Infrared Singularities of QCD Amplitudes and Resummation

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based on 0901.0722, 0903.1126, 0904.1021 with Matthias Neubert; 0808.3008, 0809.4283 with Valentin Ahrens, Li-Lin Yang and Matthias Neubert; work with Matt Schwartz (in preparation)

Overview

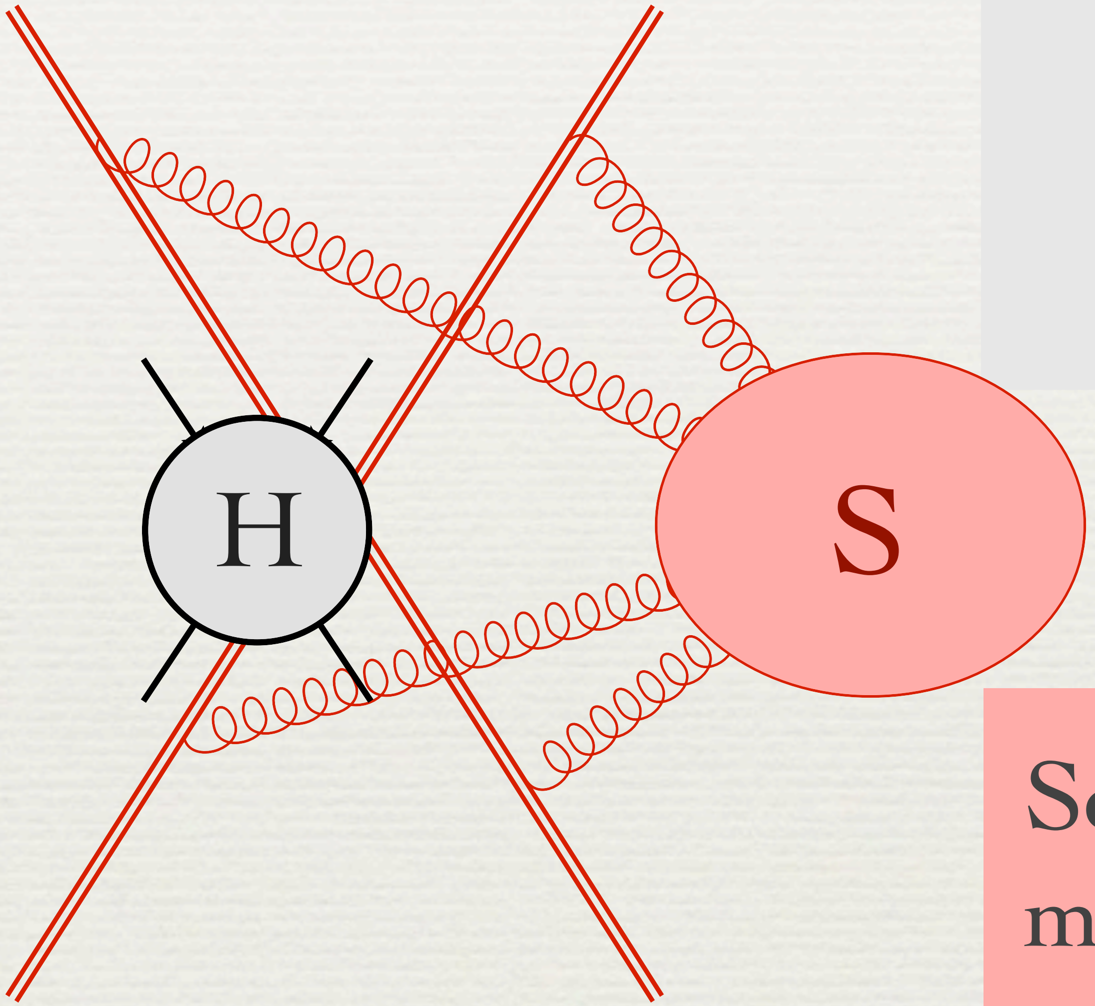
- Collider physics with effective theory methods
  - Soft-Collinear Factorization and Soft-Collinear Effective Theory
  - Sudakov resummation by RG evolution
- Towards resummation for n-jet processes
  - Anomalous dimension $\mathbf{\Gamma}$ of n-jet operators in SCET and connection to IR singularities of scattering amplitudes.
  - All-order conjecture for $\mathbf{\Gamma}$ (for arguments, see M. Neubert's talk)
  - An application: $pp \rightarrow \gamma + X$ at large p_T

Collider physics with effective theory methods

- Collider processes characterized by many scales: s, s_{ij}, M_i, Λ_{QCD}, ...
- Large Sudakov logarithms arise, which need to be resummed (e.g. parton showers)
- Effective field theories provide modern, elegant approach to this problem based on scale separation (factorization theorems) and RG evolution (resummation)

Soft emissions from heavy particles

Yennie, Frautschi, Suura 1961, Weinberg 1965



Hard function H depends on large momentum transfers s_{ij} between jets

Soft function S depends on maximum energy E_0 of unobserved soft emissions.

Factorization:

$$d\sigma = H(\{s_{ij}\}, \{m_i\}, \mu) \times S(\{v_i \cdot v_j\}, \mu)$$

Effective theory treatment

- Heavy quark (or electron) effective theory

$$\mathcal{L}_Q = \sum_{i=1}^{4} \bar{h}_i \, \underbrace{\overset{\substack{\text{quark velocity}\\ \downarrow}}{v_i} \cdot D}_{\text{eikonal interaction} \,\equiv\, \text{Wilson line}} \, h_i$$

