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# Multi-Loop Calculations in the MSSM

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in collaboration with

R. Harlander, P. Kant, J. Salomon, M. Steinhauser



# Outline

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- Motivation
- Framework
- Precision tests of MSSM
  - $\alpha_s(M_{\text{GUT}})$  and  $m_b(M_{\text{GUT}})$  with **3-loop** accuracy
  - $m_h$ : **3-loop** SQCD corrections
- Conclusions

# Motivation

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Precision tests of the MSSM

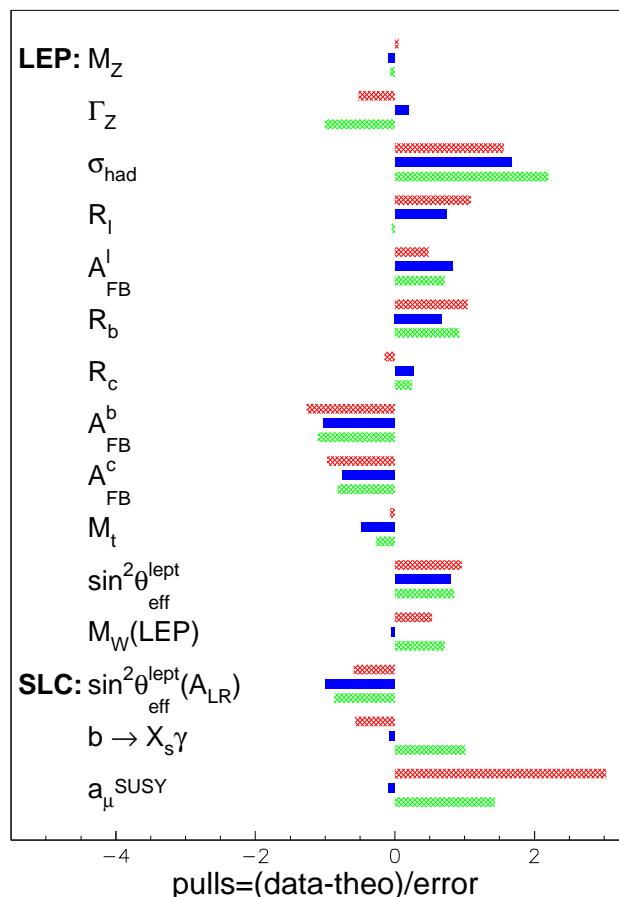
# Motivation

Precision tests of the MSSM

- Electroweak precision data

W. de Boer & C. Sander '03

SM:  $\chi^2/\text{d.o.f} = 21.0/16$   
MSSM:  $\chi^2/\text{d.o.f} = 10.1/12$   
CMSSM:  $\chi^2/\text{d.o.f} = 17.1/16$



# Motivation

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Precision tests of the MSSM

- LHC& ILC: **2-loops** SQCD and SEW corrections
  - $M_W$  and  $\sin \theta_{\text{eff}}$  [Chankowski et al '94], [Djouadi et al '96], [Heinemeyer & Weiglein '02,'04], ...
  - $a_\mu$  [Moroi '96], [Degrassi & Giudice '98], [Heinemeyer,Stöckinger,Weiglein '03,'04], ...
  - $b \rightarrow s\gamma$  [Gordbahn,Haisch,Misiak'05], [Misiak & Steinhauser'06], [Czakon,Haisch,Misiak'06], [Misiak et al'07], ...
  - $m_h$ : **+ 3-loop SQCD** (see next)

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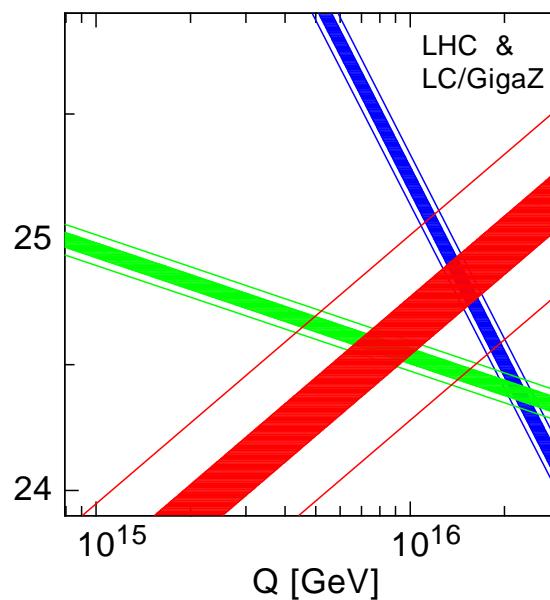
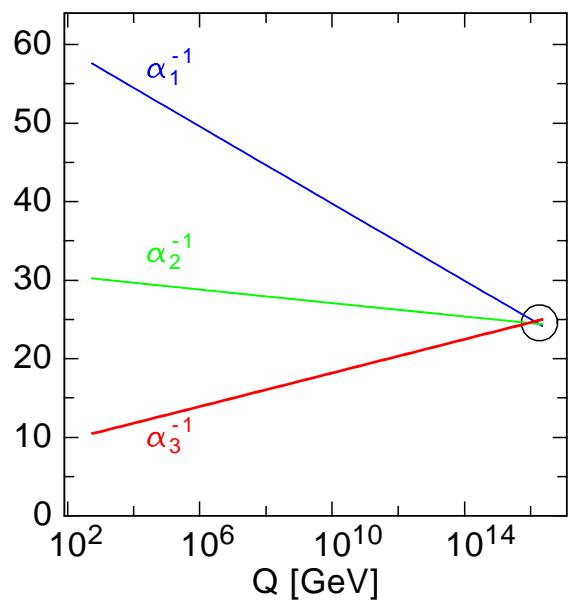
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  - unification of couplings [ISAJET](#), [SuSpect](#), [SPHENO](#), [SOFTSUSY](#)

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[Blair, Porod, Zerwas '04]

# Framework

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Regularization Scheme: gauge and SUSY invariant

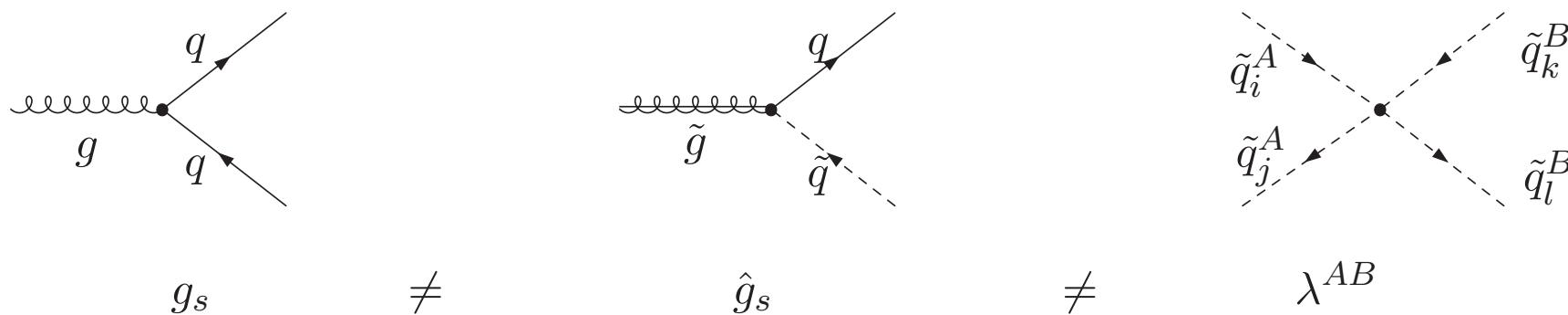
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SQCD & DREG



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SQCD & DRED

The diagram illustrates four Feynman diagrams representing different regularization schemes:

- Diagram 1: A gluon line labeled  $g$  and a quark line labeled  $q$  meet at a vertex.
- Diagram 2: A gluon line labeled  $\tilde{g}$  and a quark line labeled  $q$  meet at a vertex, with a dashed line labeled  $\tilde{q}$  extending from the vertex.
- Diagram 3: Four dashed lines labeled  $\tilde{q}_i^A$ ,  $\tilde{q}_j^A$ ,  $\tilde{q}_k^B$ , and  $\tilde{q}_l^B$  meet at a central vertex.
- Diagram 4: A dashed line labeled  $\varepsilon$  and a quark line labeled  $q$  meet at a vertex.

Below the diagrams, the corresponding coupling constants are equated:

$$g_s = \hat{g}_s = \lambda^{AB} = g_e$$

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SQCD & DRED

The figure shows four Feynman diagrams illustrating the relationship between the coupling constant  $g_s$ , the supercoupling constant  $\hat{g}_s$ , the supercharge  $\lambda^{AB}$ , and the electric coupling constant  $g_e$ .

The first diagram shows a gluon line (wavy line) with momentum  $q$  and a quark line with momentum  $q$ . The coupling is labeled  $g$ .

The second diagram shows a gluon line (wavy line) with momentum  $q$  and a gluino line with momentum  $\tilde{q}$ . The coupling is labeled  $\tilde{g}$ .

The third diagram shows a quark loop with two external gluino lines. The top line is  $\tilde{q}_i^A$  and the bottom line is  $\tilde{q}_j^A$ . The right line is  $\tilde{q}_k^B$  and the left line is  $\tilde{q}_l^B$ . The coupling is labeled  $\lambda^{AB}$ .

The fourth diagram shows a quark line with momentum  $q$  and a gluino line with momentum  $\tilde{q}$ . The coupling is labeled  $\varepsilon$ .

