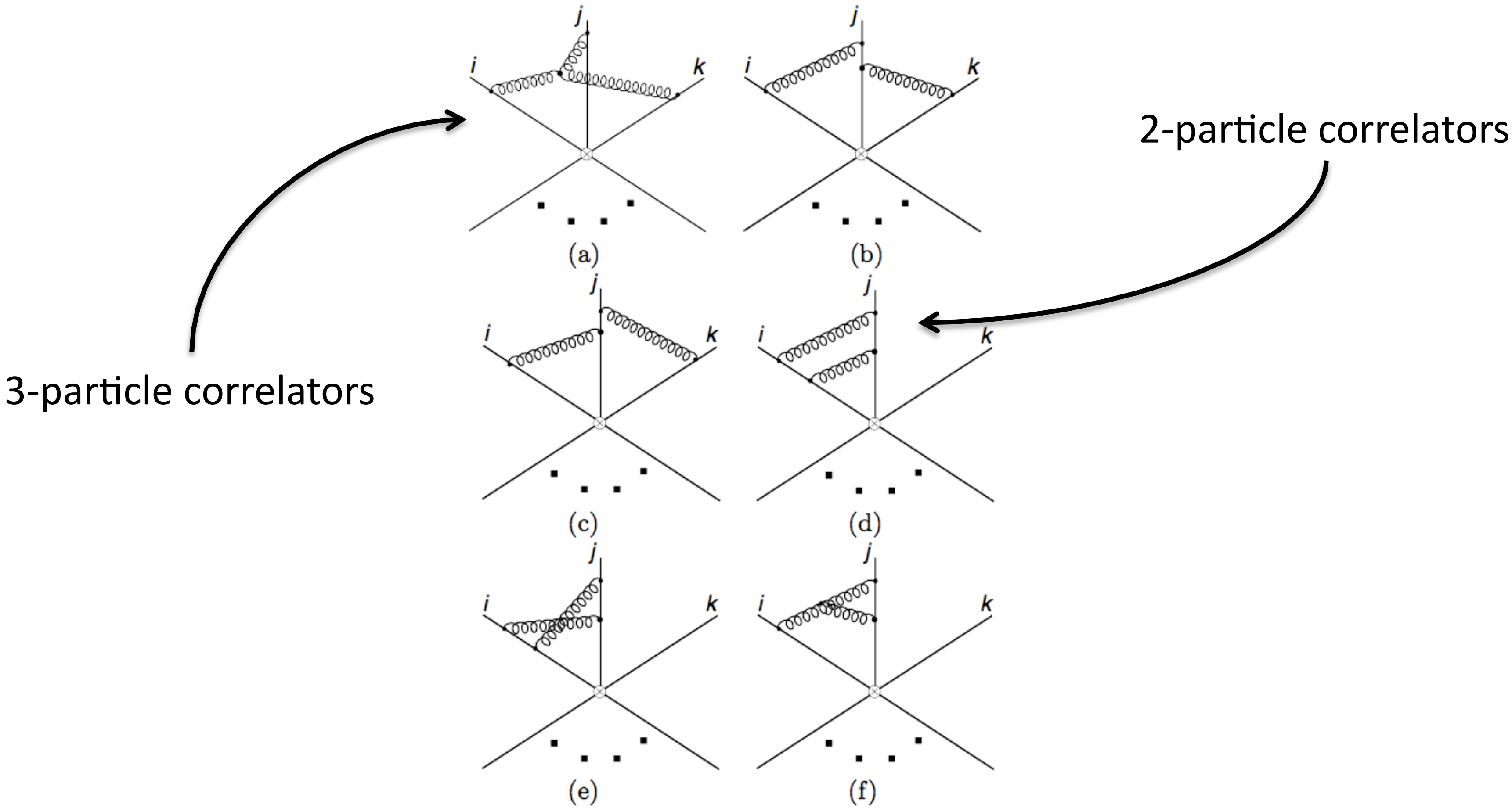
- The boundary is crucial for NNLL

$$\mathbf{S}\left(1,\alpha_s\left(Q^2/N^2\right)\right)=\mathbf{S}^{(0)}\left[1+\frac{\alpha_s\left(Q^2/N^2\right)}{\pi}C_A\begin{pmatrix}0&0&0\\0&1&0\\0&0&1\end{pmatrix}+\ldots\right]$$
$$=\mathbf{S}^{(0)}\left[1+C_A\frac{\alpha_s\left(\mu^2\right)}{\pi}\left\{1+\frac{\alpha_s\left(\mu^2\right)}{\pi}\frac{\beta_0}{4}\ln\left(\frac{N^2\mu^2}{Q^2}\right)\right\}\begin{pmatrix}0&0&0\\0&1&0\\0&0&1\end{pmatrix}+\ldots\right]$$

MC, Mitov, Sterman `09

How to Get the Anomalous Dims?

- The soft anomalous dimensions are obtained from calculations in the eikonal approximation (scattering of Wilson lines)



How to Get the Anomalous Dims?

