SUSY QCD Corrections to Higgs Production via Gluon Gluon Fusion

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Outline

◇ **Introduction**

◇ **QCD corrections to squark loops for $M_{\tilde{g}} \longrightarrow \infty$**

◇ **Full SUSY-QCD corrections**

◇ **Decoupling of $M_{\tilde{g}}$ contributions for $M_{\tilde{g}} \longrightarrow \infty$**

◇ **Conclusions**

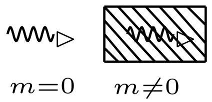
Higgs Physics

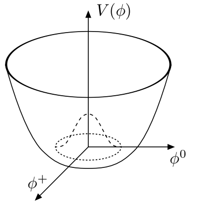
Higgs physics at future colliders:
Establish experimentally the Higgs Mechanism

The Higgs mechanism:
Creation of particle masses in a gauge-invariant way

Test of the Higgs mechanism

- Discovery – m
- Spin and CP properties – J^{PC}
- Interaction with the scalar Higgs with $v = 246$ GeV$\neq 0$ $\rightsquigarrow$ $g_{HXX} \sim m_X^{(2)}$
- EWSB requires Higgs potential $\leftrightarrow$ $\lambda_{HHH}, \lambda_{HHHH}$





The MSSM Higgs Sector

MSSM Higgs sector – supersymmetry & anomaly free theory ⇒ 2 complex Higgs doublets

$\xrightarrow{\text{EWSB}}$ neutral, CP-even h, H neutral, CP-odd A charged H^+, H^-

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Higgs masses

$$M_h \lesssim 140 \text{ GeV}$$

$$M_{A,H,H^\pm} \sim \mathcal{O}(v)...1 \text{ TeV}$$

Ellis et al;Okada et al;Haber,Hempfling;
Hoang et al;Carena et al;Heinemeyer et al;
Zhang et al;Brignole et al;...

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Decoupling limit:

$M_A \sim M_H \sim M_{H^\pm} \gtrsim v$

$M_h \rightarrow$ max. value, $\tan\beta$ fixed; h becomes SM-like

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Modified couplings with respect to the SM: (decoupling limit Gunion,Haber)

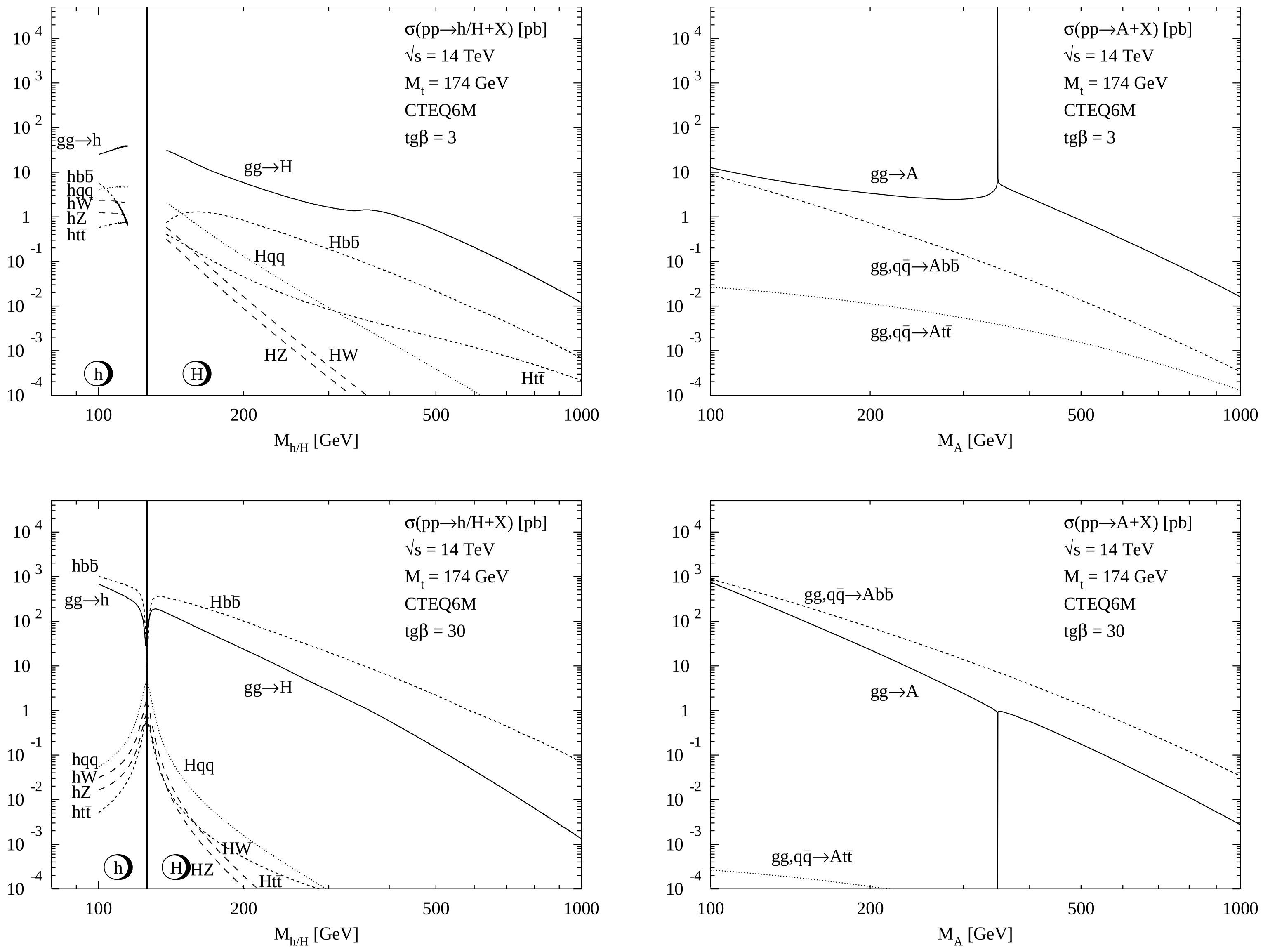
Φ	$g_{\Phi u\bar{u}}$	$g_{\phi d\bar{d}}$	$g_{\Phi VV}$
h	$c_\alpha/s_\beta \to 1$	$-s_\alpha/c_\beta \to 1$	$s_{\beta-\alpha} \to 1$
H	$s_\alpha/s_\beta \to 1/\text{tg}\beta$	$c_\alpha/c_\beta \to \text{tg}\beta$	$c_{\beta-\alpha} \to 0$
A	$1/\text{tg}\beta$	$\text{tg}\beta$	0

$\tan\beta \uparrow \;\Rightarrow g_{\Phi uu} \downarrow$

$g_{\Phi dd} \uparrow$

$g_{\Phi VV}^{MSSM} \lesssim g_{\Phi VV}^{SM}$

MSSM Higgs Boson Production at the LHC

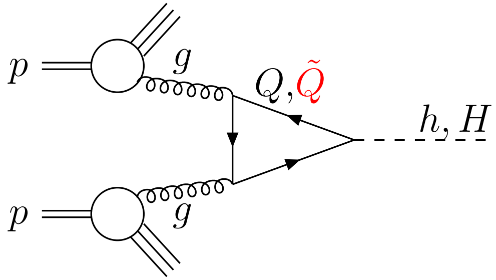


Spira

$gg \rightarrow H, h$ at leading order

Lowest order - 1 loop

Georgi,...;Gamberini,...



$$\boxed{\sigma(pp \rightarrow \Phi + X) \quad = \quad \sigma_0^\Phi \tau_\Phi \frac{d\mathcal{L}^{gg}}{d\tau_\Phi}}$$

$$\sigma_0^{h/H} = \frac{G_F \alpha_S^2(\mu_R)}{288\sqrt{2}\pi} \left| \sum_Q g_Q^{h/H} F_Q^{h/H}(\tau_Q) + \sum_{\tilde{Q}} g_{\tilde{Q}}^{h/H} F_{\tilde{Q}}^{h/H}(\tau_{\tilde{Q}}) \right|^2 \qquad \sigma_0^A = \frac{G_F \alpha_s^2}{128\sqrt{2}\pi} \left| \sum_Q g_Q^A F_Q^A(\tau_Q) \right|^2$$

$$F_Q^{h/H}(\tau_Q) \quad = \quad \tfrac{3}{2}\tau_Q \Big[1 + (1-\tau_Q) f(\tau_Q) \Big] \qquad F_Q^A(\tau_Q) = \tau_Q f(\tau_Q)$$

$$F_{\tilde{Q}}^{h/H}(\tau_{\tilde{Q}}) \quad = \quad -\tfrac{3}{4}\tau_{\tilde{Q}} \Big[1 - \tau_{\tilde{Q}} f(\tau_{\tilde{Q}}) \Big]$$

$$f(\tau) \quad = \quad \begin{cases} \arcsin^2 \frac{1}{\sqrt{\tau}} & \tau \geq 1 \\ -\frac{1}{4}\left[\log\left(\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} \right) - i\pi \right]^2 & \tau < 1 \end{cases} \qquad \tau_\Phi = \frac{M_\Phi^2}{s}, \quad \tau_{Q,\tilde{Q}} = \frac{4m_{Q,\tilde{Q}}^2}{M_\Phi^2}$$

$\mathrm{gg} \to \mathrm{H}, \mathrm{h}$ at leading order

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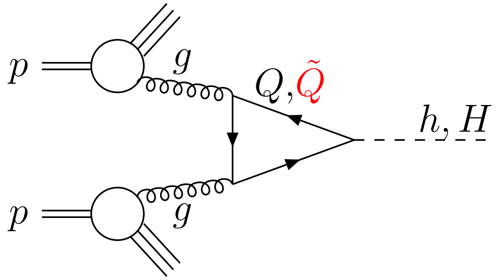

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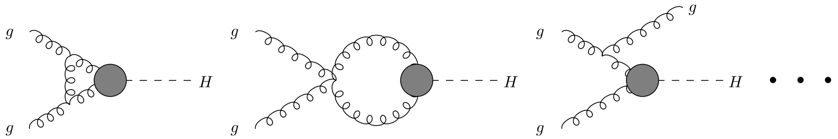
Remarks:

- $gg \to A$ no $\tilde{Q}$ contribution at LO
- MSSM: $\tan\beta \uparrow \Rightarrow b|\tilde{b} \uparrow$, and $t|\tilde{t} \downarrow$
- 3rd generation dominant, $\tilde{t}, \tilde{b}$ contributions important for $m_{\tilde{q}} \lesssim 400$ GeV.

