NNLO predictions for event shapes and jet rates in electron-positron annihilation

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The bright side of NNLO calculations:

What all experts are happy to tell you

The dark side of NNLO calculations:

What your mother never told you

Phys. Rev. Lett. 101, (2008), 162001, arxiv:0807.3241;

JHEP 0906, (2009), 041, arxiv:0904.1077;

JHEP 0907, (2009), 009, arxiv:0904.1145;

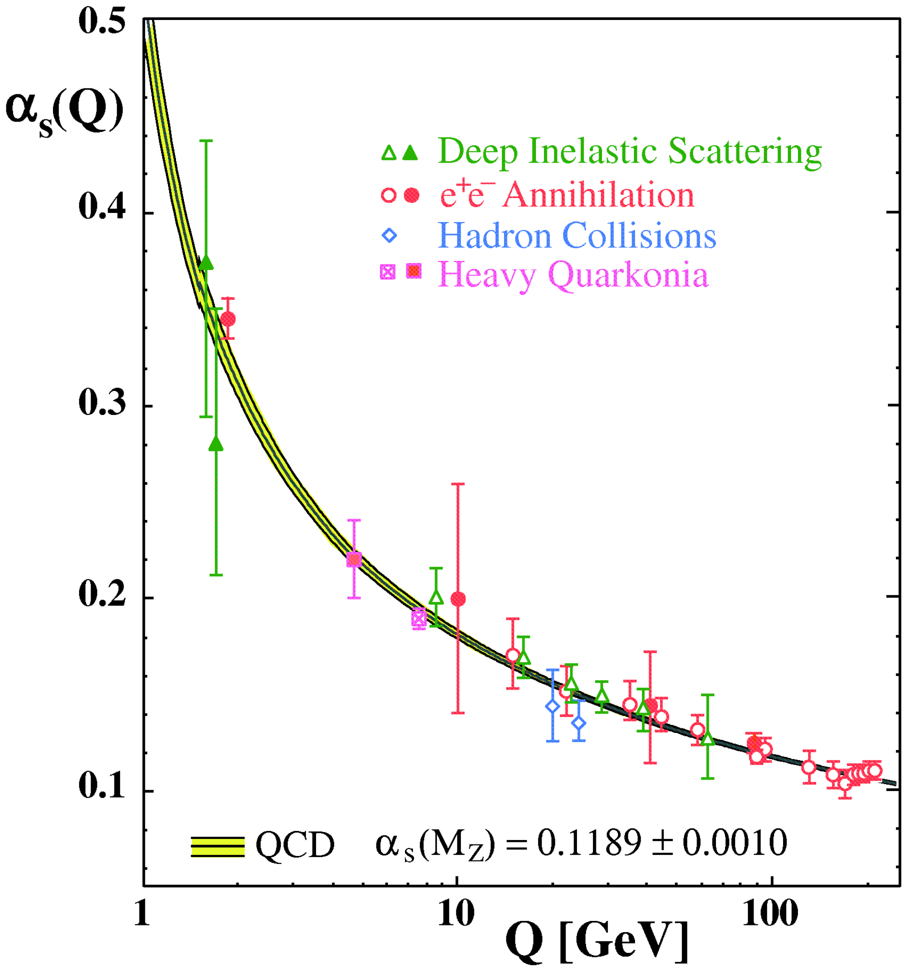
arxiv:0909.5056.

The strong coupling

The fundamental parameter of QCD:
The strong coupling α_s.

Objectives for LHC:
Extract fundamental quantities like α_s to high precision.

α_s can be measured in a variety of processes:
Deep inelastic scattering, τ-decays, heavy quarkonium, electron-positron annihilation, hadron collisions, ...



(S. Bethke, '06.)

Perturbation theory

Due to the smallness of the coupling constants α and α_s, we may compute an observable at high energies reliable in perturbation theory,

$$\langle O \rangle = \frac{\alpha_s}{2\pi}\langle O \rangle_{LO} + \left(\frac{\alpha_s}{2\pi}\right)^2 \langle O \rangle_{NLO} + \left(\frac{\alpha_s}{2\pi}\right)^3 \langle O \rangle_{NNLO} + \dots$$

Feynman diagrams contributing to the leading order:

Leading order proportional to α_s !

The amplitudes

Perturbative expansion of the amplitude (LO, NLO, NNLO):

$$|\mathcal{A}_n|^2 = \underbrace{\mathcal{A}_n^{(0)\,*}\mathcal{A}_n^{(0)}}_{\text{Born}} + \underbrace{\left(\mathcal{A}_n^{(0)\,*}\mathcal{A}_n^{(1)} + \mathcal{A}_n^{(1)\,*}\mathcal{A}_n^{(0)}\right)}_{\text{virtual}} + \underbrace{\left(\mathcal{A}_n^{(0)\,*}\mathcal{A}_n^{(2)} + \mathcal{A}_n^{(2)\,*}\mathcal{A}_n^{(0)} + \mathcal{A}_n^{(1)\,*}\mathcal{A}_n^{(1)}\right)}_{\text{two-loop and loop-loop}},$$

$$|\mathcal{A}_{n+1}|^2 = \underbrace{\mathcal{A}_{n+1}^{(0)\;*}\mathcal{A}_{n+1}^{(0)}}_{\text{real}} + \underbrace{\left(\mathcal{A}_{n+1}^{(0)\;*}\mathcal{A}_{n+1}^{(1)} + \mathcal{A}_{n+1}^{(1)\;*}\mathcal{A}_{n+1}^{(0)}\right)}_{\text{loop+unresolved}},$$

$$|\mathcal{A}_{n+2}|^2 = \underbrace{\mathcal{A}_{n+2}^{(0)\;*}\mathcal{A}_{n+2}^{(0)}}_{\text{double unresolved}}.$$

$\mathcal{A}_n^{(l)}$: amplitude with n external particles and l loops.