- As long as only two particle correlations are needed it is sufficient to use form factors and similar results at two loops

$$\mathbf{\Gamma}_S^{(2)} = \frac{1}{2}\sum_{(i\neq j)=1}^{n} T_i \cdot T_j \, \frac{K}{2} \ln\left(-\frac{\mu^2}{\sigma_{ij}}\right) + \frac{1}{2}\sum_{(i\neq j)\in\mathcal{N}_m} T_i \cdot T_j \, P_{ij}^{(2)} + 3E \text{ terms}$$

$$x_{ij} = \frac{\sqrt{1-\frac{4m^2}{s_{ij}}}-1}{\sqrt{1-\frac{4m^2}{s_{ij}}}+1}$$

$$P^{(2)} = \frac{K}{2}P^{(1)} + P^{(2),\mathrm{m}}$$

MC, Mitov, Sterman `09

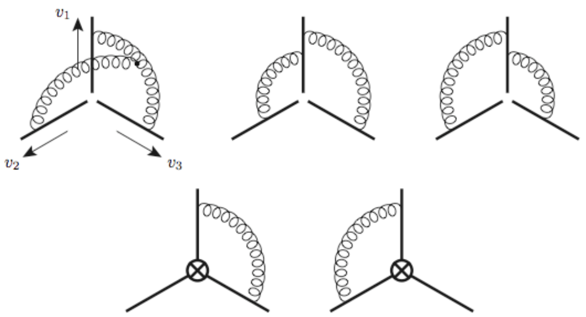
$$\begin{aligned} P^{(2),\mathrm{m}}(x) \; = \; & \frac{C_A}{(1-x^2)^2}\left\{ -\frac{(1+x^2)^2}{2}\mathrm{Li}_3(x^2) + \left(\frac{(1+x^2)^2}{2}\ln(x) - \frac{1-x^4}{2}\right)\mathrm{Li}_2(x^2) \right. \\ & + \frac{x^2(1+x^2)}{3}\ln^3(x) + x^2(1-x^2)\ln^2(x) \\ & \left. + \left(-(1-x^4)\ln\left(1-x^2\right) + x^2(1+x^2)\zeta_2\right)\ln(x) + x^2(1-x^2)\zeta_2 + 2x^2\zeta_3 \right\}, \end{aligned}$$

- Others obtained this result with different methods

Kidonakis `09 (trivial color case)
Becher, Neubert `09 (general, based on old results by Korchemsky and Radyushkin)

Checks and Predictions

- Checked the qq -> tt divergences against explicit calculation
  MC, Mitov, Sterman `09
- Given explicit formulae for divergences in both channels
  Ferroglia, Neubert, Pecjak, Yang `09
- Confirmed the gg -> tt poles
  Baernreuther, MC, in preparation
- The divergences are in priciple known for any massive 2-loop amplitude, thanks to the evaluation of triple correlators



Ferroglia, Neubert, Pecjak, Yang `09

NNLL Resummation

- The resummation looks now rather like

$$\frac{\sigma^P(N,m^2,\mu^2)}{\sigma^P_{\rm Born}(N)} = \mathrm{Tr}\left[\hat{\mathbf{H}}^P(m^2,\mu^2)\mathbf{S}_P^{(0)}\left[1+\frac{\alpha_s\left(Q^2/N^2\right)}{\pi}\,C_A\,\mathbf{\Pi}_8\right]\exp\left\{\int_0^1 dx\frac{x^{N-1}-1}{1-x}\right.\right.$$
$$\left.\left.\times\left(\int_{\mu_F^2}^{4m^2(1-x)^2}\frac{dq^2}{q^2}\,2\,A_P\left(\alpha_s\left[q^2\right]\right)\,\mathbf{1}+\hat{D}^P_{Q\overline{Q}}\left(\alpha_s\left[4m^2(1-x)^2\right]\right)\right)\right\}\right]+\mathcal{O}(1/N,N^3LL)$$

- Can be cast into the traditional formula with a modified D_{QQ}

  MC, Mitov, Sterman `09

$$D^P_{Q\overline{Q}} = \frac{\alpha_s(\mu^2)}{\pi}(-C_A)\begin{pmatrix}0&0\\0&1\end{pmatrix}+\left(\frac{\alpha_s(\mu^2)}{\pi}\right)^2\left\{D_{\rm P}^{(2)}\begin{pmatrix}1&0\\0&1\end{pmatrix}+\left(-C_A\frac{K}{2}-\frac{\zeta_3-1}{2}C_A^2-C_A\frac{\beta_0}{2}\right)\begin{pmatrix}0&0\\0&1\end{pmatrix}\right\}$$

- The same result obtained with SCET

  Beneke, Falgari, Schwinn `09 (separation of Coulomb effects made transparent)

- The hard function contains Coulomb effects, the mixing generates terms of the form log β x 1/β

Potential Contributions

- Apart from soft gluon effects, there are enhanced contributions coming gluon exchanges between the propagating heavy quarks

- These are described by potentials, but not only Coulomb!

- Can be computed in NRQCD, but also obtainable from results of

  Czarnecki, Melnikov '97 '01

  on top quark pair production in e^+e^- and $\gamma\gamma$ collisions

$$R_S^{++} = 6Q_t^4 N_c \beta \left(1 - \frac{\beta^2}{3}\right) \cdot \left[1 + C_F\left(\frac{\alpha_s}{\pi}\right)\Delta^{(1)} + C_F\left(\frac{\alpha_s}{\pi}\right)^2 \Delta^{(2)}\right]$$

$$\Delta^{(2)} = C_F\Delta_A + C_A\Delta_{NA} + T_R N_L \Delta_L + T_R N_H \Delta_H, \quad (5)$$

$$\Delta_A = \frac{\pi^4}{12\beta^2} + \left(-\frac{5}{2} + \frac{1}{8}\pi^2\right)\frac{\pi^2}{\beta} + \frac{27}{8}\pi^2 + \frac{25}{4} + \frac{35}{192}\pi^4 - 2\pi^2\ln(2\beta) + 2x_A;$$