Comments

QCD corrections to top & bottom loops (2-loop)

- ◇ NLO (SM, MSSM): increase σ by $\sim$ 10...100% — Spira,Djouadi,Graudenz,Zerwas Dawson;Kauffman,Schaffer
- ◇ SM; tg$\beta \lesssim 5$: $M_\Phi \ll m_t$ approximation for K-factor [$\Delta \lesssim$ 25%] — Krämer,Laenen,Spira



- ◇ NNLO @ $M_\Phi \ll m_t \Rightarrow$ further increase by 20-30%
  scale dependence: $\Delta \lesssim 10-15\%$ — Harlander,Kilgore Anastasiou,Melnikov Ravindran,Smith,van Neerven
- ◇ Mass effects on NNLO corrections small in interm. mass region — Marzani,Ball,DelDuca,Forte, Vicini,Harlander,Ozeren Pag,Rogat,Steinhauser
- ◇ Estimate of NNNLO effects $\rightsquigarrow$ improved convergence
  scale dependence $\Delta \lesssim 10-15\%$ — Moch,Vogt Ravindran
- ◇ Soft gluon resummation: $\sim 10\%$ — Catani,de Florian,Grazzini,Nason

Comments

EW/QCD corrections

◇ EW 2-loop effects $\sim -4-6$ % enhancement — Aglietti eal; Degrassi,Maltoni;Actis eal

◇ mixed EW-QCD corrections — Anastasiou,Boughezal,Petriello

NLO corrections to squark loops

◇ heavy squark limit — Dawson,Djouadi,Spira

◇ full SUSY-QCD corrections in heavy mass limit — Harlander,Steinhauser; Harlander,Hofmann

$\mathbf{m_{\tilde{Q}} \lesssim 400}$ **GeV:** squarks play a significant role $\rightsquigarrow$

◇ NLO squark mass effects — Anastasiou,Beerli,Bucherer, Daleo,Kunszt;Aglietti,Bonciani, Degrassi,Vicini;MMM,Spira

◇ full NLO SUSY QCD calculation — Anastasiou,Beerli,Daleo; MMM,Rzehak,Spira

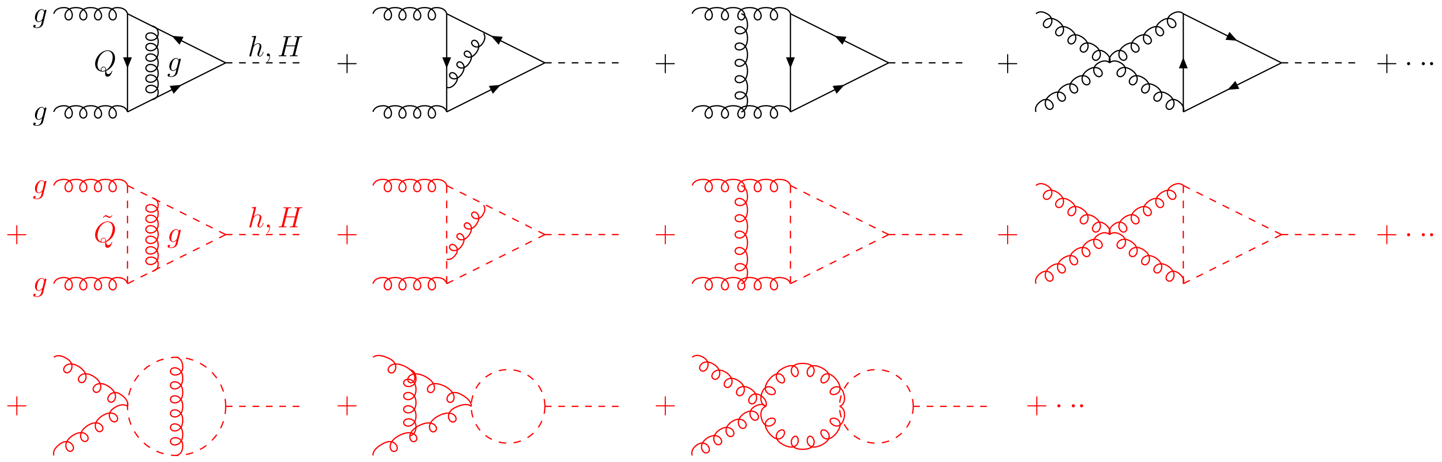
$\mathcal{F}$irst $\mathcal{S}$tep: QCD corrections

$$\boxed{\Delta\hat{\sigma}_{ij} = \sigma_0 \Big\{ C_{ij}\delta(1-\hat{\tau}) + D_{ij}\Theta(1-\hat{\tau}) \Big\} \frac{\alpha_s}{\pi}} \qquad \hat{\tau} = \frac{M_\Phi^2}{\hat{s}}$$

↗ virtual+soft corrections

↑ real corrections

Virtual corrections [2 loops, first step: no gluino contributions]



UV-,IR-,Coll-singularities in $n = 4 - 2\epsilon$ dimensions.

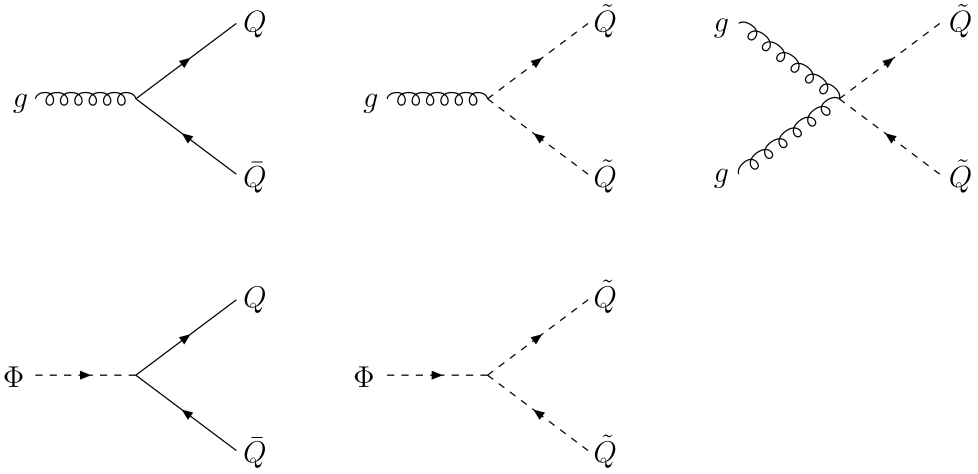
Renormalization

$\mathcal{L}$agrangian separates gluon and gluino exchange contributions in a renormalizable way

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}G^{a\mu\nu}G^{a}_{\mu\nu} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}(\partial_\mu\Phi)^2 - \frac{M_\Phi^2}{2}\Phi^2 \\ &+ \sum_Q \left[\bar{Q}(i\not{D} - m_Q)Q - g_Q^\Phi \frac{m_Q}{v}\bar{Q}Q\Phi\right] + \sum_{\tilde{Q}}\left[|D_\mu\tilde{Q}|^2 - m_{\tilde{Q}}^2|\tilde{Q}|^2 - g_{\tilde{Q}}^\Phi\frac{m_{\tilde{Q}}^2}{v}|\tilde{Q}|^2\Phi\right]\end{aligned}$$

$$iD_\mu = i\partial_\mu - g_S G_\mu^a T^a - eA_\mu \mathcal{Q}$$

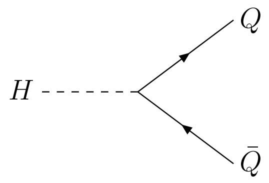
Gluon, $\Phi = \mathrm{H}/\mathrm{h}$ interaction vertices:



Renormalization - cont'd

- ◇ **Quark/Squark mass $\mathrm{m}_{\mathrm{Q},\tilde{\mathrm{Q}}}$:** on-shell
- ◇ **$\mathrm{g}_{\tilde{\mathrm{Q}}}^{\mathrm{H}}$ not renormalized** [MSSM: $g_{\tilde{Q}}^{h/H} = \frac{m_Q^2}{m_{\tilde{Q}}^2} g_Q^{h,H}$ + mixing terms + D-terms]
- ◇ **α_{S} $\overline{\mathrm{MS}}$ (5 active flavours)**
- ◇ **$\mathrm{HQ\bar{Q}}$ vertex:**

$$\mathcal{L}_{\text{int}} = -g_Q^H \frac{m_{Q_0}}{v} \bar{Q}_0 Q_0 H = -g_Q^H \frac{m_Q}{v} \bar{Q} Q H \underbrace{\left[Z_2 - \frac{\delta m_Q}{m_Q} \right]}_{Z_{HQQ}} + \mathcal{O}(\alpha_S^2)$$

