# Kunsztwerk und Wissenschaft* 

Keith Ellis<br>Fermilab

* Art and Science of Zoltan Kunszt

Colloquium in honour of the retirement of Zoltan Kunszt

## In the beginning....

- I met Zoltan in 1979 when he visited Caltech
- Later in June 19801 met him again at DESY
- This was the beginning of a professional relationship and friendship which has spanned three decades.



## Jets

- 1978-I979 was a period of intense activity at the $\mathrm{e}^{+} \mathrm{e}^{-}$colliders, DORIS and PETRA.
- Establish the quark and gluon degrees of freedom. If there were three jet events, then there should also be four jet events....
- Ali et al, provided a tree graph

- Shown is the prediction for the Acoplanarity, showing that the branching into gluons pairs is much more copious than in to quark pairs.

A. Ali et al. / QCD predictions


[^0]
## Thrust distribution in $\mathrm{e}^{+} \mathrm{e}^{-}$

- Use the precision jet data at colliders to make QCD measurements, eg $\alpha_{\mathrm{s} \text {. }}$
- In June 1980 Ross, Terrano and I finished our NLO calculation of the C parameter in $\mathrm{e}^{+} \mathrm{e}^{-}$ annihilation.
- Zoltan was the first to calculate the NLO thrust distribution in $\mathrm{e}^{+} \mathrm{e}^{-}$and it confirmed that the corrections were large.


ZK PLIO7B, I98I

## Jets in hadron-hadron collisions

- (Steve)Ellis, ZK and Soper took results from RKE \& Sexton and came up with a way to make NLO predictions for a physical jet cross section.



EKS Phys.Rev.Lett 62:726,1989

## Jet physics 2009

- In the intervening 20 years, jet physics has become more sophisticated.
- No longer compared with the EKS code, instead it is compared with the NLO++ code of Zoltan Nagy.



## Predictions for Physics at LEP

## Cern yellow report I989

- A comprehensive treatment. Laid out the full program of QCD measurements at LEP.
- The beginning of NLO parton integrators.
- NLO predictions for Thrust, oblateness, C-parameter and energy-energy correlation.
- Would certainly have 1000 's of citations if it had been published.


## Next-to-leading Monte Carlo

- Generically all radiative corrections can be considered as a plus distribution.


## event <br> $\downarrow$

- Realization that NLO $\int_{0}^{1} d x \frac{f(x)}{(1-x)_{+}}=\int_{0}^{1} d x \frac{f(x)-f(1)}{(1-x) \uparrow}$ correction formula could be included in a
counterevent Monte Carlo as event and counter-event.


## FKS subtraction

- Development of experience with Soper and Frixione, MNR experience with NLO Heavy quarks.
- Introduce a set of FKS pairs PFKS which induce soft or collinear in the $n+I$ dimensional matrix elements.
- Partition the phase space so that in each partition at most one soft and one collinear singularity are present.
- Introduce $\mathrm{S}_{\mathrm{i}, \mathrm{a}}$ positive definite partition function, such that each term only contains singularities for $\mathrm{i} \| \mathrm{j}$ or i soft.

$$
\mathcal{M}^{(n+1,0)}(r)=\sum_{(i, j) \in \mathcal{P}_{\mathrm{FKS}}} \mathcal{S}_{i j}(r) \mathcal{M}^{(n+1,0)}(r) .
$$

Method of choice for POWHEG (Nason) and MadFKS (Frederix)

## CS subtraction

- Partial fractioning of eikonal expression to associate soft singularities with a particular emitter. Subtraction valid throughout phase space.
- Exact factorization of $n+1$ dimensional phase space


## Spinor techniques

- Modern version of spinor techniques began with PL 103B, I98I CALKUL collaboration.

$$
\begin{gathered}
q^{ \pm}(k)=\frac{N}{2 \sqrt{2}}\left[k q_{-} q_{+}\left(1 \pm \gamma_{5}\right)-k q_{-} q_{+} \not k\left(1 \mp \gamma_{5}\right)\right] \\
N=\left[2\left(q_{+} \cdot q_{-}\right)\left(k \cdot q_{-}\right)\left(k \cdot q_{+}\right)\right]^{-\frac{1}{2}}
\end{gathered}
$$

- Making a particular choice of phase, dependence on the second gauge vector drops out. Xu, Zhang and Chang, preprint TUTP-84/3. Not published until NPB 29I, 392 (I987)!
- Useful because the spinor products make manifest the square root singularities of QCD.

$$
\not^{ \pm}(k)=\sqrt{2}\left[\left|k_{\mp}\right\rangle\left\langle q_{\mp}\right|+\left|q_{ \pm}\right\rangle\left\langle k_{ \pm}\right|\right] /\left\langle q_{\mp} \mid k_{ \pm}\right\rangle
$$

## Applications to six jet processes

- In a series papers with Jack Gunion, explicit expressions for १৭৭৭৭৭ and gg৭৭৭৭ processes were presented.
- Using the supersymmetry trick of Grisaru, Pendleton and van Nieuwenhuizen (1977) 6 g processes were related to processes with 4 g and 2 gluinos

$$
\begin{array}{r}
A^{6 g}\left(1_{g}^{+}, 2_{g}^{+}, 3_{g}^{+}, 4_{g}^{-}, 5_{g}^{-}, 6_{g}^{+}\right)=-\frac{\langle 45\rangle}{\langle 46\rangle} A^{4 g 2 \tilde{g}}\left(1_{g}^{+}, 2_{g}^{+}, 3_{g}^{+}, 4_{\tilde{g}}^{-}, 5_{\tilde{g}}^{+}, 6_{g}^{+}\right) \\
\uparrow{\underset{\tilde{g}}{ }}_{\uparrow}
\end{array}
$$

