Soft and Collinear gluon corrections to Higgs production and DIS beyond two loop

V. Ravindran

Harish-Chandra Research Institute, Allahabad, India

- Soft gluons at N^3LO for Higgs and Drell-Yan Production
- Structure of QCD amplitudes
- Leading logs beyond 1-x
- Conclusions

Outline

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Structure multi-parton amplitudes in QCD

- Infra-red structure of $F^a_{\mu\nu} F^{\mu\nu a}$ (Drell-Yan) and $\overline{\psi}\gamma_\mu\psi$ (Higgs production) in $SU(N)$ gauge theory using dimensional regularisation ($n - 4 = \varepsilon$)
- Universal structure of single pole $\frac{1}{\varepsilon}$ and connection to AdS/CFT.

QCD effects at LHC

- Fixed order N^3LO Soft gluon corrections to
  1. Higgs production
  2. Drell-Yan (di-lepton) production

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Threshold resummation beyond $1 - x$

- Renormalisation and Factorisation invariant approach beyond leading $1 - x$ order.
- Predictions from Single and Two-scale ansaetze

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- Renormalised version of parton model:

$$\boxed{2S\, d\sigma^{P_1 P_2}\left(\tau, m_h^2\right) = \sum_{ab} f_a(\tau, \mu_F) \otimes f_b(\tau, \mu_F) \otimes 2\hat{s}\, d\hat{\sigma}^{ab}\left(\tau, m_h^2, \mu_F\right),}$$

where $$d\hat{\sigma}_{ab}\left(\tau, m_h^2, \mu_F\right) = \sum_{i=0}^{\infty} \left(\frac{\alpha_s(\mu_R)}{4\pi}\right)^i d\hat{\sigma}^{(i)}_{ab}\left(\tau, m_h^2, \mu_F, \mu_R\right)$$

Higgs production at LHC

Harlander, Kilgore, Anastasiou, Melnikov, Smith, van Neerven, VR

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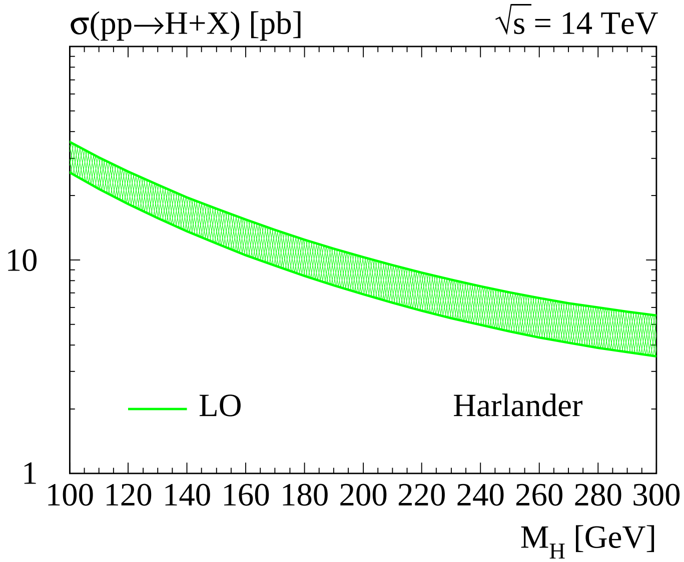

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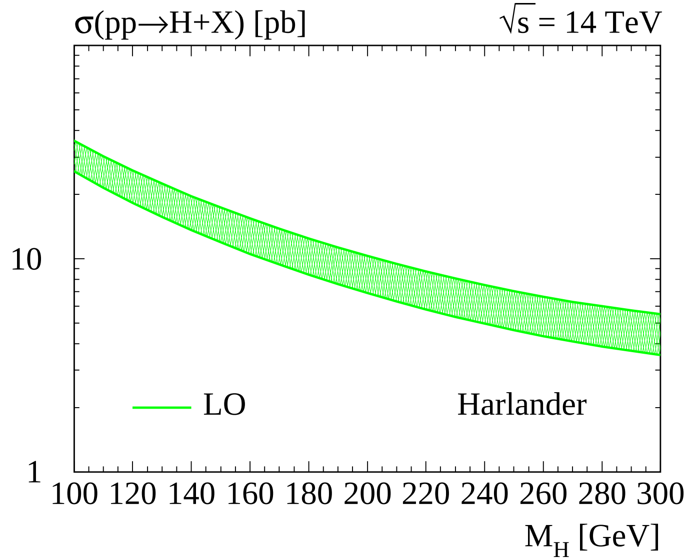