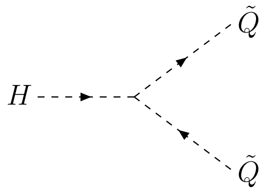


$$\Gamma_{H\bar{Q}Q}(q^2 = 0) \neq Z_{HQQ}$$

Braaten, Leveille

- ◇ **$\mathrm{H\tilde{Q}\tilde{Q}}$ vertex:**

$$\mathcal{L}_{\text{int}} = -g_{\tilde{Q}}^H \frac{m_{\tilde{Q}_0}^2}{v} \tilde{Q}_0^* \tilde{Q}_0 H = -g_{\tilde{Q}}^H \frac{m_{\tilde{Q}}^2}{v} \tilde{Q}^* \tilde{Q} H \underbrace{\left[Z_2^{\tilde{Q}} - \frac{\delta m_{\tilde{Q}}^2}{m_{\tilde{Q}}^2} \right]}_{Z_{H\tilde{Q}\tilde{Q}}} + \mathcal{O}(\alpha_S^2)$$



$$\Gamma_{H\tilde{Q}\tilde{Q}}(q^2 = 0) \neq Z_{H\tilde{Q}\tilde{Q}}$$

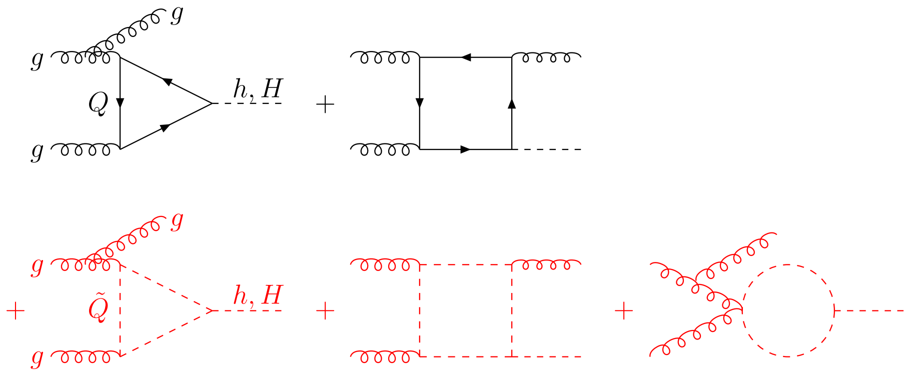
disregard renorm. of $g_{\tilde{Q}}^H$!

Real Corrections

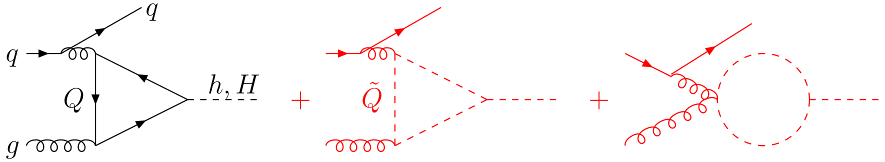
After renormalization: IR & coll. singularities $\rightsquigarrow$ real corrections have to be added.

3 incoherent processes:

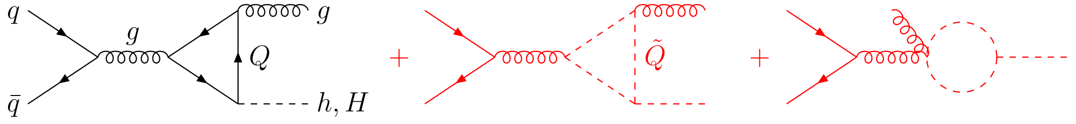
gg → Hg:



gq → Hq:



q̄q → Hg:



Phase space integration in $n = 4 - 2\epsilon$ dimensions $\rightsquigarrow$ IR, Coll. singularities: poles in ϵ

Result

- α_S: $\overline{\mathrm{MS}}$ scheme, 5 active flavours
- μ=Ren. scale, Q=Fact. scale, $\mu^2 = Q^2 = M_\phi^2$

$$\begin{aligned}
\sigma(pp \to \phi + X) &= \sigma_0^\phi [1 + C^\phi \tfrac{\alpha_S}{\pi}] \tau_\phi \tfrac{d\mathcal{L}_{gg}}{d\tau_\phi} + \Delta\sigma_{gg}^\phi + \Delta\sigma_{gq}^\phi + \Delta\sigma_{q\bar{q}}^\phi \\
C^\phi(\tau_Q, \tau_{\tilde{Q}}) &= \pi^2 + C_1^\phi(\tau_Q, \tau_{\tilde{Q}}) + \tfrac{33-2N_F}{6} \log \tfrac{\mu^2}{M_\phi^2} \\
\Delta\sigma_{gg}^\phi &= \int_{\tau_\phi}^1 d\tau \tfrac{d\mathcal{L}_{gg}}{d\tau} \tfrac{\alpha_S}{\pi} \sigma_0^\phi \Big\{ -\hat{\tau} P_{gg}(\hat{\tau}) \log \tfrac{Q^2}{\hat{s}} + d_{gg}^\phi(\hat{\tau}, \tau_Q, \tau_{\tilde{Q}}) \\
&\quad +12 \Big[\Big(\tfrac{\log(1-\hat{\tau})}{1-\hat{\tau}} \Big)_+ - \hat{\tau}[2 - \hat{\tau}(1-\hat{\tau})] \log(1-\hat{\tau}) \Big] \Big\} \\
\Delta\sigma_{gq}^\phi &= \int_{\tau_\phi}^1 d\tau \sum_{q,\bar{q}} \tfrac{d\mathcal{L}_{gq}}{d\tau} \tfrac{\alpha_S}{\pi} \sigma_0^\phi \Big\{ -\tfrac{\hat{\tau}}{2} P_{gq}(\hat{\tau}) \Big[\log \tfrac{Q^2}{\hat{s}(1-\hat{\tau})^2} \Big] d_{gq}^\phi(\hat{\tau}, \tau_Q, \tau_{\tilde{Q}}) \Big\} \\
\Delta\sigma_{q\bar{q}}^\phi &= \int_{\tau_\phi}^1 d\tau \sum_q \tfrac{d\mathcal{L}_{q\bar{q}}}{d\tau} \tfrac{\alpha_S}{\pi} \sigma_0^\phi \, d_{q\bar{q}}^\phi(\hat{\tau}, \tau_Q, \tau_{\tilde{Q}})
\end{aligned}$$

- $\tau_{Q,\tilde{Q}} = \frac{4m_{Q,\tilde{Q}}^2}{M_\Phi^2}$, $\hat{\tau} = \frac{m_\phi^2}{\hat{s}}$

The Scenario

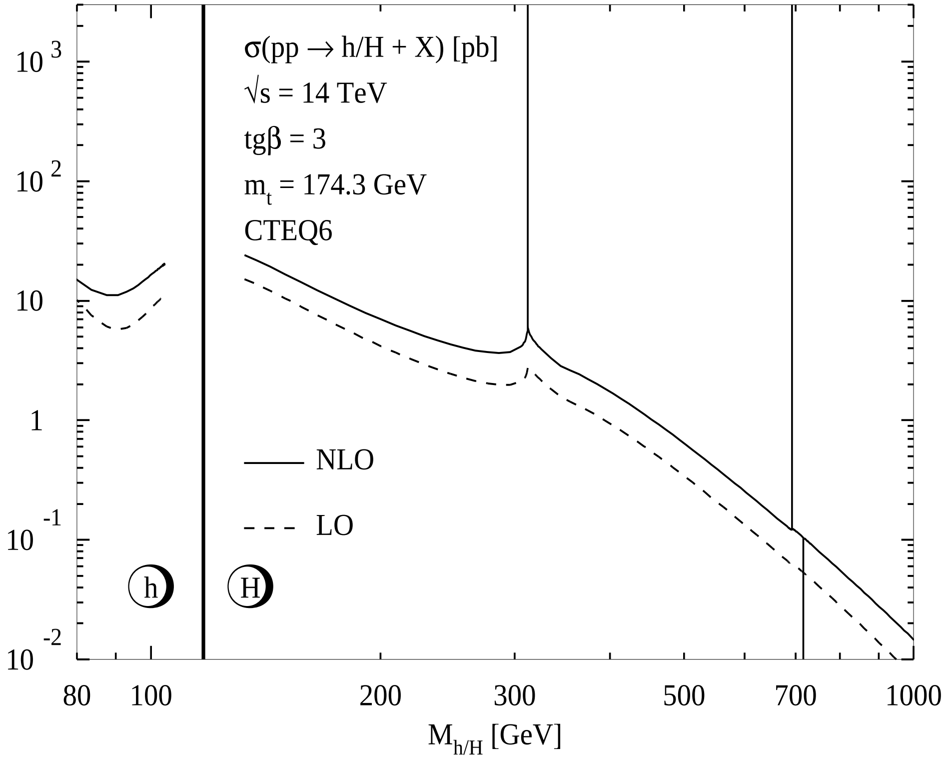
The gluophobic Higgs scenario $[m_t = 174.3$ GeV$]$ Carena,Heinemeyer,Wagner,Weiglein

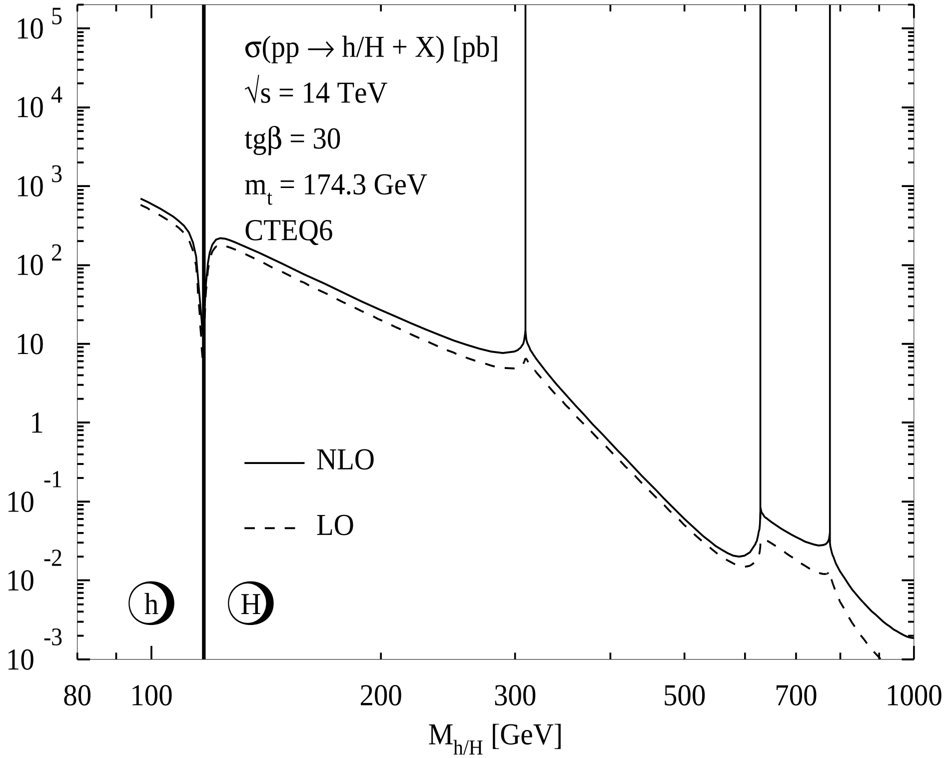
$$M_{SUSY} = 350 \text{ GeV},\ \mu = M_2 = 300 \text{ GeV},\ X_t = -770 \text{ GeV},\ A_b = A_t,\ m_{\tilde{g}} = 500 \text{ GeV}$$