Below the diagrams, the equations are:

$$g_s = \hat{g}_s = \lambda^{AB} = g_e$$

- 1- and 2-loops: [[Jack, Jones '94](#)], [[Martin, Vaughn '94](#)], [[Yamada '94](#)], [[Hollik, Stöckinger'05](#)]

# Framework

Regularization Scheme: gauge and SUSY invariant

- Dimensional Regularization (DREG) [['t Hooft and Veltmann'72](#)] **breaks SUSY**
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SQCD & DRED

The diagram shows four Feynman-like diagrams side-by-side. The first two are for SQCD: the left one shows a gluon (wavy line) interacting with a quark-antiquark pair (q-qbar), and the right one shows a gluino-gluino interaction (tilde g-tilde g-bar). The third diagram is for DRED and shows a vertex where three gluinos (tilde q\_i^A, tilde q\_j^A, tilde q\_k^B) interact via dashed lines. The fourth diagram is for DRED and shows a vertex where a gluon (q) and a gluino (tilde q) interact via dashed lines.

$$g_s = \hat{g}_s = \lambda^{AB} = g_e$$

- 1- and 2-loops: [[Jack, Jones '94](#)], [[Martin, Vaughn '94](#)], [[Yamada '94](#)], [[Hollik, Stöckinger'05](#)]

This talk:  $g_s = \hat{g}_s = g_e$  at **3-loops** [[Harlander, L.M., Steinhauser '09](#) ]

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Regularization Scheme: gauge and SUSY invariant

- Dimensional Regularization (DREG) [['t Hooft and Veltmann'72](#)] **breaks SUSY**
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- DREG-DRED parameter conversion
  - 1-loop MSSM [[Martin, Vaughn '94](#)], [[Beenaker, Hopker, Zervas '96](#)]
  - 2-loop SQCD [[L.M. '09](#)]

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$$\alpha_s^{\overline{\text{MS}}} = \alpha_s^{\overline{\text{DR}}} \left[ 1 - \frac{\alpha_s^{\overline{\text{DR}}}}{\pi} \frac{C_A}{3} + \left( \frac{\alpha_s^{\overline{\text{DR}}}}{\pi} \right)^2 \left( -\frac{11}{9} C_A^2 + 2T_F N_f C_F \right) \right],$$

$$m_q^{\overline{\text{MS}}} = m_q^{\overline{\text{DR}}} \left[ 1 + \frac{\alpha_s^{\overline{\text{DR}}}}{\pi} C_F + \left( \frac{\alpha_s^{\overline{\text{DR}}}}{\pi} \right)^2 \left( \frac{7}{12} C_A C_F + \frac{7}{4} C_F^2 - \frac{1}{2} C_F T_F N_f \right) \right]$$

# Renormalization Constants

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$$Z_{g_i} = \frac{Z_{\phi_1 \phi_2 \phi_3}^{(i)}}{\sqrt{Z_{\phi_1} Z_{\phi_2} Z_{\phi_3}}}$$

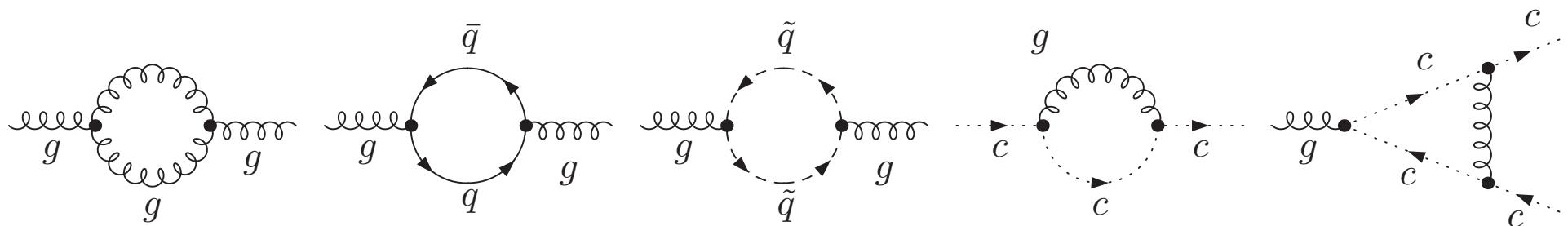
- Minimal Subtraction Scheme & DRED ( $\overline{\text{DR}}$ )

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1-loop:

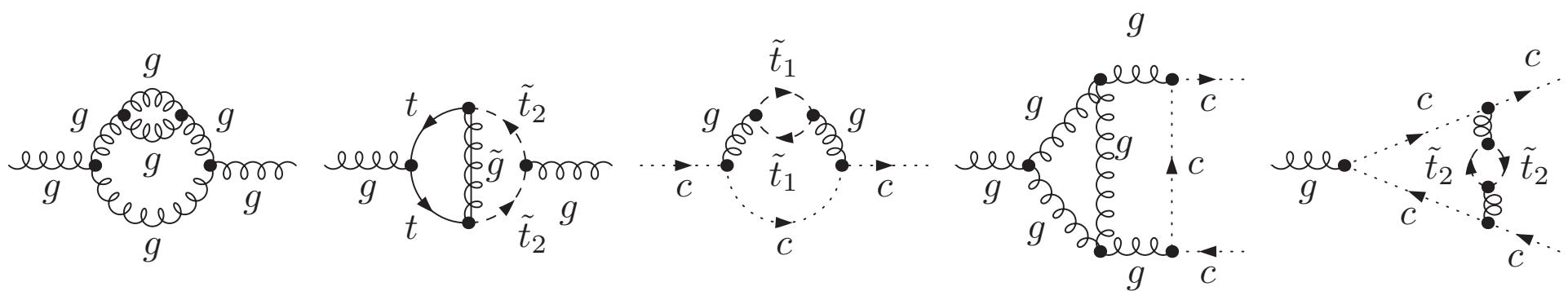


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2-loops:

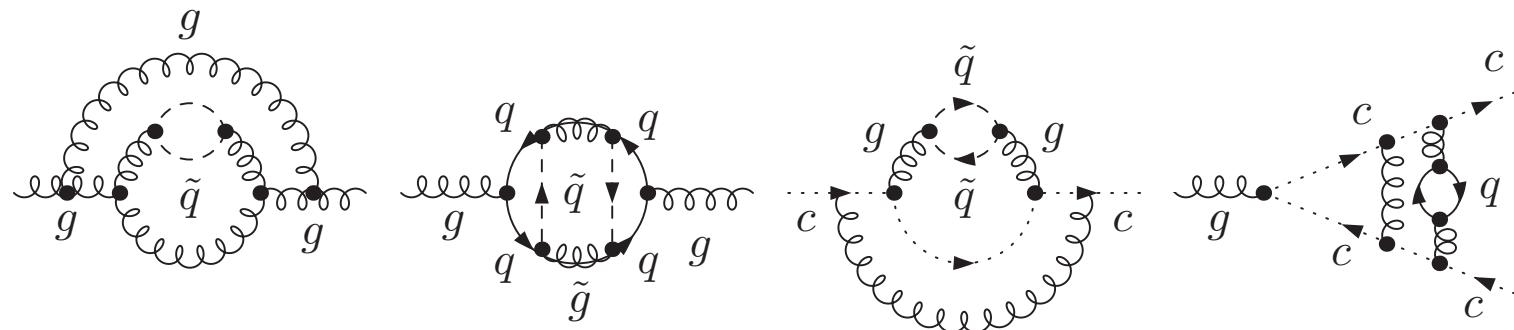


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# Renormalization Constants

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$$Z_{g_i} = \frac{Z_{\phi_1 \phi_2 \phi_3}^{(i)}}{\sqrt{Z_{\phi_1} Z_{\phi_2} Z_{\phi_3}}}$$

- Minimal Subtraction Scheme & DRED ( $\overline{\text{DR}}$ )
- 3-loop computation
  - $\simeq 200.000$  diagrams
  - Computer programs: QGRAF, FORM, MINCER, MATAD, EXP, ...  
[Noguiera; Vermaseren; Larin, Tkachov; Steinhauser; Seidensticker, Harlander; ...]