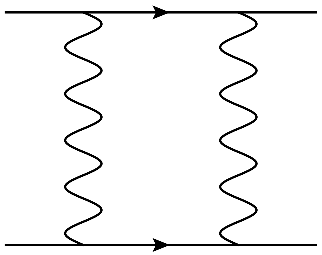
$$\Delta_{NA} = \left(\frac{31}{72} - \frac{11}{12}\ln(2\beta)\right)\frac{\pi^2}{\beta} + \pi^2\left(\frac{5}{4} - \ln(2\beta)\right) + 2x_{NA};$$

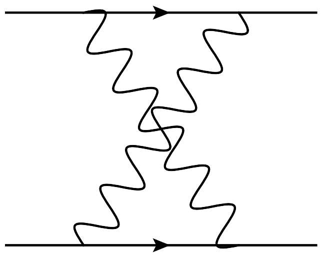
$$\Delta_L = \left(-\frac{5}{18} + \frac{1}{3}\ln(2\beta)\right)\frac{\pi^2}{\beta} + 2x_L;$$

$$\Delta_H = 2x_H. \quad (6)$$

Tricky Points

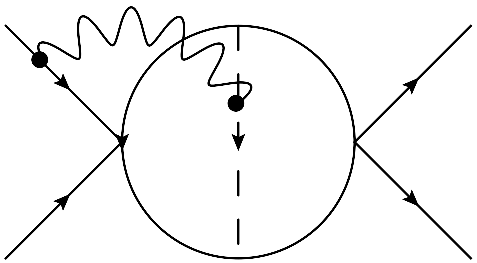
- Transition from singlet to octet



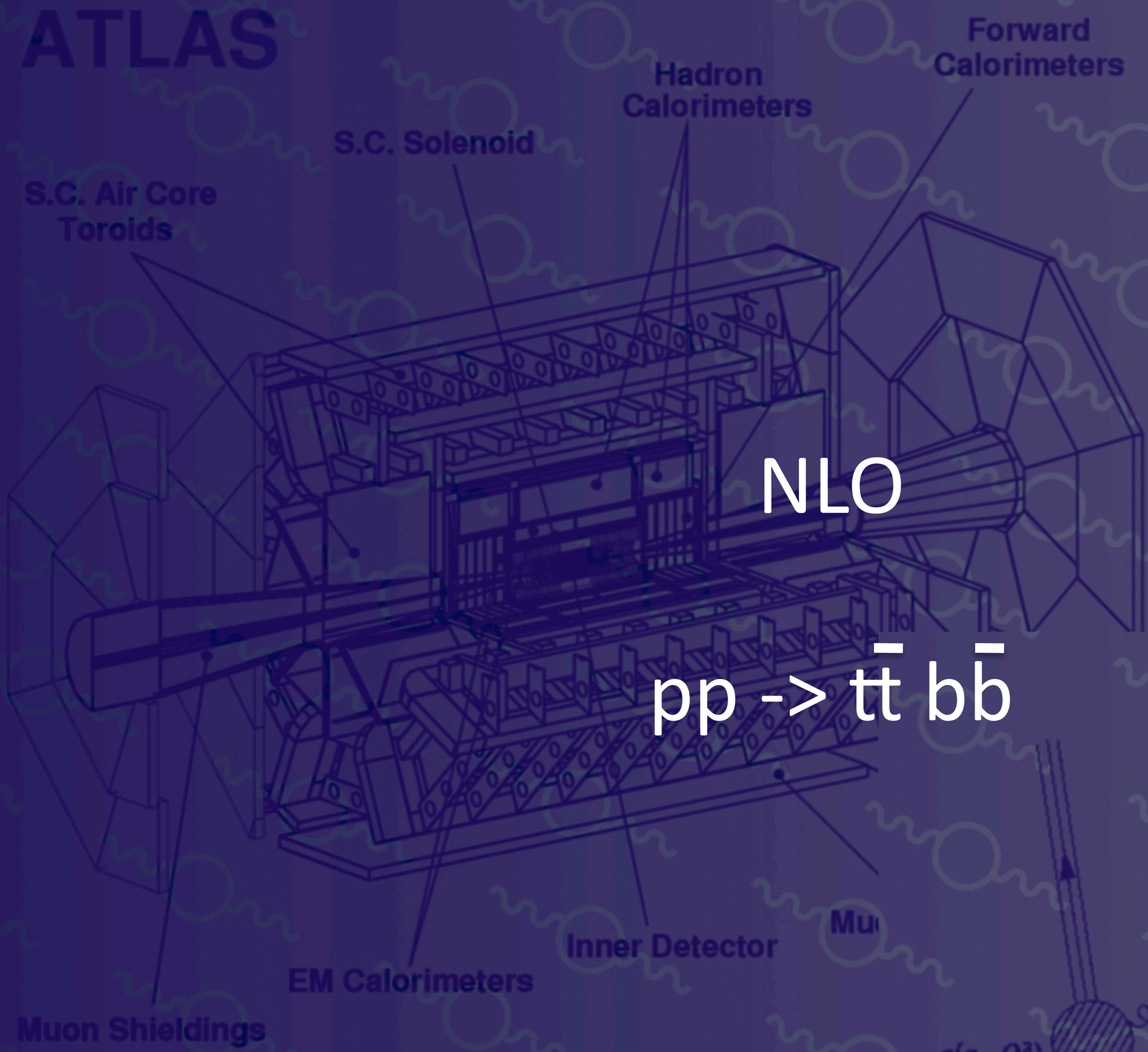


CF -> CF - CA/2 works in this case too !