$\tan\beta = 3$		$\tan\beta = 30$	
$m_{\tilde{t}_1} = 156$ GeV	$m_{\tilde{t}_2} = 517$ GeV	$m_{\tilde{t}_1} = 155$ GeV	$m_{\tilde{t}_2} = 516$ GeV
$m_{\tilde{b}_1} = 346$ GeV	$m_{\tilde{b}_2} = 358$ GeV	$m_{\tilde{b}_1} = 314$ GeV	$m_{\tilde{b}_2} = 388$ GeV

NLO cross section →

The LO and NLO cross section w/ Squarks





$\Delta \sim 20 - 100\%$

Kinks, bumps, spikes: $\tilde{t}_1\bar{\tilde{t}}_1, \tilde{b}_1\bar{\tilde{b}}_1, \tilde{b}_2\bar{\tilde{b}}_2$ thresholds in consecutive order with rising Higgs mass.

Coulomb singularities

$\tilde{Q}\bar{\tilde{Q}}$ thresholds: Formation of 0^{++} states ⇝ Coulomb singularities

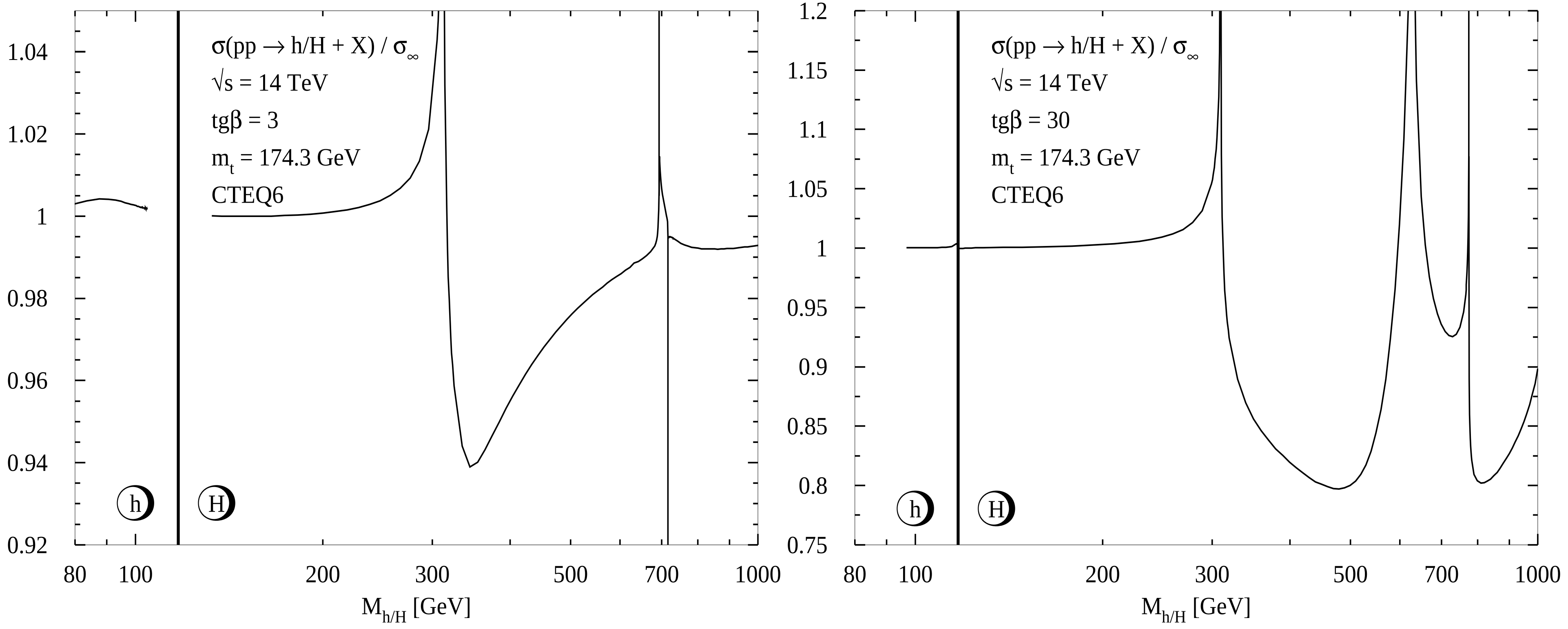
Singular behaviour can be derived from the Sommerfeld rescattering corrections ⇝

At each specific $\tilde{Q}_0\bar{\tilde{Q}}_0$ threshold:

$$C_1(\tau_Q, \tau_{\tilde{Q}}) \to \mathrm{Re}\left\{ \frac{g^{\Phi}_{\tilde{Q}_0} \tilde{F}(\tilde{Q}_0) \frac{16\pi^2}{3(\pi^2-4)} \left[-\ln\left(\tau^{-1}_{\tilde{Q}_0} - 1\right) + i\pi + const \right]}{\sum_Q g^{\Phi}_Q F(\tau_Q) + \sum_{\tilde{Q}} g^{\Phi}_{\tilde{Q}} \tilde{F}(\tau_{\tilde{Q}})} \right\}$$

Agrees quantitatively with numerical results.

σ_{NLO} w/ full squark mass dependence / σ_{NLO} in the heavy squark limit

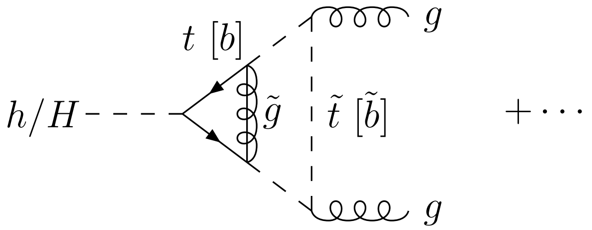


$\sigma(pp \to h/H + X)/\sigma_\infty$ up to 20%

Kinks, bumps, spikes: $\tilde{t}_1\bar{\tilde{t}}_1, \tilde{b}_1\bar{\tilde{b}}_1, \tilde{b}_2\bar{\tilde{b}}_2$ thresholds in consecutive order with rising Higgs mass.

Genuine SUSY-QCD corrections

- **Limit heavy SUSY masses** $\rightarrow \mathcal{O}(10\ \%)$ Harlander,Steinhauser,Hofmann



- **Numerical analysis:** $F_Q^{h/H}(\tau_Q) \rightarrow F_Q^{h/H}(\tau_Q)[1 + C_{SUSY}^{Q} \frac{\alpha_S}{\pi}]$

- $m^2_{Q/\tilde{Q}} \rightarrow m^2_{Q/\tilde{Q}}(1 - i\epsilon)$

- **5-dimensional Feynman integral** $\rightarrow$ **endpoint substractions:**

$$\int_0^1 dx\, \frac{f(x)}{x(1-x)} \rightarrow \int_0^1 dx \left\{ \frac{f(x)}{x(1-x)} - \frac{f(0)}{x} - \frac{f(1)}{(1-x)} \right\}$$

$\Rightarrow$ isolation of singularities

Genuine SUSY-QCD corrections

- **Thresholds for $M_H > 2m_Q$** → numerical instabilities → partial integration

$$\begin{aligned}\int_0^1 dz \frac{f(z)}{(a+bz)^2} &= -\frac{f(z)}{b(a+bz)}\bigg|_0^1 + \int_0^1 dz\, \frac{f'(z)}{b(a+bz)} \\ \int_0^1 dz \frac{f(z)}{a+bz} &= \frac{f(z)}{b}\ln(a+bz)\bigg|_0^1 - \int_0^1 dz\, \frac{f'(z)}{b}\ln(a+bz)\end{aligned}$$

$\Rightarrow$ thresholds in arguments of logs $\Rightarrow$ stabilization

[more involved for quadratic polynomials]

- **Renormalization**

α_S : $\overline{\text{MS}}$ scheme [5 flavours]

$m_Q, m_{\tilde{Q}}, A_t$: on-shell

A_b : on-shell $\leftrightarrow$ $\overline{\text{MS}}$

A_t, A_b : anomalous SUSY-restoring counter-terms

$$\begin{aligned}A_b &= \frac{\sin 2\theta_b}{2m_b}(m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2}) + \mu \tan\beta \\ \delta\theta_b &= \frac{1}{2}\text{Re}\frac{\Sigma_{12}(m^2_{\tilde{b}_1}) + \Sigma_{12}(m^2_{\tilde{b}_2})}{m^2_{\tilde{b}_2} - m^2_{\tilde{b}_1}}\end{aligned}$$