# Renormalization Constants

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$$Z_{g_i} = \frac{Z_{\phi_1 \phi_2 \phi_3}^{(i)}}{\sqrt{Z_{\phi_1} Z_{\phi_2} Z_{\phi_3}}}$$

- Minimal Subtraction Scheme & DRED ( $\overline{\text{DR}}$ )
- 3-loop anomalous dimensions in SQCD [Harlander, L.M., Steinhauser '09 ]

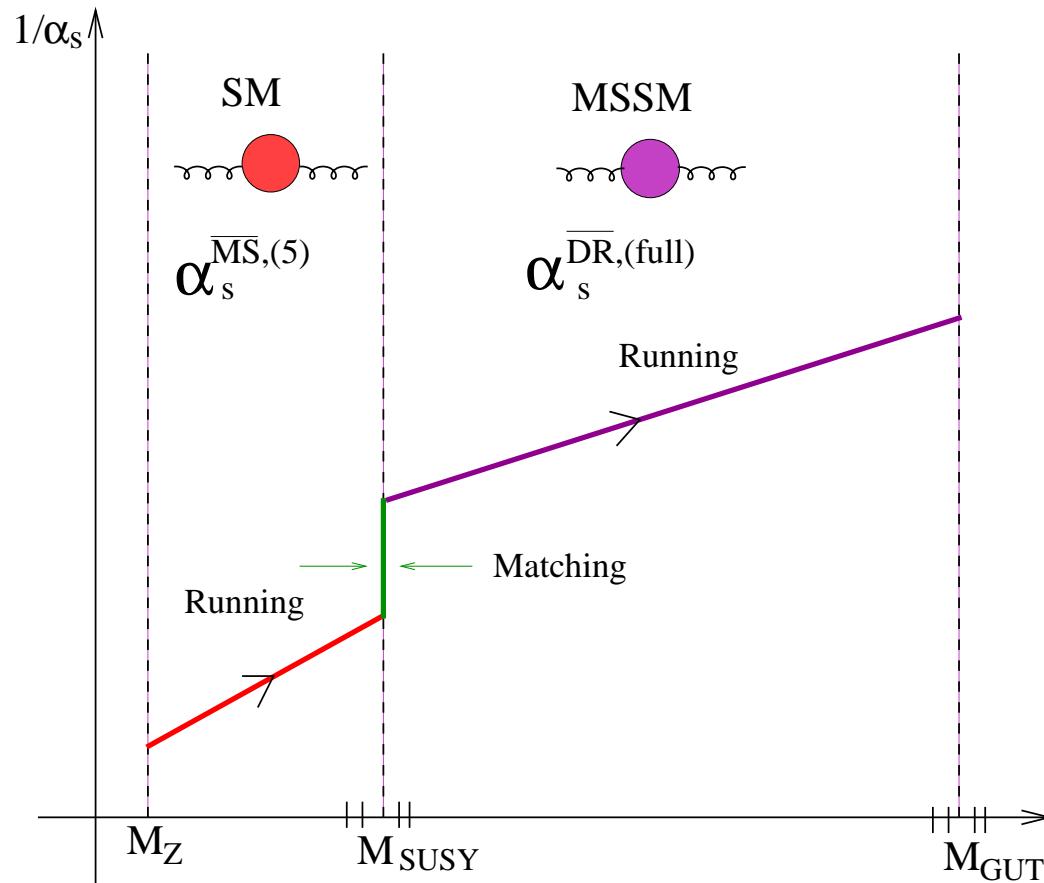
$$\mu^2 \frac{d}{d\mu^2} \alpha_s(\mu) = \beta(\alpha_s) \quad \beta(\alpha_s) = - \sum_{n \geq 0} \left( \frac{\alpha_s}{\pi} \right)^{n+2} \beta_n$$

$$\begin{aligned} \beta_0 &= \frac{3}{4} C_A - \frac{1}{2} T_f, \quad C_F = 4/3, C_A = 3, 2T_f = n_f, \\ \beta_1 &= \frac{3}{8} C_A^2 - T_f \left( \frac{1}{2} C_F + \frac{1}{4} C_A \right), \\ \beta_2 &= \frac{21}{64} C_A^3 + T_f \left( \frac{1}{4} C_F^2 - \frac{13}{16} C_A C_F - \frac{5}{16} C_A^2 \right) + T_f^2 \left( \frac{3}{8} C_F + \frac{1}{16} C_A \right). \end{aligned}$$

- complete **agreement** with [Jack, Jones, North '96], [Pickering, Gracey, Jones '01]

# Evolution of the couplings

Effective Field Theory:

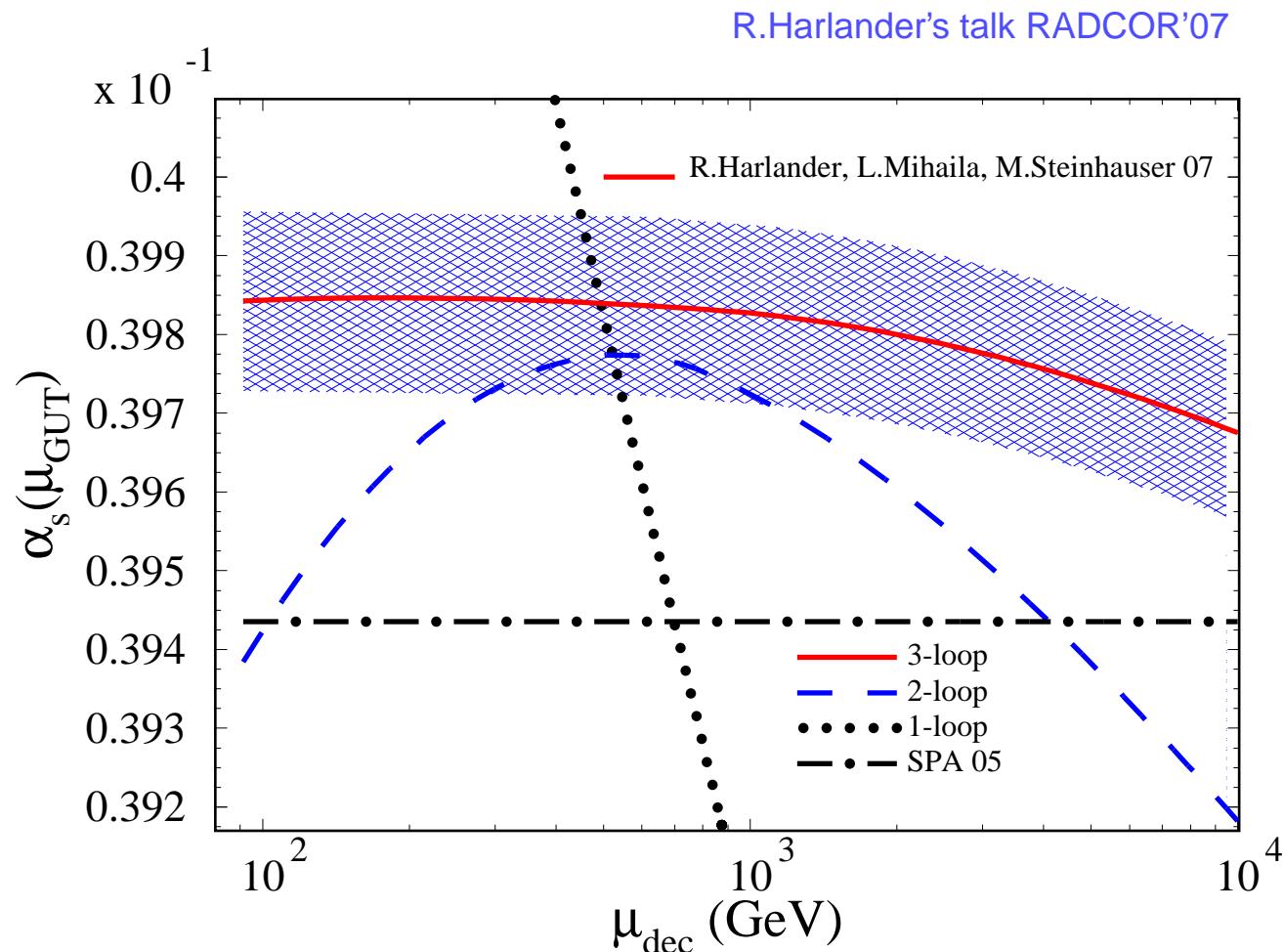


Matching: 1-loop [Pierce et al '95]

2-loops [R. Harlander, L. M., M. Steinhauser '05, '07] [A. Bauer, L. M., J. Salomon '08]

# $\alpha_s(M_{\text{GUT}})$

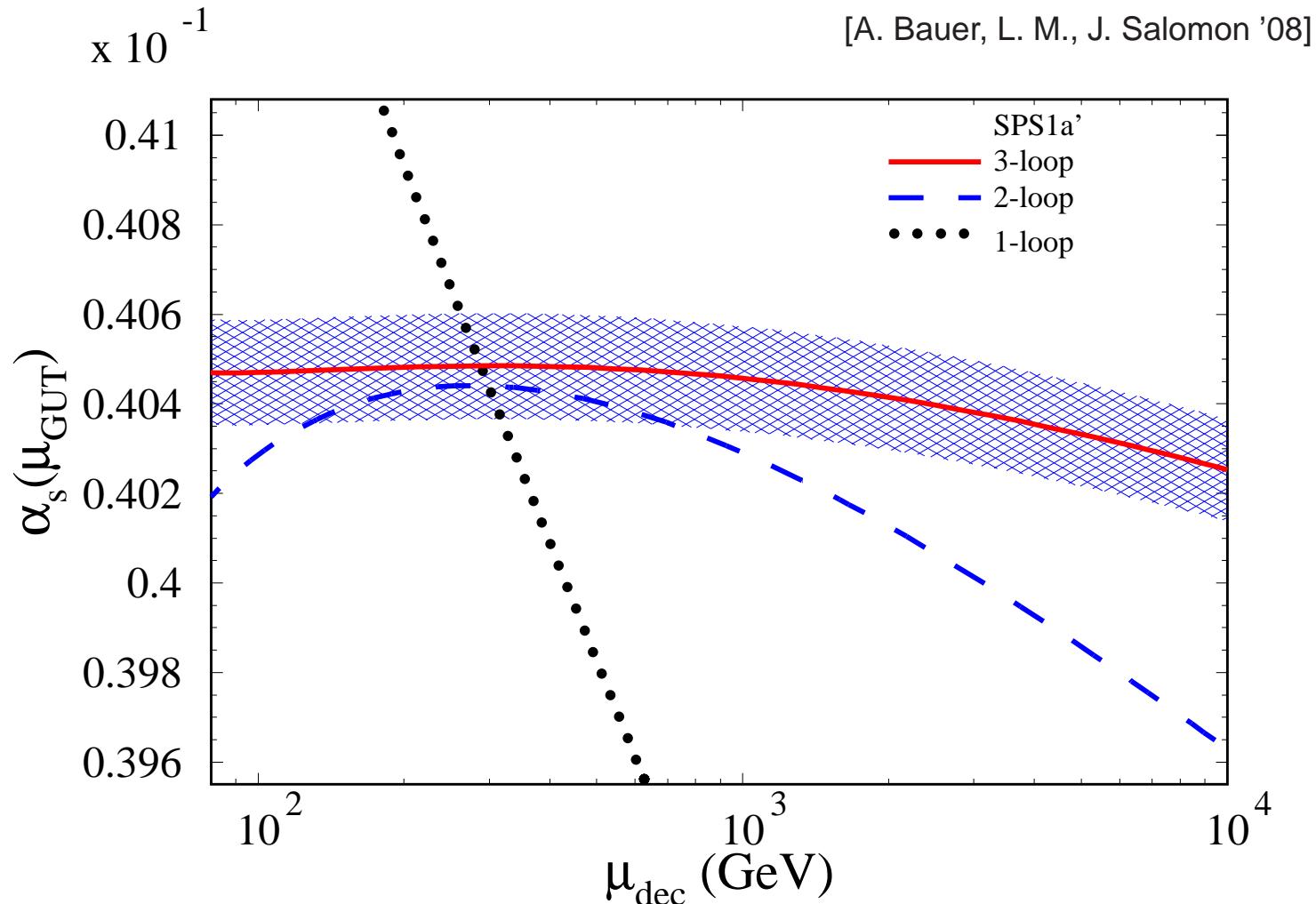
Input:  $\alpha_s^{\overline{\text{MS}},(5)}(M_Z) = 0.1189 \pm 0.001$  [Bethke '06],  $M_Z = 91.1876 \text{ GeV}$ ,  
 $\tilde{M} = m_{\tilde{q}} = m_{\tilde{g}} = 1000 \text{ GeV}$