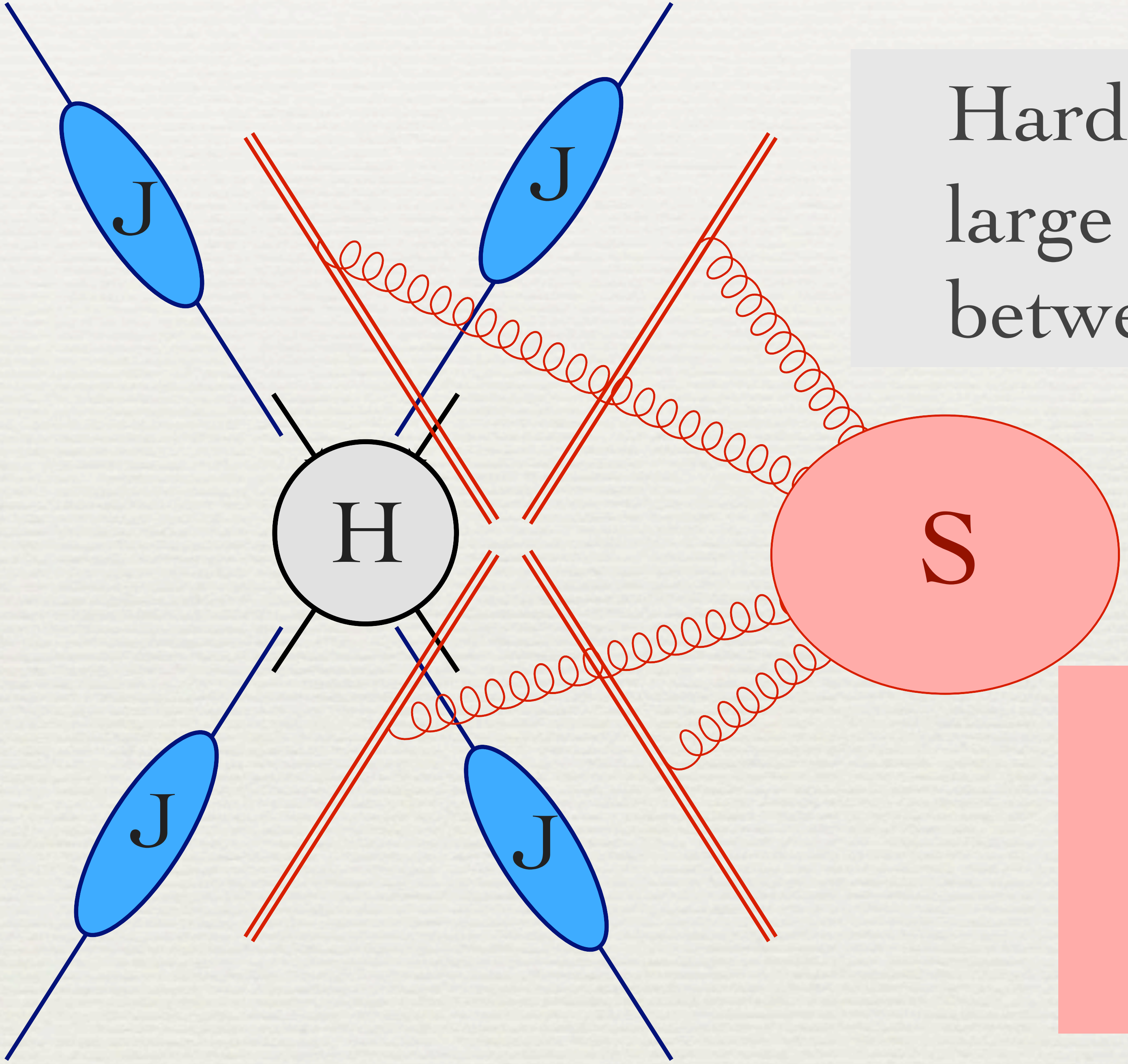
- Hard function from Wilson coefficients

$$\mathcal{L}_{\text{int}} = \sum_{AB} C_{AB}(\{s_{ij}\}, \{m_i\}, \mu) \; \bar{h}_1 \Gamma_A h_2 \; \bar{h}_3 \Gamma_B h_4 + \ldots$$

- Sudakov resummation by RG evolution of Wilson coefficient from hard to soft scale.

Massless case: Soft-collinear factorization

Sen 1983; Kidonakis, Oderda, Sterman 1998



Hard function H depends on large momentum transfers s_{ij} between jets

Soft function S depends on scales $\Lambda_{ij}^2 = \frac{M_i^2 M_j^2}{-s_{ij}}$

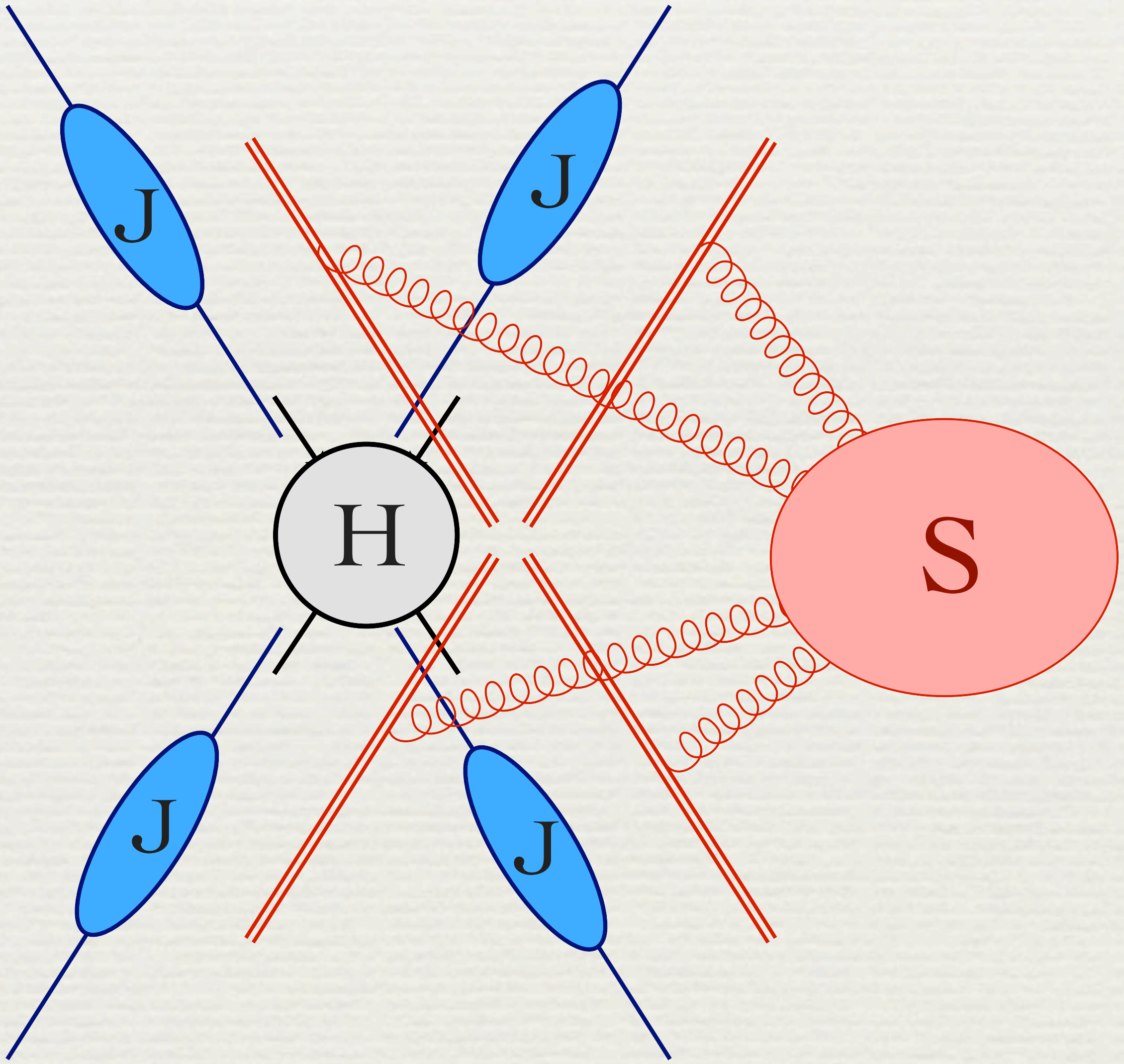
Jet functions $J_i = J_i(M_i^2)$

Soft-collinear factorization

- Factorize cross section:

$$d\sigma \sim H(\{s_{ij}\},\mu)\prod_i J_i(M_i^2,\mu)\otimes S(\{\Lambda_{ij}^2\},\mu)$$

