Multi-Loop Calculations in the MSSM

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in collaboration with

R. Harlander, P. Kant, J. Salomon, M. Steinhauser

Outline

- Motivation
- Framework
- Precision tests of MSSM
  - $\alpha_s(M_{\rm GUT})$ and $m_b(M_{\rm GUT})$ with 3-loop accuracy
  - m_h: 3-loop SQCD corrections
- Conclusions

Motivation

Precision tests of the MSSM

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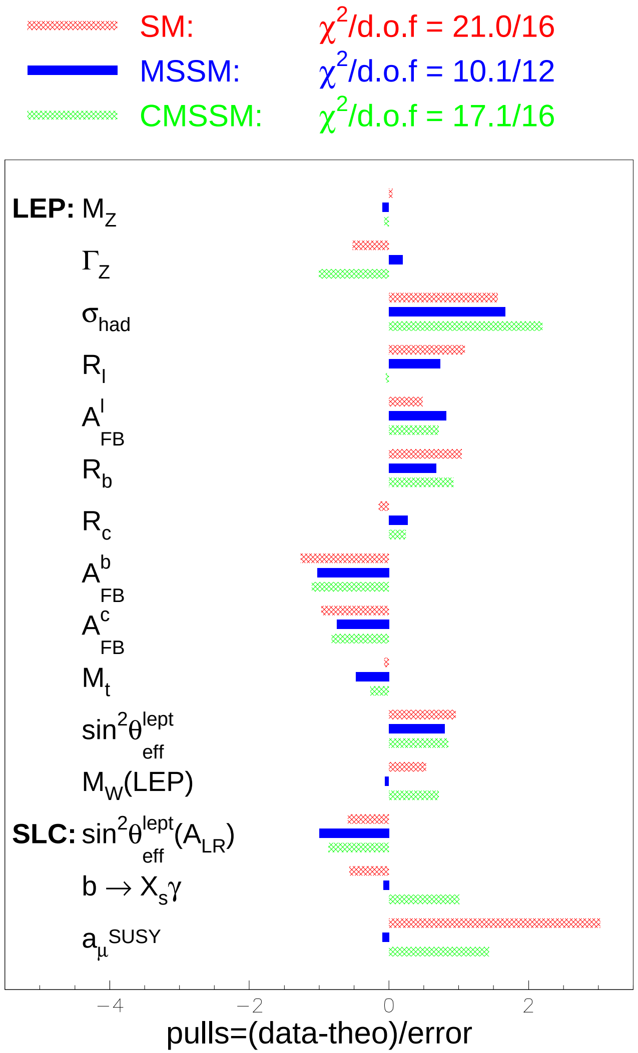
- Electroweak precision data

W. de Boer & C. Sander '03



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Precision tests of the MSSM

- LHC& ILC: 2-loops SQCD and SEW corrections
  - M_W and $\sin\theta_{\text{eff}}$ [Chankowski et al '94], [Djouadi et al '96], [Heinemeyer & Weiglein '02,'04], . . .
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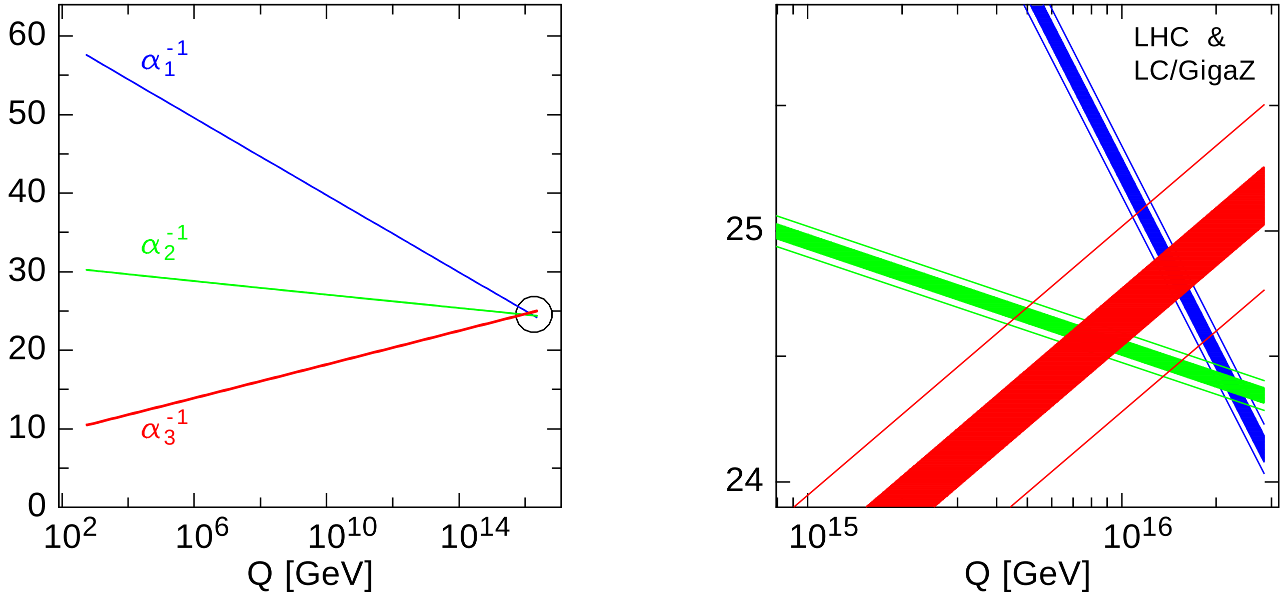

[Blair, Porod, Zerwas '04]

Framework

Regularization Scheme: gauge and SUSY invariant

- Dimensional Regularization (DREG) ['t Hooft and Veltmann'72] breaks **SUSY**

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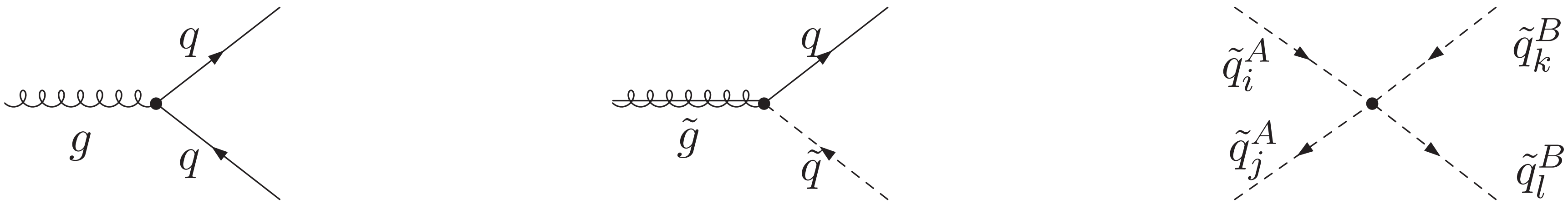
SQCD & DREG



$$g_s \neq \hat{g}_s \neq \lambda^{AB}$$

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- 1- and 2-loops: [Jack, Jones '94], [Martin, Vaughn '94], [Yamada '94], [Hollik, Stöckinger'05]

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This talk: $g_s = \hat{g}_s = g_e$ at 3-loops [Harlander, L.M., Steinhauser '09]

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- DREG-DRED parameter conversion
  - 1-loop MSSM [Martin, Vaughn '94], [Beenaker, Hopker, Zervas '96]
  - 2-loop SQCD [L.M. '09]

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$$\alpha_s^{\overline{\rm MS}} = \alpha_s^{\overline{\rm DR}}\left[1 - \frac{\alpha_s^{\overline{\rm DR}}}{\pi}\frac{C_A}{3} + \left(\frac{\alpha_s^{\overline{\rm DR}}}{\pi}\right)^2\left(-\frac{11}{9}C_A^2 + 2T_F N_f C_F\right)\right],$$

$$m_q^{\overline{\rm MS}} = m_q^{\overline{\rm DR}}\left[1 + \frac{\alpha_s^{\overline{\rm DR}}}{\pi}C_F + \left(\frac{\alpha_s^{\overline{\rm DR}}}{\pi}\right)^2\left(\frac{7}{12}C_A C_F + \frac{7}{4}C_F^2 - \frac{1}{2}C_F T_F N_f\right)\right]$$

Renormalization Constants

$$Z_{g_i} = \frac{Z^{(i)}_{\phi_1\phi_2\phi_3}}{\sqrt{Z_{\phi_1} Z_{\phi_2} Z_{\phi_3}}}$$

- Minimal Subtraction Scheme & DRED ($\overline{\text{DR}}$)