Challenges

What are the bottle-necks ?

- Amplitudes: At one-loop and beyond, the occuring integrals cannot be simply looked up in an integral table.
- Divergences: At NLO and beyond, infrared divergences occur in intermediate stages, if massless particles are involved.
- Numerics: Stable and efficient numerical methods are required for the Monte Carlo integration.

The amplitudes for $e^+e^- \longrightarrow 3$ jets at NNLO

A NNLO calculation of $e^+e^- \longrightarrow 3$ jets requires the following amplitudes:

- Born amplitudes for $e^+e^- \longrightarrow 5$ jets:

  F. Berends, W. Giele and H. Kuijf, 1989;

  K. Hagiwara and D. Zeppenfeld, 1989.

- One-loop amplitudes for $e^+e^- \longrightarrow 4$ jets:

  Z. Bern, L. Dixon, D.A. Kosower and S.W., 1996;

  J. Campbell, N. Glover and D. Miller, 1996.

- Two-loop amplitudes for $e^+e^- \longrightarrow 3$ jets:

  L. Garland, T. Gehrmann, N. Glover, A. Koukoutsakis and E. Remiddi, 2002;

  S. Moch, P. Uwer and S.W., 2002.

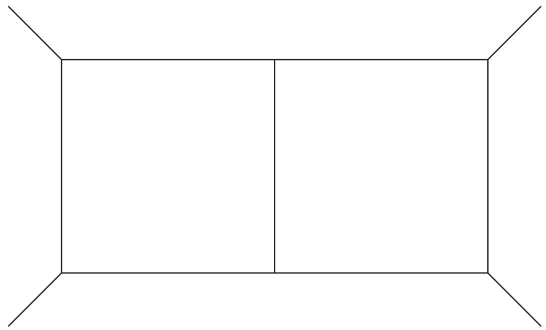
The calculation of two-loop integrals

- Techniques to calculate two-loop integrals
  - Mellin-Barnes transformation, Smirnov ’99, Tausk ’99.
  - Differential equations, Gehrmann, Remiddi ‘00.
  - Nested sums, Moch, Uwer, S.W. ‘01.
  - Sector decomposition (numerical), Binoth, Heinrich, ’00.

- Methods to reduce the work-load:
  - Integration-by-parts, Chetyrkin, Kataev, Tkachov ‘81.
  - Reduction algorithms, Tarasov ‘96, Laporta ’01.
  - Cut technique Bern, Dixon, Kosower, ’00

The double-box integral

Two-loop amplitudes for $2 \longrightarrow 2$ processes involve the double-box integral:



- First calculated by Smirnov (planar) and Tausk (non-planar) in 1999.
- Calculation based on Mellin-Barnes representation.
- Result expressed in harmonic polylogarithms.

$$H_{m_1,\ldots,m_k}(x) = \sum_{i_1>i_2>\ldots>i_k>0} \frac{x^{i_1}}{i_1^{m_1} i_2^{m_2} \ldots i_k^{m_k}}, \qquad x = \frac{s}{t}.$$

Multiple polylogarithms

- Definition:

$$\mathrm{Li}_{m_1,\ldots,m_k}(x_1,\ldots,x_k) \quad = \quad \sum_{i_1>i_2>\ldots>i_k>0} \frac{x_1^{i_1}}{i_1^{m_1}} \frac{x_2^{i_2}}{i_2^{m_2}} \cdots \frac{x_k^{i_k}}{i_k^{m_k}}.$$

  (Goncharov; Borwein, Bradley, Broadhurst and Lisonek)

- Special subsets: Harmonic polylogs, Nielsen polylogs, classical polylogs (Remiddi and Vermaseren, Gehrmann and Remiddi).

- Have also an integral representation.

- Obey two Hopf algebras (Moch, Uwer, S.W.).

- Can be evaluated numerically for all complex values of the arguments (Gehrmann and Remiddi, Vollinga and S.W.).

Infrared divergences: General methods at NLO

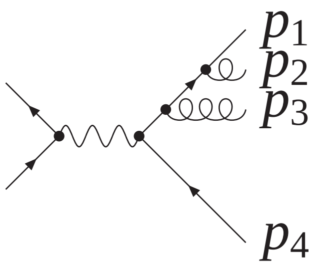
Fully differential NLO Monte Carlo programs need a general method to handle the cancelation of infrared divergencies.