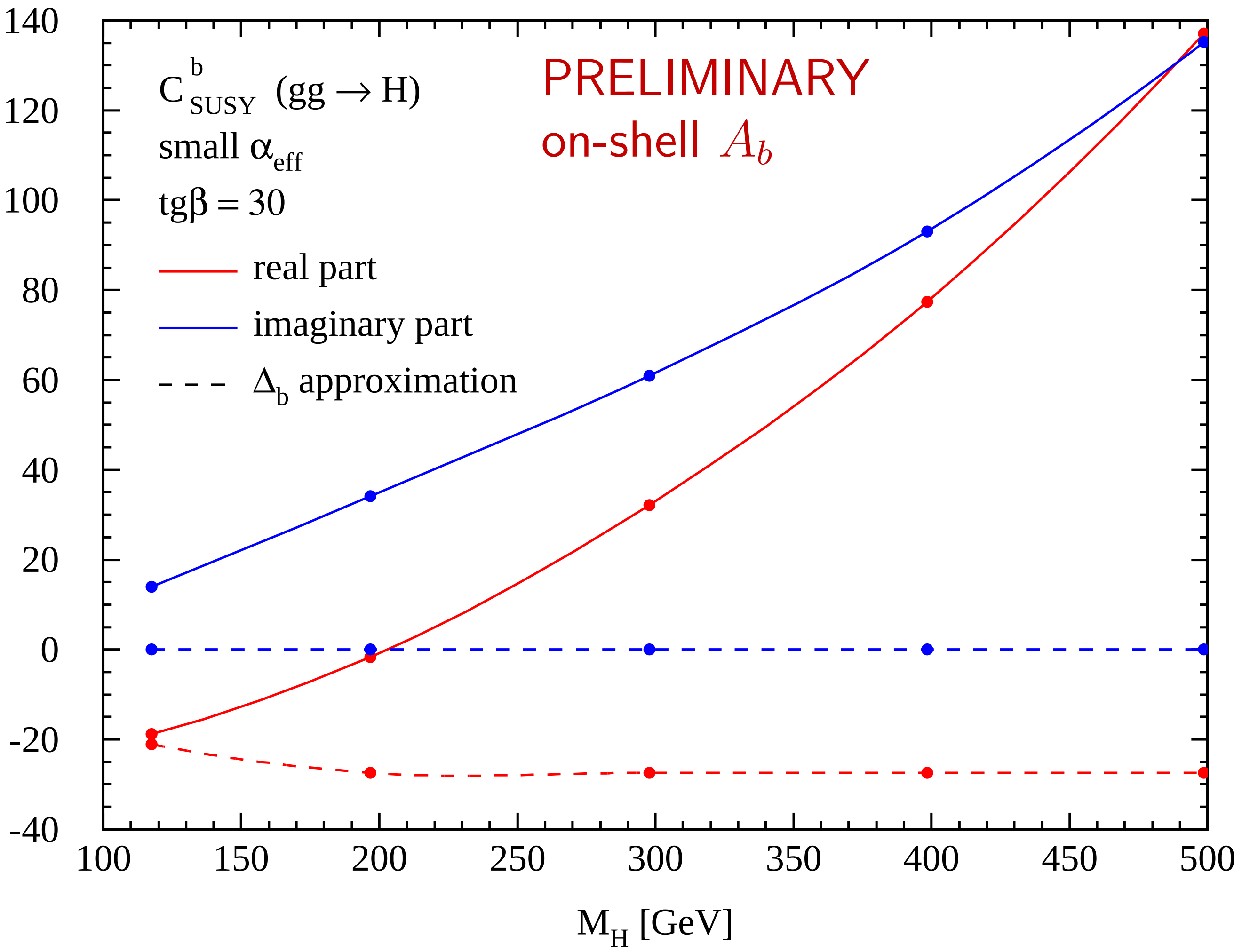
The Scenario

Small α_{eff} scenario [modified]

$$\begin{array}{lcll}
\tan\beta & = & 30 & \\
M_{\tilde{Q}} & = & 800\ \text{GeV} & \\
M_{\tilde{g}} & = & 1000\ \text{GeV} & \longleftarrow \\
M_2 & = & 500\ \text{GeV} & \\
A_b = A_t & = & -1.133\ \text{TeV} & \\
\mu & = & 2\ \text{TeV} &
\end{array}$$

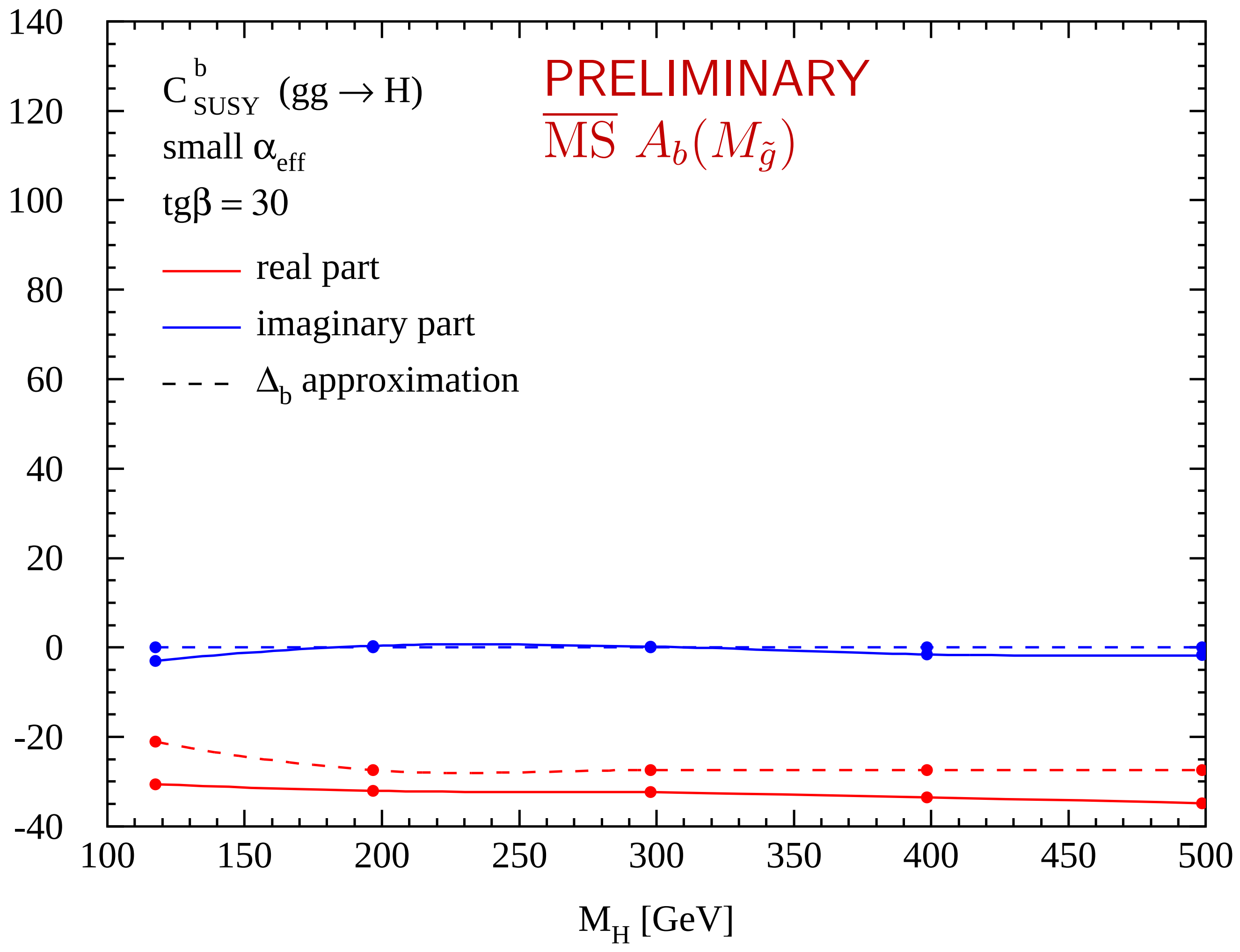
$$\begin{array}{lclclcl}
m_{\tilde{t}_1} & = & 679\ \text{GeV} & \qquad & m_{\tilde{t}_2} & = & 935\ \text{GeV} \\
m_{\tilde{b}_1} & = & 601\ \text{GeV} & \qquad & m_{\tilde{b}_2} & = & 961\ \text{GeV}
\end{array}$$

Preliminary results



MMM, Rzehak, Spira

Preliminary results



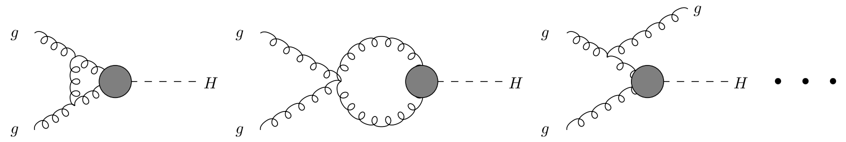
MMM, Rzehak, Spira

Heavy loop particle mass limit

- **Heavy quarks/squarks and very heavy gluinos** $[m_{\tilde{g}}^2 \gg m_{Q,\tilde{Q}}^2 \gg M_\phi^2]$

$$\begin{array}{llllll} c_Q^\phi & \to & \frac{11}{2} & d_{gg}^\phi & \to & -\frac{11}{2}(1-\hat{\tau})^3 \\ d_{gq}^\phi & \to & -1+2\hat{\tau}-\frac{\hat{\tau}^2}{3} & d_{q\bar{q}}^\phi & \to & \frac{32}{27}(1-\hat{\tau})^3 \\ c_{\tilde{Q}}^\phi & \to & 9 & c_{SUSY}^\phi & \to & \frac{10}{3} \end{array}$$

- **c_{SUSY}^ϕ: proper decoupling of gluinos** → non-supersymmetric $\mathcal{L}_{eff}$ MMM,Rzehak,Spira