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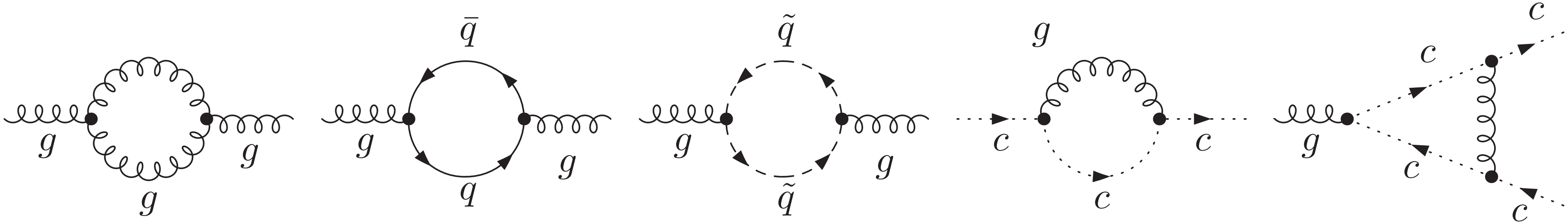

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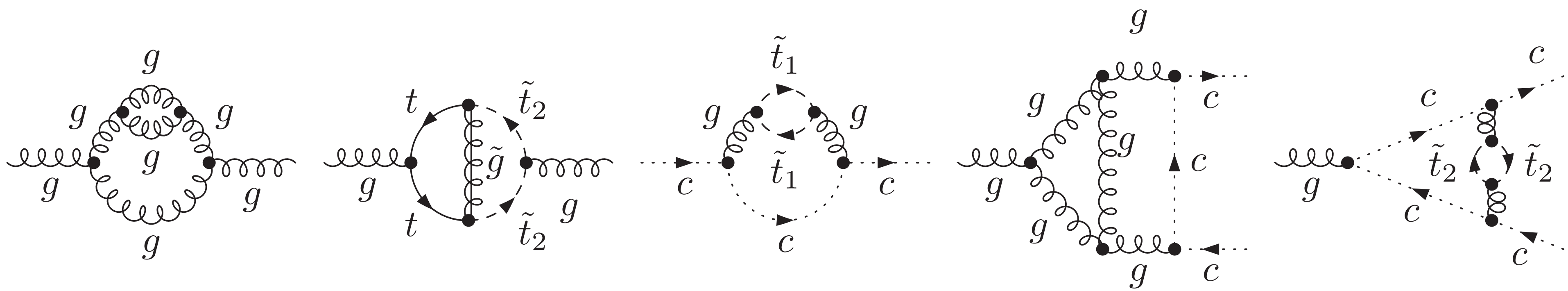

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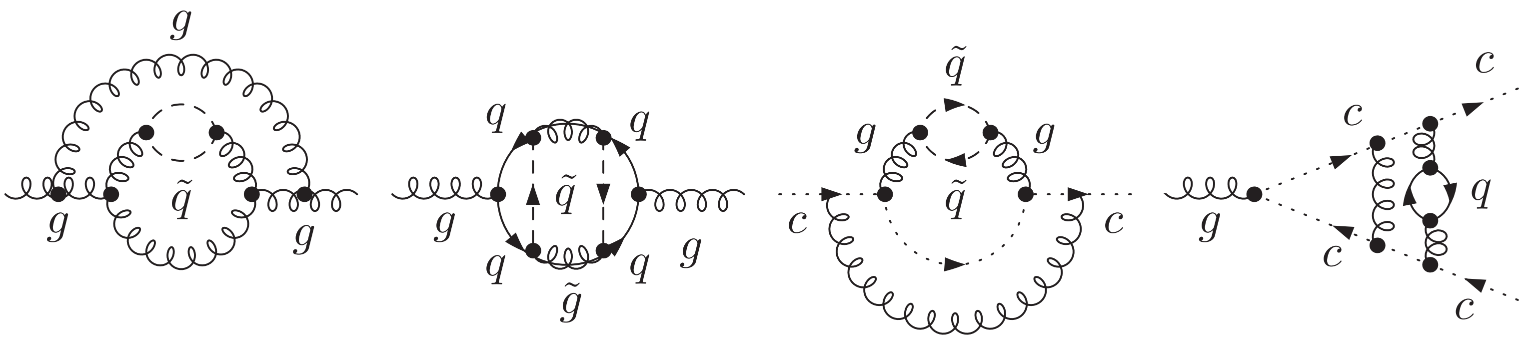

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- Minimal Subtraction Scheme & DRED ($\overline{\text{DR}}$)
- 3-loop computation
  - $\simeq$ 200.000 diagrams
  - Computer programs: QGRAF, FORM, MINCER, MATAD, EXP, . . .

    [Noguiera; Vermaseren; Larin, Tkachov; Steinhauser; Seidensticker, Harlander; . . .]

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- Minimal Subtraction Scheme & DRED ($\overline{\text{DR}}$)
- 3-loop anomalous dimensions in SQCD [Harlander, L.M., Steinhauser '09]

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \alpha_s(\mu) = \beta(\alpha_s) \qquad \beta(\alpha_s) = -\sum_{n\geq 0} \left(\frac{\alpha_s}{\pi}\right)^{n+2} \beta_n$$

$$\beta_0 = \frac{3}{4} C_A - \frac{1}{2} T_f \,, \qquad C_F = 4/3,\; C_A = 3,\; 2T_f = n_f \,,$$

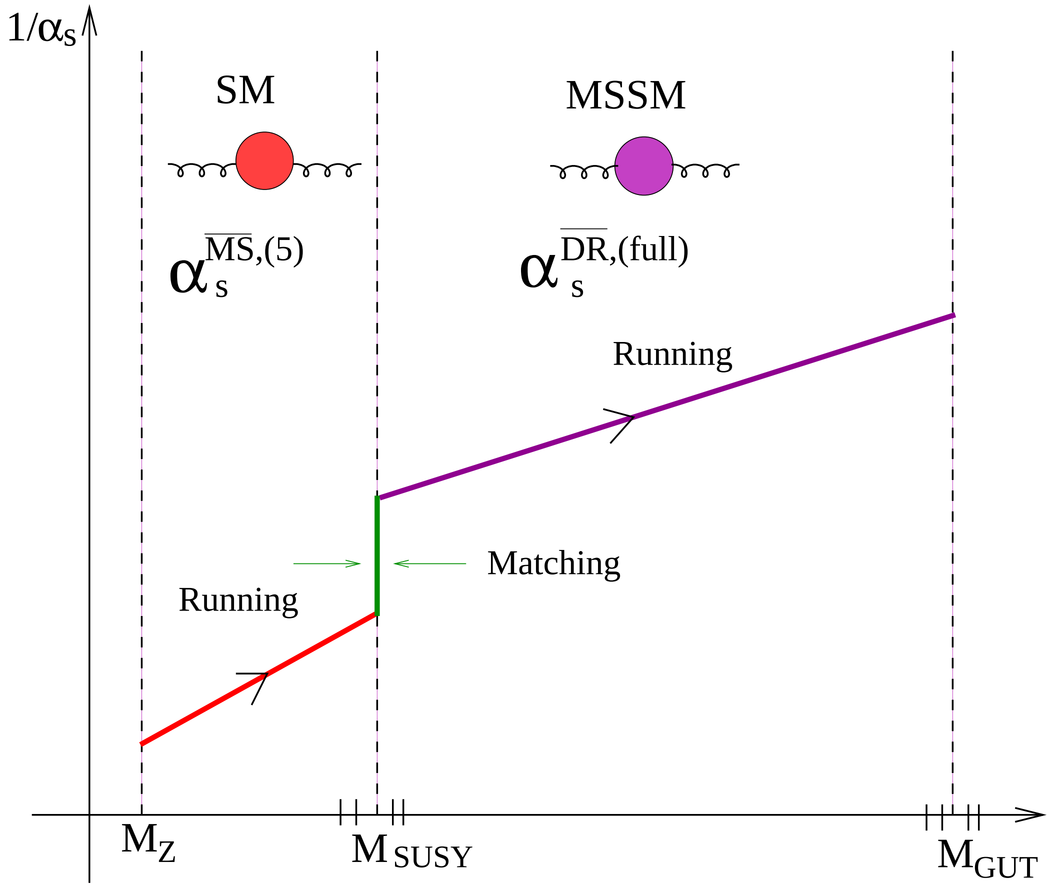
$$\beta_1 = \frac{3}{8} C_A^2 - T_f \left(\frac{1}{2} C_F + \frac{1}{4} C_A \right) ,$$

$$\beta_2 = \frac{21}{64} C_A^3 + T_f \left(\frac{1}{4} C_F^2 - \frac{13}{16} C_A C_F - \frac{5}{16} C_A^2 \right) + T_f^2 \left(\frac{3}{8} C_F + \frac{1}{16} C_A \right) .$$