- Subleading mixing between soft and potential interactions

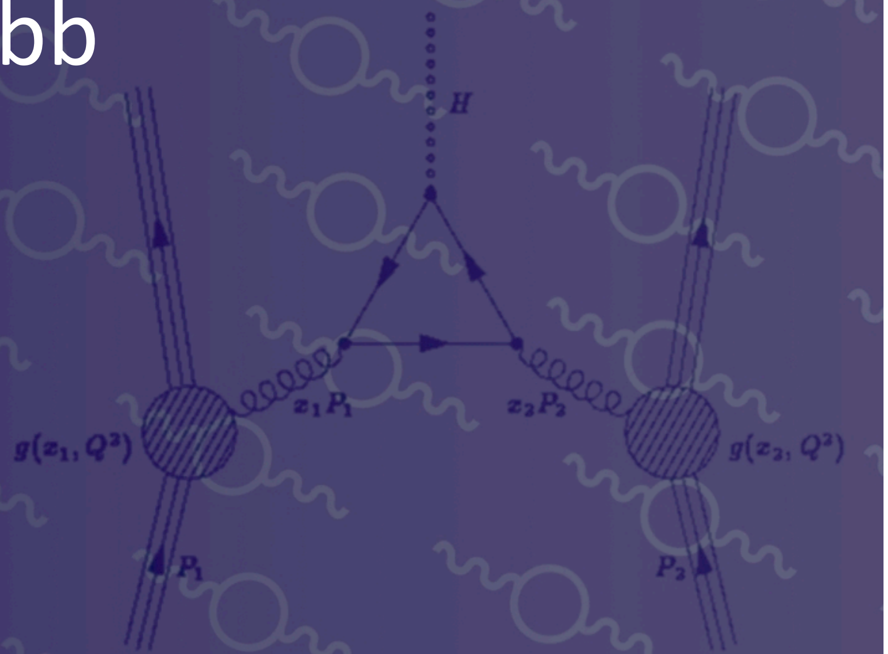


Corrections jump by two powers of the velocity – harmless !
(analysis of the ultrasoft and potential regions to arbitrary order)



NLO

pp -> $t\bar{t}$ $b\bar{b}$



Motivation

- Background to ttH production, with Higgs boson decays into a bb pair → $M_H < 135 GeV$

- Early studies at ATLAS and CMS suggested discovery potential

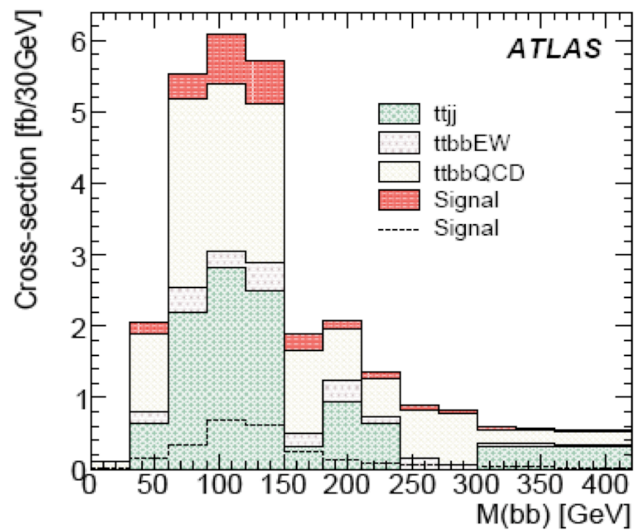
- Analyses with realistic backgrounds show problems if backgrounds not controlled

- ttjj - 'reducible' background

- ttbb - 'irreducible' background

- Problem: misassociation of b-jets to the orginal partons

ATLAS TDR, CERN-OPEN-2008-020



•Reconstructed mass distribution

•All samples, contributions stacked

•Signal contribution also shown separately at the bottom.

Goals

- NLO corrections to 2 -> 4 is current technical frontier
- Complexity of calculations triggered creation of prioritized wishlist
- ttbb production ranges among the most wanted candidates

- NLO QCD corrections to ttH

  Beenakker, Dittmaier, Krämer, Plümper, Spira, Zerwas, '01
  Reina, Dawson, '01; Dawson, Orr, Reina, Wackeroth, '03

- NLO QCD corrections to ttbb

  Bredenstein, Denner, Dittmaier, Pozzorini, '08 '09

- Confirm published results
- Demonstrate power of system based on HELAC-PHEGAS, HELAC-1LOOP, CUTTOOLS, ONELOOP and HELAC-DIPOLES in realistic computation with 6 external legs and massive partons