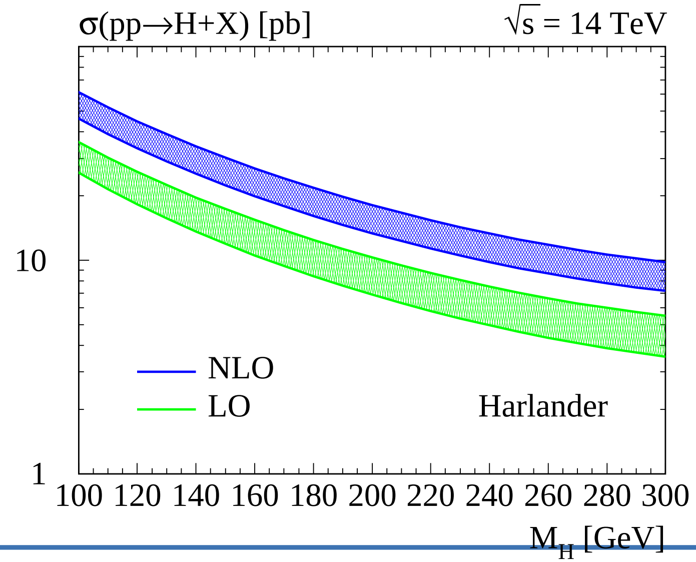

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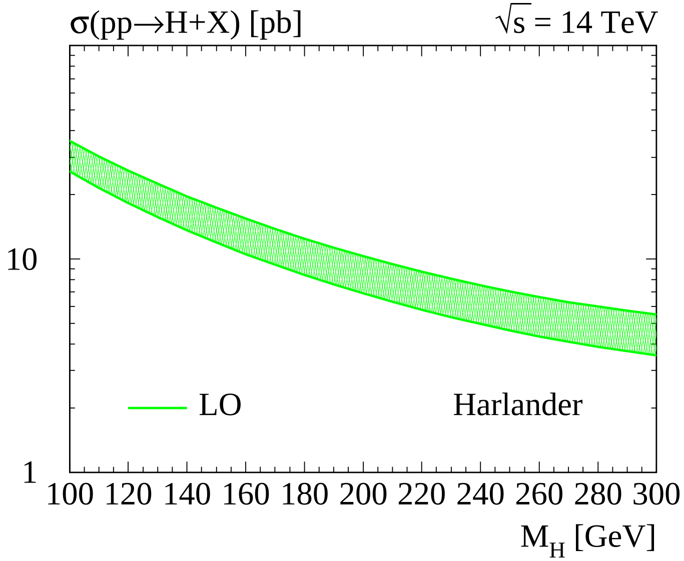

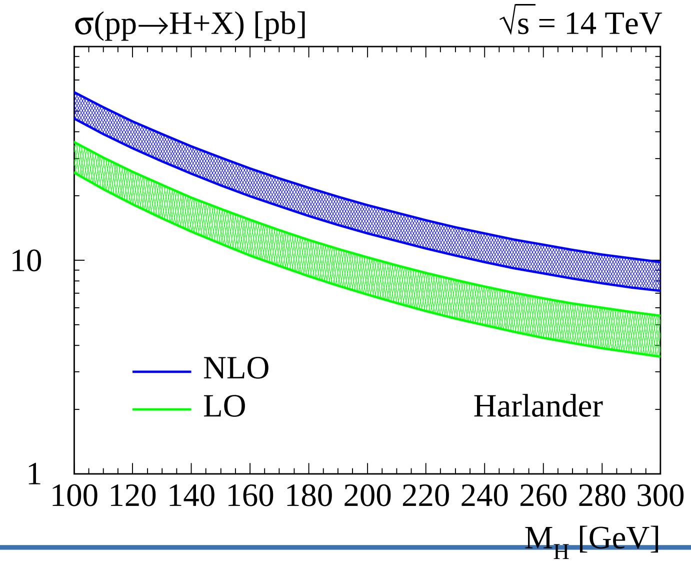

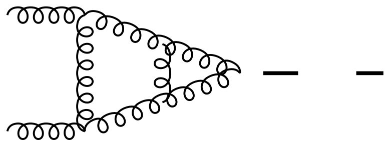

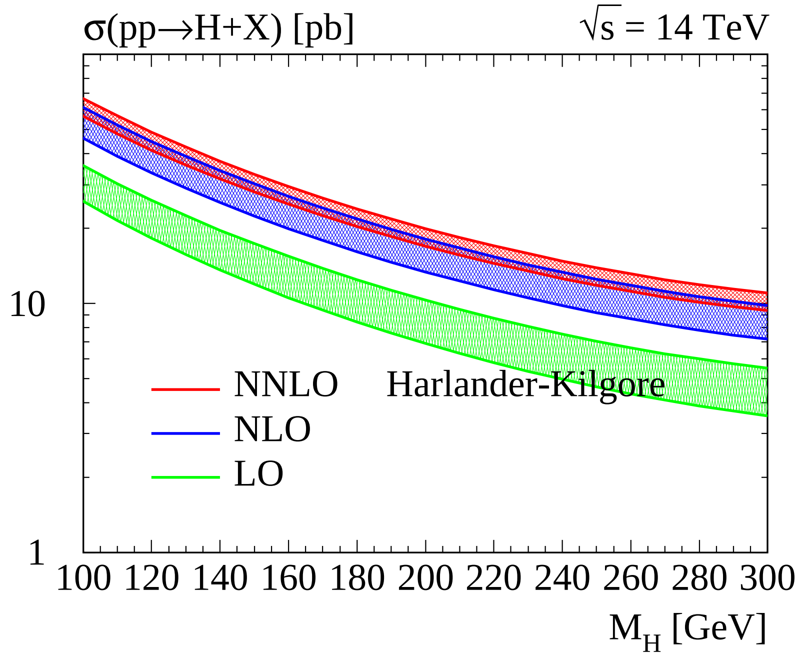

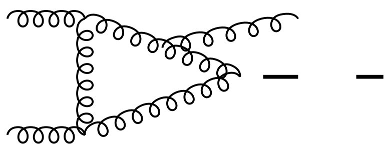

Infra-red structure of QCD amplitudes in $n = 4 + \varepsilon$ dimensions

Ashoke Sen, Mueller, Collins

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Quark and Gluon Form factors:

$$\textbf{Drell} - \textbf{Yan}: \quad \hat{F}^q(\hat{a}_s, Q^2, \mu^2, \varepsilon) = \langle q(p) \,|\, \overline{\psi}\gamma^\mu \psi \,|\, q(p') \rangle \Big[\overline{u}(p')\gamma_\mu u(p)\Big]^{-1}$$

$$\textbf{Higgs}: \quad \hat{F}^g(\hat{a}_s, Q^2, \mu^2, \varepsilon) = -\frac{1}{2} \langle g(p) \,|\, Tr\,[G^{\mu\nu} G_{\mu\nu}] \,|\, g(p') \rangle$$

