Radiative Corrections to Quantum Hall and Josephson Effects

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Topics Discussed

- Exact results in quantum mechanics
  - *Flux quantization*
  - *Quantum Hall effect*
  - *Josephson effect*
- Vacuum polarization in strong magnetic field
- QED corrections to quantum Hall and Josephson effect
  - *Theory*
  - *Experiment*

Exact result: an example

Determination of the fine structure constant α

- electron $g-2$
  - *experimental error:* 10^{-10}
  - *theoretical error:* 10^{-10} - *requires 4-loop calculation in QED*
- quantum Hall effect
  - *experimental error:* 10^{-8} *(could be* 10^{-12}*)*
  - *theoretical error:* $\mathbf{0}$ - *for free!*

Gauge invariance and quantum phase

$\hbar = c = 1, \ \alpha = \frac{e^2}{4\pi}$

- ✔ Schrödinger equation $\qquad (i\partial_t - \mathcal{H})\,\Psi = 0$
- ✔ Wave function $\qquad \Psi = |\Psi| e^{i\theta}$
- ✔ Hamiltonian $\qquad \mathcal{H} = qA_0 + F(\boldsymbol{B}, \boldsymbol{E}, \boldsymbol{D})$

$$\boldsymbol{D} = \boldsymbol{\partial} - iq\boldsymbol{A}$$

↪ If $\boldsymbol{\nabla} \times \boldsymbol{A} = 0$ then

$$\theta(\boldsymbol{r}_1) - \theta(\boldsymbol{r}_2) = q \int_{\boldsymbol{r}_2}^{\boldsymbol{r}_1} \boldsymbol{A}(\boldsymbol{r}') \cdot d\boldsymbol{r}'$$

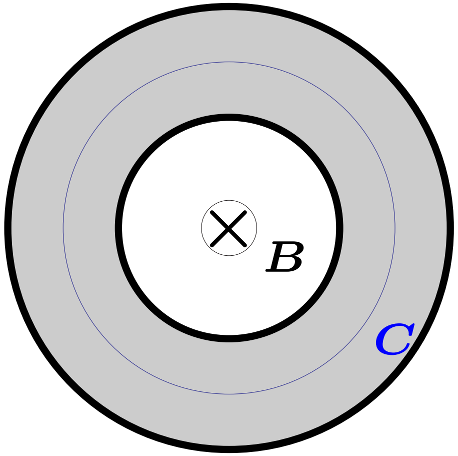
Flux quantization

F. London (1948)

Superconductivity ⇨ *coherent state of Cooper pairs* ⇨ $q = 2e$

Meissner effect ⇨ $\boldsymbol{B} = 0$ inside superconductor

Superconducting ring:



Single-valued wave function ⇨ $\Delta\theta = 2e \oint_C \boldsymbol{A} \cdot d\boldsymbol{x} = 2\pi n$

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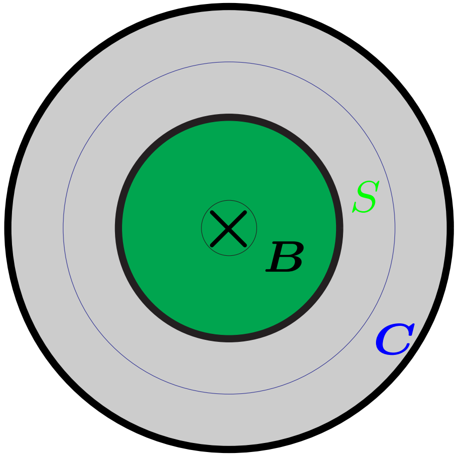


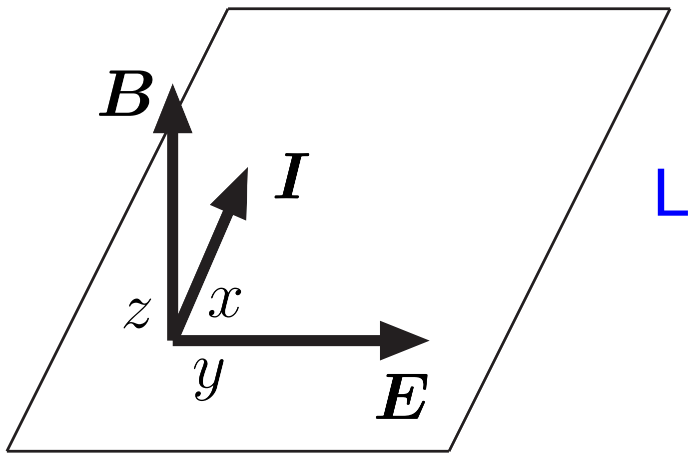
Single-valued wave function ➪ $\Delta\theta = 2e \oint_C \boldsymbol{A} \cdot d\boldsymbol{x} = 2\pi n$