HELAC-1LOOP

CutTools & OPP

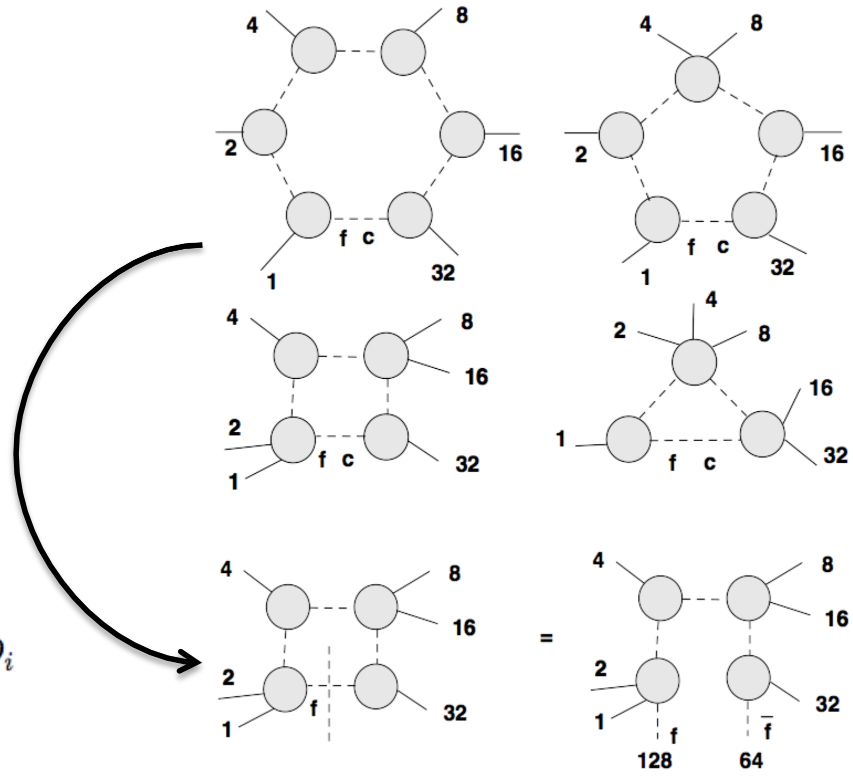
- Reduction at integrand level

  Ossola, Papadopoulos, Pittau `07 `08

- Rational terms with special Feynman rules (not a problem!)

  Draggiotis, Garzelli, Papadopoulos, Pittau `09

$$
\begin{aligned}
N(q) = & \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i \\
& + \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2) \right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\
& + \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1) \right] \prod_{i \neq i_0, i_1}^{m-1} D_i \\
& + \sum_{i_0}^{m-1} \left[a(i_0) + \tilde{a}(q; i_0) \right] \prod_{i \neq i_0}^{m-1} D_i \\
& + \tilde{P}(q) \prod_{i}^{m-1} D_i \,.
\end{aligned}
$$



HELAC-1Loop

van Hameren, Papadopoulos, Pittau `09

- Numerator functions for fixed loop momentum (tree level!)

Efficiency & Precision

- Monte Carlo over color (full color available! 0.5 seconds per event)

- Reweighting

$$\sigma_{ab}^{LO+V} = \int dx_1 dx_2 d\Phi_m f_a(x_1) f_b(x_2) |\mathcal{M}|^2 \left(1 + \frac{\mathcal{M}\mathcal{L}^* + \mathcal{M}^*\mathcal{L}}{|\mathcal{M}|^2}\right)$$

1. much less points to evaluate
   (200.000 for 1 permille accuracy in our case!)

2. based on smoothness arguments

3. avoids numerical instabilities

- Gauge check for every event to certify precision

HELAC-DIPOLES

http://helac-phegas.web.cern.ch/helac-phegas/

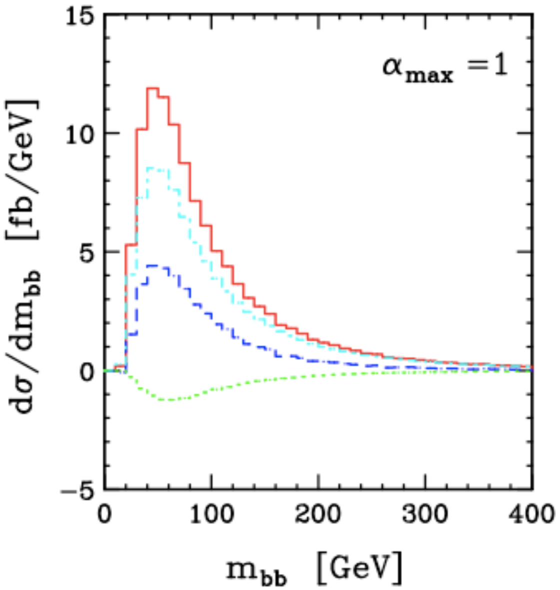
- Complete, public, automatic Catani-Seymour dipoles

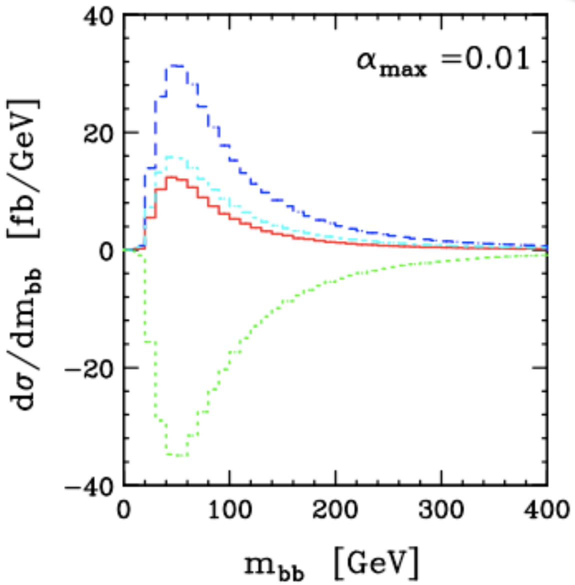
- Phase space integration of subtracted real radiation and integrated dipoles in both massless and massive cases

- Extended for arbitrary polarizations MC, Papadopoulos, Worek, '09
  Monte Carlo over polarization states of external particles