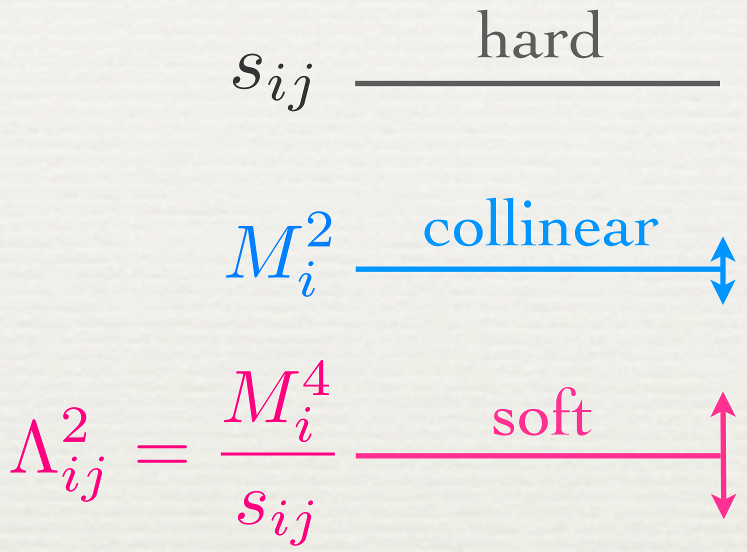
- Define components in terms of field theory objects in SCET
- Resum large Sudakov logarithms directly in momentum space using RG equations



Soft-collinear effective theory

Bauer, Pirjol, Stewart et al. 2001, 2002; Beneke et al. 2002; ...

- Typical scale hierarchy:



hard >> collinear >> soft

- Integrate out hard quantum fluctuations, and describe collinear and soft modes by fields in SCET
- In second step, integrate out collinear modes (if perturbative) and match onto SET (soft effective theory)

SCET for n-jet processes

- n different types of collinear quark and gluon fields (jet functions J_i), interacting only via soft gluons (soft function S)
  - → operator definitions for J_i and S
- Hard contributions (Q ~ √s) are integrated out and absorbed into Wilson coefficients:

$$\mathcal{H}_n = \sum_i \mathcal{C}_{n,i}(\mu)\, O_{n,i}^{\text{ren}}(\mu)$$

Bauer, Schwartz 2006

- Scale dependence controlled by RGE:

$$\frac{d}{d\ln\mu}\,|\mathcal{C}_n(\{\underline{p}\},\mu)\rangle = \boldsymbol{\Gamma}(\mu,\{\underline{p}\})\,|\mathcal{C}_n(\{\underline{p}\},\mu)\rangle$$

anomalous-dimension matrix of n-jet SCET operators

Decoupling of soft interactions

- At leading power only a single component of the soft gluon field interacts with each collinear field.

$n_i \sim p_i$ light-like reference vector

$$\mathcal{L}_{c_i+s} = \bar{\chi}_i(x)\,\frac{\bar{\not{n}}_i}{2}\,n_i\cdot A_s(x_-)\,\chi_i(x)$$

collinear quark field in *i*th direction

- Can decoupled by field redefinition

$$\chi_i(x) = S_i(x_-)\,\chi_i^{(0)}(x)$$

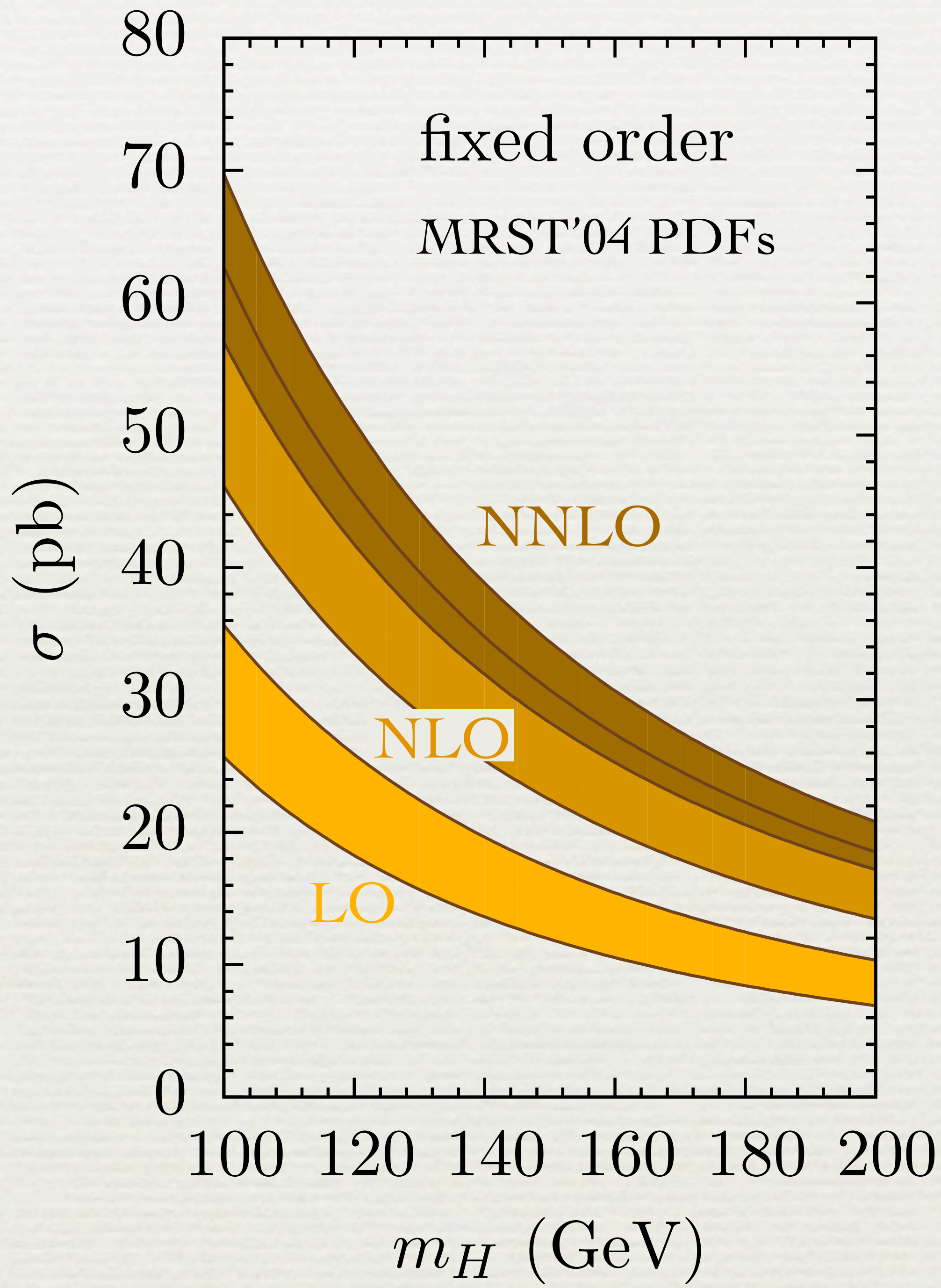
$$S_i(x) = \mathbf{P}\exp\left(ig\int_{-\infty}^{0} dt\,n_i\cdot A_s^a(x+tn_i)\,t^a\right)$$