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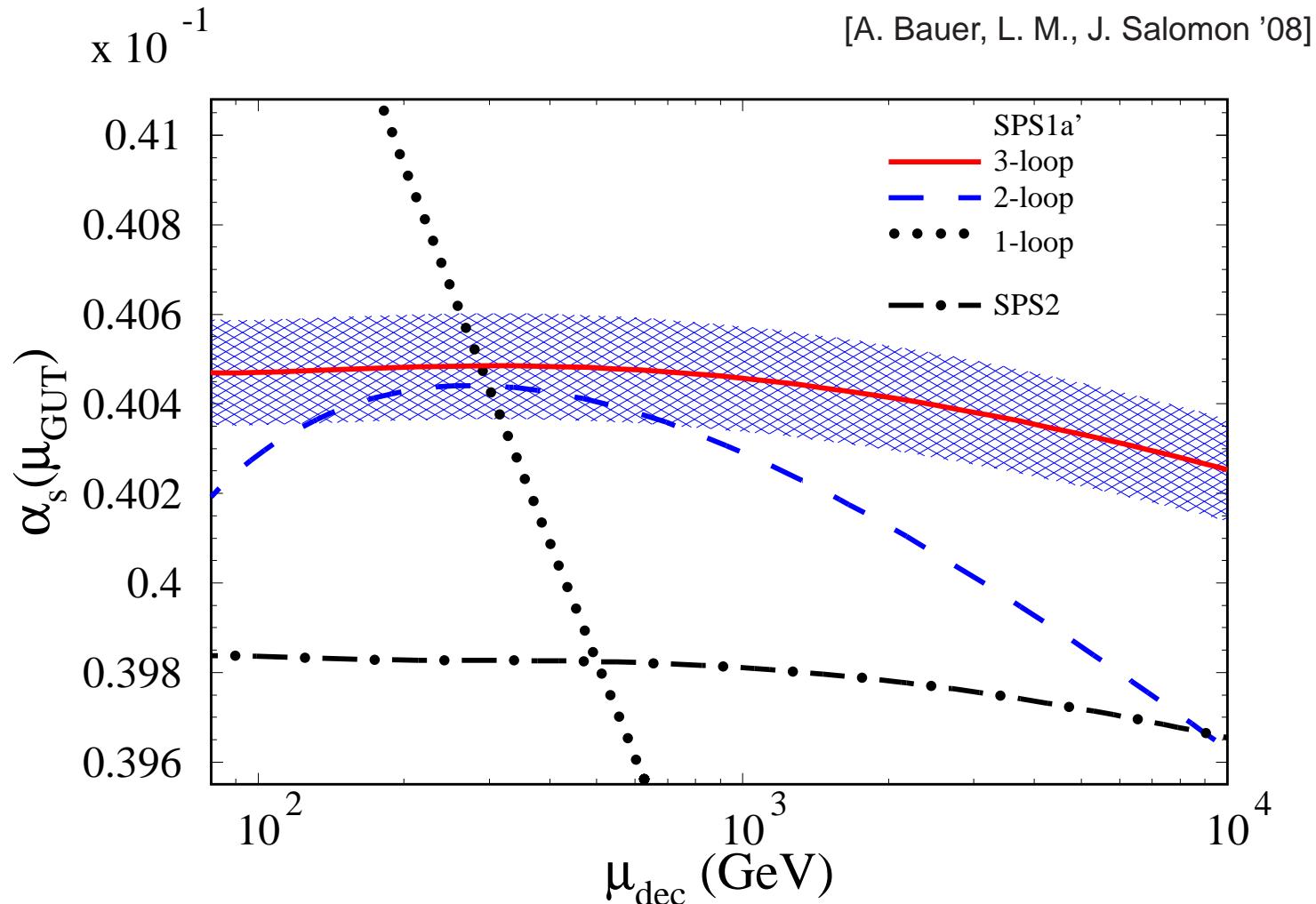
MSSM parameters: SPS1a' point



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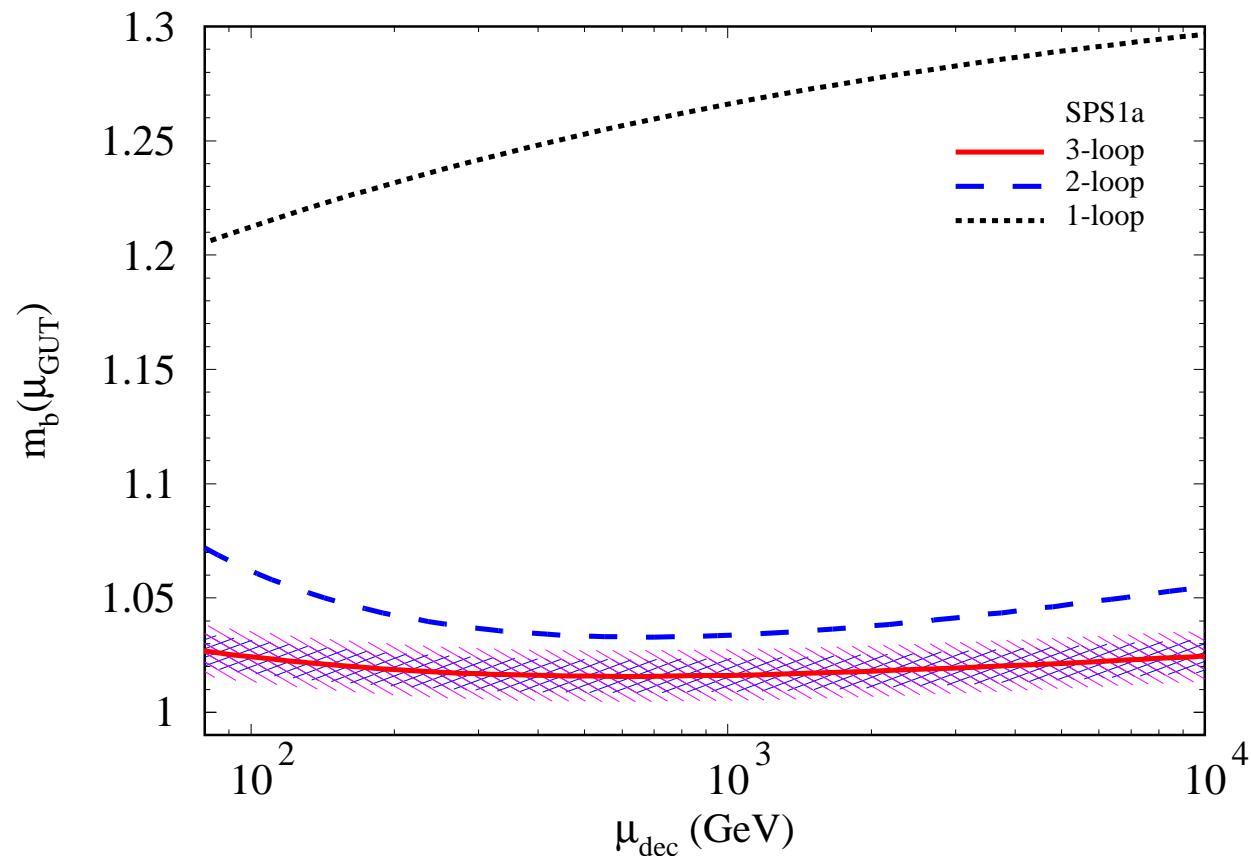


# $m_b(M_{\text{GUT}})$

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 $m_b^{\overline{\text{MS}}}(m_b) = 4.164 \pm 0.025 \text{ GeV}$  [Kühn, Steinhauser, Sturm '07]