- Phase space slicing
  - e^+e^-: W. Giele and N. Glover, (1992)
  - initial hadrons: W. Giele, N. Glover and D.A. Kosower, (1993)
  - massive partons, fragmentation: S. Keller and E. Laenen, (1999)

- Subtraction method
  - residue approach: S. Frixione, Z. Kunzst and A. Signer, (1995)
  - dipole formalism: S. Catani and M. Seymour, (1996)
  - massive partons: L. Phaf and S.W. (2001), S. Catani, S. Dittmaier, M. Seymour and Z. Trócsányi, (2002)

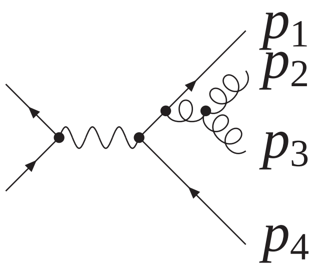
Infrared divergences at NNLO

The leading-colour contributions to $e^+e^- \longrightarrow qgg\overline{q}$.



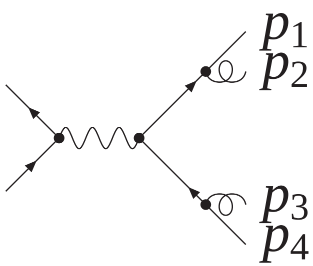
Double unresolved configurations:

- Two pairs of separately collinear particles
- Three particles collinear
- Two particles collinear and a third soft particle
- Two soft particles
- Coplanar degeneracy



Single unresolved configurations:

- Two collinear particles
- One soft particle



The subtraction method at NNLO

- Singular behaviour
  - Factorization of tree amplitudes in double unresolved limits, Berends, Giele, Cambell, Glover, Catani, Grazzini, Del Duca, Frizzo, Maltoni, Kosower '99
  - Factorization of one-loop amplitudes in single unresolved limits, Bern, Del Duca, Kilgore, Schmidt, Kosower, Uwer, Catani, Grazzini, '99

- Extension of the subtraction method to NNLO Kosower; S.W.; Kilgore; Gehrmann-De Ridder, Gehrmann, Glover, Heinrich; Frixione, Grazzini; Somogyi, Trócsányi and Del Duca;

- Cancellation based on sector decomposition Anastasiou, Melnikov, Petriello; Heinrich;

- Applications:
  - $pp \longrightarrow W$, Anastasiou, Dixon, Melnikov, Petriello
  - $pp \longrightarrow H$, Harlander Kilgore; Ravindran, Smith, van Neerven; Anastasiou, Dixon, Melnikov, Petriello; Catani, Grazzini
  - $e^+e^- \longrightarrow$ 2 jets, Anastasiou, Melnikov, Petriello; S.W.
  - $e^+e^- \longrightarrow$ 3 jets, Gehrmann-De Ridder, Gehrmann, Glover, Heinrich; S.W.

Spin and colour correlations

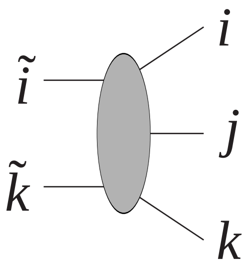
- In the soft limit, amplitudes factorize completely in spin space, but colour correlations remain.
- In the collinear limit, amplitudes factorize completely in colour space, but spin correlations remain.

Spin-correlations occur for the splittings $g \longrightarrow gg$ and $g \longrightarrow q\bar{q}$, but not for $q \longrightarrow qg$.

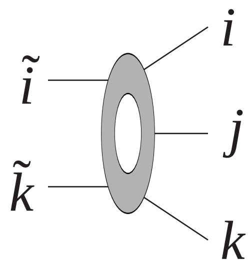
If one uses spin-averaged subtraction terms, one has a local counterterm only after the average over the azimuthal angle.

Alternative: Use combination of subtraction and slicing.

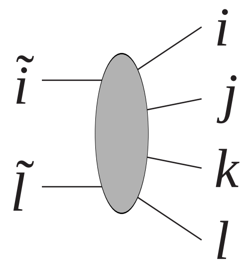
Antenna subtraction terms at NNLO



NLO



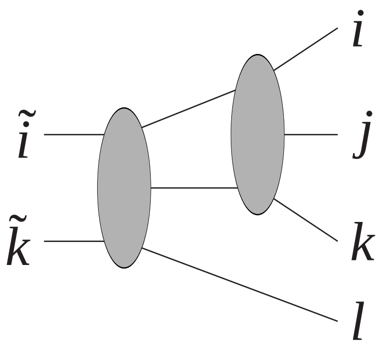
one-loop unresolved



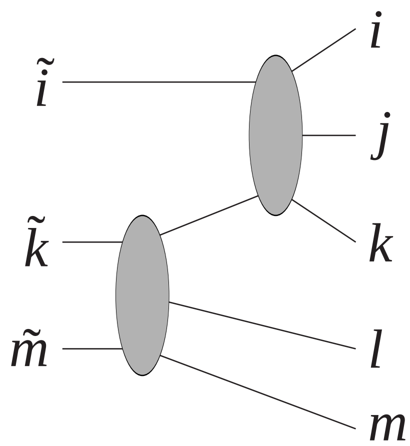
double unresolved

Gehrmann-De Ridder, Gehrmann, Glover, ’05

At NNLO also iterated structures:



colour connected



almost colour connected

The hybrid method

Strict subtraction:

3 parton:

- explicit poles cancel point-by-point

4 parton:

- explicit poles cancel point-by-point
- phase space singularities cancel point-by-point