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$$\boxed{\ln \hat{F}^I(\hat{a}_s, Q^2, \mu^2, \varepsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{Q^2}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \, \hat{\mathcal{L}}_F^{I,(i)}(\varepsilon)} \qquad Q^2 = (p - p')^2$$

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$$\hat{\mathcal{L}}_F^{I,(1)} = \frac{1}{\varepsilon^2}\Big(-2A_1^I\Big) + \frac{1}{\varepsilon}\Big(G_1^I(\varepsilon)\Big)$$

$$\hat{\mathcal{L}}_F^{I,(2)} = \frac{1}{\varepsilon^3}\Big(\beta_0 A_1^I\Big) + \frac{1}{\varepsilon^2}\Big(-\frac{1}{2}A_2^I - \beta_0 G_1^I(\varepsilon)\Big) + \frac{1}{2\varepsilon} G_2^I(\varepsilon)$$

....

Universality of single pole

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G^Is have interesting structure:

$$G_1^I(\varepsilon) = 2(B_1^I - \gamma_1^I) + f_1^I + \sum_{k=1}^{\infty} \varepsilon^k g_1^{I,k}$$

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$$\boxed{G_i^I(\varepsilon) = 2(B_i^I - \gamma_i^I) + f_i^I + \mathcal{O}(\beta, \varepsilon) \qquad i = 1, 2, 3}$$

Moch, Vermaseren and Vogt

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Moch, Vermaseren and Vogt

- γ_i^I are UV anomalous dimensions of F^I and
- B_i^I are collinear anomalous dimensions of twist-2 quark and gluon composite operators.
- The soft anomalous dimensions f_i^I satisfy

$$f_i^g = \frac{C_A}{C_F} f_i^q \qquad i = 1, 2$$

- It is confirmed by explicit three loop ($i = 3$) results by Moch, Vermaseren and Vogt

Drell-Yan and Wilson loops

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via the Fourier transform, $\widetilde{W}_{\rm DY}(z)$, of the vacuum average of the Wilson loop

$$\widetilde{W}_{\rm DY}(z) = \frac{Q}{2} \int_{-\infty}^{\infty} \frac{dy_0}{2\pi} e^{iy_0\omega} W_{\rm DY}(y),$$

$$where \quad W_{\rm DY}(y) = \frac{1}{N_c} \langle 0 | {\rm Tr} \mathcal{T}\mathcal{P} \exp\left(ig \oint_{C_{\rm DY}} dx_\mu B_\mu(x) \right) |0\rangle,$$

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Renormalisation group equation:

$$\left(\mu \frac{\partial}{\partial\mu} + \beta(g) \frac{\partial}{\partial g}\right) \ln W_{\rm DY}(y) = 2\Gamma_{\rm cusp}(g) \ln L(y) + \Gamma_{\rm DY}(g),$$

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Soft anomalous dimension $\Gamma_{\rm DY}$ is now known upto two loop level:

$$\Gamma_{\rm DY}(g) = \sum_{i=1} a_s^i \Gamma^{(i)}_{\rm DY} \qquad\qquad i = 1, 2$$

We find

$$\boxed{\Gamma^{(i)}_{\rm DY} = f_i^q \qquad\qquad i = 1, 2}$$

Multi-parton amplitudes (MPA) in QCD

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- The peculiar combination $\boxed{2(B^I - \gamma^I) + f^I}$ appears in $G^I(\varepsilon)$.

Universality in the strong coupling region from AdS/CFT

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- $\mathcal{N} = 4$ Yang-Mills theory is conformal ($C_A = 2C_F = N_c$)

$$\mu_R^2 \frac{dg_s}{d\mu_R^2} = \beta(g_s) = 0.$$

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$$\mathcal{A}_{div}(\varepsilon) = \prod_{I=legs} \exp\left(\frac{1}{\varepsilon^2} A_I(\lambda) + \frac{1}{\varepsilon} G_I(\lambda, \varepsilon)\right)$$

where $\lambda = g_s^2 N_c/8/\pi$, $A_I(\lambda)$ are cusp anomalous dimensions and $G_I(\lambda)$ are collinear anomalous dimensions.

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- $B_I(\lambda)$ are related to energy of the classical strings spinning on AdS and are related to anomalous dimensions of twist-2 operators with high spin.

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- $B_I(\lambda)$ are related to energy of the classical strings spinning on AdS and are related to anomalous dimensions of twist-2 operators with high spin.
- In the strong coupling using AdS/CFT, Alday has computed $G(\lambda)$ and $B(\lambda)$ to show the universality

$$G_I(\lambda) = 2B_I(\lambda) + c_I \frac{\sqrt{\lambda}}{2\pi}\Big(-1 - 2\gamma_E + 5\log(2) + 2\log(\pi) - \log(\lambda)\Big)$$

Soft gluon cancellation

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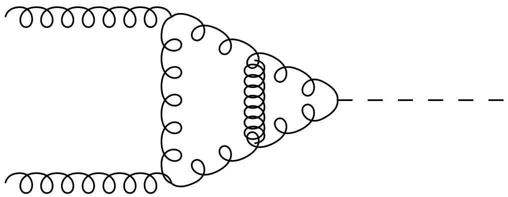
$\boldsymbol{\sigma^{V}_{ab}(z, Q^2, \varepsilon_s, \varepsilon_c)}$ - Virtual soft gluon
$(|\boldsymbol{F^I}|^2)$