- complete agreement with [Jack, Jones, North '96], [Pickering, Gracey, Jones '01]

Evolution of the couplings

Effective Field Theory:



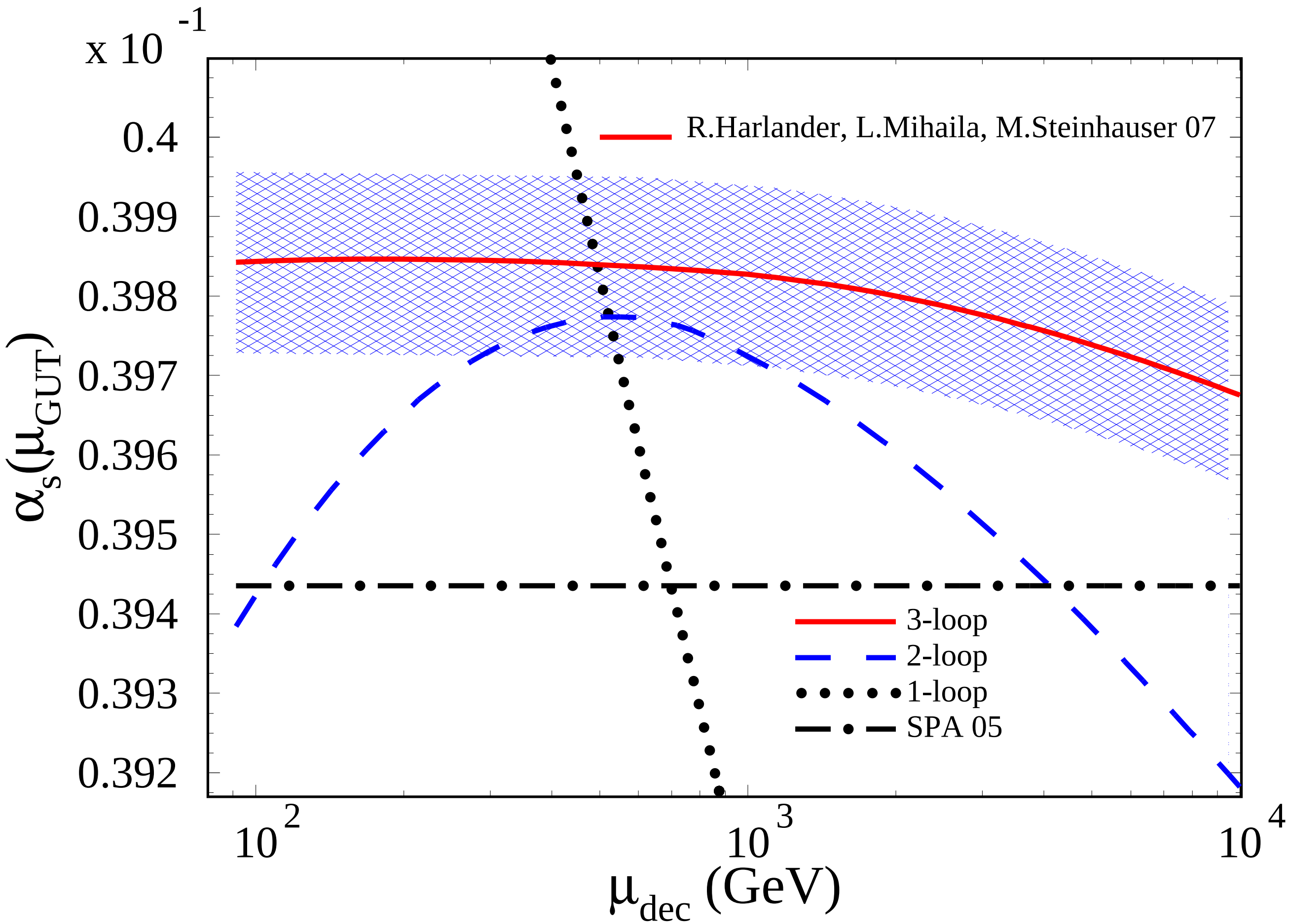
Matching: 1-loop [Pierce et al '95]

2-loops [R. Harlander, L. M., M. Steinhauser '05, '07] [A. Bauer, L. M., J. Salomon '08]

$\alpha_s(M_{\text{GUT}})$

Input: $\alpha_s^{\overline{\text{MS}},(5)}(M_Z) = 0.1189 \pm 0.001$ [Bethke '06], $M_Z = 91.1876$ GeV,
$\tilde{M} = m_{\tilde{q}} = m_{\tilde{g}} = 1000$ GeV

R.Harlander's talk RADCOR'07



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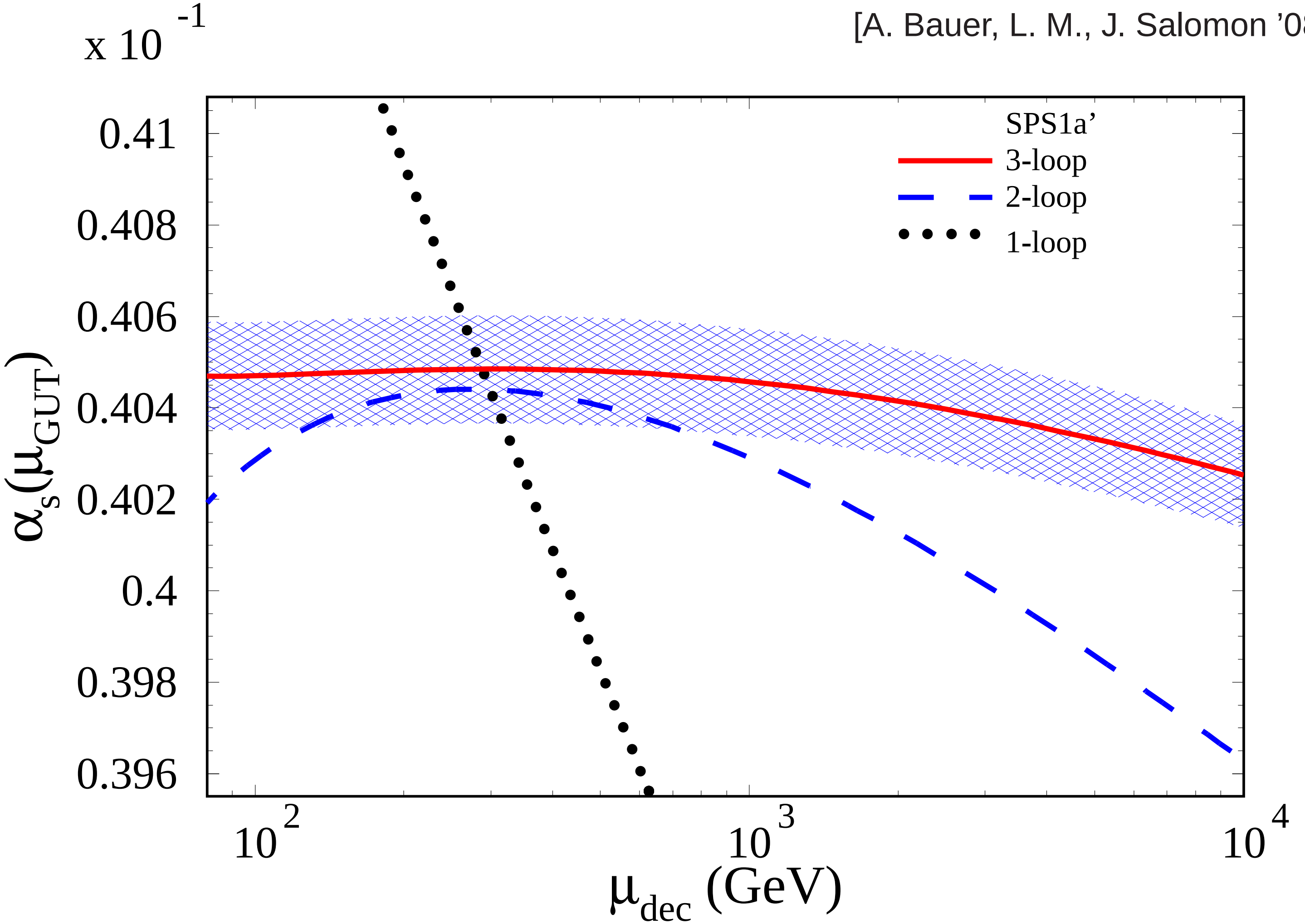