Sudakov resummation with SCET

- Many collider physics applications of SCET in the past few years. Resummations up to N^{3}LL, however only for two jet observables, e.g.
  - Drell-Yan rapidity dist. TB, Neubert, Xu ‘07
  - inclusive Higgs production Idilbi, Ji, Ma and Yuan ‘06 ; Ahrens, TB, Neubert, Yang ‘08
  - thrust distribution in e^+e^- TB, Schwartz ’08
- Need to generalize the method to
  - observables with > 2 jets
  - hadronically more exclusive observables

    Stewart, Tackmann, Waalewijn, 0910.0467; Cheung, Luke, Zuberi, 0910.2479

2-jet example: Higgs production



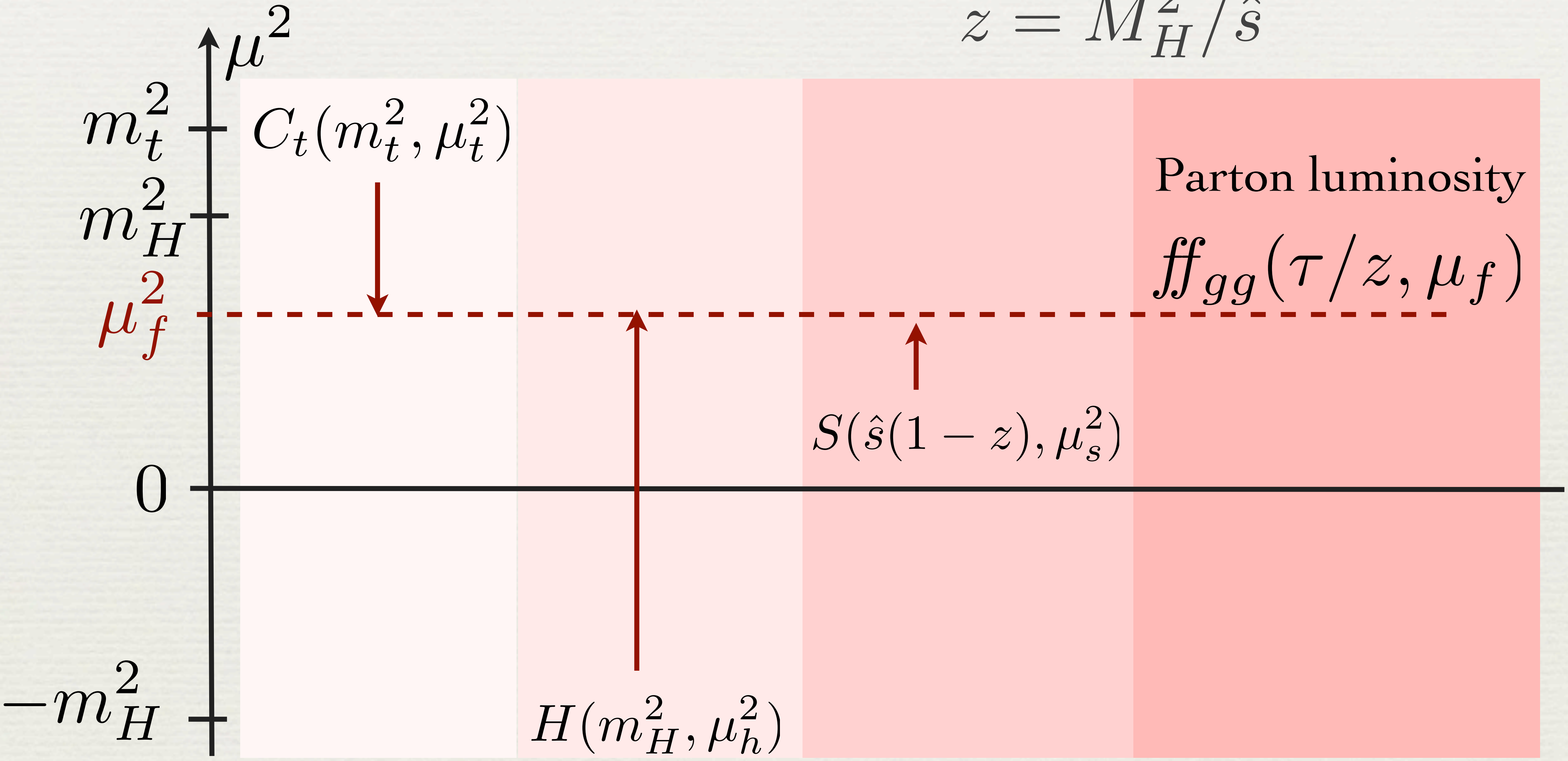
- Corrections are large: 70% at NLO + 30% at NNLO [130% and 80% if PDFs and α_s are held fixed]
- Only C_{gg} contains leading singular terms, which give 90% of NLO and 94% of NNLO correction
- Contributions of C_{qg} and C_{qq} are small: -1% and -8% of the NLO correction

Harlander, Kilgore 2002; Anastasiou, Melnikov 2002
Ravindran, Smith, van Neerven 2003