"In this way we have avoided the direct calculation of 220 Feynman diagrams" ZK, NPB27I, I 986 , (see also Parke \& Taylor)

## And on to MHV...

$$
\begin{aligned}
& A^{4 g}\left(1_{g}^{-}, 2_{g}^{-}, 3_{g}^{+}, 4_{g}^{+}\right)=\frac{\langle 12\rangle^{4}}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} \\
& A^{5 g}\left(1_{g}^{-}, 2_{g}^{-}, 3_{g}^{+}, 4_{g}^{+}, 5_{g}^{+}\right)=\frac{\langle 12\rangle^{4}}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle} \\
& A^{6 g}\left(1_{g}^{-}, 2_{g}^{-}, 3_{g}^{+}, 4_{g}^{+}, 5_{g}^{+}, 6_{g}^{+}\right)=\frac{\langle 12\rangle^{4}}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 56\rangle\langle 61\rangle}
\end{aligned}
$$

With confirmed numerical results in hand, Parke and Taylor conjectured that this pattern would hold for all $n$

## Arsenal of tools for tree amplitude

- Helicity methods, spinor products
- Supersymmetry
- Colour stripped amplitudes (Chan-Paton)

All of the techniques which were useful at tree graph level would prove invaluable as one went on to loops.

- Recursion relations (eg. Berends \& Giele)


## One-loop helicity amplitudes

- One loop 4-parton scattering matrix elements squared (gggg, qqgg, १q৭৭) were calculated in 1986 (RKE, Sexton)
- One-loop gggg helicity amplitudes Bern, Kosower 1990
- Remaining q৭gg, q৭qq helicity amplitudes obtained by (ZK, Signer and Trocsanyi, I993
- Provided the structure and confidence in the methods to go onto 5 partons (Bern, Dixon and Kosower, ZK, Signer, Trocsanyi)(1993-1994)


## One-loop four parton processes

- Extremely simple formula for one-loop amplitudes revealing simplicity in supersymmetric limit.

$$
\begin{aligned}
c_{4 ; 1}^{\mathrm{HV}}(+,+;+,-) & =-\frac{\mathrm{i}}{48 \pi^{2}} \frac{\langle 34\rangle[12][13][23]}{s_{12} s_{14}}\left[\frac{3}{2} \frac{N_{c}^{2}+1}{N_{c}}+\left(N_{c}-N_{f}\right) \frac{s_{14}}{s_{12}}\right] . \\
c_{4 ; 1}^{\mathrm{HV}}(-,-;+,-) & =\frac{\mathrm{i}}{48 \pi^{2}} \frac{\langle 12\rangle\langle 14\rangle\langle 24\rangle[34]}{s_{12} s_{14}}\left[\frac{3}{2} \frac{N_{c}^{2}+1}{N_{c}}+\left(N_{c}-N_{f}\right) \frac{s_{14}}{s_{12}}\right] .
\end{aligned}
$$

One-loop helicity amplitudes for all $2 \rightarrow 2$ processes in QCD and $N=1$ supersymmetric Yang-Mills theory ${ }^{1}$

Zoltan Kunszt, Adrian Signer and Zoltán Trócsányi
Theoretical Physics, ETH,
Zürich, Switzerland

## Unitarity and one-loop diagrams

- Important steps include:-
- First modern use of the idea Bern, Dixon,Kosower
- Cuts w.r.t. to loop momenta give (box) coefficients directly Cachazo, Britto, Feng
- OPP tensor reduction scheme, Ossola, Pittau,

Papadopoulos

- Integrating the OPP procedure with unitarity Ellis,
Giele, Kunszt
- D-dimensional unitarity Giele, Kunszt, Melnikov


## Unitarity in D-dimensions

- The theory contains divergences which we regulate dimensionally. Divergences give poles as $=(4-D) / 2->0$
- Calculate the unitarity cuts numerically in integer dimensions
$D>4$. Internal degrees of freedom are taken to be $D_{s}$ dimensional.
- Dependence on $D_{s}$ is linear so we calculating in a two different integer dimensions and extrapolate to 0
- Only the length of the loop momentum in the extra dimension is relevant so we can treat the loop momentum as fivedimensional.


## One-loop: the extension to n -legs

- Time to calculate oneloop amplitude scales as $\mathrm{N}^{9}$ as expected.
- For small numbers of legs $\mathrm{N}=4,5,6$ the times are of the order of IO's of milliseconds


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4g:Ellis-Sexton(I985)
5g:Bern-Dixon-Kosower(I993)
6g:Ellis-Giele-Zanderighi(2006)
```


## Zoltan and ETH

- For 23 years Zoltan has kept the flame of accelerator-based particle physics alive at ETH Zurich.
- He has nurtured students: Peter Bamert, Stefan Beerli, Stefan Bucherer, Gudrun Heinrich,Francesco Knechtli, Martin Puchwein, Adrian Signer ( $\rightarrow$ Darren Forde, scientific grandchild)
- Many of us have been happy to accept the hospitality at ETH, for example ESW to write parts of our book, QCD and Collider physics


## Zoltan's secret

- Zoltan has an infectious sense of excitement about physics, which he communicates to others.
- He knows that the ultimate arbiter of all that we do is experiment, and that until a theory makes a prediction for the experiment, nothing has really been done.


## Key ideas of perturbative QCD

- Asymptotic Freedom
- Factorization/Resummation
- DGLAP evolution
- Parton Shower Monte Carlo
- Jets
- Monte Carlo programs beyond leading order
- Spinor methods
- Unitarity for loop diagrams

Zoltan has been a prime mover in half of these...and it ain't over yet

So please join with me in wishing Zoltan and Marika many happy years hiking through life together.



[^0]:    ${ }^{1}$ On leave from L. Eötvös University, Budapest.