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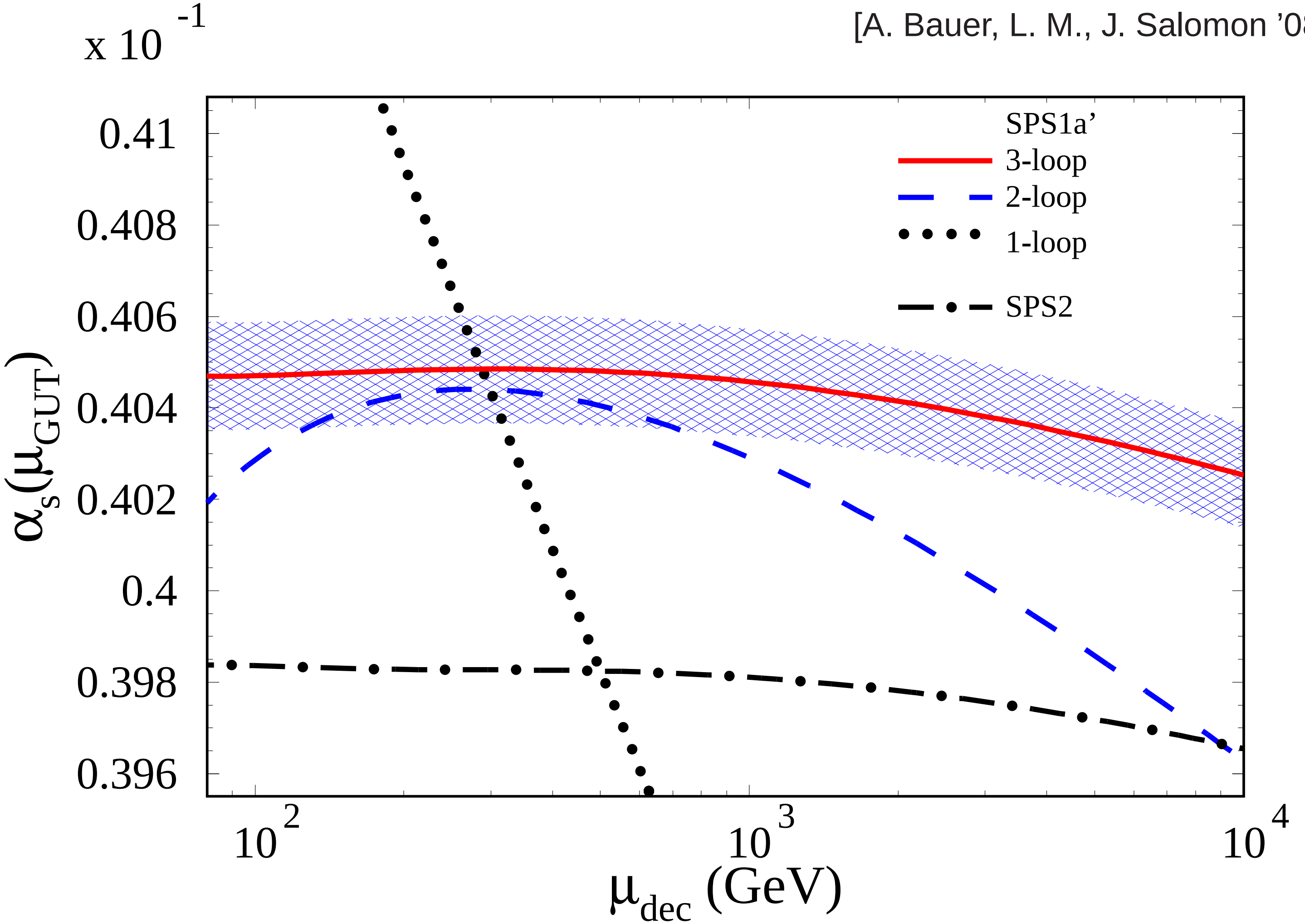

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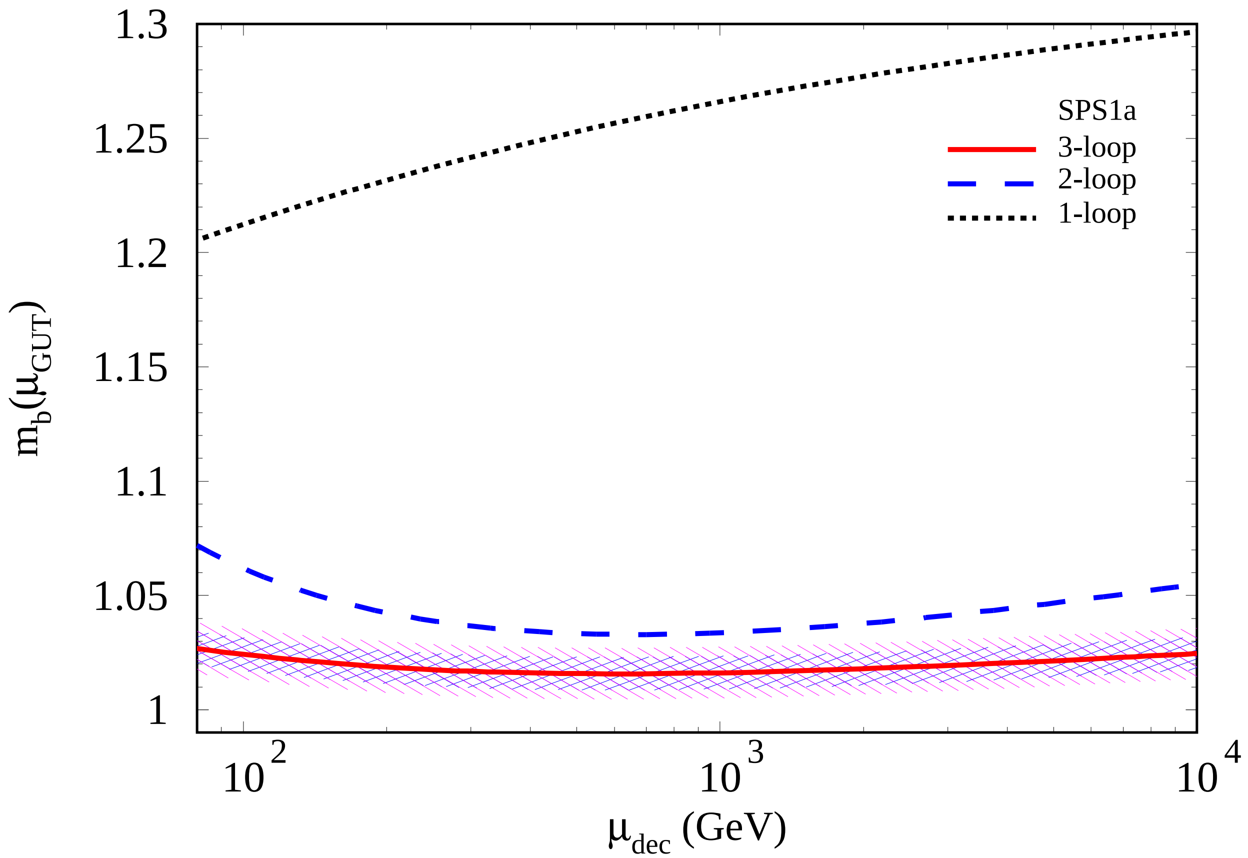