- Phase space restriction on the dipole phase space $\alpha_{max} \in]0,1]$
  1. Less dipole subtraction terms needed per event
  2. Increased numerical stability by decreasing size of dipole phase space
  3. Reduced missed binning problem
  4. Large cancellations between dipole subtracted real radiation and integrated dipoles

Cutoff Dependence in Real Emission



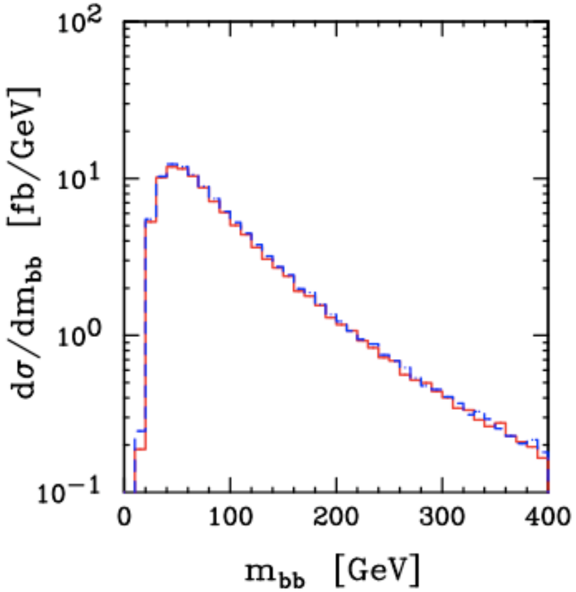


Dipole subtracted real emission

K+P operators

I operator

Full result



Internal check:

Cutoff independence in distributions

Results

Bevilacqua, MC, Papadopoulos, Pittau, Worek `09

Process	$\sigma^{\text{LO}}_{[23,24]}$ [fb]	σ^{LO} [fb]	$\sigma^{\text{NLO}}_{[23,24]}$ [fb]	$\sigma^{\text{NLO}}_{\alpha_{max}=1}$ [fb]	$\sigma^{\text{NLO}}_{\alpha_{max}=0.01}$ [fb]
$q\bar{q} \to t\bar{t}b\bar{b}$	85.522(26)	85.489(46)	87.698(56)	87.545(91)	87.581(134)
$pp \to t\bar{t}b\bar{b}$	1488.8(1.2)	1489.2(0.9)	2638(6)	2642(3)	2636(3)

Permille level agreement!

$\xi \cdot m_t$	$1/8 \cdot m_t$	$1/2 \cdot m_t$	$1 \cdot m_t$	$2 \cdot m_t$	$8 \cdot m_t$
σ^{LO} [fb]	8885(36)	2526(10)	1489.2(0.9)	923.4(3.8)	388.8(1.4)
σ^{NLO} [fb]	4213(65)	3498(11)	2636(3)	1933.0(3.8)	1044.7(1.7)

$$\sigma^{\text{LO}}_{t\bar{t}b\bar{b}}(\text{LHC}, m_t = 176.2 \text{ GeV}, \text{CTEQ6L1}) = 1489.2 \begin{array}{l} +1036.8\ (70\%) \\ -\ \ 565.8\ (38\%) \end{array} \text{ fb} ,$$

$$\sigma^{\text{NLO}}_{t\bar{t}b\bar{b}}(\text{LHC}, m_t = 176.2 \text{ GeV}, \text{CTEQ6M}) = 2636 \begin{array}{l} +862\ (33\%) \\ -703\ (27\%) \end{array} \text{ fb} .$$

Scale dependence reduced:

70% (LO) -> 33% (NLO)