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Soft gluon cancellation

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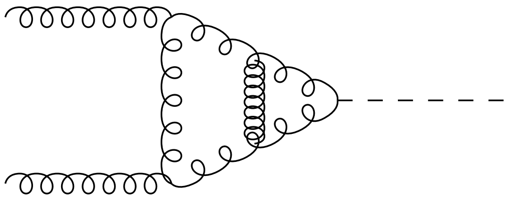
$\sigma^V_{ab}(z, Q^2, \varepsilon_s, \varepsilon_c)$ - Virtual soft gluon ($|F^I|^2$)



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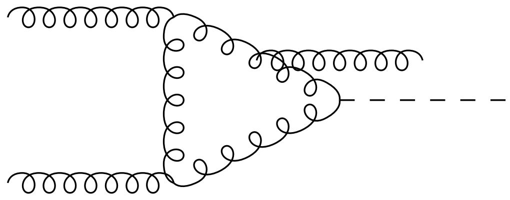

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Soft gluon cancellation

V. Ravindran

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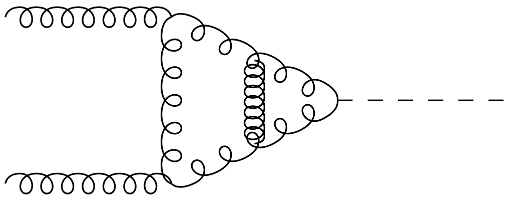

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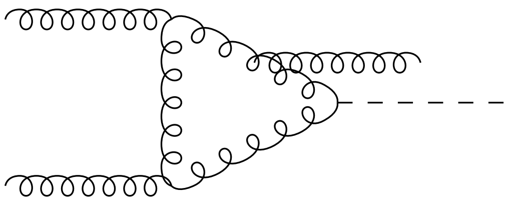

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- ε_s - soft gluon regulator
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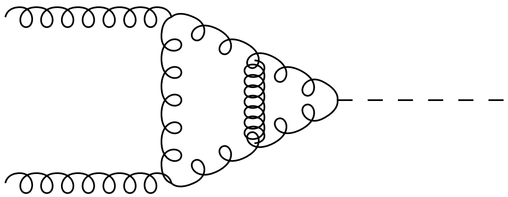

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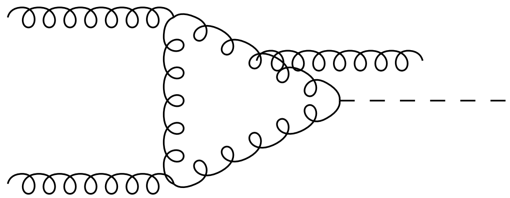

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Only collinear partons remain

Factorisation of Collinear partons

Due to the massless partons, collinear singularities appear in

- the phase space of the real emission processes
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Soft plus Virtual at N^3LO and beyond

Catani, Sterman, VR

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Using "factorisation" of UV, Soft and Collinear:

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$$\hat a_s = \frac{\hat g_s^2}{16\pi^2} \qquad m=\frac{1}{2}\quad \text{for}\quad \mathrm{DIS}/e^+e^-, \quad m=1 \quad \text{for} \quad \mathrm{DY},\mathrm{Higgs}$$

Scale variation at N^3LO_{pSV} for Higgs production

Moch, Vogt, V. Ravindran

$$\boldsymbol{R = \frac{\sigma_{N^iLO}(\mu)}{\sigma_{N^iLO}(\mu_0)}}$$

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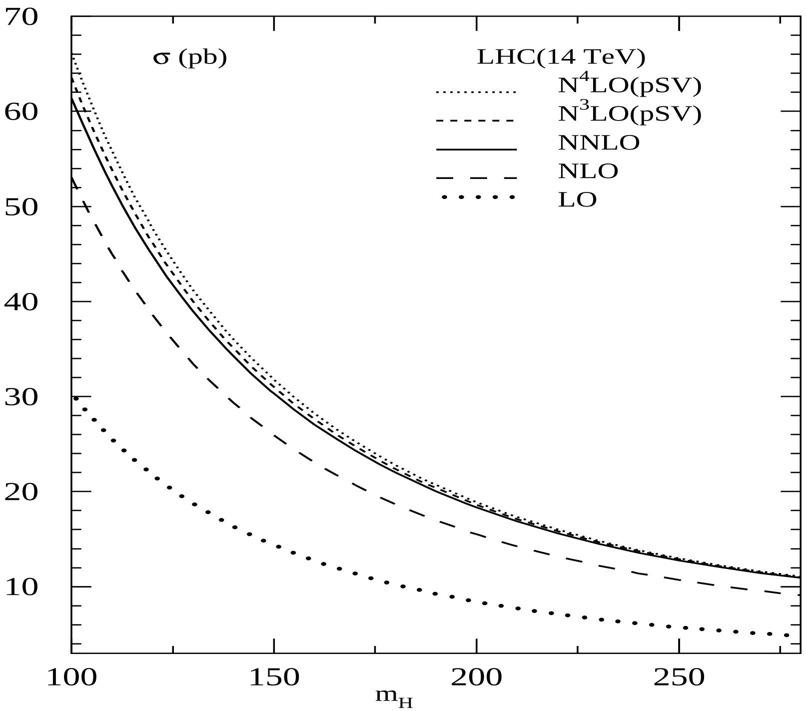