$\oint_C \boldsymbol{A}(\boldsymbol{r}) \cdot \boldsymbol{r} = \int_S \boldsymbol{B}(\boldsymbol{r}) \cdot d^2\boldsymbol{s} \equiv \Phi$ ➪ $\Phi = \frac{\pi n}{e} \equiv n\Phi_0$

Flux quantum: $\Phi_0 = \frac{h}{2e}$

Hall effect

E. Hall (1879)



Lorentz force *vs* electrostatic force

$$e\boldsymbol{E} = -e\boldsymbol{v} \times \boldsymbol{B}$$

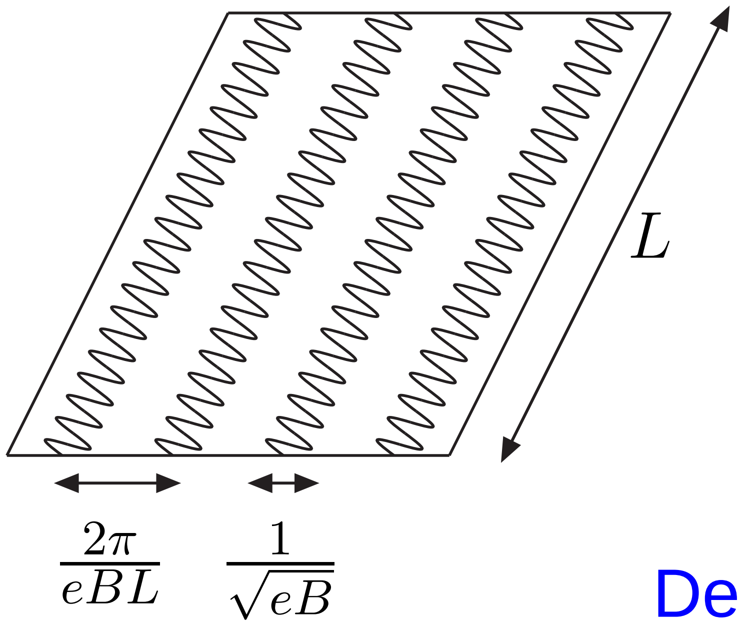
current density $$j = \rho e v = \frac{\rho e}{B} E$$

total current per length $$I = \frac{\rho e}{B} V$$

Hall conductivity $$R^{-1} = \frac{\rho e}{B}$$

Quantum Hall effect

K. von Klitzing, G. Dorda, M. Pepper (1980)



Wave function:

$$\Psi(x,y) = e^{i2\pi m \frac{x}{L}}\ \psi(y - y_m)$$

$\psi(y - y_m)$ ➪ *harmonic oscillator*
centered at $\quad y_m = \frac{2\pi m}{eBL}$

Density of quantum states with

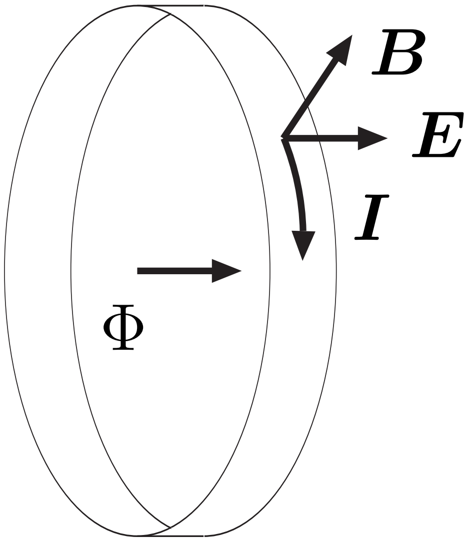
$\boldsymbol{n}$ Landau levels filled: $\quad \rho = n\frac{eB}{2\pi}$

Quantum Hall conductivity: $\quad R^{-1} = 2n\alpha = n/R_K$

von Klitzing constant: $\quad R_K = \frac{h}{e^2}$

Gauge invariance argument

R.B. Laughlin (1981)



current density $\boldsymbol{j} \propto \frac{\delta \mathcal{H}}{\delta \boldsymbol{A}}$

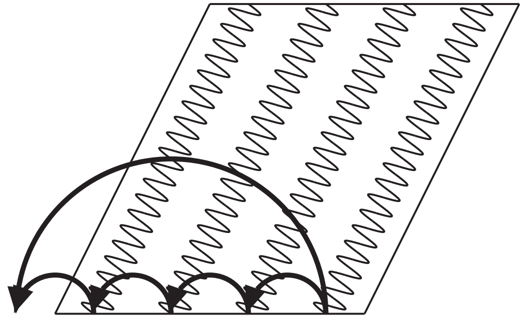
total current $I = \frac{d\mathcal{E}}{d\Phi}$