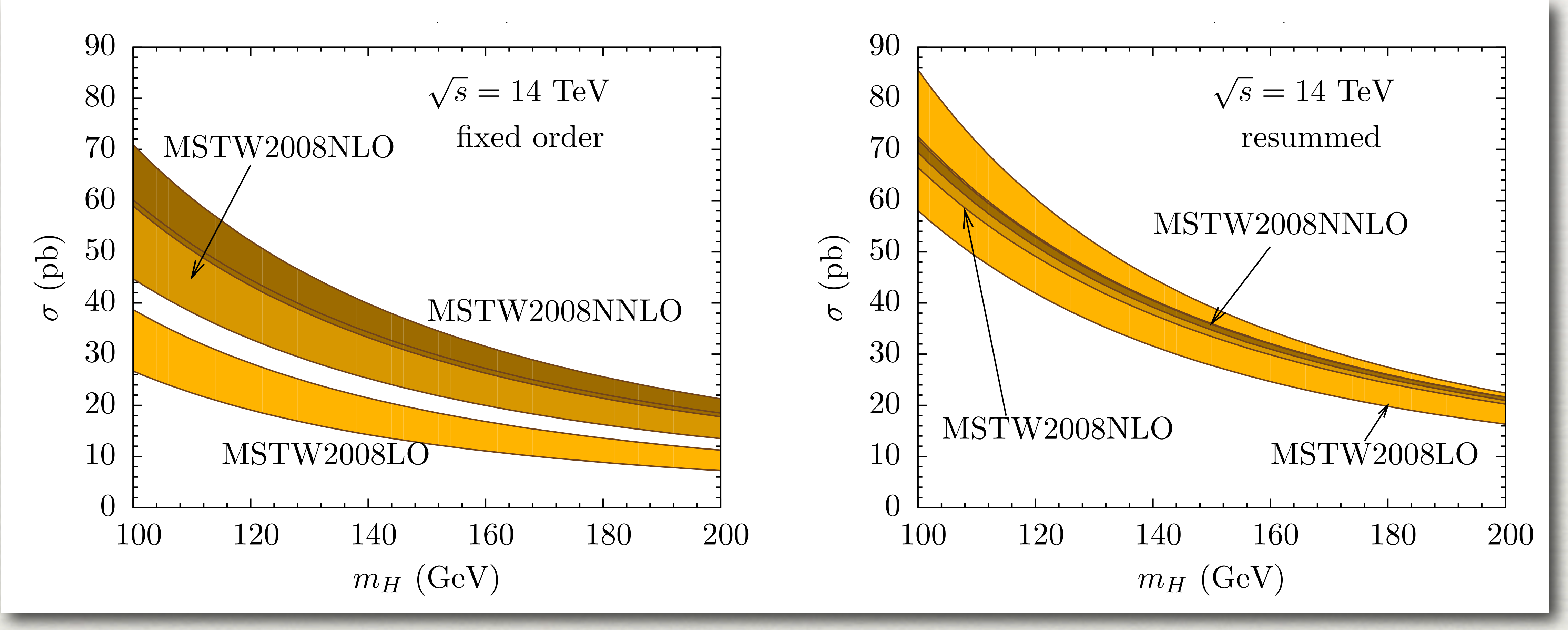
Resummation by RG evolution

- Factorize cross section, evaluate each part at its characteristic scale, evolve to common scale:

$z = M_H^2/\hat{s}$



Numerical results Ahrens, TB, Neubert, Yang '08



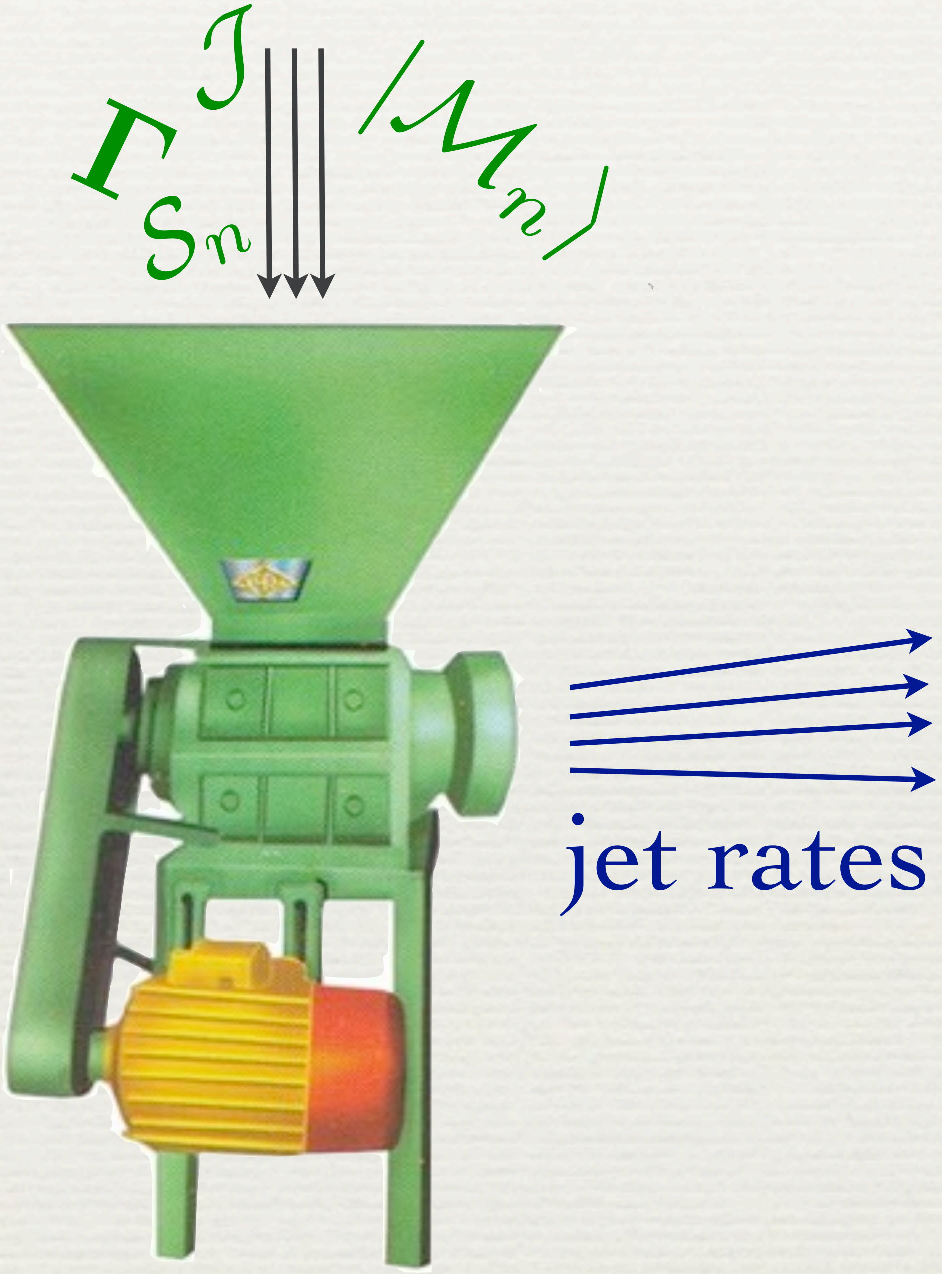
- Includes soft-gluon resummation, but the main effect arises from resumming large corrections due to time-like kinematics by setting $\mu^2 = -m_H^2$ in hard function.
  - RG improved NNLO result is 8% larger than fixed order (13% at Tevatron).

Toward n-jet processes at LHC

Goal: NLO+NNLL resummation

- Necessary ingredients:
  - **Hard functions:** from fixed-order results for on-shell amplitudes; new unitarity methods allow calculation of NLO amplitudes with many legs (→ match with NNLL resummation)
  - **Jet functions:** from imaginary parts of two-point functions; needed at one-loop order (depend on cuts, jet definitions)
  - **Soft functions:** from matrix elements of Wilson-line operators; one-loop calculations comparatively simple
- Then resum logarithms using RG evolution eqns.

Ultimate goal: Automatization



- in the longer term, this will hopefully lead to automated higher-log resummations for jet rates
- goes beyond parton showers, which are only accurate at LL even after matching
- predicts jet cross sections, not parton cross sections!