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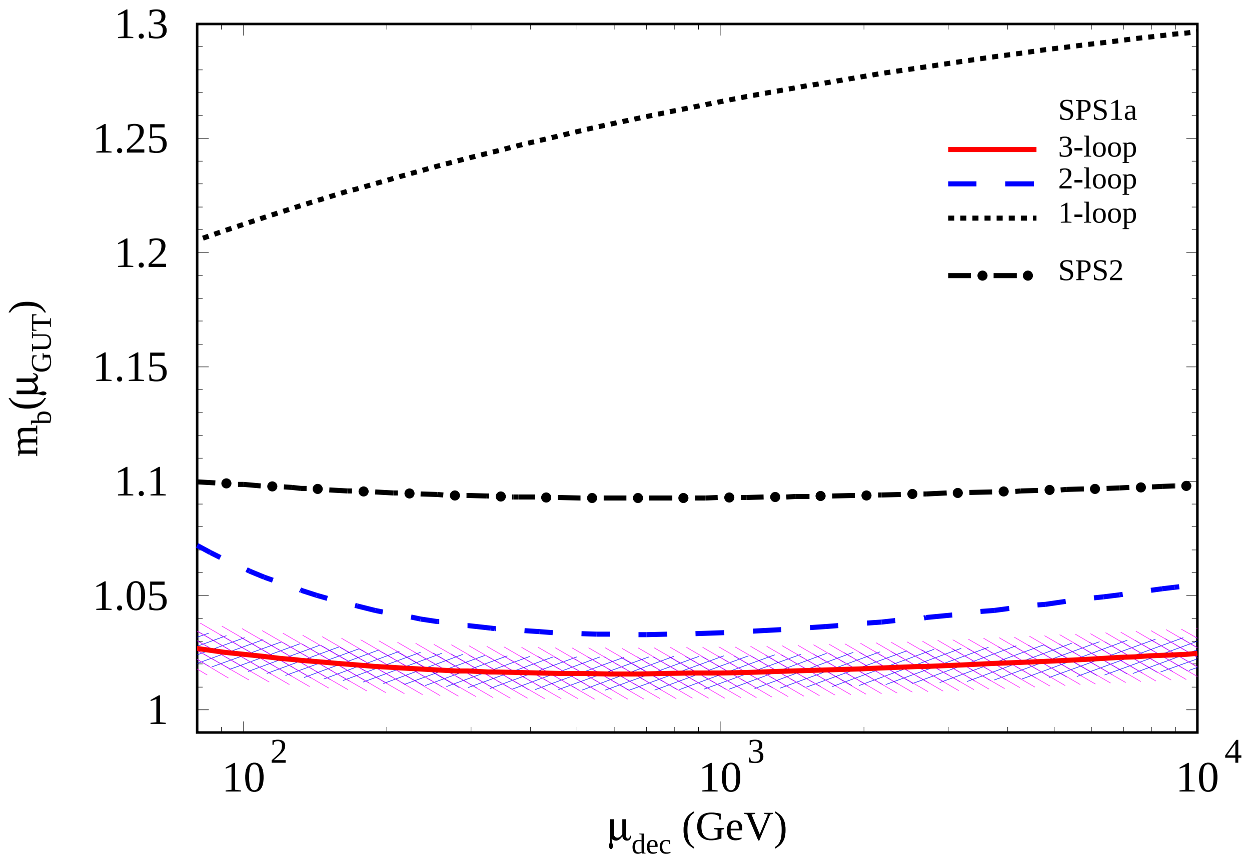

Light Higgs boson mass in the MSSM

Motivation

Experiment : $\delta m_h^{\text{exp}} = 100 - 200\,\text{MeV}$ CERN-LHCC-2006-21

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Theory:

- exact 1-loop [Chankowski, Pokorski and Rosiek '92], [Brignole '92], [Dabelstein '94]
- 2-loop $\mathcal{O}(\alpha_t\alpha_s, \alpha_t^2, \alpha_b\alpha_s, \alpha_b\alpha_t)$ in effective potential approximation ($p^2 = 0$)
  [Haber, Hempfling, Hoang '96], [Heinemeyer, Hollik and Weiglein '98], [Degrassi, Slavich, Zwirner '01], [Espinosa and Zang '00], [Brignole, Degrassi, Slavich, Zwirner '02] , [Carena et al '00], [Heinemeyer et al '05] , [S. Martin '03]
- $p^2 = m_h^2$: 2-loop SUSY-QCD [S.Martin '05]
- 3-loop LL and NLL $\mathcal{O}(\alpha_t\alpha_s^2,\, \alpha_t^2\alpha_s,\, \alpha_t^3)$ [S. Martin '07]
- Missing contributions: $\delta m_h^{\rm th} \simeq 3 - 5\,{\rm GeV}$ [G. Degrassi et al '02], [Allanach et al '04]

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This talk: 3-loop SUSY-QCD corrections

Framework

SUSY $\Rightarrow$ two free parameters: $\tan\beta = v_2/v_1, \quad M_A^2 = -m_{12}^2(\tan\beta + \cot\beta)$

CP-even Higgs $\phi_{1,2}$:

$$\mathcal{M}^2_{H,\text{tree}} = \frac{\sin 2\beta}{2} \times \begin{pmatrix} M_Z^2 \cot\beta + M_A^2 \tan\beta & -M_Z^2 - M_A^2 \\ -M_Z^2 - M_A^2 & M_Z^2 \tan\beta + M_A^2 \cot\beta \end{pmatrix}$$

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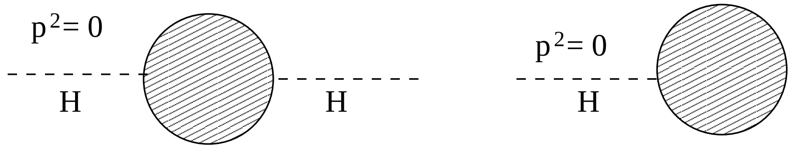
Higher order corrections

$$\mathcal{M}_H^2 = \mathcal{M}_{H,\text{tree}}^2 - \begin{pmatrix} \hat{\Sigma}_{\phi_1} & \hat{\Sigma}_{\phi_1\phi_2} \\ \hat{\Sigma}_{\phi_1\phi_2} & \hat{\Sigma}_{\phi_2} \end{pmatrix}$$

V_{eff}-approximation: $p^2 = 0 \Rightarrow \quad \hat{\Sigma}_i(0) = \Sigma_i(0) - \delta V_i$

$$\hat{\Sigma}_i(0) \quad =$$



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Approximations: $\mathcal{O}(\alpha_t \alpha_s^2)$ corrections $\rightsquigarrow$ $\mathcal{O}(M_{\rm top}^4)$

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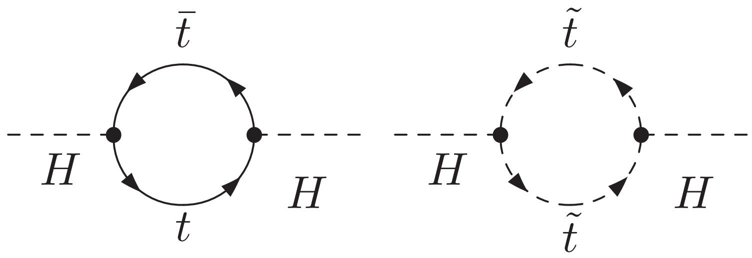
Computation of $\hat{\Sigma}_{\phi_{ij}}(0)$ at 3-loops:

1-loop:



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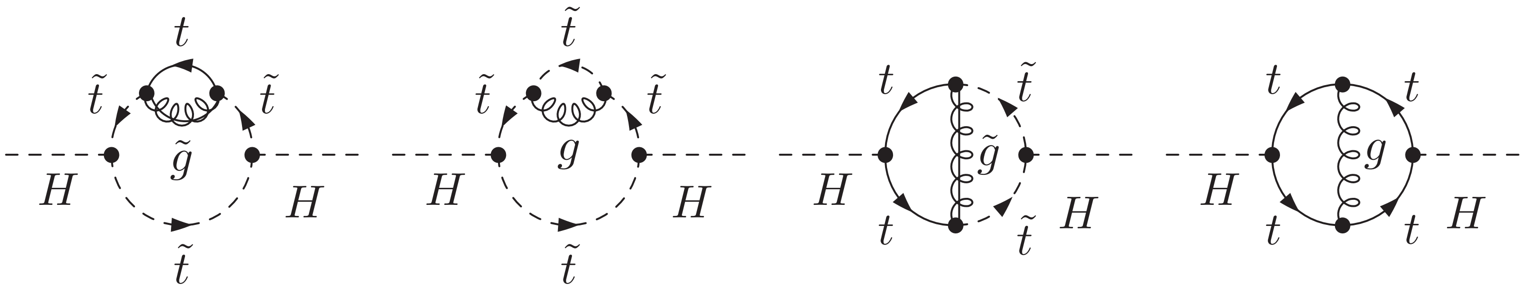