Flux quantization $\Phi = 4\pi|\boldsymbol{A}|/L = n2\Phi_0$

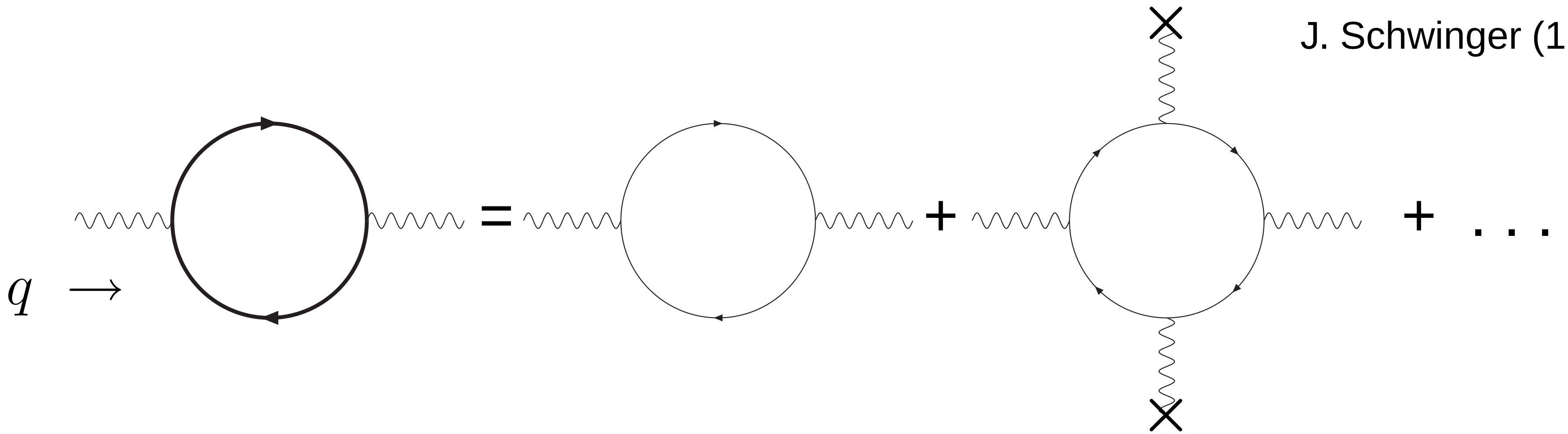
Flux dependence $y_m(\Phi + 2\Phi_0) = y_{m+1}(\Phi)$

for $d\Phi = 2\Phi_0$ ⇨ $d\mathcal{E} = neV$

↪ $I = \frac{neV}{2\Phi_0} = \frac{nV}{R_K}$



Vacuum polarization



J. Schwinger (1951)

Charge renormalization $\delta e \propto \Pi_{\mu\nu}(q)/q^2|_{q^2\to 0}$

Vacuum polarization in magnetic field

$$\delta\Pi_{\mu\nu}(q) = -\frac{\alpha}{\pi}\left(\frac{eB}{m^2}\right)^2 \frac{1}{45}\left[2\left(g_{\mu\nu}q^2 - q_\mu q_\nu\right) - 7\left(g_{\mu\nu}q^2 - q_\mu q_\nu\right)_{\|} + 4\left(g_{\mu\nu}q^2 - q_\mu q_\nu\right)_{\perp}\right]$$

Lorentz invariance broken ⇨ different couplings

QED corrections to electromagnetic couplings

Coulomb potential

$$V_C(\boldsymbol{r}) = e^2 \int \left(1 - \frac{\delta\Pi_{00}(q)}{q^2}\right) e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} \frac{d^3\boldsymbol{q}}{(2\pi)^3} = \frac{\alpha}{|\boldsymbol{r}|}\left[1 + \frac{\alpha}{\pi}\left(\frac{eB}{m^2}\right)^2 \left(\frac{2}{45} - \frac{7}{90}\sin^2\theta\right)\right]$$

Interaction with the homogeneous electric field

$$\left(1 + \frac{\alpha}{\pi}\left(\frac{eB}{m^2}\right)^2 \frac{11}{180}\right) e\, \boldsymbol{r}\cdot\boldsymbol{E} \equiv e^* \boldsymbol{r}\cdot\boldsymbol{E}$$

Hall current

$$\left(1 + \frac{\alpha}{\pi}\left(\frac{eB}{m^2}\right)^2 \frac{1}{15}\right) \boldsymbol{I} \equiv \frac{e''}{e}\boldsymbol{I}$$