K factor of 1.77

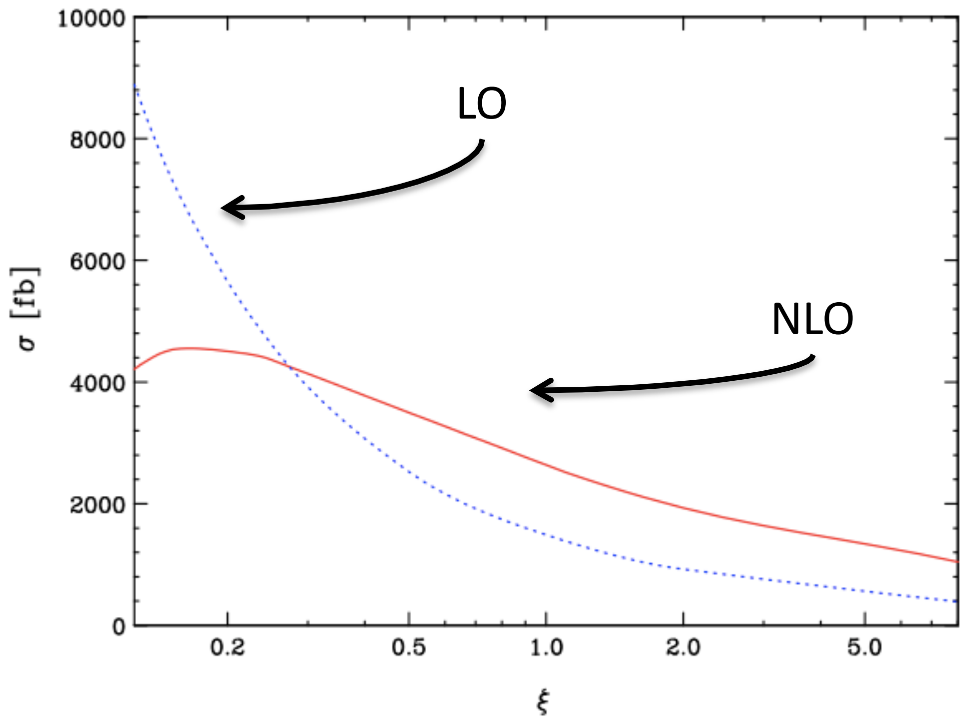
for quarks only 1.03

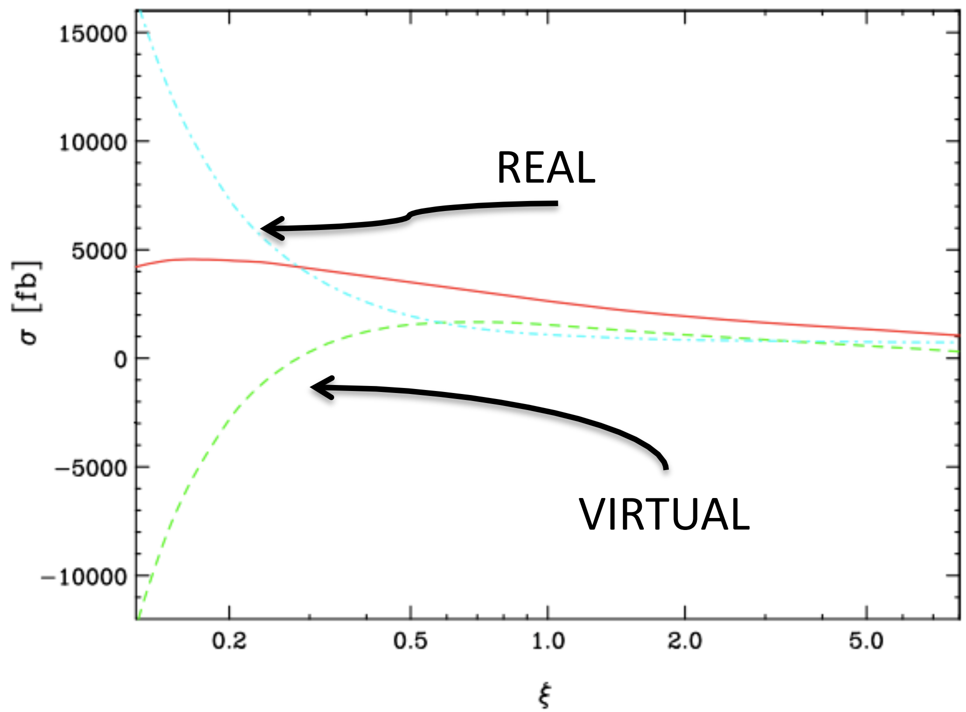
Scale Dependence

Bevilacqua, MC, Papadopoulos, Pittau, Worek `09

Scale dependence for

$$\mu_R = \mu_F = \xi m_t$$

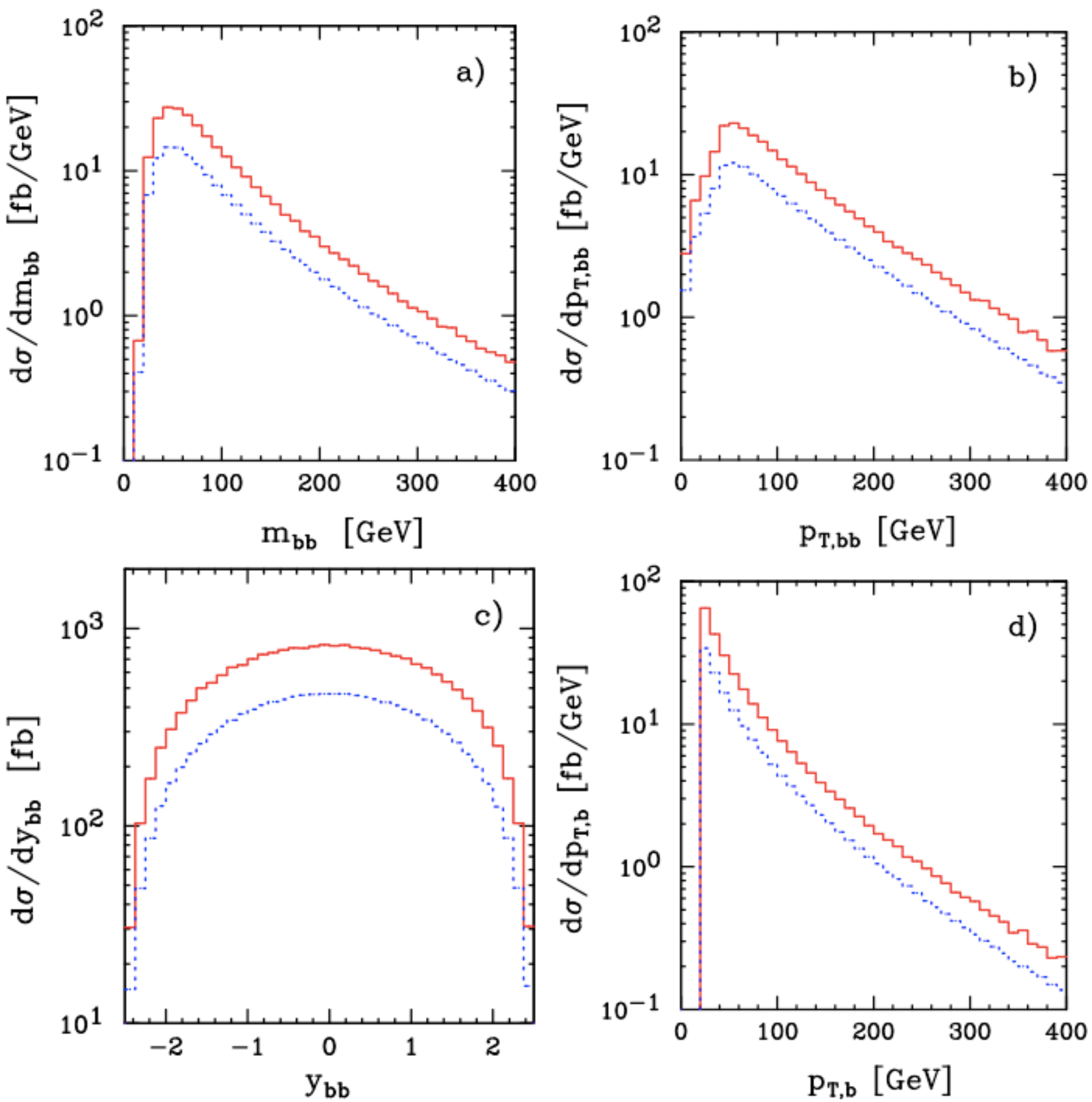




Scale dependence decomposed into contributions from real and virtual corrections

Distributions

Bevilacqua, MC, Papadopoulos, Pittau, Worek `09



b-jet pair kinematics

1. Invariant mass
2. Transverse momentum
3. Rapidity

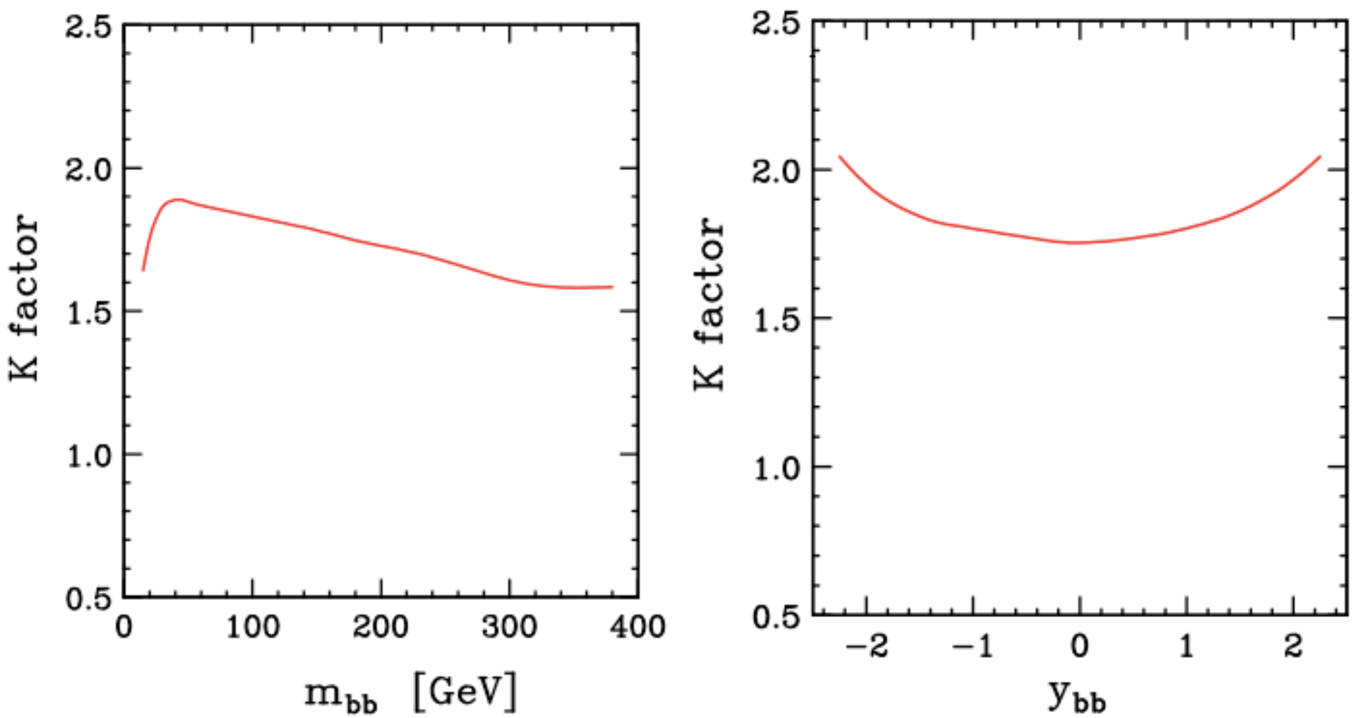
single b-jet kinematics

4. Transverse momentum

LO & NLO

Dynamical K Factors

Bevilacqua, MC, Papadopoulos, Pittau, Worek `09



$$K(m_{b\bar{b}}) = \frac{d\sigma^{NLO}/dm_{b\bar{b}}}{d\sigma^{LO}/dm_{b\bar{b}}}$$

$$K(y_{b\bar{b}}) = \frac{d\sigma^{NLO}/dy_{b\bar{b}}}{d\sigma^{LO}/dy_{b\bar{b}}}$$

Relatively small variation compared to the size, but shape changes important

Conclusions

- NNLO threshold expansion for pp -> tt known
- Complete NNLO calculations under way
- NLO pp -> ttbb completed by two groups
- Complete tool at NLO built around HELAC-PHEGAS:
  HELAC-1LOOP, CUTTOOLS, ONELOOP and HELAC-DIPOLES
- Other automatic systems:
  1. BlackHat
  2. Rocket
  3. Golem