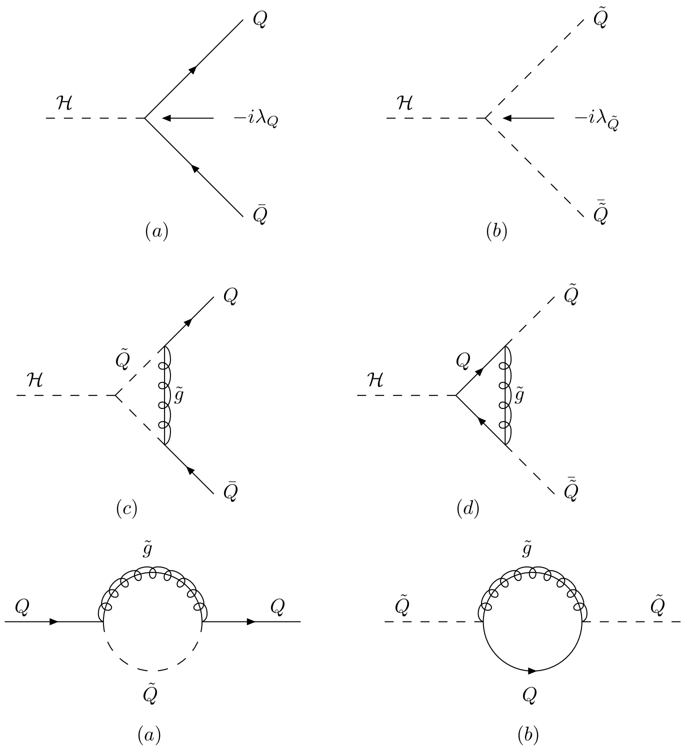
- Harlander,Steinhauser: **mass degenerate squarks, no mixing, supersymmetric renormalization**

$M_{\tilde{g}} \gg m_{\tilde{Q}}, m_Q$:

$$C_{SQCD}^{HS} = \frac{11}{2} - \frac{4}{3}\ln\frac{M_{\tilde{g}}^2}{m_{\tilde{Q}}^2} - 2\ln\frac{m_{\tilde{Q}}^2}{m_Q^2} \qquad [\text{SUSY: } g_{\tilde{Q}}^{\mathcal{H}} = 2g_Q^{\mathcal{H}}\frac{m_Q^2}{m_{\tilde{Q}}^2}]$$

Heavy loop particle mass limit

- **$M_{\tilde{g}} \gg m_{\tilde{Q}}, m_Q$:** Supersymmetry lost due to decoupled gluino→ integrate gluinos out



- **No mixing at LO:**

$$\lambda_Q = g_Q^{\mathcal{H}} \frac{m_Q}{v} \qquad \lambda_{\tilde{Q}} = 2 g_Q^{\mathcal{H}} \frac{m_Q^2}{v} = \kappa \lambda_Q^2 \qquad \kappa = 2 \frac{v}{g_Q^{\mathcal{H}}}$$

Heavy loop particle mass limit

- **SUSY beyond LO:** $\overline{\text{MS}}$ couplings $[\mu_R > M_{\tilde{g}}]$

$$\bar{\lambda}_{\tilde{Q}}(\mu_R) = \kappa \bar{\lambda}^2_Q(\mu_R)$$

- $\mu_R < M_{\tilde{g}}$: (i) threshold corrections

  (ii) different RGEs [decoupled $\tilde{g}$]

- $\mu_R < M_{\tilde{g}}$: momentum-substracted coupling → threshold correction:

  (i) threshold correction:

$$\bar{\lambda}_{Q,MO}(M_{\tilde{g}}) = \bar{\lambda}_Q(M_{\tilde{g}}) \left\{ 1 - \tfrac{3}{8} C_F \frac{\alpha_S(M_{\tilde{g}})}{\pi} \right\}$$

  (ii) different RGEs:

$$\mu_R^2 \frac{\partial \bar{\lambda}_Q(\mu_R)}{\partial \mu_R^2} = -\frac{C_F}{2} \frac{\alpha_S(\mu_R)}{\pi} \bar{\lambda}_Q(\mu_R) \qquad [\mu_R > M_{\tilde{g}}]$$

$$\mu_R^2 \frac{\partial \bar{\lambda}_{Q,MO}(\mu_R)}{\partial \mu_R^2} = -\tfrac{3}{4} C_F \frac{\alpha_S(\mu_R)}{\pi} \bar{\lambda}_{Q,MO}(\mu_R) \qquad [\mu_R < M_{\tilde{g}}]$$

Heavy loop particle mass limit

- **analogously for $\lambda_{\tilde{Q}}$:**

  (i) threshold correction:

$$\bar{\lambda}_{\tilde{Q},MO}(M_{\tilde{g}}) = \bar{\lambda}_{\tilde{Q}}(M_{\tilde{g}}) \Big\{ 1 + \tfrac{3}{4} C_F \frac{\alpha_S(M_{\tilde{g}})}{\pi} \Big\}$$

  (ii) different RGEs:

$$\begin{aligned} \mu_R^2 \frac{\partial \bar{\lambda}_{\tilde{Q}}(\mu_R)}{\partial \mu_R^2} &= -C_F \frac{\alpha_S(\mu_R)}{\pi} \bar{\lambda}_{\tilde{Q}}(\mu_R) && [\mu_R > M_{\tilde{g}}] \\ \mu_R^2 \frac{\partial \bar{\lambda}_{\tilde{Q},MO}(\mu_R)}{\partial \mu_R^2} &= -\frac{C_F}{2} \frac{\alpha_S(\mu_R)}{\pi} \bar{\lambda}_{\tilde{Q},MO}(\mu_R) && [\mu_R < M_{\tilde{g}}] \end{aligned}$$

- **Relation to quark pole mass:** Gray,Broadhurst,Grafe,Schilcher

$$g_Q^\phi \frac{m_Q}{v} = \bar{\lambda}_{Q,MO}(m_Q) \Big\{ 1 + C_F \frac{\alpha_S(m_Q)}{\pi} \Big\}$$

Heavy loop particle mass limit

$$\begin{aligned}
2g_Q^{\mathcal{H}}\frac{m_Q^2}{v} &= \bar{\lambda}_{\tilde{Q},MO}(m_{\tilde{Q}})\Big\{1+C_F\frac{\alpha_S}{\pi}\Big(\ln\frac{M_{\tilde{g}}^2}{m_{\tilde{Q}}^2}+\frac{3}{2}\ln\frac{m_{\tilde{Q}}^2}{m_Q^2}+\frac{1}{2}\Big)\Big\} \\
\mathcal{L}_{eff} &= \frac{\alpha_S}{12\pi}G^{a\mu\nu}G^a_{\mu\nu}\frac{\mathcal{H}}{v}\Big\{\sum_Q g_Q^{\mathcal{H}}\Big[1+\frac{11}{4}\frac{\alpha_S}{\pi}\Big]+\sum_{\tilde{Q}}\frac{g_{\tilde{Q}}^{\mathcal{H}}}{4}\Big[1+C_{SQCD}\frac{\alpha_S}{\pi}\Big]\Big\} \\
g_{\tilde{Q}}^{\mathcal{H}} &= v\frac{\bar{\lambda}_{\tilde{Q},MO}(m_{\tilde{Q}})}{m_{\tilde{Q}}^2} \\
\Delta C_{SQCD} &= \frac{4}{3}\ln\frac{M_{\tilde{g}}^2}{m_{\tilde{Q}}^2}+2\ln\frac{m_{\tilde{Q}}^2}{m_Q^2}+\frac{2}{3} \qquad \Longrightarrow \qquad \boxed{C_{SQCD}=\frac{37}{6}}
\end{aligned}$$

- **Solution to RGEs** $[\beta_0=(33-2N_F-N_{\tilde{F}})/12]$

$$\boxed{\bar{\lambda}_{\tilde{Q},MO}(m_{\tilde{Q}})=2g_Q^{\mathcal{H}}\frac{m_Q^2}{v}\frac{1+\frac{3}{2}C_F\frac{\alpha_S(M_{\tilde{g}})}{\pi}}{1+2C_F\frac{\alpha_S(m_Q)}{\pi}}\left(\frac{\alpha_S(M_{\tilde{g}})}{\alpha_S(m_{\tilde{Q}})}\right)^{\frac{C_F}{\beta_0}}\left(\frac{\alpha_S(m_{\tilde{Q}})}{\alpha_S(m_Q)}\right)^{\frac{3C_F}{2\beta_0}}}$$

- **No $\tilde{Q}$ loops to $gg \to A$ at LO** $\Rightarrow$ no $\ln M_{\tilde{g}}$

Harlander,Hofmann

Conclusions

- ◇ SUSY QCD corrections at NLO to $gg \to h/H$ including the full squark mass dependence.

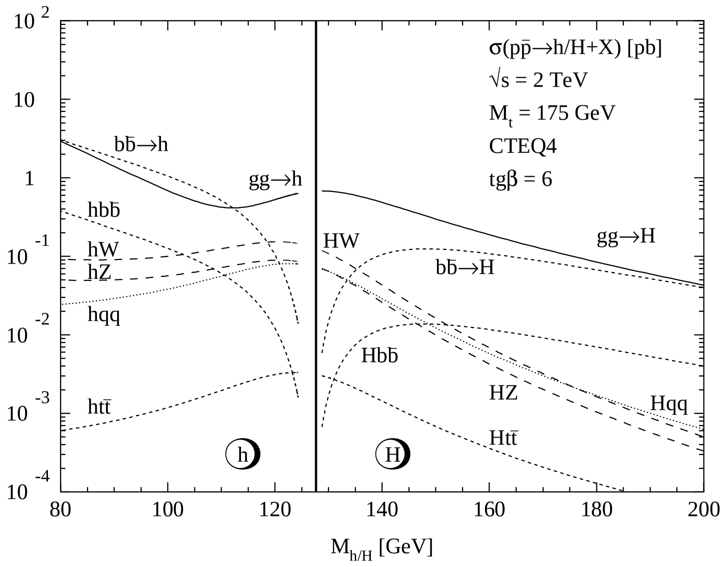
- ◇ Inclusion of full squark mass dependence has significant effects on the K-factor compared to the heavy squark mass limit. The deviation can be as large as $\mathcal{O}(20\%)$ for $gg \to h/H$.