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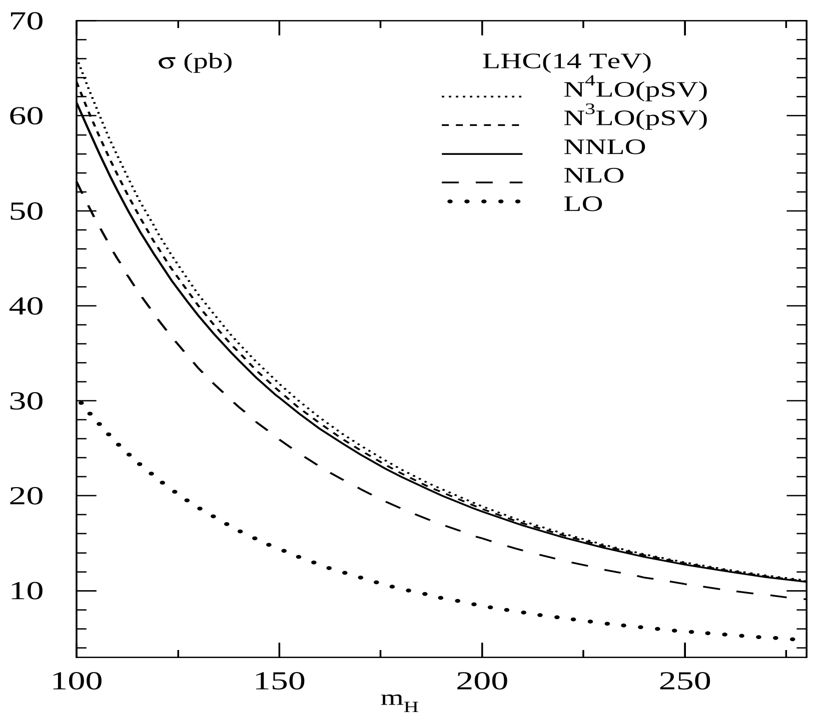

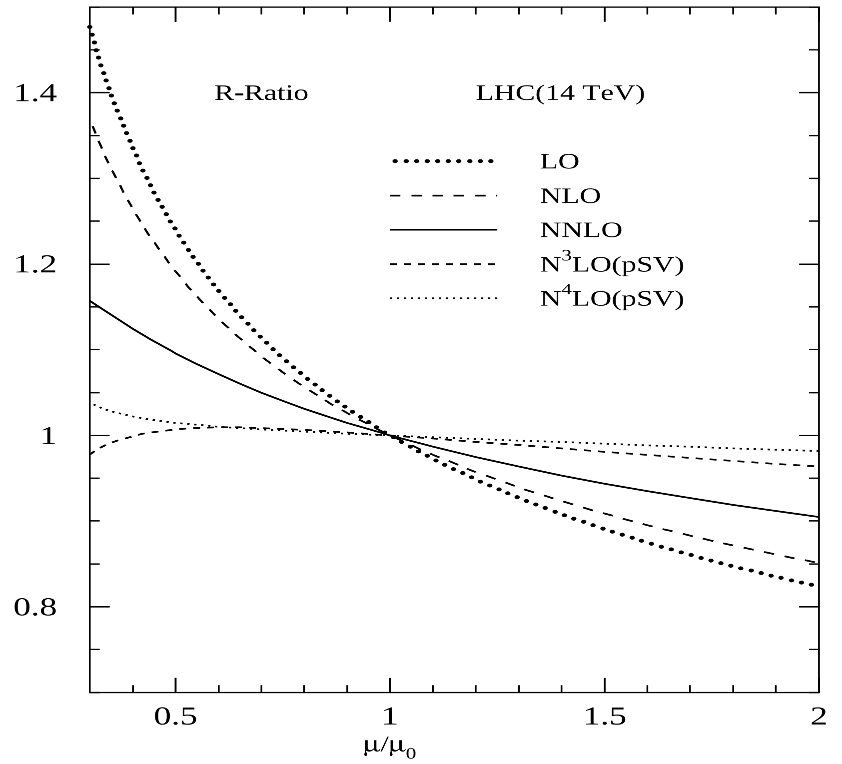

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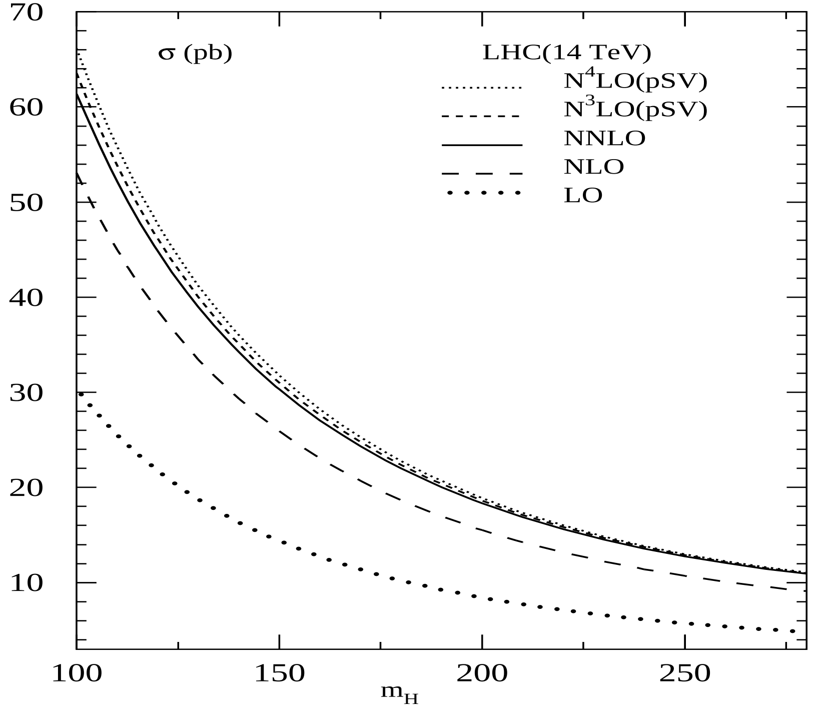