MSSM parameters: SPS1a' point ( $\tan \beta = 10$ )

[A. Bauer, L. M., J. Salomon '08]

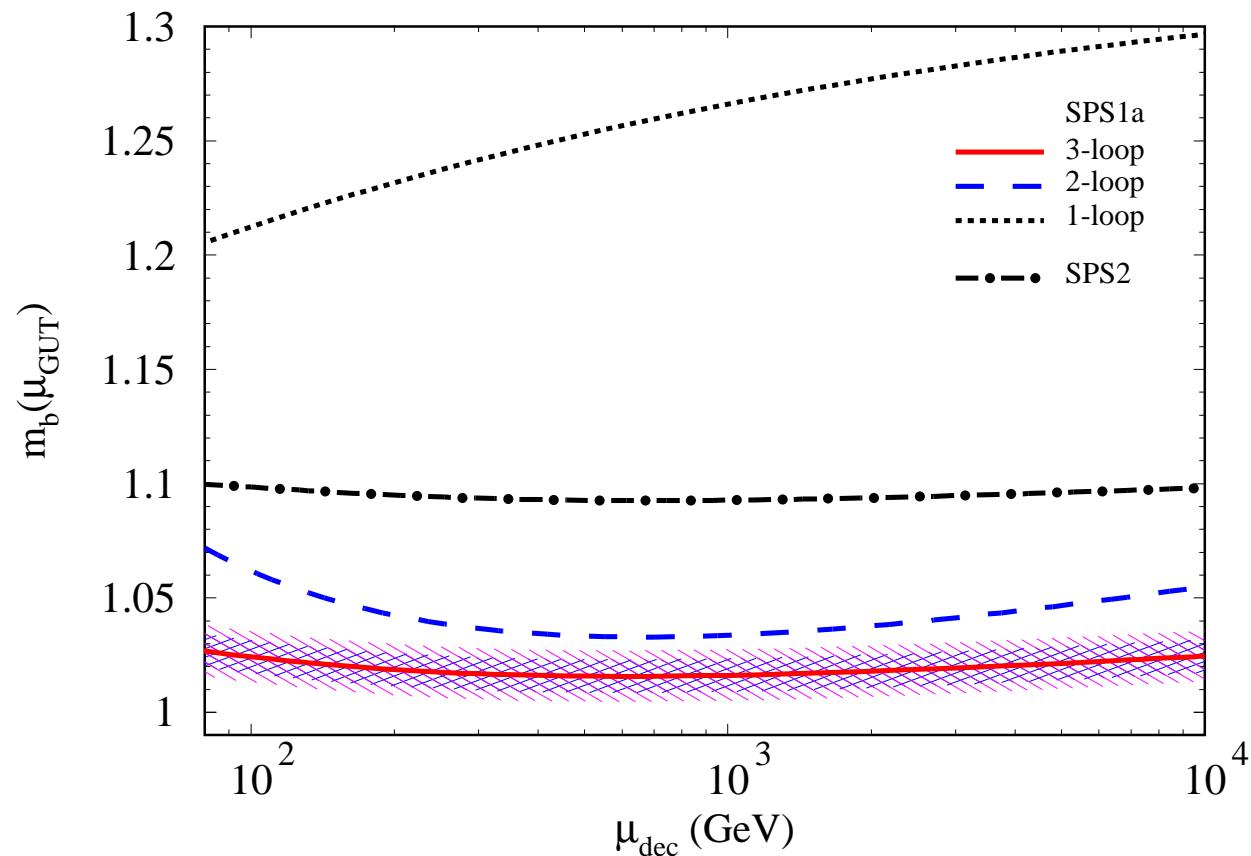


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# Light Higgs boson mass in the MSSM

# Motivation

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Experiment :  $\delta m_h^{\text{exp}} = 100 - 200 \text{ MeV}$  CERN-LHCC-2006-21

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Theory:

- exact 1-loop [Chankowski, Pokorski and Rosiek '92], [Brignole '92], [Dabelstein '94]
- 2-loop  $\mathcal{O}(\alpha_t \alpha_s, \alpha_t^2, \alpha_b \alpha_s, \alpha_b \alpha_t)$  in effective potential approximation ( $p^2 = 0$ )  
[Haber, Hempfling, Hoang '96], [Heinemeyer, Hollik and Weiglein '98], [Degrassi, Slavich, Zwirner '01], [Espinosa and Zang '00], [Brignole, Degrassi, Slavich, Zwirner '02] , [Carena et al '00], [Heinemeyer et al '05] , [S. Martin '03]
- $p^2 = m_h^2$ : 2-loop SUSY-QCD [S.Martin '05]
- 3-loop LL and NLL  $\mathcal{O}(\alpha_t \alpha_s^2, \alpha_t^2 \alpha_s, \alpha_t^3)$  [S. Martin '07]
- Missing contributions:  $\delta m_h^{\text{th}} \simeq 3 - 5 \text{ GeV}$  [G. Degrassi et al '02], [Allanach et al '04]

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This talk: 3-loop SUSY-QCD corrections

# Framework

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SUSY  $\Rightarrow$  two free parameters:  $\tan \beta = v_2/v_1$ ,  $M_A^2 = -m_{12}^2(\tan \beta + \cot \beta)$

CP-even Higgs  $\phi_{1,2}$ :

$$\mathcal{M}_{H,\text{tree}}^2 = \frac{\sin 2\beta}{2} \times \begin{pmatrix} M_Z^2 \cot \beta + M_A^2 \tan \beta & -M_Z^2 - M_A^2 \\ -M_Z^2 - M_A^2 & M_Z^2 \tan \beta + M_A^2 \cot \beta \end{pmatrix}$$

# Framework

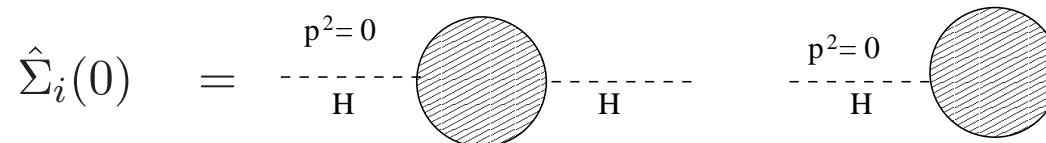
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Higher order corrections

$$\mathcal{M}_H^2 = \mathcal{M}_{H,\text{tree}}^2 - \begin{pmatrix} \hat{\Sigma}_{\phi_1} & \hat{\Sigma}_{\phi_1\phi_2} \\ \hat{\Sigma}_{\phi_1\phi_2} & \hat{\Sigma}_{\phi_2} \end{pmatrix}$$

$V_{\text{eff}}$ -approximation:  $p^2 = 0 \Rightarrow \hat{\Sigma}_i(0) = \Sigma_i(0) - \delta V_i$



# Framework (2)

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Approximations:  $\mathcal{O}(\alpha_t \alpha_s^2)$  corrections  $\rightsquigarrow \mathcal{O}(M_{\text{top}}^4)$

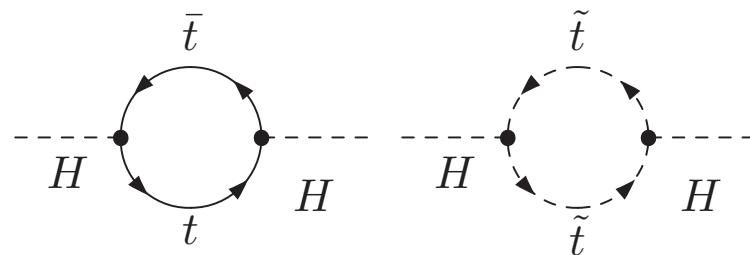
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Computation of  $\hat{\Sigma}_{\phi_{ij}}(0)$  at 3-loops:

1-loop:



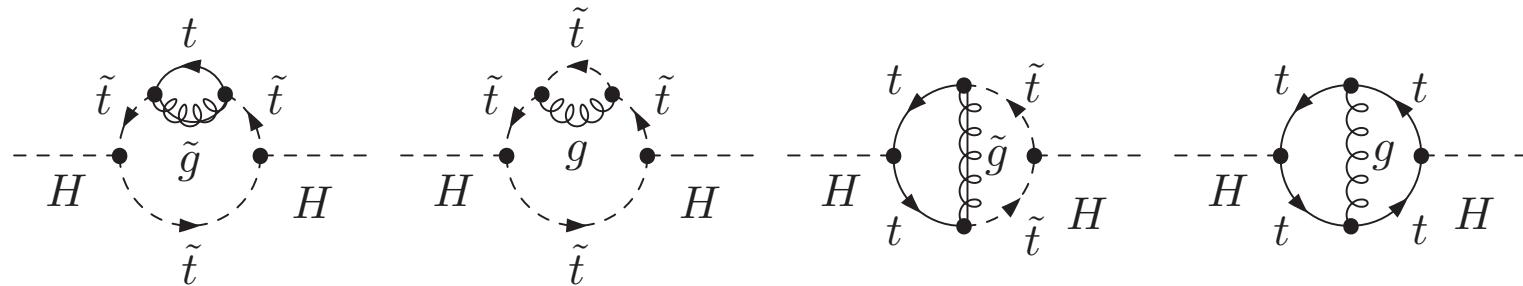
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Computation of  $\hat{\Sigma}_{\phi_{ij}}(0)$  at 3-loops:

2-loops:



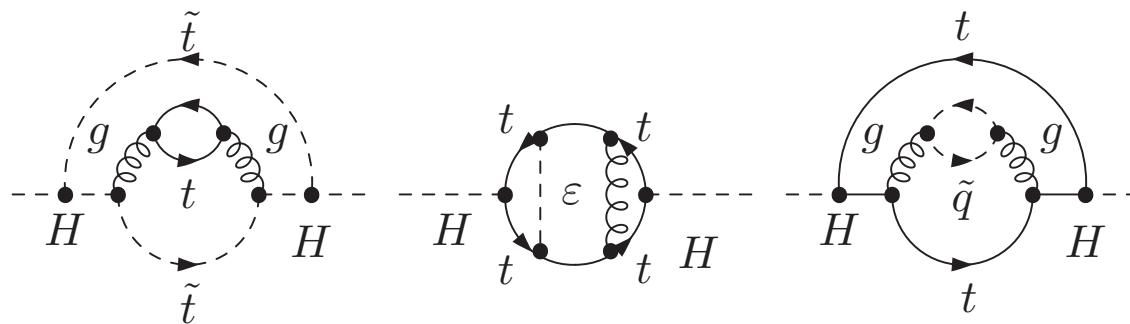
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- $\simeq 28.000$  diagrams

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$$m_t \ll m_{\tilde{t}_1} \approx m_{\tilde{t}_2} \approx m_{\tilde{g}} \approx m_{\tilde{q}}$$

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Computation of  $\hat{\Sigma}_{\phi_{ij}}(0)$  at 3-loops:

- $\simeq 28.000$  diagrams
- Assume fixed mass hierarchies  $\rightsquigarrow$  asymptotic expansions
- Renormalization Scheme:  $\overline{\text{DR}}$  -scheme
  - $\alpha_s$  in  $\overline{\text{DR}}$  to 1-loop
  - $M_t, M_{\tilde{t}_1}, M_{\tilde{t}_2}$  in  $\overline{\text{DR}}$  to 2-loops
  - $M_{\tilde{g}}$  in  $\overline{\text{DR}}$  to 1-loop
  - $\theta_{\tilde{t}}$  in  $\overline{\text{DR}}$  to 2-loops
  - $M_\varepsilon$  in OS to 1-loop

Agreement with

[Pierce, Bagger, Matchev and Zahng '96], [Jack and Jones '94], [Martin and Vaughn '94], [S. Martin '03,'05]

# Framework (2)

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Approximations:  $\mathcal{O}(\alpha_t \alpha_s^2)$  corrections  $\rightsquigarrow \mathcal{O}(M_{\text{top}}^4)$

Computation of  $\hat{\Sigma}_{\phi_{ij}}(0)$  at 3-loops:

- $\simeq 28.000$  diagrams
- Assume fixed mass hierarchies  $\rightsquigarrow$  asymptotic expansions
- Renormalization Scheme:  $\overline{\text{DR}}$  -scheme
- Computer programs: QGRAF, PERL, FORM, MINCER, MATAD, EXP, ...

[Noguiera; Vermaseren; Harlander; Larin, Tkachov; Steinhauser; Seidensticker, Harlander; ... ]

# Numerical Results (no stop-mixing)

---

Input SM parameters:  $\mu = M_t = 172.4 \text{ GeV}$   $G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2}$   
 $M_Z = 91.1876 \text{ GeV}$   $\alpha_s^{(5)}(M_Z) = 0.1189 \Rightarrow \alpha_s(M_t) = 0.0926$

MSSM parameters:  $M_A = 1 \text{ TeV}$   $\tan \beta = 40$   $A_t = 0$   $M_{\tilde{q}} = 2 \text{ TeV}$   $M_{\tilde{t}_2} = M_{\tilde{t}_1} = M_{\tilde{g}} = M_{\text{SUSY}}$

Renormalization scheme dependence

# Numerical Results (no stop-mixing)

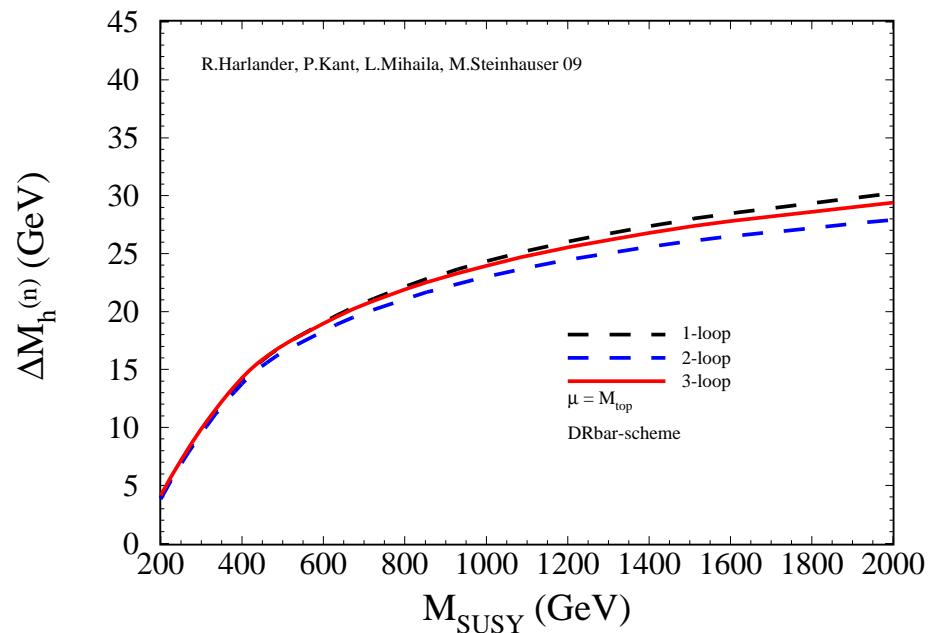
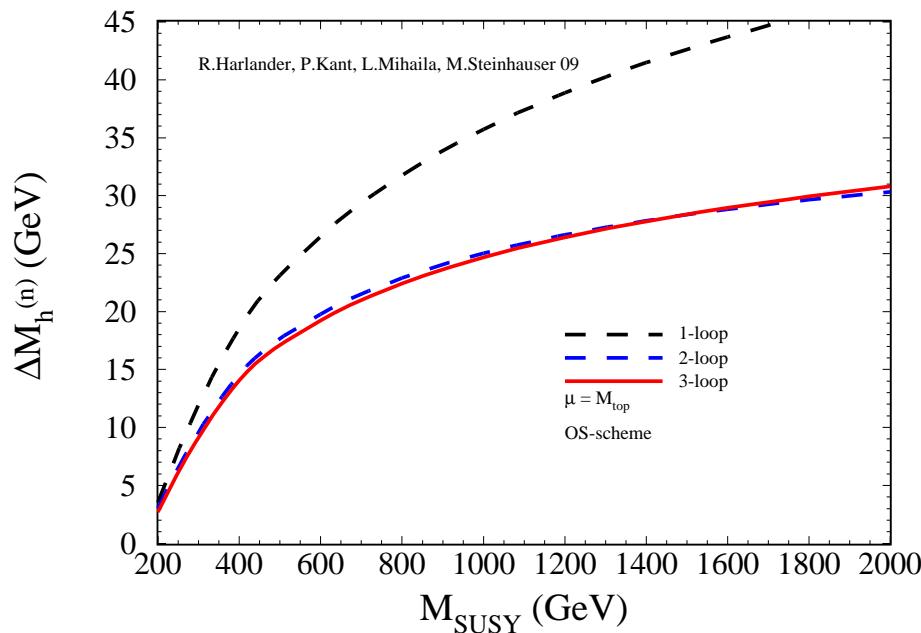
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MSSM parameters:  $M_A = 1 \text{ TeV}$   $\tan \beta = 40$   $A_t = 0$   $M_{\tilde{q}} = 2 \text{ TeV}$   $M_{\tilde{t}_2} = M_{\tilde{t}_1} = M_{\tilde{g}} = M_{\text{SUSY}}$

Renormalization scheme dependence:  $\Delta M_h^{(n)} \equiv M_h^{(n-\text{loop})} - M_h^{\text{tree}}$

OS:  $\delta M_h^{(3)} \simeq 500 \text{ MeV}$

$\overline{\text{DR}}$ :  $\delta M_h^{(3)} \simeq 1.2 \text{ GeV}$



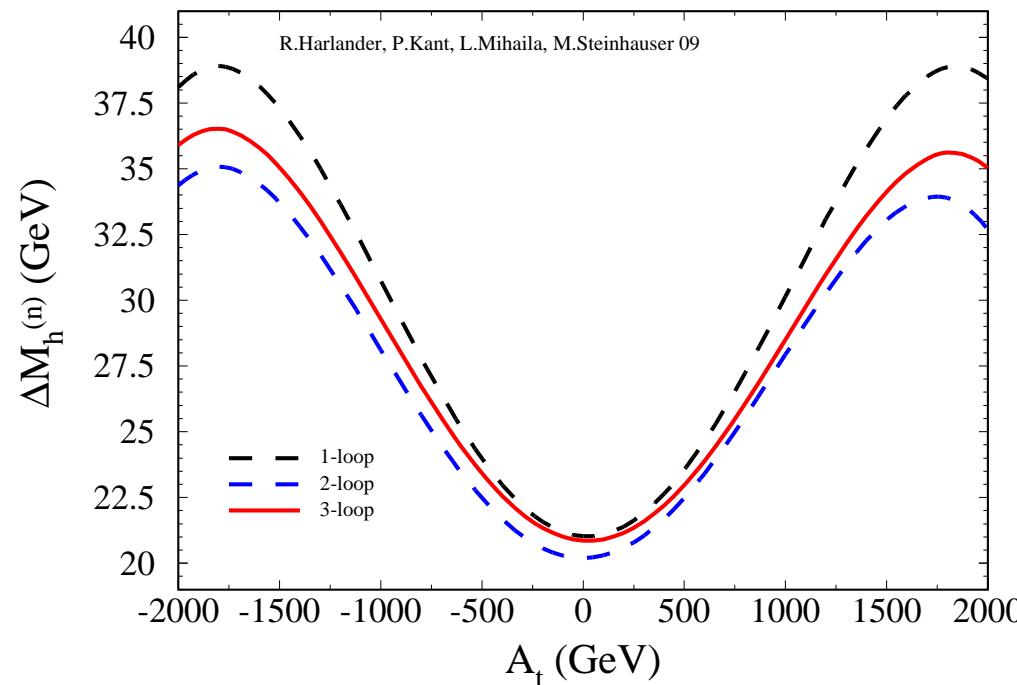
# Numerical Results (stop-mixing)