5 parton:

- phase space singularities cancel point-by-point

Hybrid subtraction:

3 parton:

- explicit poles cancel point-by-point

4 parton:

- explicit poles cancel after integration over unresolved phase space
- phase space singularities cancel after azimuthal average

5 parton:

- phase space singularities cancel after azimuthal average

Cancellation of explicit poles after integration

$$\int d\phi_3 \left| \mathcal{A}_3^{(0)} \right|^2 \left[\int d\phi^D_{unres}(i,j,k)\, X_3^0(i,j,k) \underbrace{\sum \mathcal{Y}_3^0(i,j,k,\ldots)}_{1/\varepsilon} \right]$$

Two cases:

1. Expression in the square bracket vanishes after integration to all orders in ε.
   That's nice :-)

2. Only the pole terms vanish after integration.
   Warning: $d\phi^D_{unres} = d\phi^4_{unres} |\sin\phi|^{-2\varepsilon}$.

Can arrange subtraction terms such that only the first case occurs !

The subtraction method at NNLO

5 partons : $d\sigma_5^{(0)} - d\alpha^{NLO} - d\alpha^{double} - d\alpha^{almost} + d\alpha^{iterated} - d\alpha^{soft},$

4 partons : $d\sigma_4^{(1)} + d\alpha^{NLO} - d\alpha^{loop} + d\alpha^{almost} - d\alpha^{iterated} - d\alpha^{product} + d\alpha^{soft},$

3 partons : $d\sigma_3^{(2)} + d\alpha^{double} + d\alpha^{loop} + d\alpha^{product}.$

$d\alpha^{double}$ contains the four-parton antenna functions,

$d\alpha^{almost}$ contains a product of two three-parton antenna functions,

$d\alpha^{iterated}$ is the approximation of $d\alpha^{NLO}$,

$d\alpha^{loop}$ is the approximation of the one-loop matrix elements,

$d\alpha^{product}$ contains a product of two three-parton antenna functions, both with $4 \to 3$ parton kinematics

$d\alpha^{soft}$ is an additional subtraction term due to soft gluons, occuring in processes with three or more hard partons.

The soft subtraction term: 4 partons

If one ignores the soft subtraction term:

$$\int d\phi_3 \left|\mathcal{A}_3^{(0)}\right|^2 \int d\phi_{unres}^D \, X_3^0(a,i,b) \frac{1}{\varepsilon}\left[\ln\frac{s_{\tilde{a}j}s_{j\tilde{b}}}{s_{\tilde{a}\tilde{b}}} - \ln\frac{s_{aj}s_{jb}}{s_{ab}}\right]$$

Integration over the unresolved phase space leads to

$$\frac{1}{\varepsilon}\int_0^{2\pi} d\phi \ln\left(\frac{(1+c_j)(1-c_b)}{2(1-c_bc_j-s_bs_j\cos\phi)}\right) = \frac{2\pi}{\varepsilon}\ln\left(\frac{1-c_bc_j+(c_j-c_b)}{1-c_bc_j+|c_j-c_b|}\right).$$

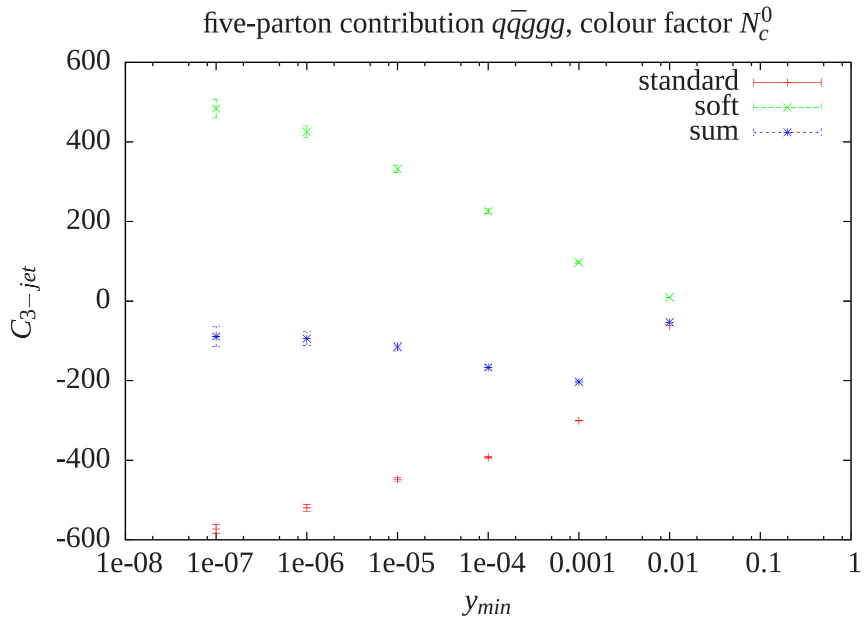
Non-zero for $c_j < c_b$!