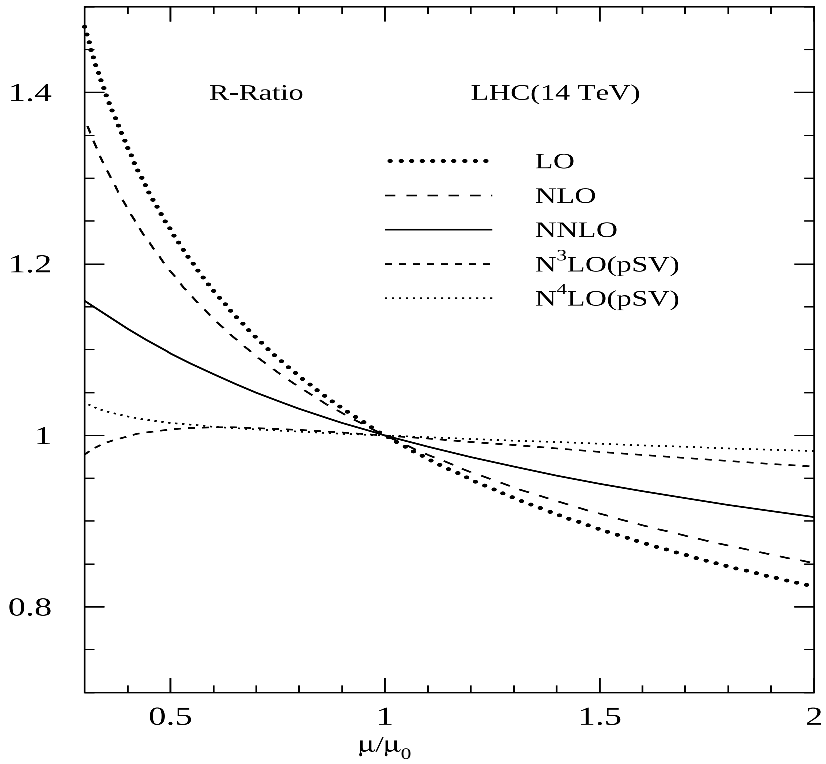

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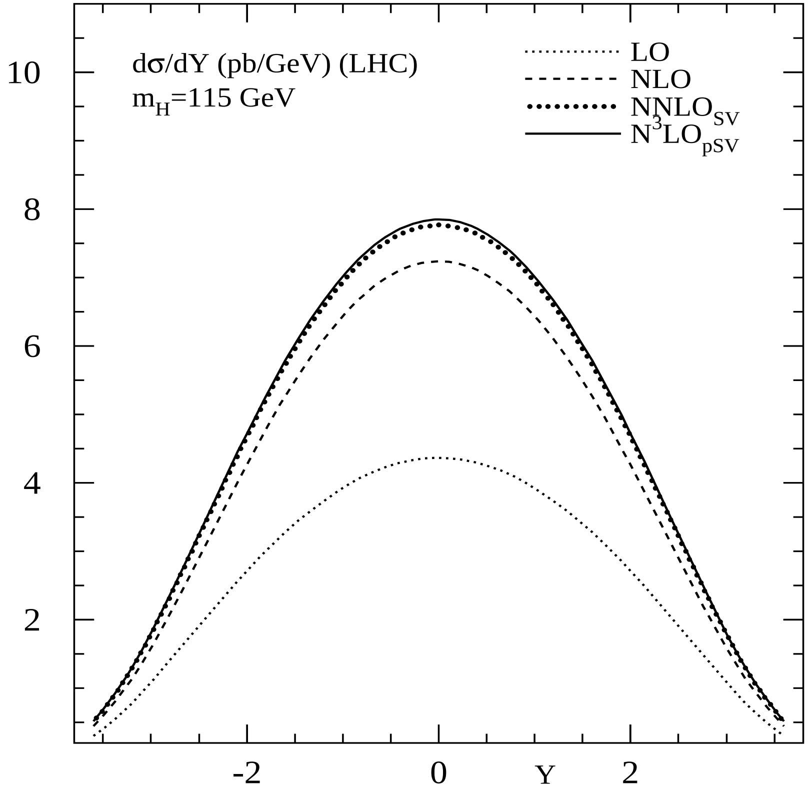

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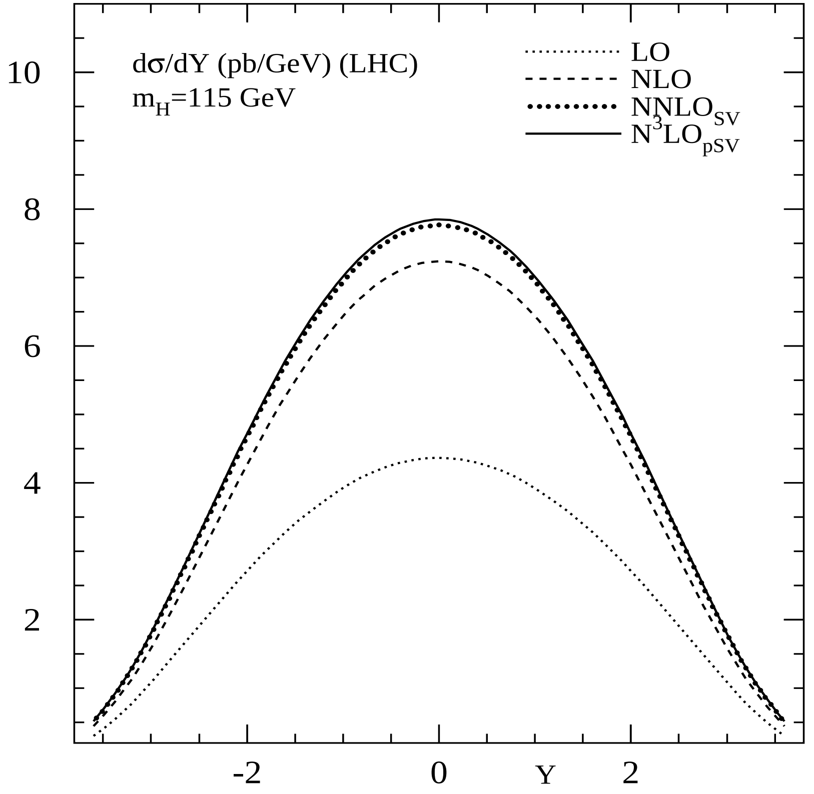

