Two-loop Corrections to the Lamb Shift

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in collaboration with
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Outline

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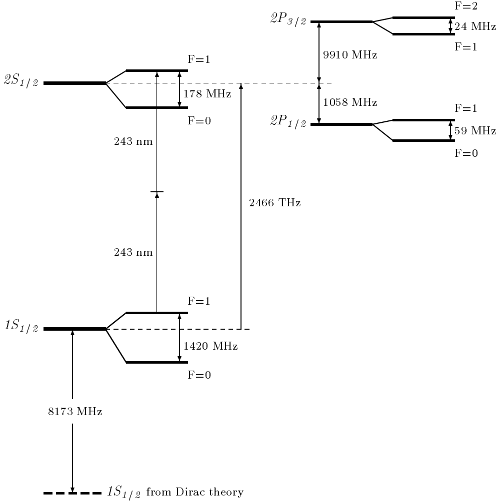
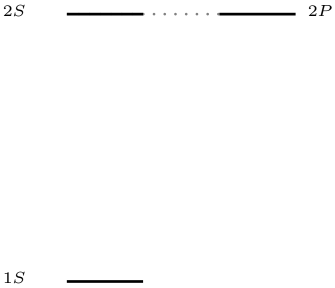
- Calculation
- Results
- Summary



[Eides et al. '01]

Hydrogen Spectrum



- Schrödinger equation:
  states with same n have same energy

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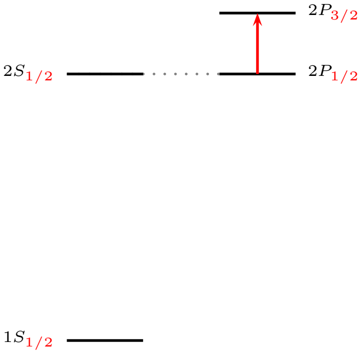

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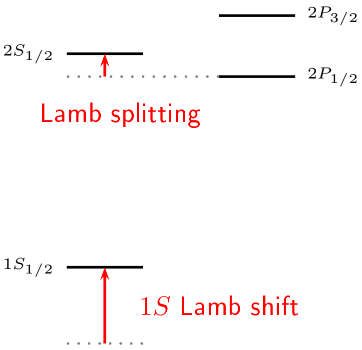
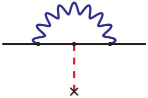
- Dirac equation with Coulomb source:
  relativistic corrections and electron spin
  energy depends on n and j

Hydrogen Spectrum

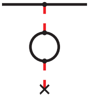


- Schrödinger equation:
  states with same n have same energy
- Dirac equation with Coulomb source:
  relativistic corrections and electron spin
  energy depends on n and j
- degeneracy lifted by:
  finite size of Coulomb source
  recoil corrections
  QED loops
- Lamb shift is field theoretic effect

Physical Origin of Lamb Shift



radiative corrections lead to finite electron charge radius



vacuum polarisation leads to charge screening

$\rightsquigarrow$ corrections to Coulomb potential induce energy shift:

$$\Delta E \;=\; \langle nS|\delta V|nS\rangle \;\sim\; |\psi(0)|^2$$

Early History of Lamb Splitting

1947 experimental observation by Lamb and Retherford:
"$2^2S_{1/2}$ state is higher than the $2^2P_{1/2}$ by about 1000 Mc/sec"

calculation by Bethe yields 1040 MHz

1949	exp. value is	1062(5) MHz	[Retherford, Lamb]
	theory gives	1051.41(15) MHz	[Bethe, Brown, Stehn]

1952/53 calculation of relativistic corrections by
Karplus, Klein, Schwinger and Baranger, Bethe, Feynman

$\rightsquigarrow$ 7.1 MHz correction

Status

exp.	1057.845(3) MHz		[Schwob et al. '98]
theory	1057.814(5) MHz	$r_p = 0.805(11)$ fm	[Eides et al. '01]
	1057.833(5) MHz	$r_p = 0.862(12)$ fm	

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extract r_p from Lamb shift: $r_p = 0.891(18)$ fm

agrees with recent analysis of e-p scattering: $r_p = 0.895(18)$ fm [Sick '03]

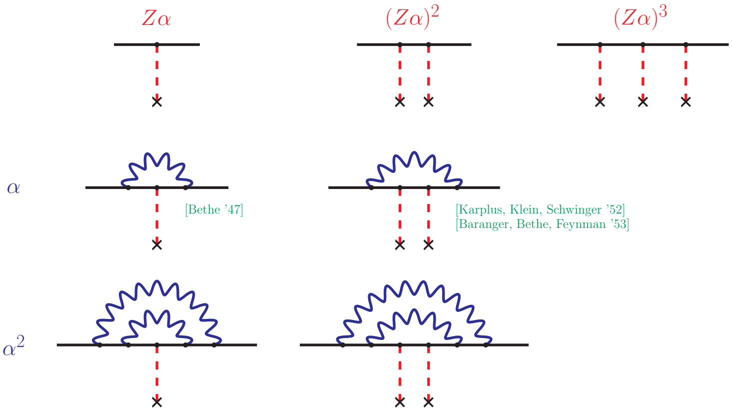
more precise value from measurement of Lamb shift
in muonic hydrogen at PSI

Structure of the Perturbative Series

double expansion:

$Z\alpha \longrightarrow$ binding corrections

$\alpha \longrightarrow$ QED loops



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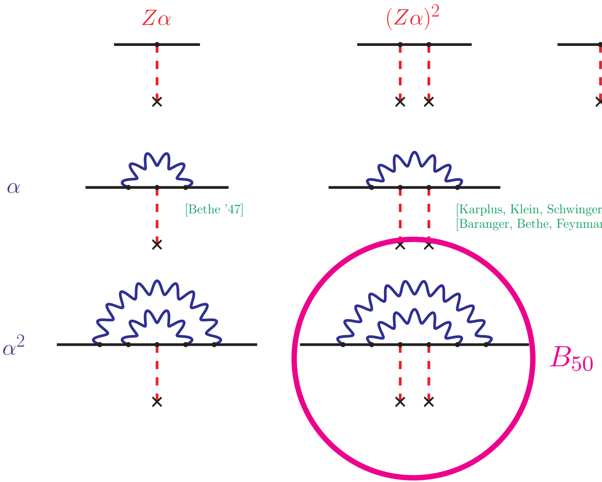
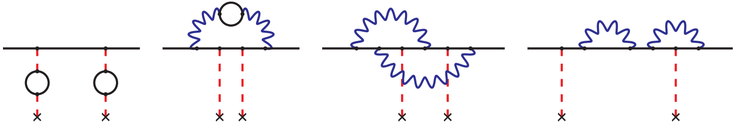


B_{50}

⇝ correction of $\mathcal{O}(\alpha^2(Z\alpha)^5)$:



calculation was done by Pachucki, and Eides and Shelyuto

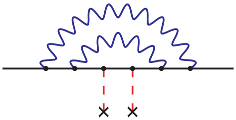
results:

$$B_{50}^{nvp} = -7.6(2) \quad \text{[Pachucki '94]}$$
$$B_{50}^{nvp} = -7.725(1) \quad \text{[Eides, Shelyuto '95]}$$

Comparison of the Calculations

previous calculations:

- effective Dirac equation
- Fried-Yennie gauge for IR divergences
- calculate UV-finite combinations



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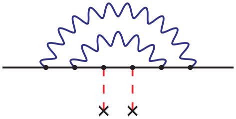
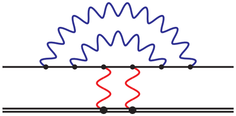


our approach:

- Feynman diagrams
- dimensional regularisation
- R_ξ gauge
- reduction to master integrals with IBP [Chetyrkin, Tkachov '81]



Interaction with the Nucleus

non-recoil corrections:

- assume that the nucleus is infinitely heavy: $M \to \infty$
- construct expansion around this limit
- discard all sub-leading terms

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$\rightsquigarrow$ effective photon propagator:

$$P^2 = M^2$$

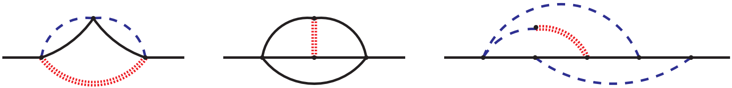
$$\begin{aligned}
&\sim \frac{1}{2P\cdot k + i\varepsilon} + \frac{1}{-2P\cdot k + i\varepsilon} \\
&= -2\pi i\,\delta(2P\cdot k) \\
&= \frac{-i\pi}{M}\,\delta\left(k^0\right)
\end{aligned}$$

Reduction to Master Integrals

$$\int \frac{\mathrm{d}^D k}{2P\cdot k + i\varepsilon}\cdots \quad - \quad \int \frac{\mathrm{d}^D k}{2P\cdot k - i\varepsilon}\cdots$$

- use Laporta algorithm [Laporta, Remiddi]
  FIRE [A. Smirnov]
- $i\varepsilon$ prescription is irrelevant for IBP relations
- discard integrals without nucleon propagator
- calculation reduces to 32 master integrals (including 7 with numerator)

Evaluation of Master Integrals



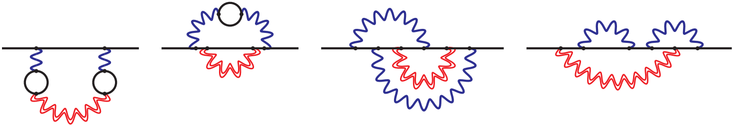
- MB representation for "easy" integrals
  MB, MBresolve
- FIESTA
- numerical integration with CUBA for most complicated integral
- use basis change as cross-check

[Czakon]
[A. Smirnov,V. Smirnov]

[A. Smirnov, Tentyukov]

[Hahn]

Overview



- calculated B_{50} contribution to Lamb shift
- explicitly checked gauge independence
- improved precision of previous calculations
- found new analytical results for several diagrams

Result for B_{50}

$$B_{50}^{nvp} = -7.6(2) \quad \text{[Pachucki '94]}$$
$$B_{50}^{nvp} = -7.725(1) \quad \text{[Eides, Shelyuto '95]}$$
$$B_{50}^{nvp} = -7.7239(5) \quad \text{preliminary}$$

vacuum-polarisation contribution:

$$B_{50}^{vp} = 0.862814(3) \quad \text{[Pachucki '93]}$$
$$B_{50}^{vp} = 0.86281422(5) \quad \text{preliminary}$$

Effect on Lamb Splitting

$$\Delta E(B_{50}) = \frac{\alpha^2\,(Z\alpha)^5}{\pi\,n^3}\,\left(\frac{\mu}{m}\right)^3\,m\,\left(B_{50}^{nvp} + B_{50}^{vp}\right)\,\delta_{l0}$$

$\Delta E(n=2)$	$=$	$-36.5(9)$ kHz	[Pachucki '94]
$\Delta E(n=2)$	$=$	$-37.112(5)$ kHz	[Eides, Shelyuto '95]
$\Delta E(n=2)$	$=$	$-37.109(3)$ kHz	preliminary

measured value:	1057845(3) kHz	[Schwob et al. '98]
prediction:	1057833(5) kHz	[Eides et al. '01]

Summary

- new calculation of B_{50} contribution to Lamb shift
- calculation uses dimensional regularisation and other techniques from multi-loop calculations
- result agrees with previous calculations
- uncertainty can be improved