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MSSM parameters:  $M_A = 1 \text{ TeV}$   $\tan \beta = 40$   $M_{\tilde{q}} = 2 \text{ TeV}$   
 $m_{\tilde{t}_2}(\mu) = 1 \text{ TeV}$   $m_{\tilde{t}_1}(\mu) = m_{\tilde{g}}(\mu) = 0.5 \text{ TeV}$

$\overline{\text{DR}}$  scheme

$$A_t = 0 : \pm 2 \text{ TeV} : \Delta M_h^{(3)} = 0.5 - 1.5 \text{ GeV}$$



# Numerical Results: SPS2

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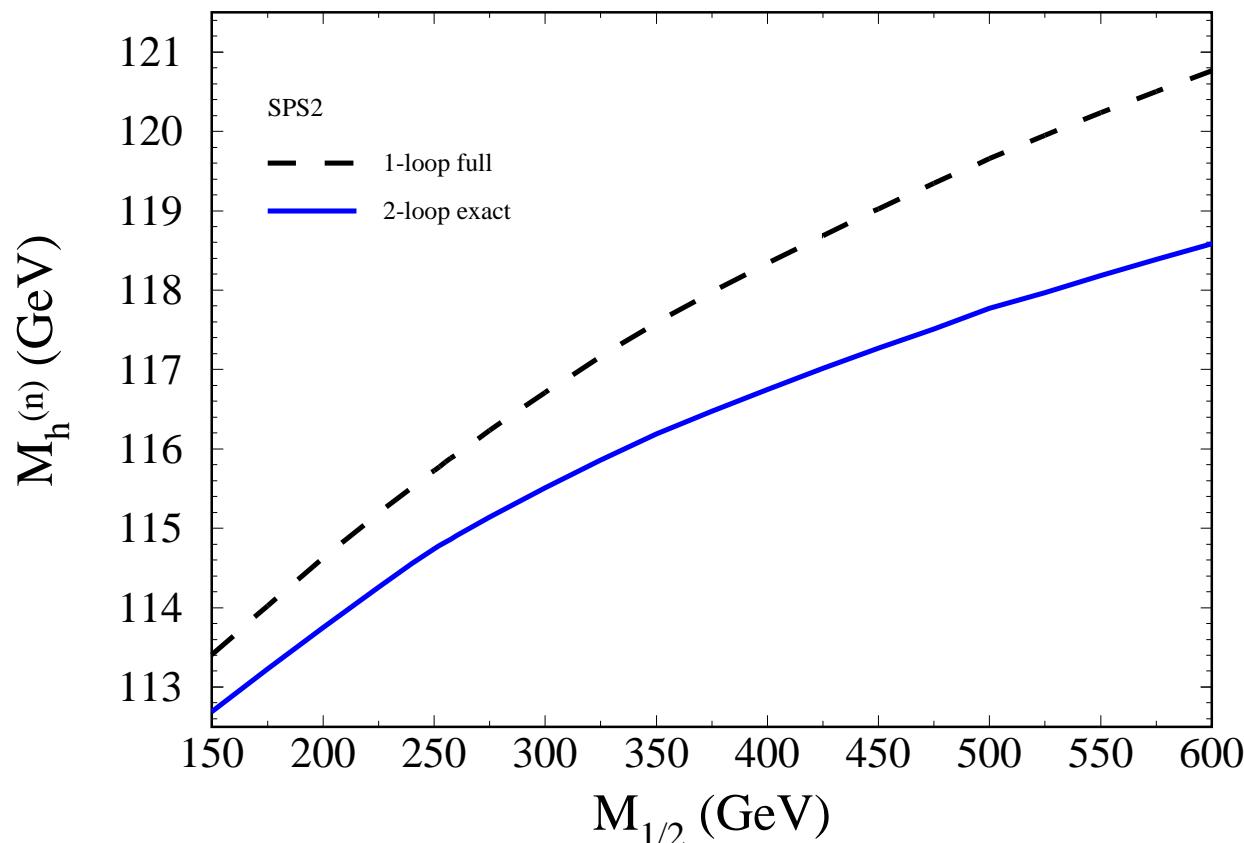
Input SM parameters:  $\mu = M_t = 172.4 \text{ GeV}$   $G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2}$   
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MSSM parameters: SOFTSUSY [W. Allanach '01](#)  $m_t^{\overline{\text{DR}}}(m_t)$ : TSILL [S. Martin '03](#)

$m_h$  (1-loop): FeynHiggs [S. Heinemeyer et al '05](#)

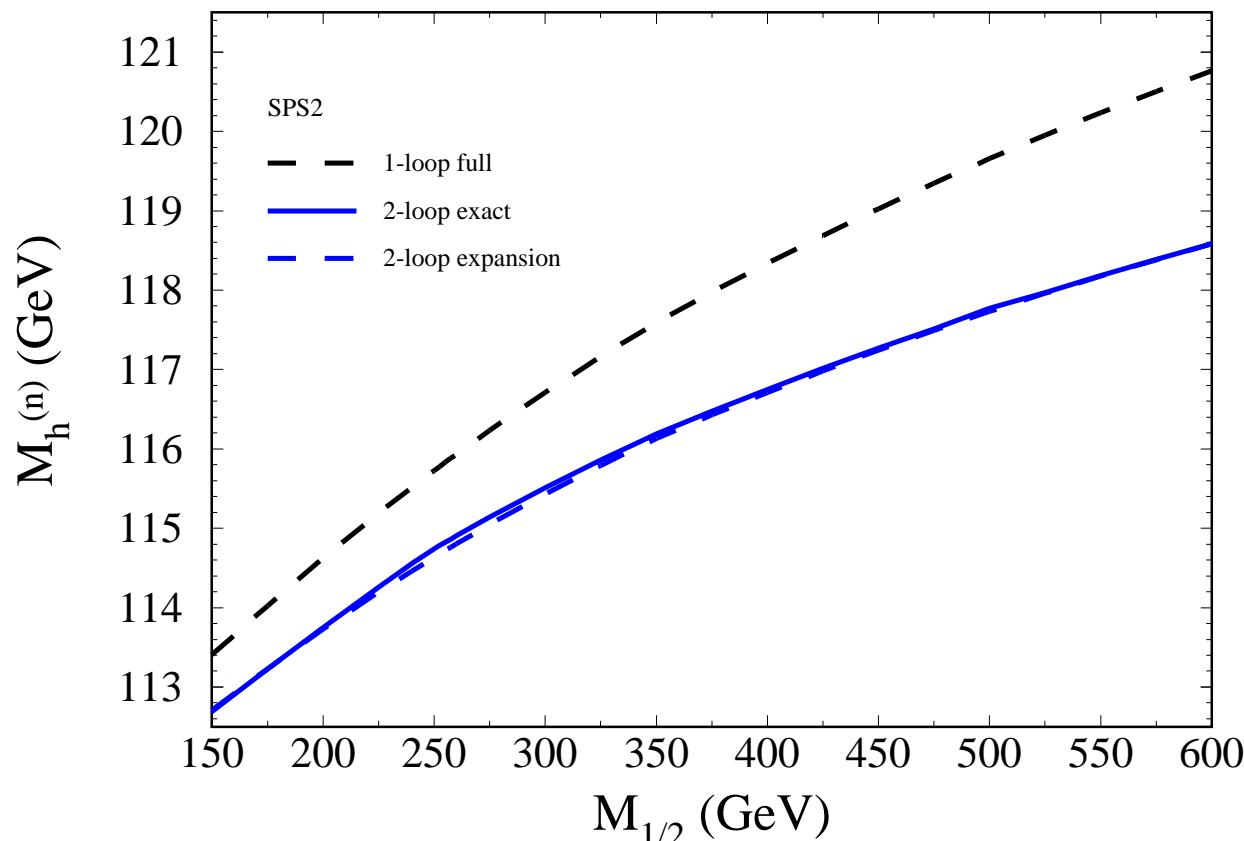
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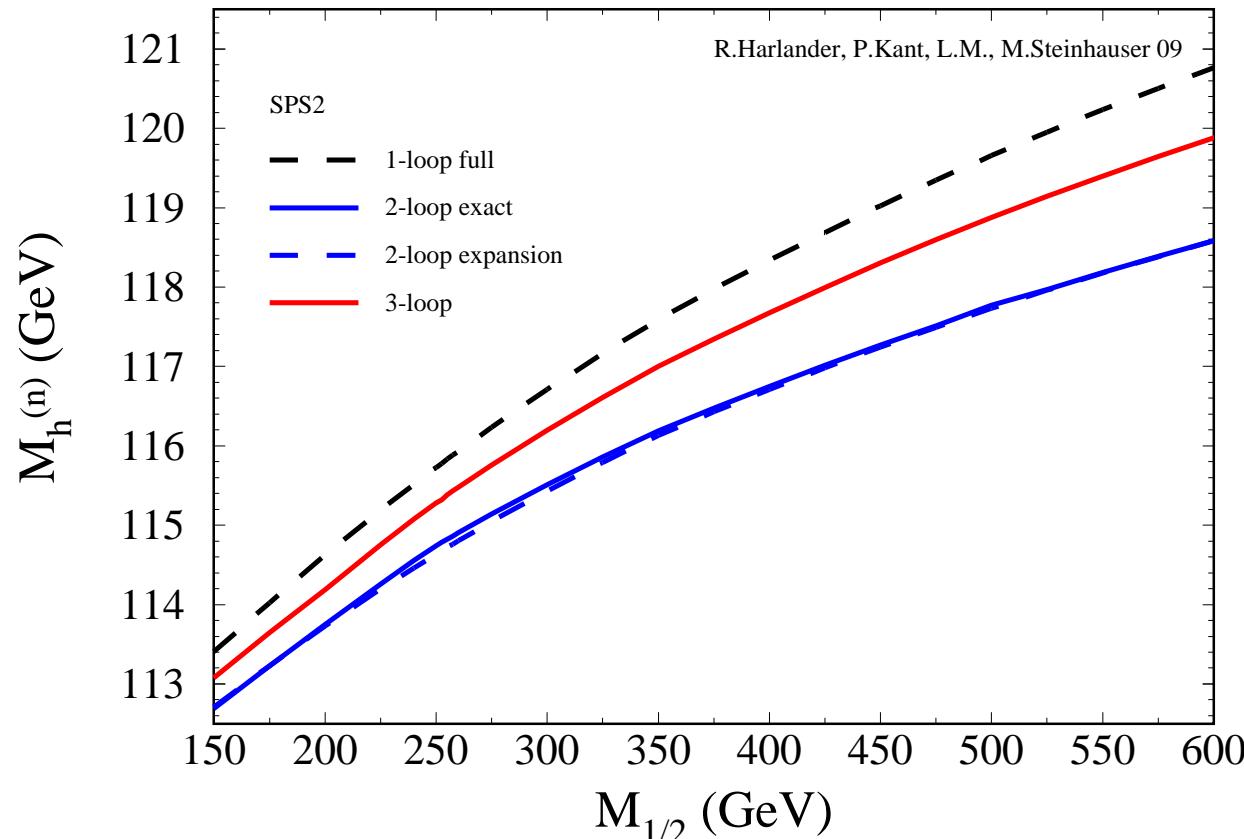