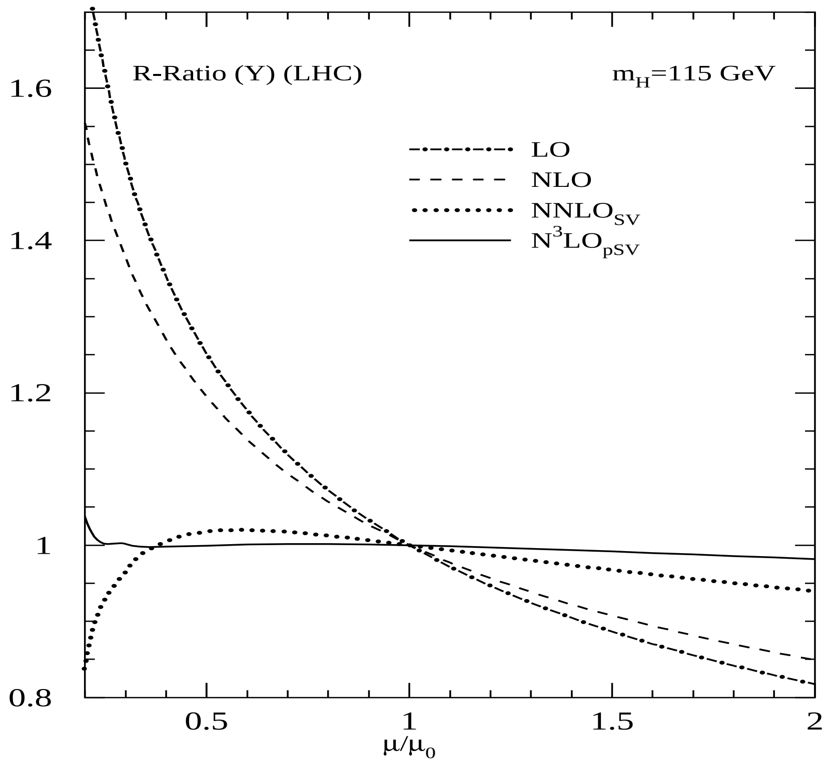

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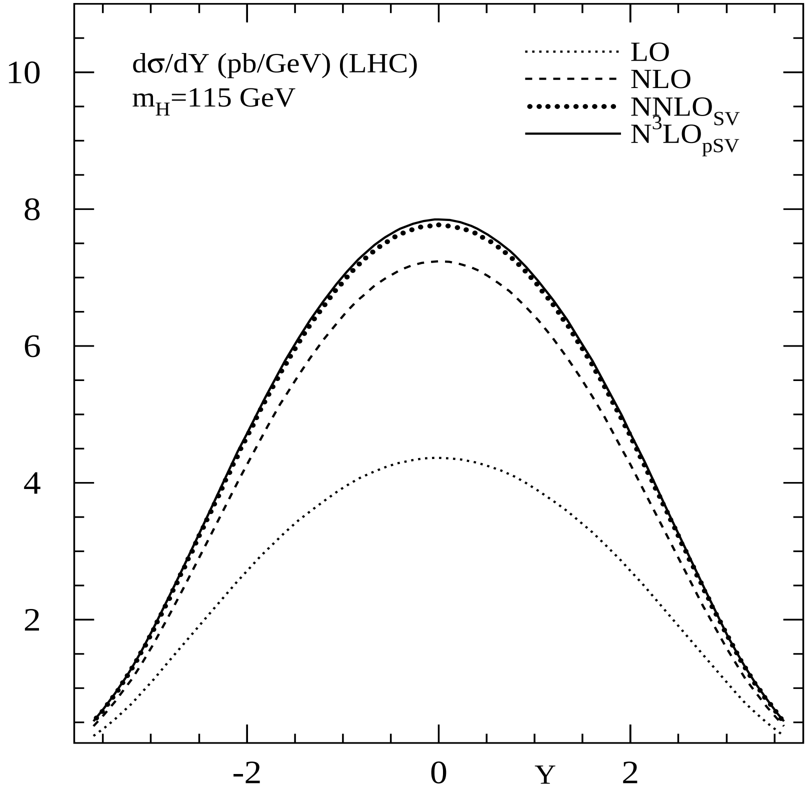