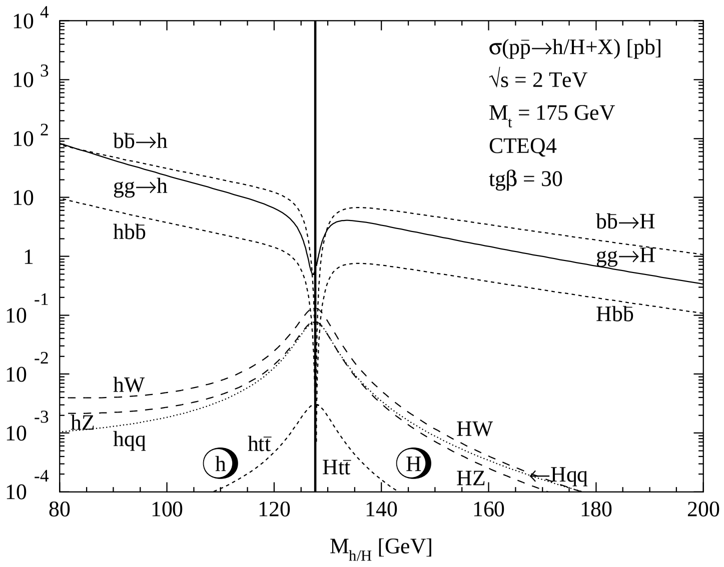
- ◇ Large QCD corrections + large genuine SUSY QCD corrections for large $\tan\beta$ in MSSM $\leftarrow \Delta_b$ approximation for $\overline{\text{MS}}$ A_b.

- ◇ $\mathcal{H}gg$ coupling: decoupling of gluinos for large $M_{\tilde{g}}$: consistent with Appelquist-Carazzone theorem if properly renormalized → effective Lagrangian.

MSSM Higgs Boson Production at the Tevatron

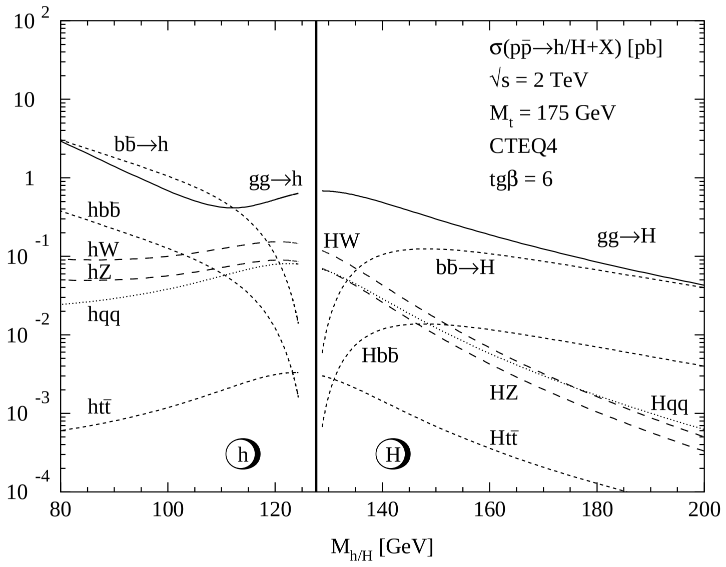
Spira

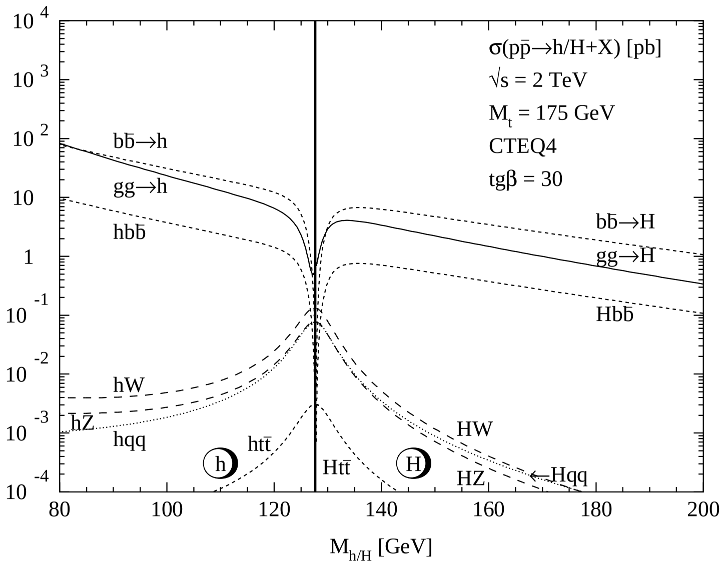




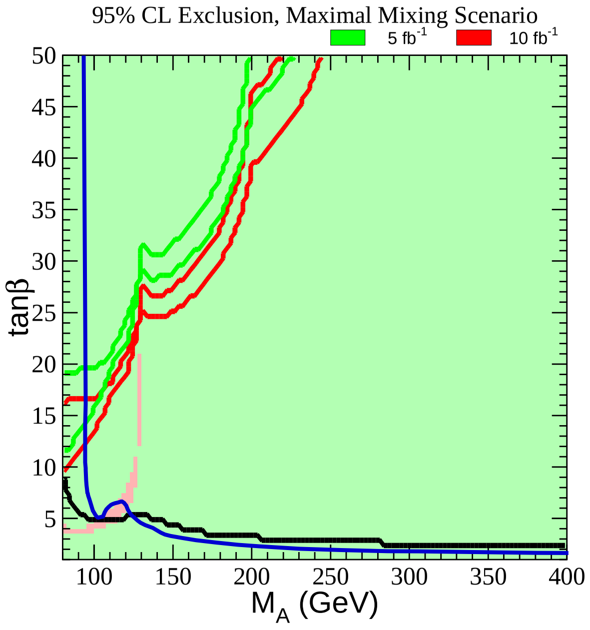
MSSM Higgs Boson Production at the Tevatron

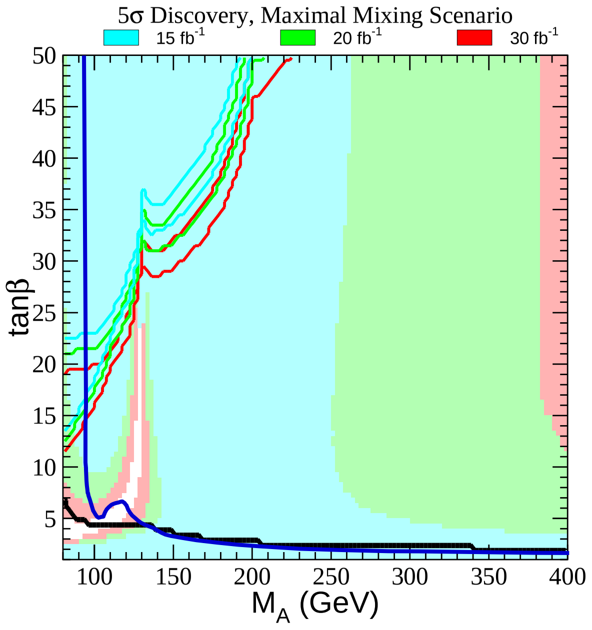
Spira



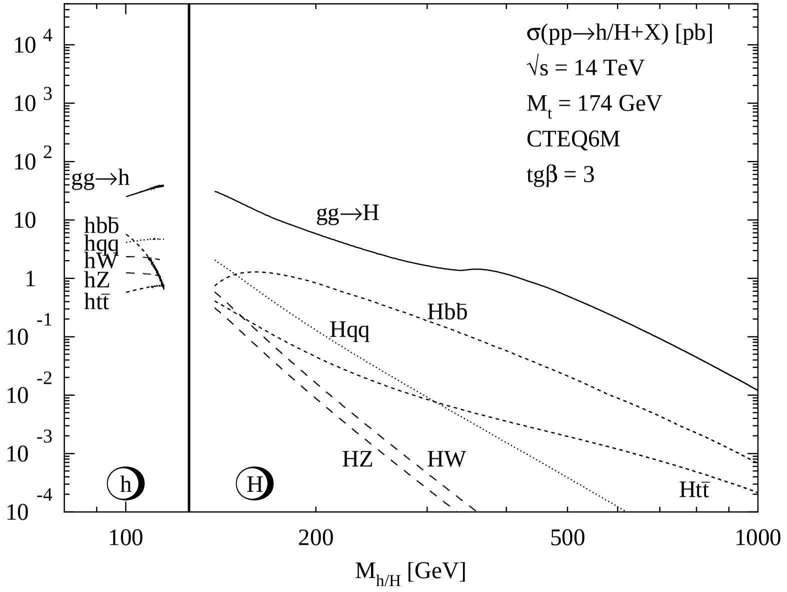


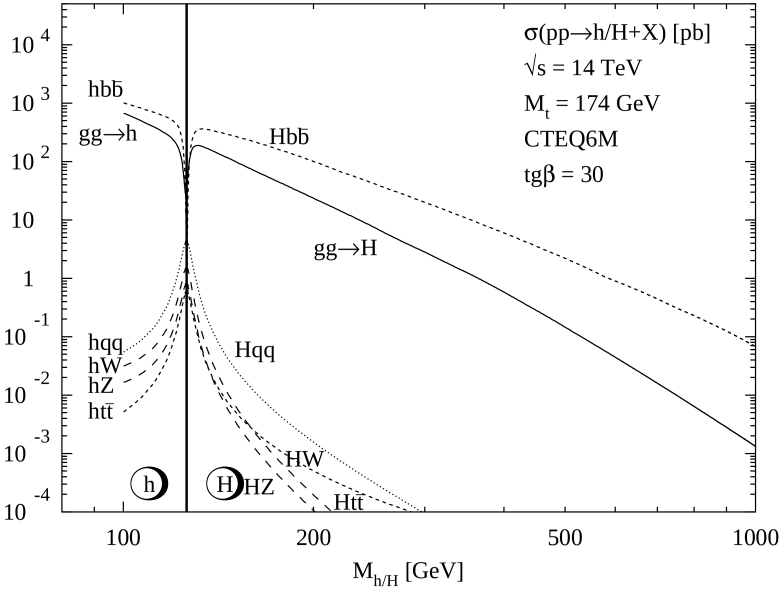
Carena et al.