Evolution of hard functions

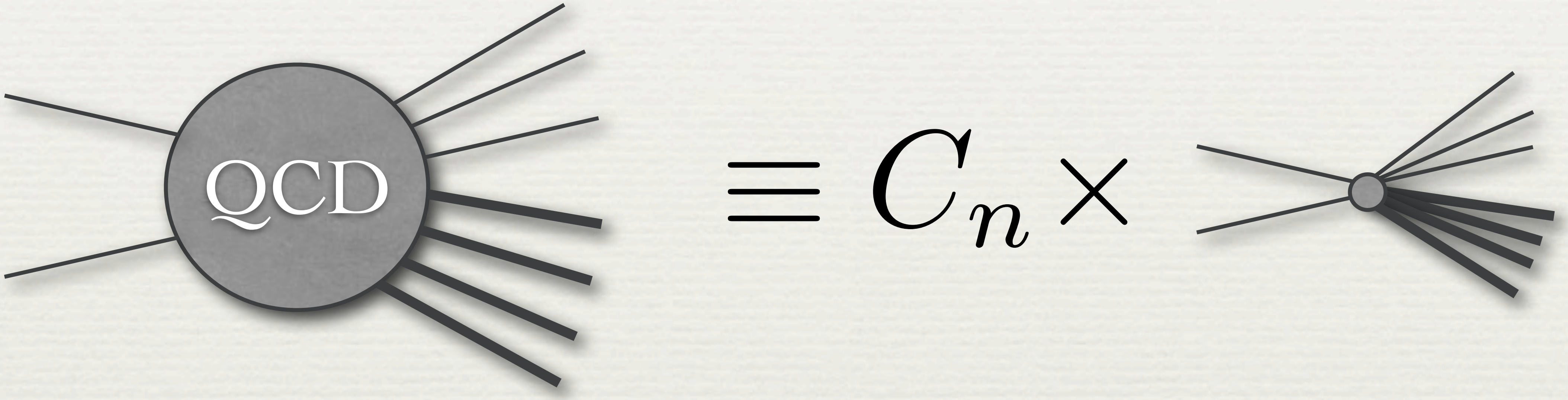
- Technically most challenging aspect besides the computation of the hard functions is their evolution, governed by anomalous-dimension matrix of n-jet operators:

$$\frac{d}{d\ln\mu}\,|\mathcal{C}_n(\{p\},\mu)\rangle = \mathbf{\Gamma}(\mu,\{p\})\,|\mathcal{C}_n(\{p\},\mu)\rangle$$

- Same anomalous-dimension matrix governs IR poles of dimensionally regularized, on-shell parton scattering amplitudes. TB, Neubert 2009

On-shell matching

- To determine hard function, calculate on-shell amplitudes in QCD and effective theory



$$\equiv C_n \times$$

- In effective theory all loop corrections vanish on-shell, because integrals are scaleless.

$$\lim_{\epsilon \longrightarrow 0} \boldsymbol{Z}^{-1}\left(\epsilon, \{\underline{p}\}, \mu\right) |\mathcal{M}_n(\epsilon, \{\underline{p}\})\rangle = |C_n(\{\underline{p}\}, \mu)\rangle$$

- IR poles in QCD map onto UV poles of n-jet operators in SCET

All-order proposal for $\mathbf{\Gamma}$ (massless case)

- Anomalous dimension is conjectured to be extremely simple: TB, Neubert 2009; Gardi, Magnea 2009; Bern et al. 2008

color charges

anom. dimensions,
known to three-loop order

$$\mathbf{\Gamma}(\{\underline{p}\},\mu) = \sum_{(i,j)} \frac{\boldsymbol{T}_i \cdot \boldsymbol{T}_j}{2}\, \gamma_{\rm cusp}(\alpha_s)\, \ln\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s)$$

sum over pairs
i≠j of partons

$(p_i + p_j)^2$

  - minimal structure, reminiscent of QED
  - IR poles determined by color charges and momenta of external partons
  - color dipole correlations, like at one-loop order

Z factor to three loops

- Explicit result:

d-dimensional β-function

$$\ln \boldsymbol{Z}(\epsilon, \{\underline{p}\}, \mu) = \int_0^{\alpha_s} \frac{d\alpha}{\alpha} \frac{1}{2\epsilon - \beta(\alpha)/\alpha} \left[\boldsymbol{\Gamma}(\{\underline{p}\}, \mu, \alpha) + \int_0^{\alpha} \frac{d\alpha'}{\alpha'} \frac{\Gamma'(\alpha')}{2\epsilon - \beta(\alpha')/\alpha'} \right]$$

where

$$\Gamma'(\alpha_s) \equiv \frac{\partial}{\partial \ln \mu} \boldsymbol{\Gamma}(\{\underline{p}\}, \mu, \alpha_s) = -\gamma_{\text{cusp}}(\alpha_s) \sum_i C_i$$

- Perturbative expansion:

all coefficients known!

$$\begin{aligned} \ln \boldsymbol{Z} &= \frac{\alpha_s}{4\pi} \left(\frac{\Gamma_0'}{4\epsilon^2} + \frac{\boldsymbol{\Gamma}_0}{2\epsilon} \right) + \left(\frac{\alpha_s}{4\pi} \right)^2 \left[-\frac{3\beta_0 \Gamma_0'}{16\epsilon^3} + \frac{\Gamma_1' - 4\beta_0 \boldsymbol{\Gamma}_0}{16\epsilon^2} + \frac{\boldsymbol{\Gamma}_1}{4\epsilon} \right] \\ &+ \left(\frac{\alpha_s}{4\pi} \right)^3 \left[\frac{11\beta_0^2\, \Gamma_0'}{72\epsilon^4} - \frac{5\beta_0 \Gamma_1' + 8\beta_1 \Gamma_0' - 12\beta_0^2\, \boldsymbol{\Gamma}_0}{72\epsilon^3} + \frac{\Gamma_2' - 6\beta_0 \boldsymbol{\Gamma}_1 - 6\beta_1 \boldsymbol{\Gamma}_0}{36\epsilon^2} + \frac{\boldsymbol{\Gamma}_2}{6\epsilon} \right] + \dots \end{aligned}$$