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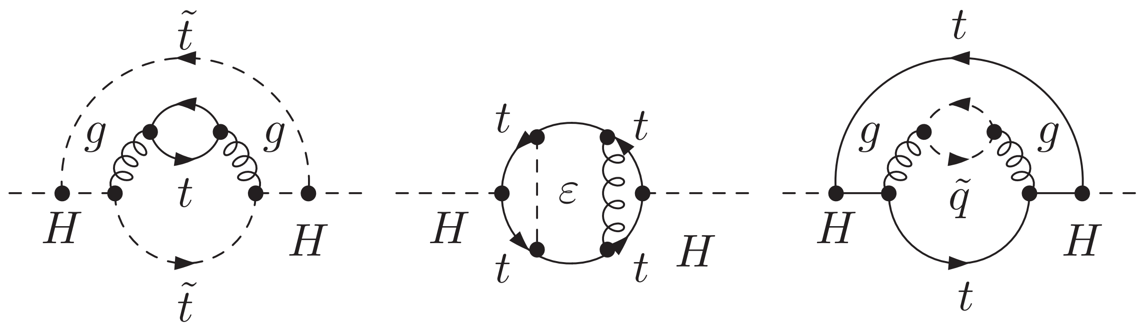

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- $\simeq$ 28.000 diagrams

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$$m_t \ll m_{\tilde{t}_1} \approx m_{\tilde{t}_2} \approx m_{\tilde{g}} \approx m_{\tilde{q}}$$
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- Renormalization Scheme: $\overline{\text{DR}}$ -scheme
  - α_s in $\overline{\text{DR}}$ to 1-loop
  - $M_t,\ M_{\tilde{t}_1},\ M_{\tilde{t}_2}$ in $\overline{\text{DR}}$ to 2-loops
  - $M_{\tilde{g}}$ in $\overline{\text{DR}}$ to 1-loop
  - $\theta_{\tilde{t}}$ in $\overline{\text{DR}}$ to 2-loops
  - M_ε in OS to 1-loop

Agreement with

[Pierce, Bagger, Matchev and Zahng '96], [Jack and Jones '94], [Martin and Vaughn '94], [S. Martin '03,'05]

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- Computer programs: QGRAF, PERL, FORM, MINCER, MATAD, EXP, . . .

  [Noguiera; Vermaseren; Harlander; Larin, Tkachov; Steinhauser; Seidensticker, Harlander; . . .]

Numerical Results (no stop-mixing)

Input SM parameters: $\mu = M_t = 172.4$ GeV $G_F = 1.16637 \times 10^{-5}$ GeV^{-2}

$M_Z = 91.1876$ GeV $\alpha_s^{(5)}(M_Z) = 0.1189 \Rightarrow \alpha_s(M_t) = 0.0926$

MSSM parameters: $M_A = 1$ TeV $\tan\beta = 40$ $A_t = 0$ $M_{\tilde{q}} = 2$ TeV $M_{\tilde{t}_2} = M_{\tilde{t}_1} = M_{\tilde{g}} = M_{\mathrm{SUSY}}$

Renormalization scheme dependence

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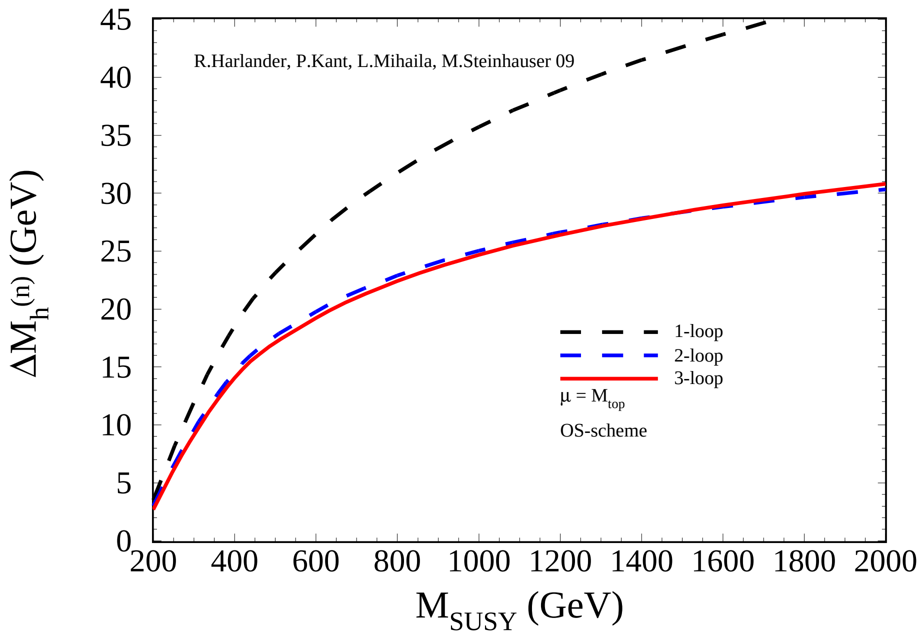
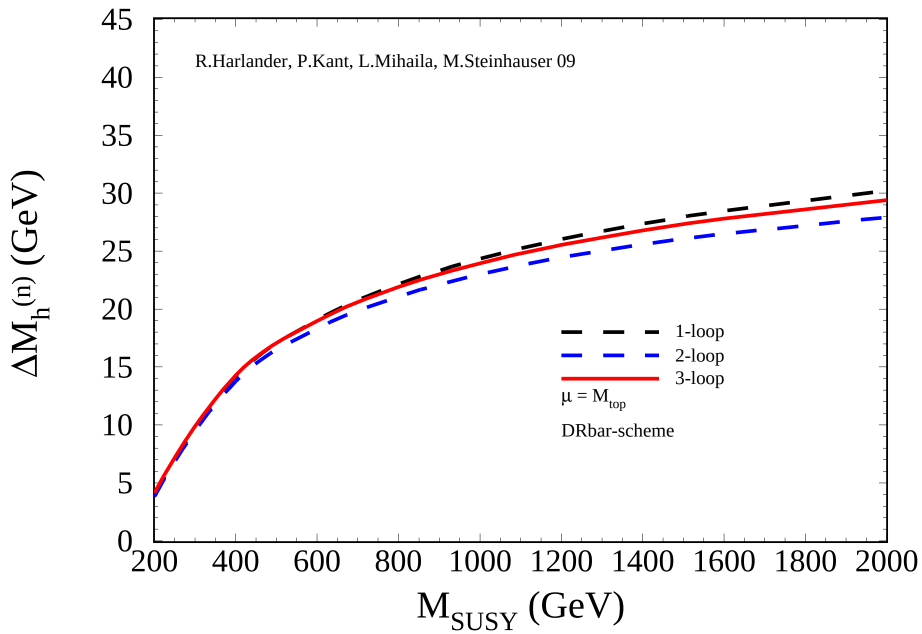
Renormalization scheme dependence: $\Delta M_h^{(n)} \equiv M_h^{(n-\text{loop})} - M_h^{\text{tree}}$

OS: $\delta M_h^{(3)} \simeq 500$MeV

$\overline{\text{DR}}$: $\delta M_h^{(3)} \simeq 1.2$GeV





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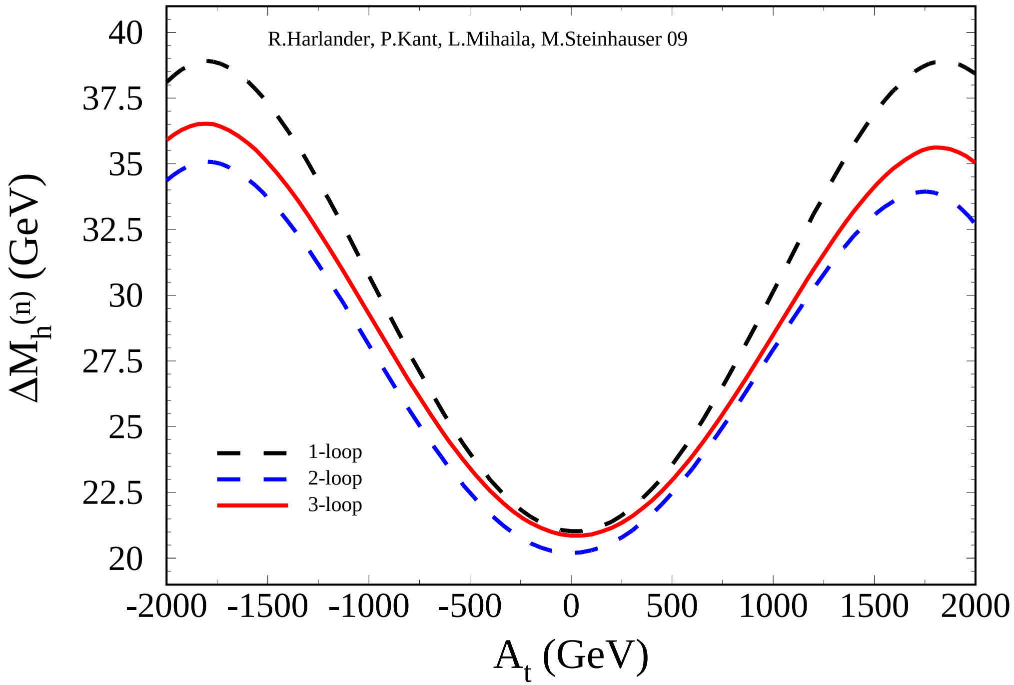
$M_Z = 91.1876$ GeV $\quad \alpha_s^{(5)}(M_Z) = 0.1189 \Rightarrow \alpha_s(M_t) = 0.0926$