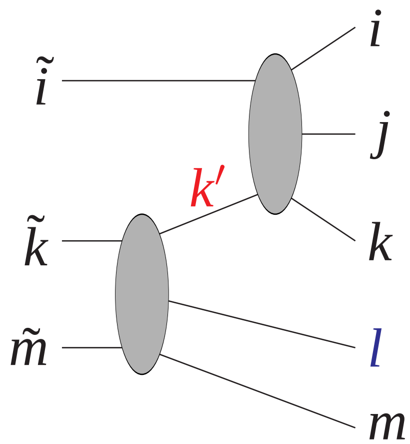
The explicit poles in the four-parton configuration have to cancel: $d\alpha^{soft}$ is needed.

The soft subtraction term: 5 partons

If one ignores the soft subtraction term:

Dependence on the slicing parameter:





Gluon l soft:

Eikonal factor

$\mathrm{Eik}(k', l, m)$

The five-parton contribution has to be independent of the slicing parameter:
$-d\alpha^{soft}$ is needed.

$e^+e^- \longrightarrow 3$ **jets at NNLO**

Fully differential Monte-Carlo programs for 3-jet observables at NNLO:

- EERAD3

  Gehrmann-De Ridder, Gehrmann, Glover, Heinrich,

  Phys.Rev.Lett.99:132002,2007,

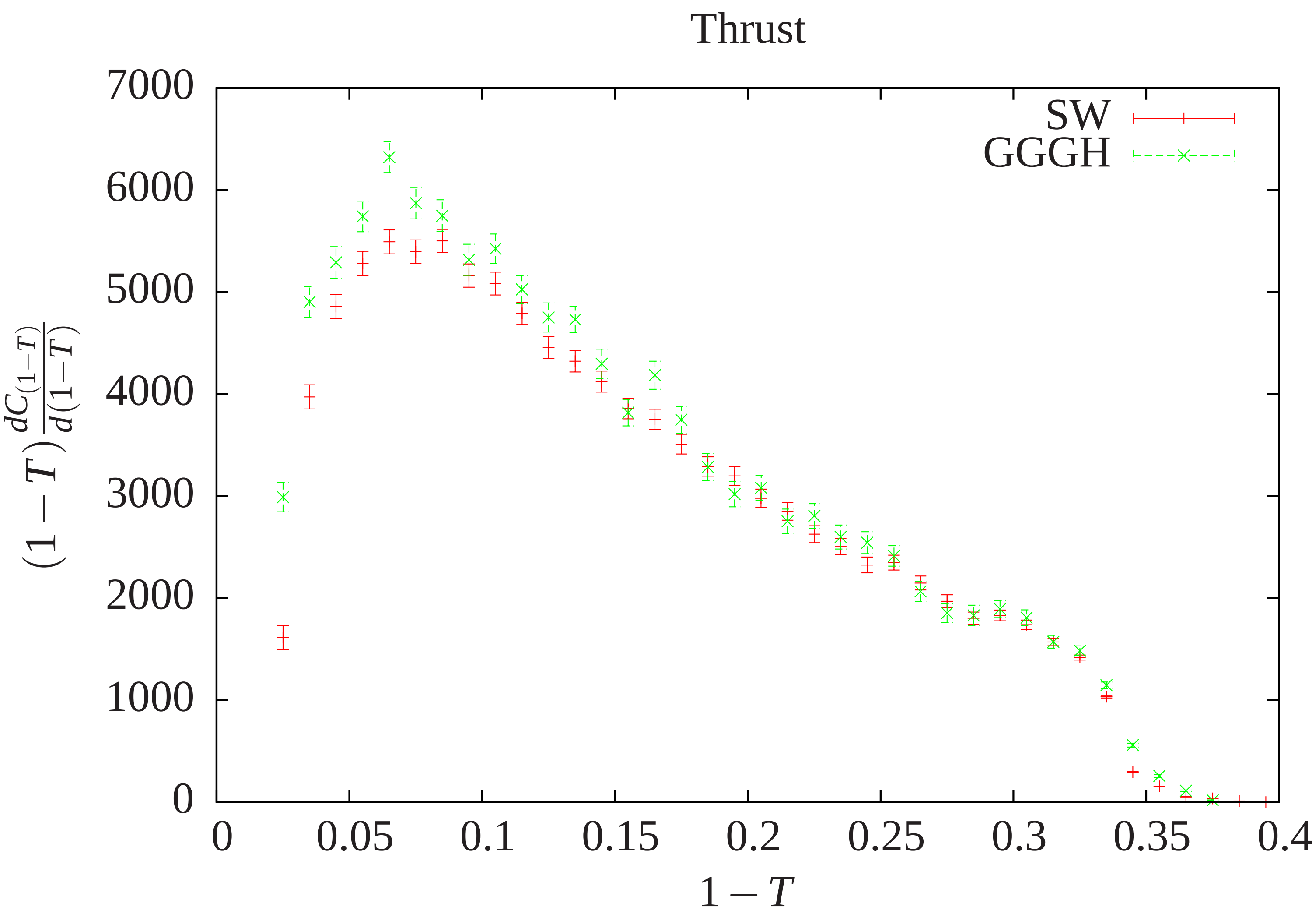
  Phys.Rev.Lett.100:172001,2008

- MERCUTIO2

  S.W.,

  Phys.Rev.Lett.101:162001,2008

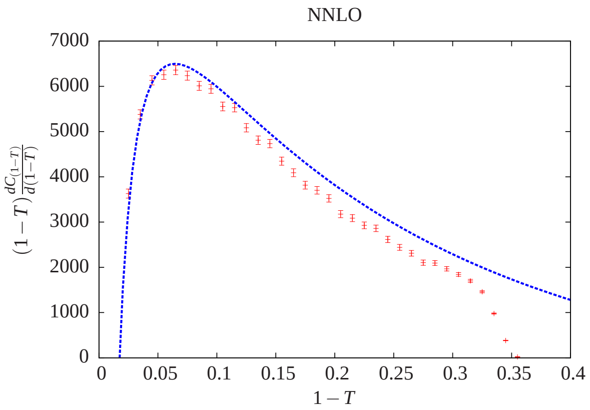
Comparison with EERAD3



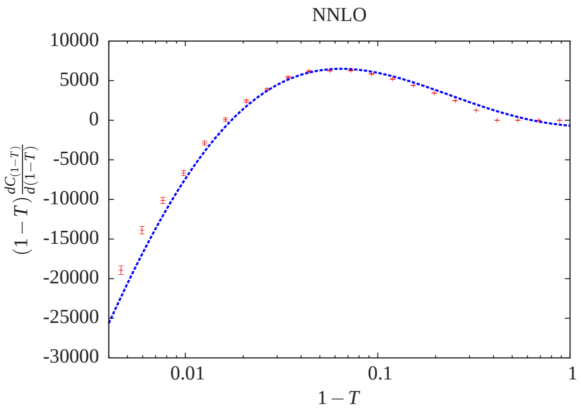
Comparison of the thrust distribution with Becher/Schwartz

Logarithmic terms:

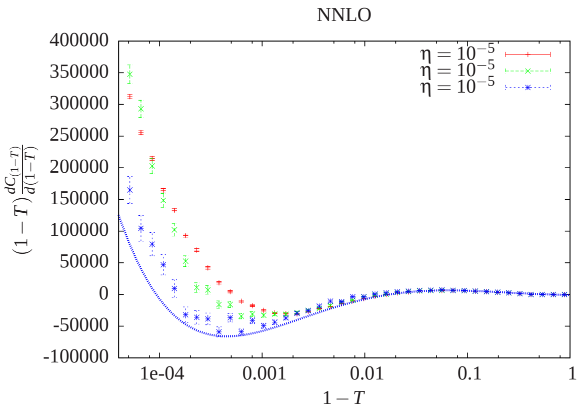
$$\frac{dC_\tau}{d\tau} = \frac{1}{\tau}\left[a_5 \ln^5\tau + a_4 \ln^4\tau + a_3 \ln^3\tau + a_2 \ln^2\tau + a_1 \ln\tau + a_0 + O(\tau)\right], \quad \tau = 1 - T.$$