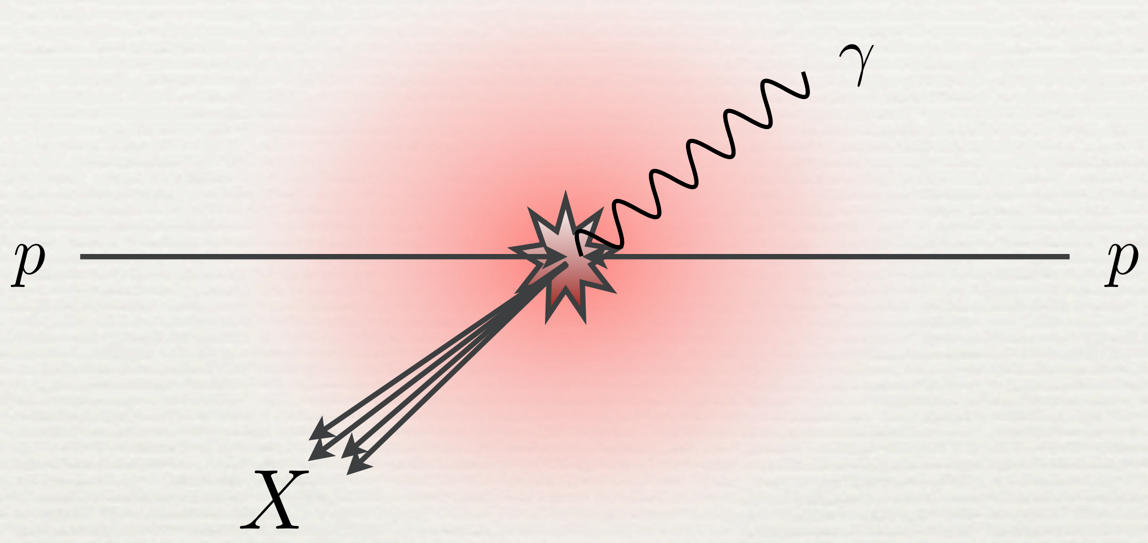
⇒ exponentiation yields **Z** factor at three loops!

Checks

- Expression for IR pole terms agrees with all known perturbative results:
  - 3-loop quark and gluon form factors, which determine the functions $\gamma^{q,g}(\alpha_s)$
    Moch, Vermaseren, Vogt 2005
  - 2-loop 3-jet qqg amplitude
    Garland, Gehrmann et al. 2002
  - 2-loop 4-jet amplitudes
    Anastasiou, Glover et al. 2001
    Bern, De Freitas, Dixon 2002, 2003
  - 3-loop 4-jet amplitudes in N=4 super Yang-Mills theory in planar limit
    Bern et al. 2005, 2007

First 3-jet application: γ production at large p_T

TB, M. Schwartz, in preparation



- Have derived factorization theorem for photon production at large $p_T \gg M_X$

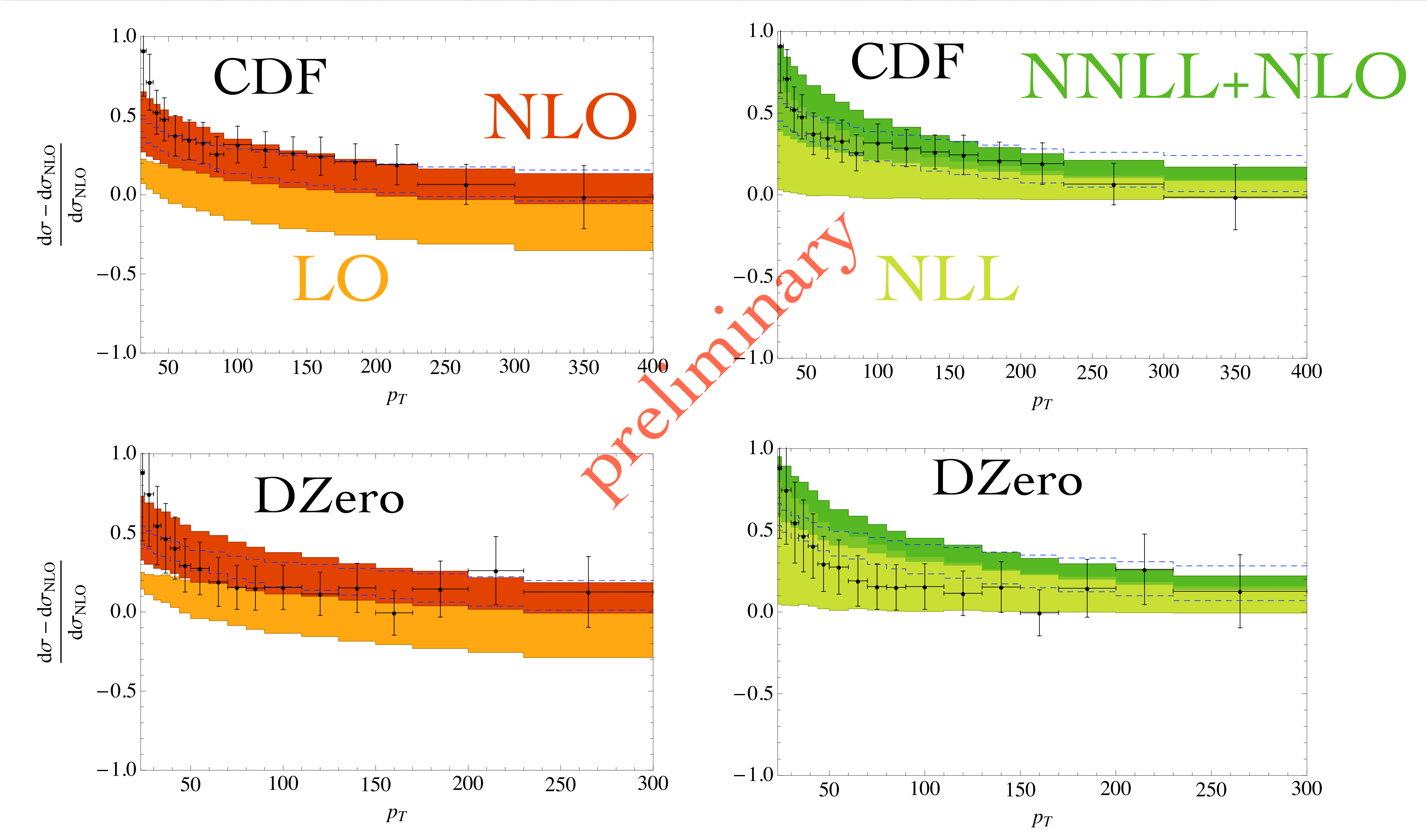
$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}y\mathrm{d}p_T} = H \otimes J \otimes S \otimes f_1 \otimes f_2$$

  - (there are different partonic channels, with different H, J, S and f's)

Photon production at large p_T

- Have calculated *H*, *J* and *S* to one loop
  - *H* from known virtual corrections to $q\bar{q} \to \gamma g$.
- Have extracted all anomalous dimensions to 3 loops
  - Use *H* from general result, Casimir scaling for *S*, RG invariance.
- Solving RG equations we obtain NNLL resummed result (NLL is known).
- For phenomenological analysis we match to fixed order and account for isolation cuts using JetPhox fixed order MC generator.