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$$\Delta M_h^{(3)} = 0.5 - 1.5 \text{ GeV}$$



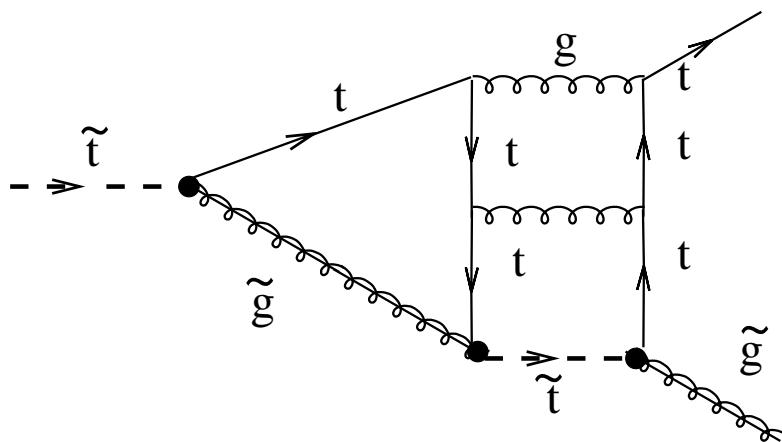
# Conclusions

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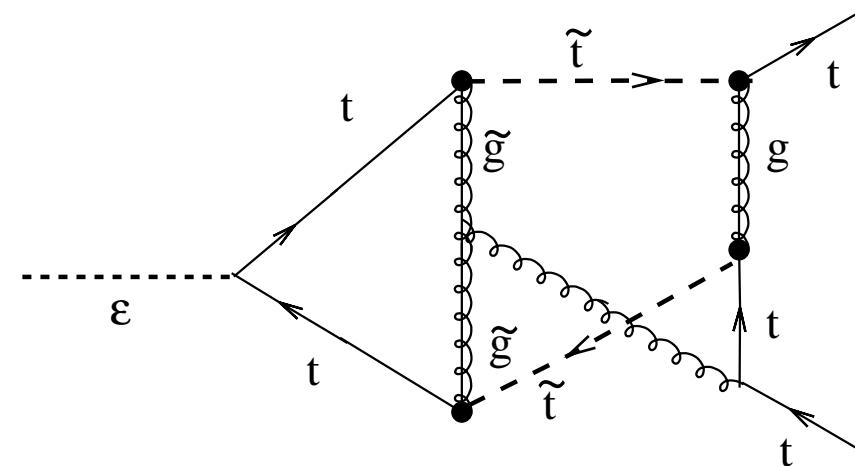
- DRED preserves SUSY through 3-loops
- Many particles and mass scales  $\rightsquigarrow$  asymptotic expansions
- Precision tests
  - $\alpha_s(M_{\text{GUT}})$  and  $m_b(M_{\text{GUT}})$ : 3-loop effects **larger** than experimental accuracy
  - $M_h$ : 3-loop effects **larger** than experimental accuracy at the LHC
  - Theoretical uncertainties under control

# Backup slides(1)

- Anti-commuting  $\gamma_5$ :  $\{\gamma_5, \gamma^\mu\} = 0, \quad \{\gamma_5, \gamma^{\tilde{\mu}}\} = 0$
- 3-loop vertex diagrams



$$\text{Tr}(\gamma_5 \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma) = 4i\varepsilon^{\mu\nu\rho\sigma}$$



$$\text{Tr}(\gamma_5 \gamma^{\tilde{\mu}} \gamma^\nu \gamma^\rho \gamma^\sigma) = 4i\varepsilon^{\tilde{\mu}\nu\rho\sigma}$$

$$m_b(M_{\text{GUT}})$$

---

- $b - \tau$  or  $t - b - \tau$  Yukawa coupling unification

$$m_b(M_{\text{GUT}})$$

---

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- SM: 4-loop accuracy

# $m_b(M_{\text{GUT}})$

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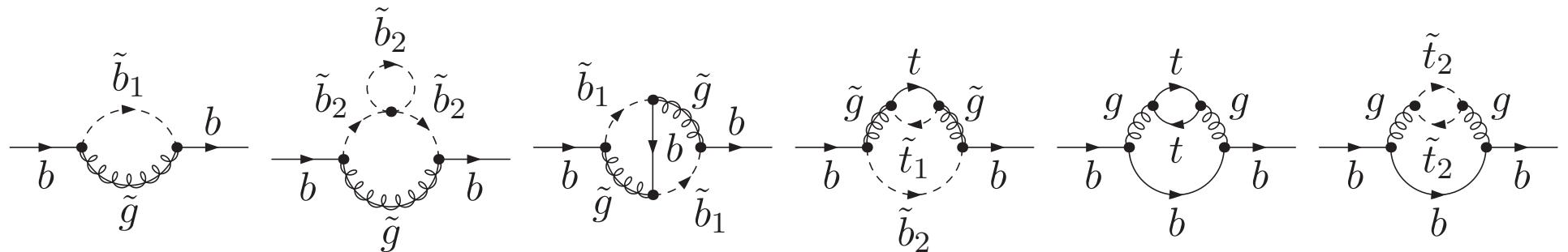
- $b - \tau$  or  $t - b - \tau$  Yukawa coupling unification
- SM: 4-loop accuracy
- MSSM

$$m_b^{\text{MSSM}}(\mu) = \frac{m_b^{\text{SM}}(\mu)}{\zeta_{m_b}(\alpha_s, M_{\text{SUSY}}, m_t, \mu)}$$

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# $m_b(M_{\text{GUT}})$

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- $b - \tau$  or  $t - b - \tau$  Yukawa coupling unification
- SM: 4-loop accuracy
- MSSM

$$m_b^{\text{MSSM}}(\mu) = \frac{m_b^{\text{SM}}(\mu)}{\zeta_{m_b}(\alpha_s, M_{\text{SUSY}}, m_t, \mu)}$$

$$\begin{aligned}\zeta_{m_b} &= 1 + \delta\zeta_{m_b}^{\tan\beta} + \delta\zeta_{m_b}^{\text{rest}}, \\ \delta\zeta_{m_b}^{\tan\beta} &= 1 + \sum_n \alpha_s^n (A_b - \mu_{\text{SUSY}} \tan\beta) C_n(M_{\text{SUSY}}, m_t, \mu)\end{aligned}$$

large  $\tan\beta$  :  $\delta\zeta_{m_b}^{\tan\beta}$  has to be resummed !!

1-loop: [Hempfling '94], [Hall, Rattazzi, Sarid '94] [Carena, Garcia, Nierste, Wagner '01]

2-loops: [Noth, Spira '08], [Bauer, L. M, Salomon '08]