Hard region

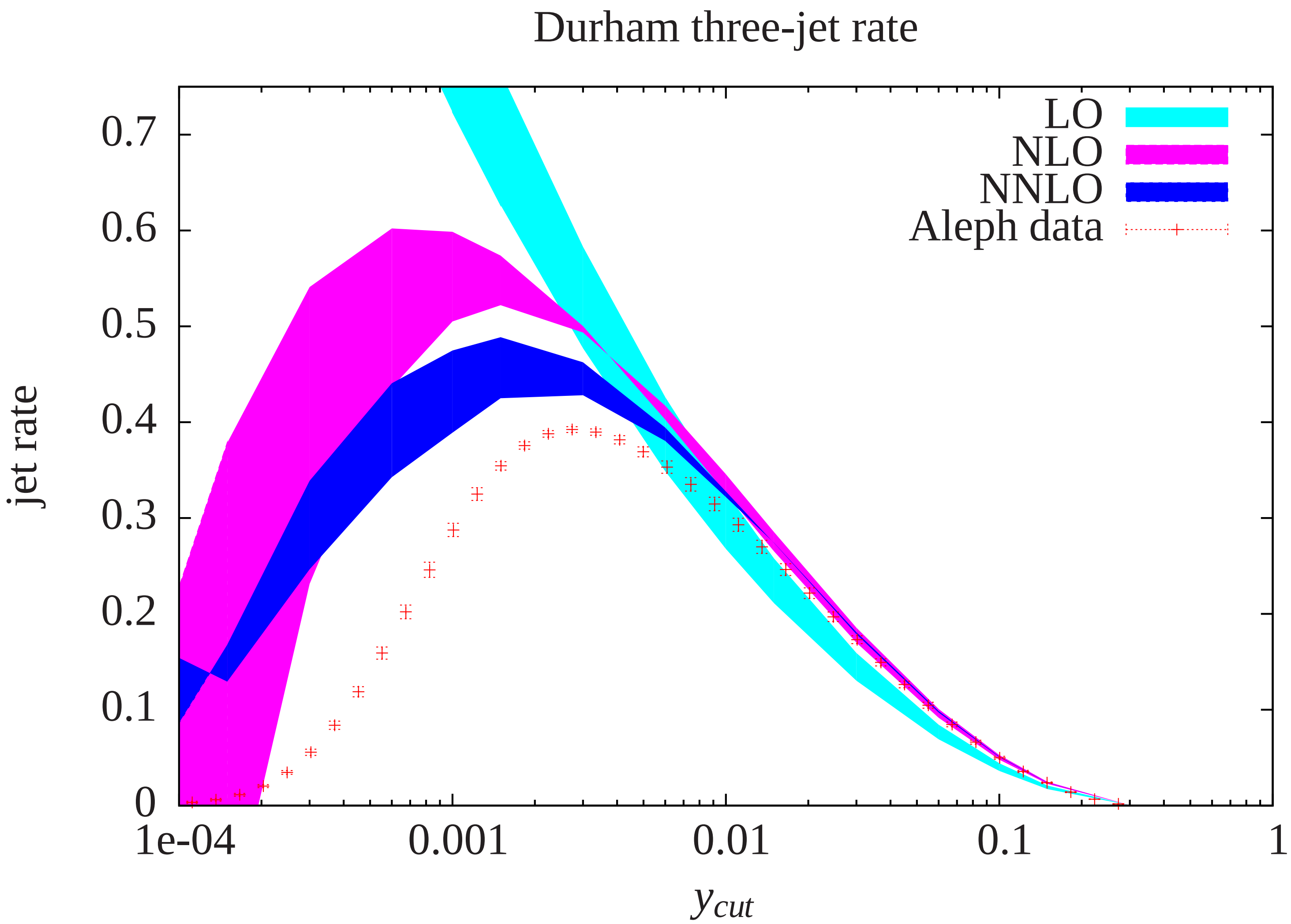


Peak region



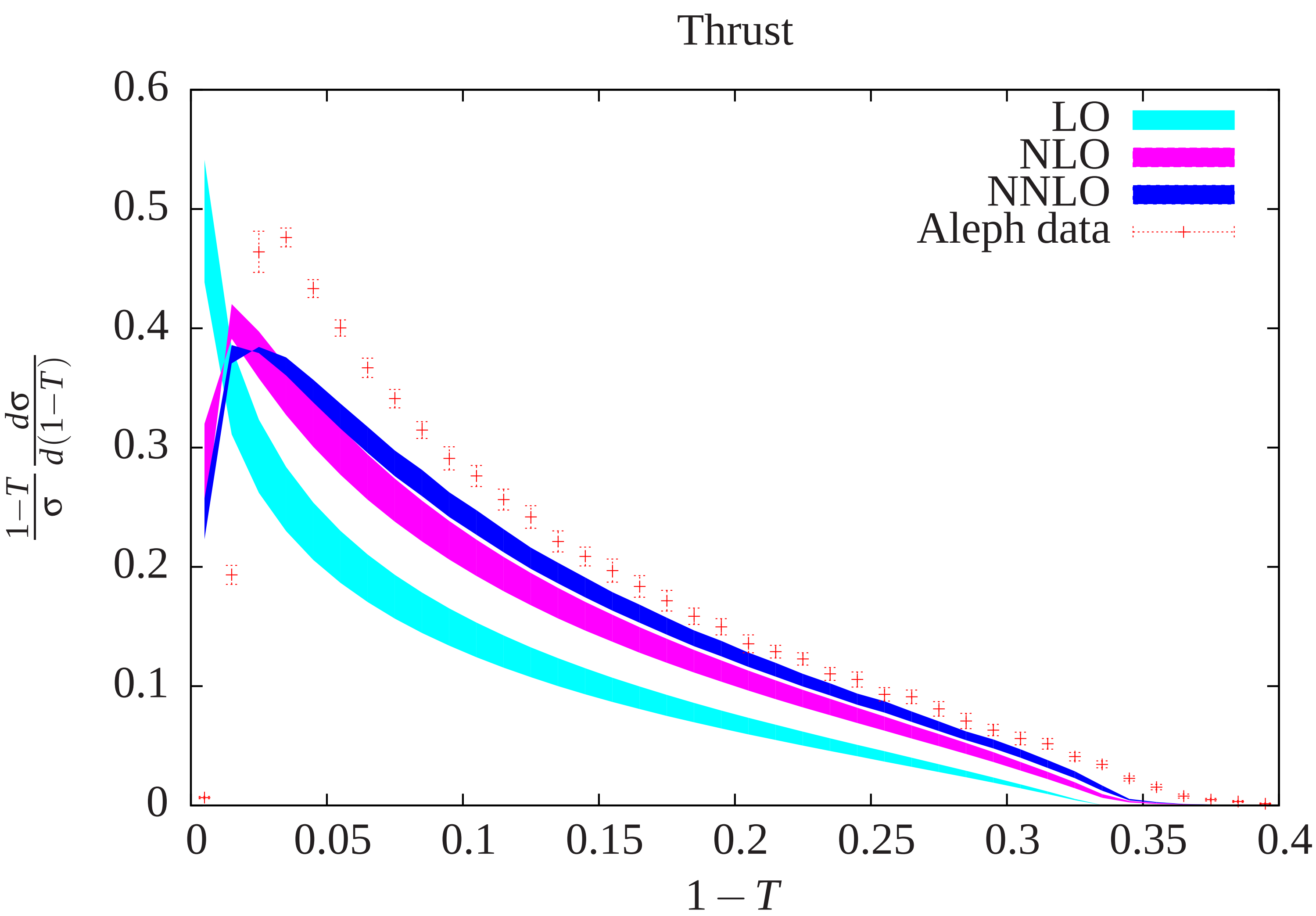
Extreme two-jet region

Results for the three-jet rate in electron-positron annihilation



S.W., Phys.Rev.Lett.101:162001,2008

Results for the thrust distribution



S.W., JHEP 0906, (2009), 041

Further refinements

Soft-gluon resummation: Perturbative expansion is of the form

$$1 + c_0 \alpha_s + c_1 \alpha_s \ln y_{cut} + c_2 \alpha_s \ln^2 y_{cut} + O(\alpha^2)$$

In the region where $\alpha_s \ln^2 y_{cut} \approx 1$ resum the large logarithms.