MSSM parameters: $M_A = 1$ TeV $\quad \tan\beta = 40 \quad M_{\tilde{q}} = 2$ TeV

$m_{\tilde{t}_2}(\mu) = 1$ TeV $\quad m_{\tilde{t}_1}(\mu) = m_{\tilde{g}}(\mu) = 0.5$ TeV

$\overline{\text{DR}}$ scheme $\quad A_t = 0 : \pm 2$ TeV : $\Delta M_h^{(3)} = 0.5 - 1.5$ GeV



Numerical Results: SPS2

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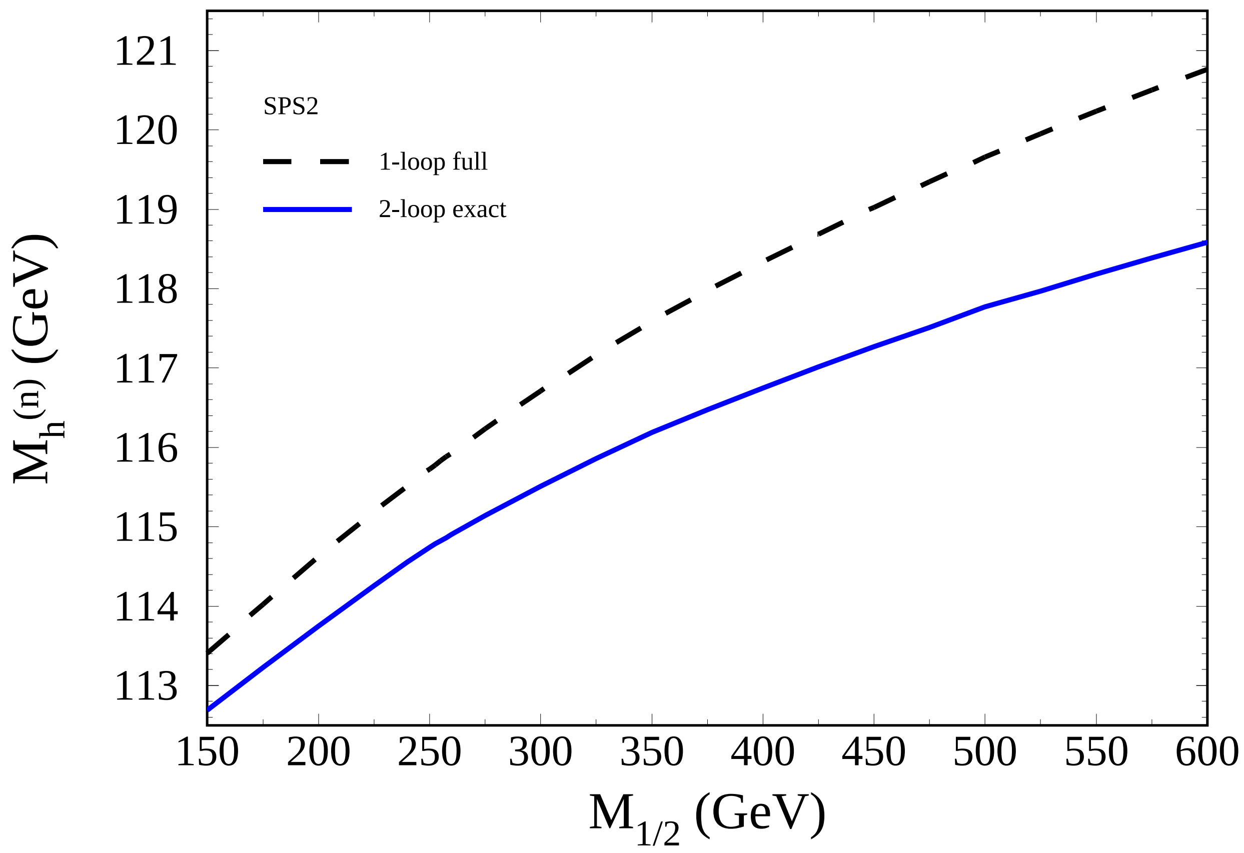

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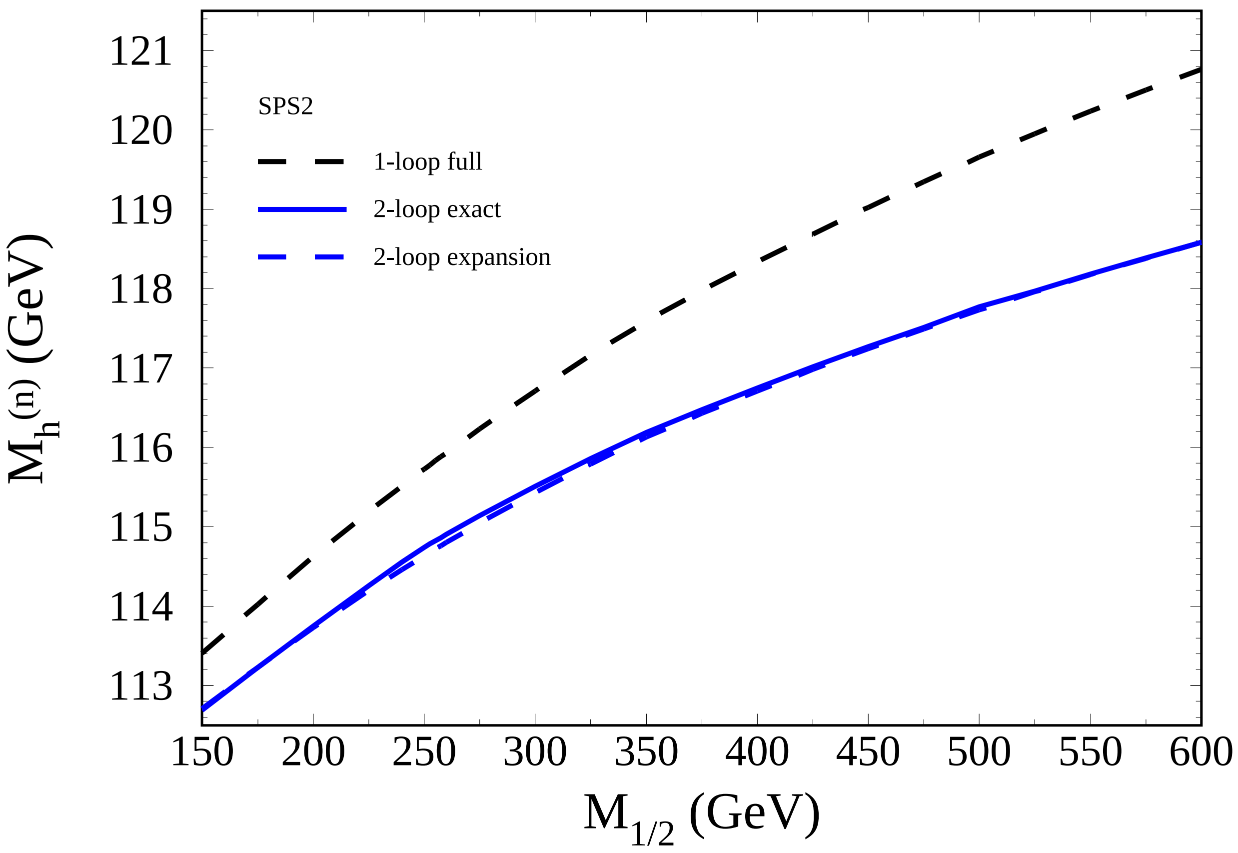