QED corrections to R_K

A.P., Phys. Rev. B 79, 113303 (2009)

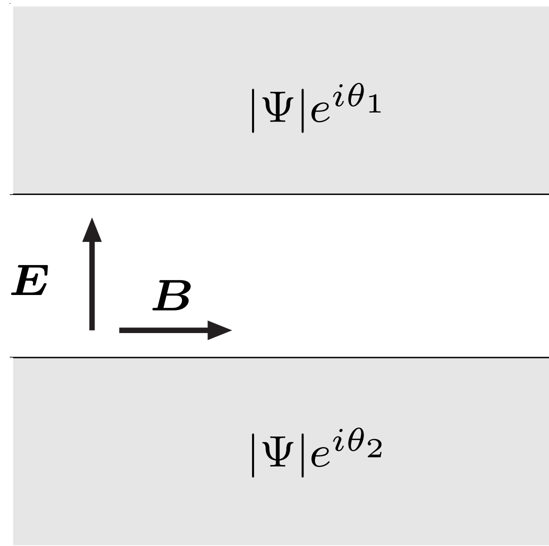
Origin of the corrections $\quad R_K^{-1} \propto e^2 \to e^* e''$

Result:

$$R_K^{-1} = \frac{e^2}{h}\left[1 + \frac{23}{180}\frac{\alpha}{\pi}\left(\frac{\hbar e B}{c^2 m^2}\right)^2\right] \approx \frac{e^2}{h}\left[1 + 10^{-20} \times \left(\frac{B}{10\,\mathrm{T}}\right)^2\right]$$

Josephson effect

B.D. Josephson (1962); S. Shapiro (1963)



Tunneling Josephson current

$$I \approx I_c \sin(\theta_1 - \theta_2)$$

Constant voltage V across the junction

➥ *the current oscillates at frequency* ν

Josephson frequency-voltage relation $$\nu = K_J V$$

Josephson constant $$K_J = \frac{2e}{h}$$

QED corrections to K_J

A.P. (preliminary result)

Origin of the corrections $\qquad K_J \propto e \to e^*$

Result:

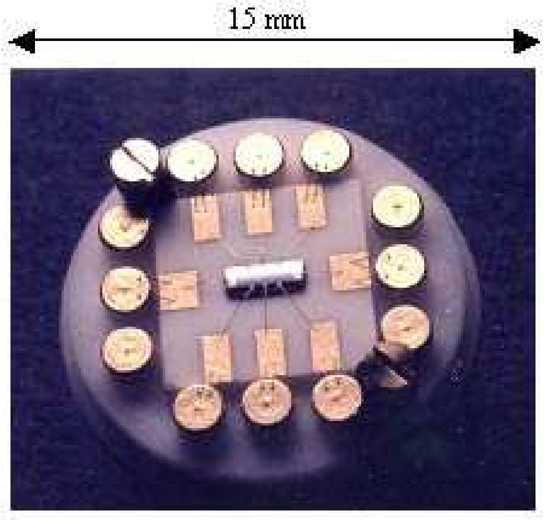
$$K_J = \frac{2e}{h}\left[1 + \frac{11}{180}\frac{\alpha}{\pi}\left(\frac{\hbar e B}{c^2 m^2}\right)^2\right] \approx \frac{2e}{h}\left[1 + 10^{-20} \times \left(\frac{B}{10\,\mathrm{T}}\right)^2\right]$$

Experiment

Quantum Hall effect (F. Schopfer, W. Poirier (2007))

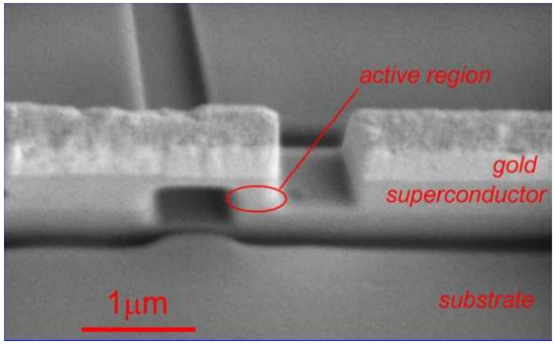
thermal Johnson-Nyquist noise ➪

✘ accuracuy limit 10^{-12}



Josephson effect (A.K. Jain, J.E. Lukens, J.-S. Tsai (1987))

✔ *accuracy* 10^{-19}



Summary

- Vacuum polarization results in a weak dependence of the Josephson and von Klitzing constant on the magnetic field strength
- This remarkable manifestation of a fine nonlinear quantum field effect in collective phenomena in condensed matter can be observed in a dedicated experiment

One literally can measure the vacuum polarization with a voltmeter!