Catani, Trentadue, Turnock, Webber, ’93; Becher, Schwartz, ’08

Power corrections: From the operator product expansion we expect power corrections of the form

$$\frac{\lambda}{Q} + O\left(\frac{1}{Q^2}\right)$$

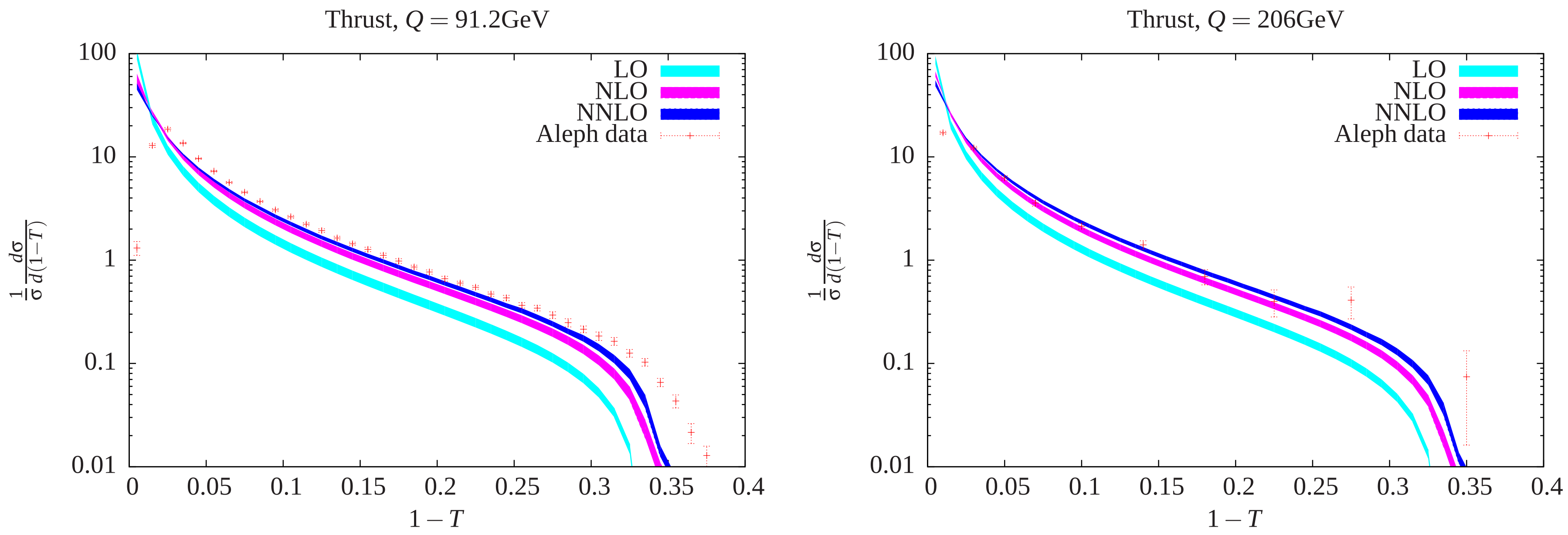
Dokshitzer, Webber, ’97; Davison, Webber, ’08

Electroweak corrections: At the per-cent level the $O(\alpha^3 \alpha_s)$ electroweak corrections are equally important.

Carloni-Calame, Moretti, Piccinin, Ross, ’09; Denner, Dittmaier, Gehrmann, Kurz, ’09

Results for the thrust distribution

Changing the centre-of-mass energy:



Moments

$$\langle O^n \rangle = \frac{1}{\sigma_{tot}} \int_0^1 O^n \frac{d\sigma}{dO} dO.$$

Moments for $n \geq 2$ unproblematic.

The first moment can receive sizeable corrections from the two-jet region.

First moment for thrust:

$$\langle 1 - T \rangle_{NNLO} = \underbrace{(1.076 \pm 0.001) \cdot 10^3}_{1-T>10^{-3}} + \underbrace{(0.029 \pm 0.025) \cdot 10^3}_{1-T<10^{-3}} = (1.10 \pm 0.03) \cdot 10^3$$

Worst case: First moment of the wide jet broadening

$$\langle B_W \rangle_{NNLO} = \underbrace{(0.356 \pm 0.004) \cdot 10^3}_{B_W>10^{-3}} + \underbrace{(1.7 \pm 1.2) \cdot 10^3}_{B_W<10^{-3}} = (2.1 \pm 1.2) \cdot 10^3$$

Summary

- α_s is one of the fundamental parameters of nature
- Error on α_s dominated by theory
- NNLO calculations reduce the theoretical uncertainty

  Re-analysis of JADE data, ...
- Calculational techniques developed for $e^+e^- \longrightarrow 3$ jets can be applied to other processes