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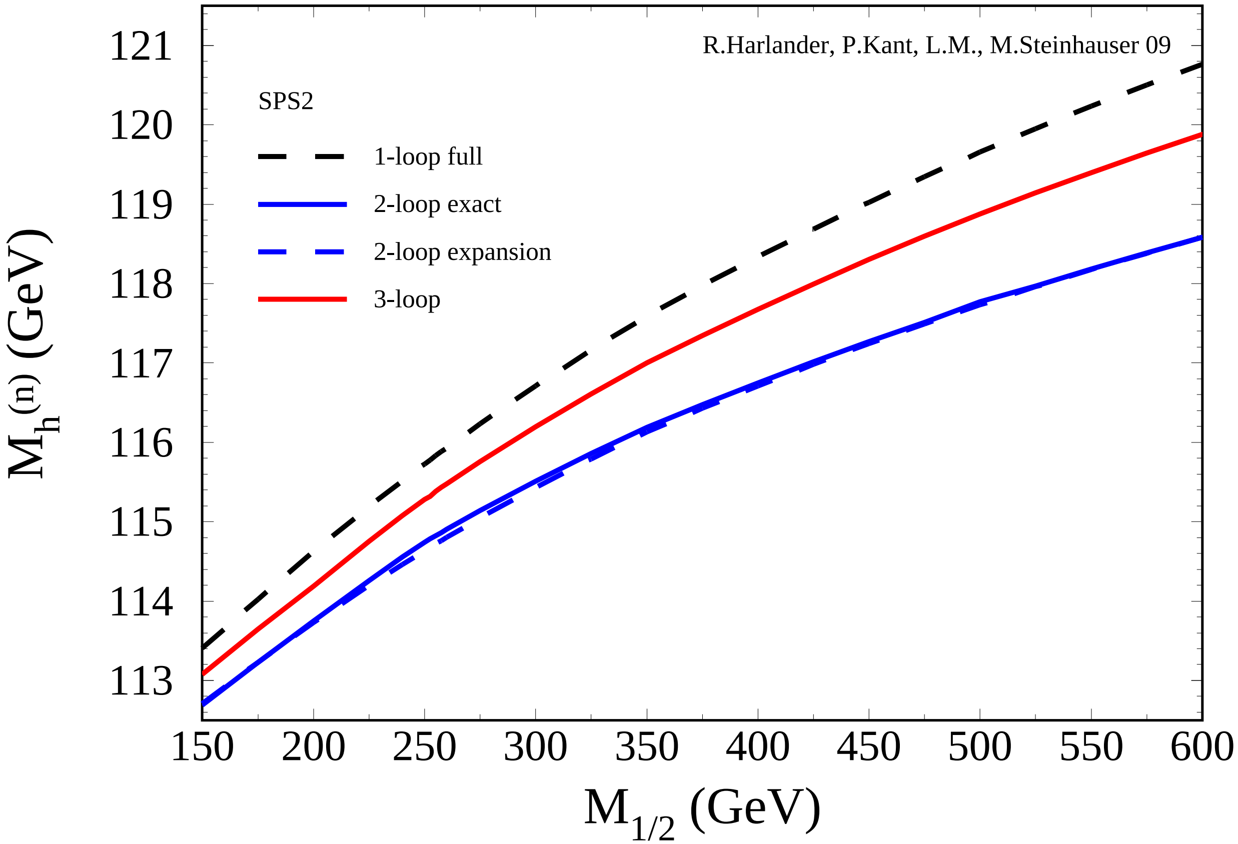
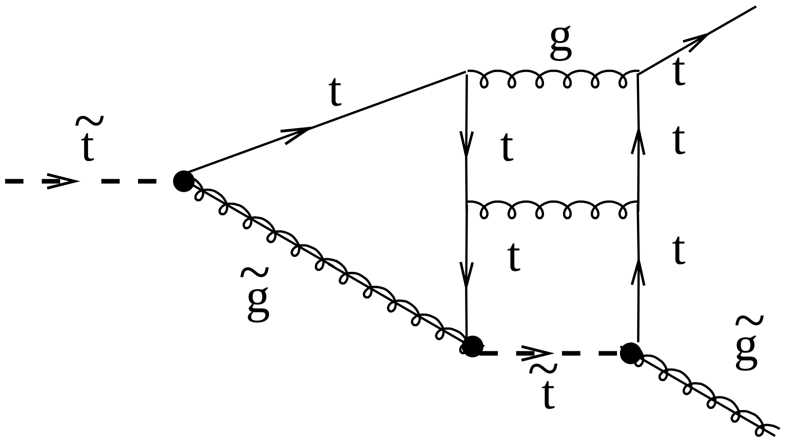


Conclusions

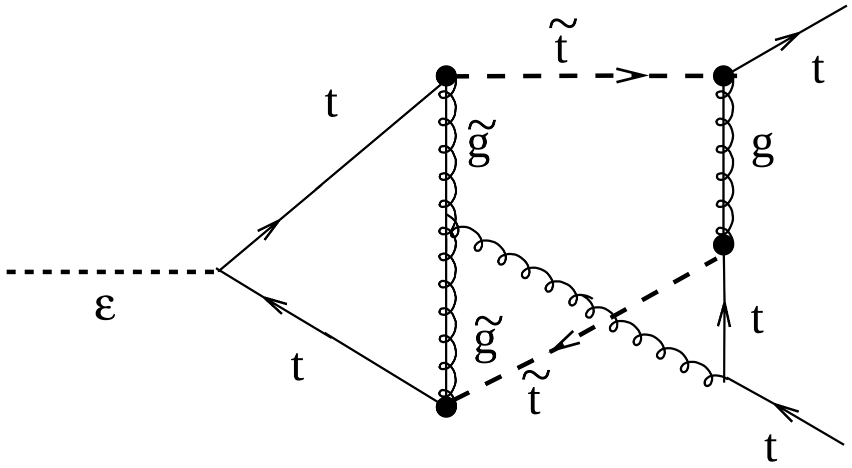
- DRED preserves SUSY through 3-loops
- Many particles and mass scales $\rightsquigarrow$ asymptotic expansions
- Precision tests
  - $\alpha_s(M_{\mathrm{GUT}})$ and $m_b(M_{\mathrm{GUT}})$: 3-loop effects larger than experimental accuracy
  - M_h: 3-loop effects larger than experimental accuracy at the LHC
  - Theoretical uncertainties under control

Backup slides(1)

- Anti-commuting γ_5: $\{\gamma_5, \gamma^\mu\} = 0\,, \quad \{\gamma_5, \gamma^{\tilde{\mu}}\} = 0$
- 3-loop vertex diagrams



$\mathrm{Tr}(\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4i\varepsilon^{\mu\nu\rho\sigma}$



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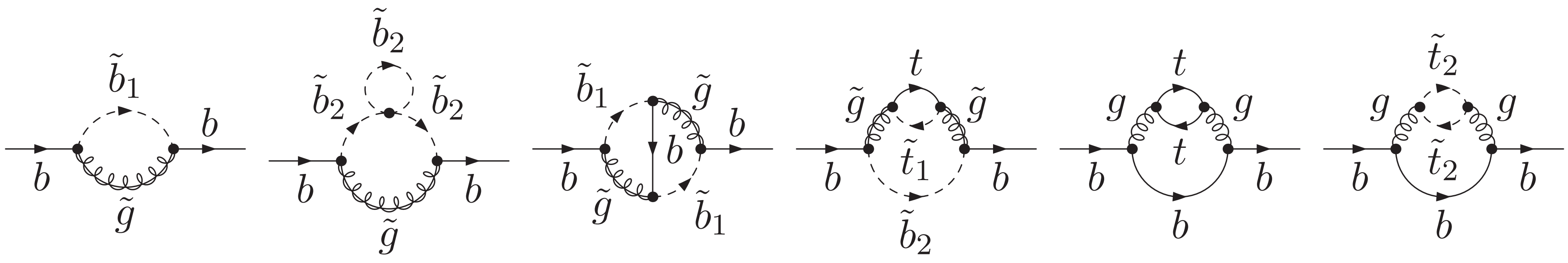

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$$\zeta_{m_b} = 1 + \delta\zeta_{m_b}^{\tan\beta} + \delta\zeta_{m_b}^{\rm rest}\,,$$

$$\delta\zeta_{m_b}^{\tan\beta} = 1 + \sum_n \alpha_s^n (A_b - \mu_{\rm SUSY}\tan\beta) C_n(M_{\rm SUSY}, m_t, \mu)$$

large $\tan\beta$: $\delta\zeta_{m_b}^{\tan\beta}$ has to be resummed !!

1-loop: [Hempfling ’94], [Hall, Rattazzi, Sarid ’94] [Carena, Garcia, Nierste, Wagner ’01]

2-loops: [Noth, Spira ’08], [Bauer, L. M, Salomon ’08]