Tevatron results



normalized to NLO w/o photon isolation cuts and fragmentation

Conclusion

- Soft collinear effective theory provides an efficient tool to
  - factorize contributions associated with different scales and
  - resum logarithms of scale ration using RG evolution in momentum space
- Are on track to perform higher-log resummation for n-jet processes at LHC using RG evolution SCET.
  - Have anomalous dimension $\mathbf{\Gamma}$ relevant for NNLL resummation of n-jet processes.
  - First application: $pp \to \gamma + X$

Backup

Analysis of Sterman and Tejeda-Yeomans '03

- Based on factorization

$$|\mathcal{M}_n\rangle = \prod_i \underset{\substack{\uparrow \\ \text{color-diagonal}}}{J_i(\alpha_s,\epsilon)}\, \underset{\substack{\uparrow \\ \text{eikonal}}}{\boldsymbol{S}(\alpha_s,\epsilon)}\, \underset{\substack{\uparrow \\ \text{finite}}}{|h_n(\alpha_s)\rangle}$$

- Define jet-function as square root of form factor $J_i(\alpha_s,\epsilon) = [F(Q^2)]^{1/2}$
- Structure of IR divergences governed by $\boldsymbol{S}$
- Same physical picture, but rather different definition of hard, jet and soft functions
  - In SCET $|\mathcal{M}_n\rangle$ is purely hard, since it only depends on hard scales.

Higgs production $pp \rightarrow H$+X

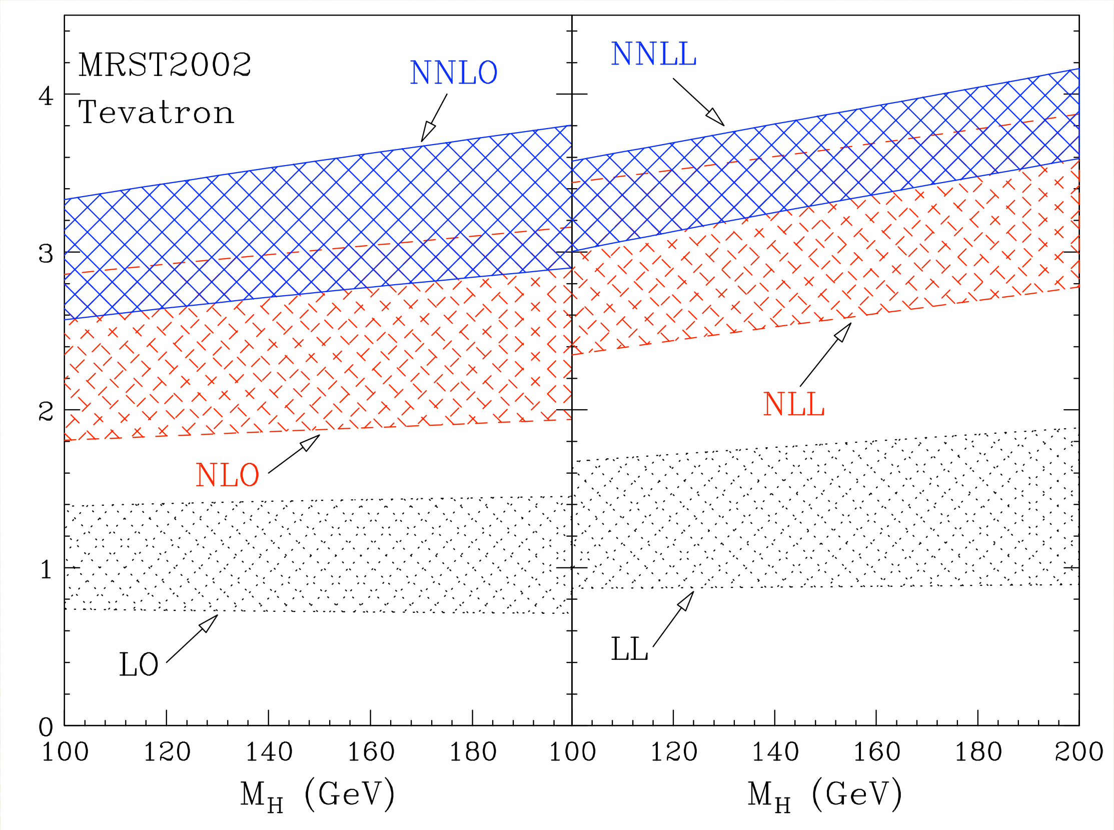
- Factorization theorem for partonic cross section near threshold $z = m_H^2/\hat{s} \rightarrow 1$

$$\sigma_{\text{part}} = C_t(m_t^2, \mu^2)\, H(m_H^2, \mu^2)\, S(m_H^2(1-z)^2, \mu^2)$$

- Can solve RG equations for the different parts: this resums log's of scale ratios.
    - equivalent to soft-gluon resummation
- Soft scale is set dynamically via the fall-off of the PDF. For m_{H}= 120 GeV,

$$\sigma_{\text{had}} \propto \int_0^1 dz\, z^{1.5} \sigma_{\text{part}}(z)$$

weight function not strongly peaked near z=1



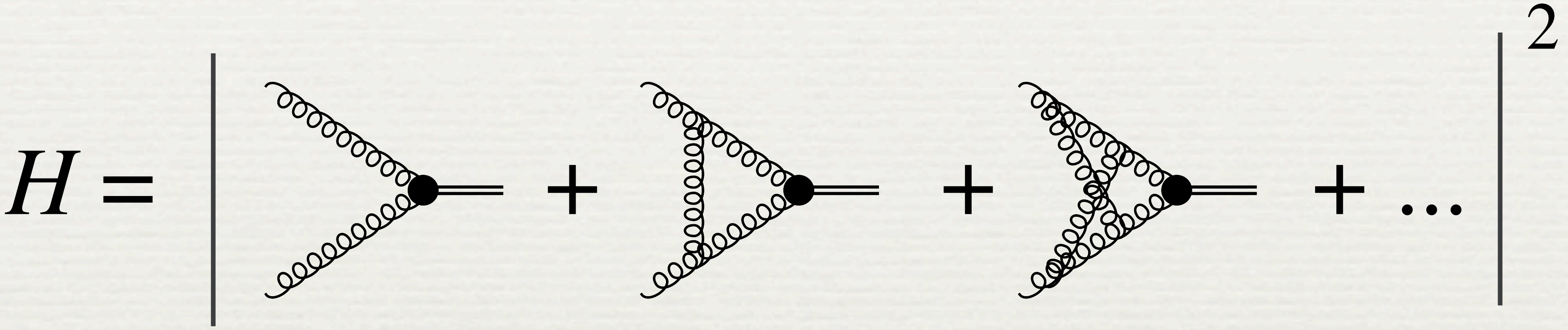
Catani, de Florian, Grazzini, Nason ’03

- Soft scale is ~ $m_H/2$, not much lower than hard scale. No large soft logarithms.
    - however, the threshold region is numerically large, gives ~ 90% of NLO and NNLO correction
- Even after resummation of log’s, higher order corrections are very large.

Origin of the large corrections

Ahrens, TB, Neubert, Yang '08;

- Hard function gets large higher order corrections



$$\begin{aligned} H(m_H^2, \mu^2 = m_H^2) &= 1 + 5.50\alpha_s(m_H^2) + 17.24\alpha_s^2(m_H^2) + \ldots \\ &= 1 + 0.623 + 0.221 + \ldots \end{aligned}$$

- The space-like form factor has well behaved expansion:

$$H(m_H^2, \mu^2 = -m_H^2) = 1 - 0.15 - 0.0012 + \ldots$$

  - use RG to evolve back to $\mu^2 = +m_H^2$