MSSM Higgs Boson Production at the LHC





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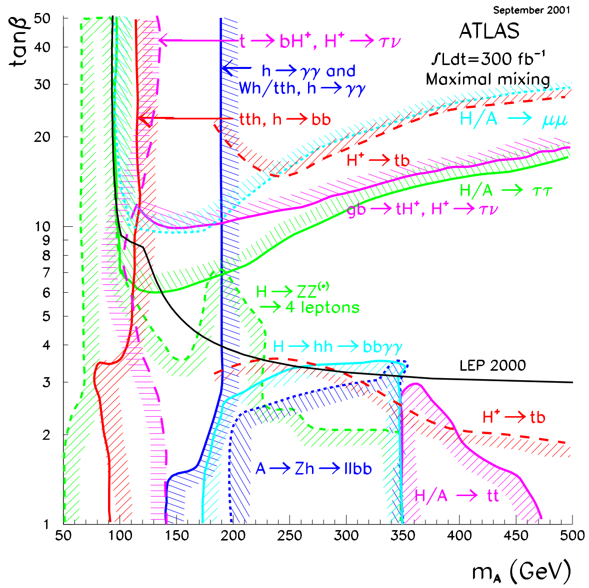
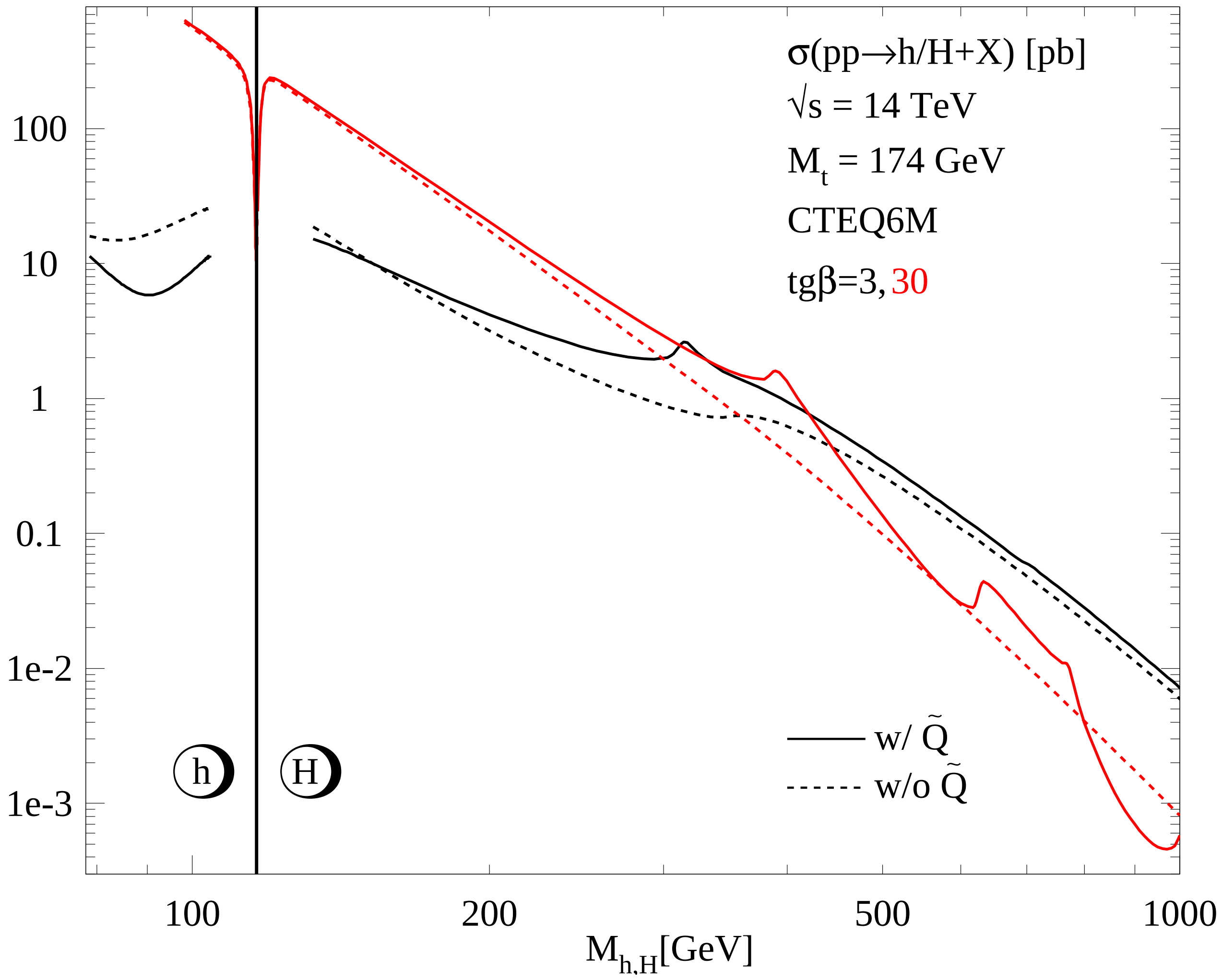


ATLAS

The LO cross section w/ and w/o Squarks



Virtual corrections - heavy loop particle mass limit

Total virtual correction [heavy squark/quark limit]:

$$\boxed{C_{\rm virt} = \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{4\pi\mu^2}{M_\Phi^2}\right)^\epsilon \left\{ -\underset{\overset{\uparrow}{\text{IR}}}{\frac{3}{\epsilon^2}} - \underset{\overset{\uparrow}{\text{Coll}}}{\frac{33-2N_F}{6\epsilon}} \left(\frac{\mu^2}{M_\Phi^2}\right)^{-\epsilon} + \pi^2 + \frac{11}{2} + \frac{7}{2}\text{Re}\left\{ \frac{\sum_{\tilde{Q}} g_{\tilde{Q}}^\Phi \tilde{F}(\tau_{\tilde{Q}})}{\sum_Q g_Q^\Phi F(\tau_Q) + \sum_{\tilde{Q}} g_{\tilde{Q}}^\Phi \tilde{F}(\tau_{\tilde{Q}})} \right\} \right\}}$$

[without squark loops only $\frac{11}{2}$]

To get a finite cross section the real corrections have to be added.

Real corrections - heavy loop particle mass limit

Total real corrections [heavy squark/quark limit]:

$$
\begin{aligned}
C_{\rm real} &= \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{4\pi\mu^2}{m_\Phi^2}\right)^{\epsilon}\left\{\frac{3}{\epsilon^2}+\frac{33-2N_F}{6\epsilon}\right\} \\
D_{gg} &= -\frac{\hat\tau}{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{4\pi\mu^2}{\hat s}\right)^{\epsilon}P_{gg}(\hat\tau)-\frac{11}{2}(1-\hat\tau)^3 \\
&\quad +12\left\{\left(\frac{\log(1-\hat\tau)}{1-\hat\tau}\right)_+ -\hat\tau[2-\hat\tau(1-\hat\tau)]\log(1-\hat\tau)\right\} \\
D_{gq} &= -\left\{\frac{1}{2\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{4\pi\mu^2}{\hat s}\right)^{\epsilon}-\log(1-\hat\tau)\right\}\hat\tau P_{gq}(\hat\tau)-1+2\hat\tau-\frac{\hat\tau^2}{3} \\
D_{q\bar q} &= \frac{32}{27}(1-\hat\tau)^3
\end{aligned}
$$

- IR, Coll. poles in $C_{\rm real}$ subtract the corresponding ones of the virtual corrections.
- Coll. poles in the real corrections (Altarelli-Parisi kernels as coefficients)
  $\rightsquigarrow$ absorbed in NLO structure functions.

Result - heavy loop particle mass limit

$$\begin{aligned}
\sigma(pp \to \Phi + X) &= \sigma_0[1 + C\tfrac{\alpha_S}{\pi}]\tau_\Phi \tfrac{d\mathcal{L}_{gg}}{d\tau_\Phi} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{qq} \\
C &= \pi^2 + \tfrac{11}{2} + \tfrac{7}{2}\mathrm{Re}\left\{\frac{\sum_{\tilde{Q}} g^\Phi_{\tilde{Q}} \tilde{F}(\tau_{\tilde{Q}})}{\sum_Q g^\Phi_Q F(\tau_Q) + \sum_{\tilde{Q}} g^\Phi_{\tilde{Q}} \tilde{F}(\tau_{\tilde{Q}})}\right\} + \tfrac{33-2N_F}{6}\log\tfrac{\mu^2}{M_\Phi^2} \\
\Delta\sigma_{gg} &= \int_{\tau_\Phi}^1 d\tau \tfrac{d\mathcal{L}_{gg}}{d\tau}\tfrac{\alpha_S}{\pi}\sigma_0\Big\{ -\hat{\tau}P_{gg}(\hat{\tau})\log\tfrac{Q^2}{\hat{s}} - \tfrac{11}{2}(1-\hat{\tau})^3 \\
&\quad +12\Big[\Big(\tfrac{\log(1-\hat{\tau})}{1-\hat{\tau}}\Big)_+ - \hat{\tau}[2-\hat{\tau}(1-\hat{\tau})]\log(1-\hat{\tau})\Big]\Big\} \\
\Delta\sigma_{gq} &= \int_{\tau_\Phi}^1 d\tau \sum_{q,\bar{q}} \tfrac{d\mathcal{L}_{gq}}{d\tau}\tfrac{\alpha_S}{\pi}\sigma_0\Big\{ -\tfrac{\hat{\tau}}{2}P_{gq}(\hat{\tau})\Big[\log\tfrac{Q^2}{\hat{s}} - 2\log(1-\hat{\tau})\Big] \\
&\quad -1 + 2\hat{\tau} - \tfrac{\hat{\tau}^2}{3}\Big\} \\
\Delta\sigma_{q\bar{q}} &= \int_{\tau_\Phi}^1 d\tau \sum_q \tfrac{d\mathcal{L}_{q\bar{q}}}{d\tau}\tfrac{\alpha_S}{\pi}\sigma_0\tfrac{32}{27}(1-\hat{\tau})^3
\end{aligned}$$

[μ=Ren. scale, Q=Fact. scale]

natural scales: $\mu^2 = Q^2 = M_\Phi^2$