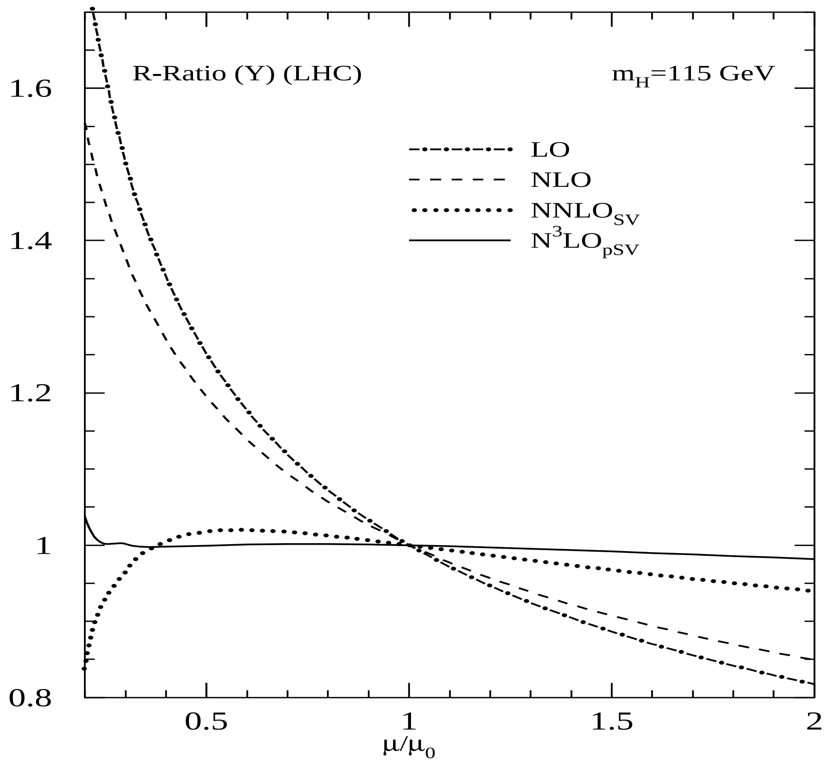

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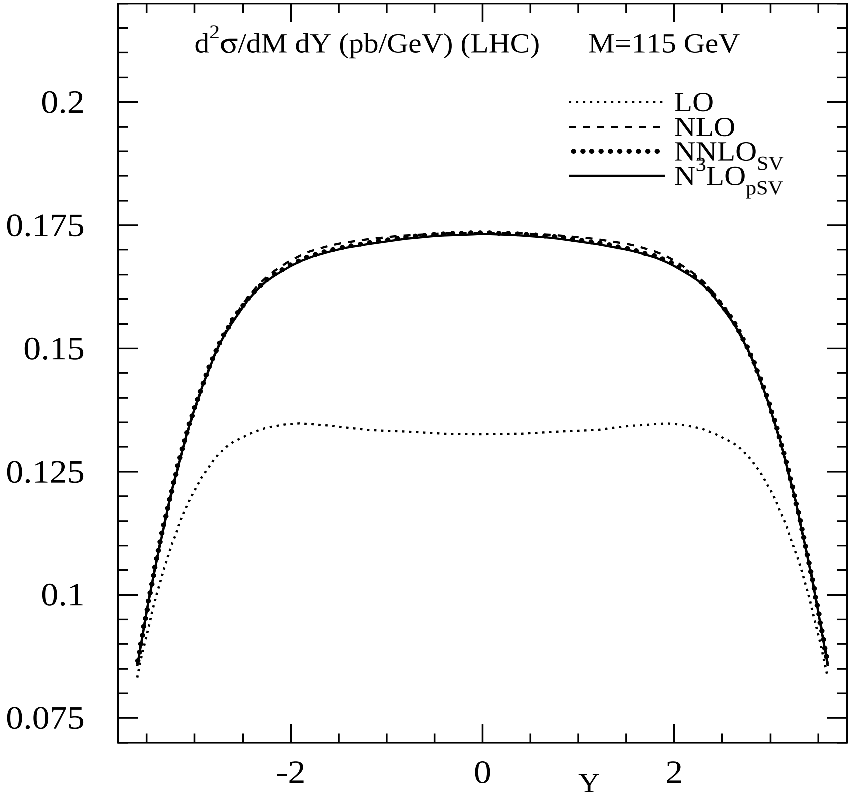

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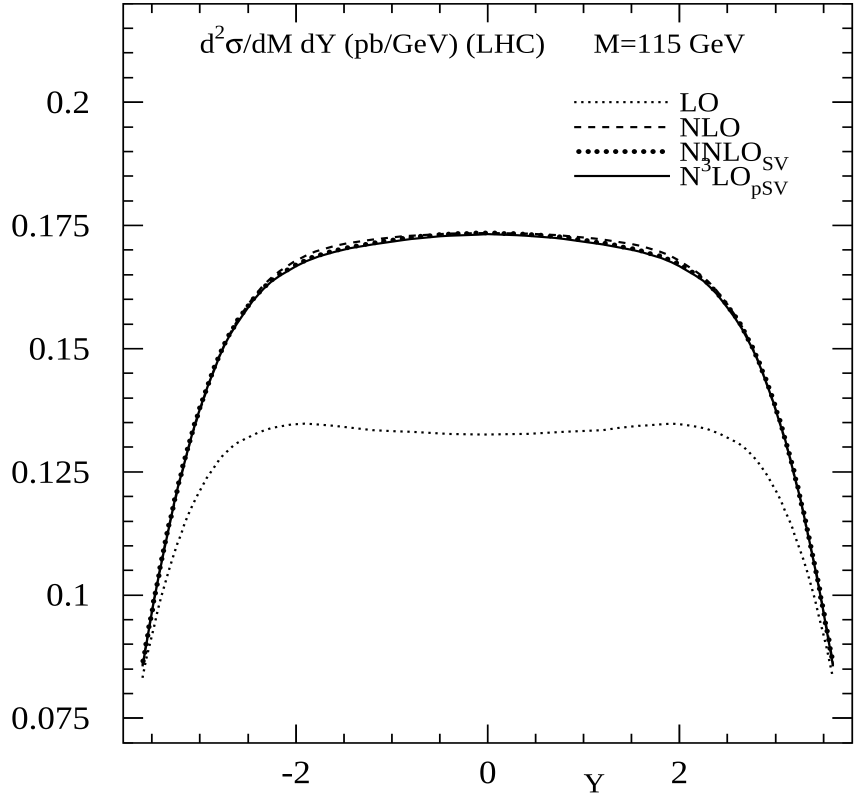

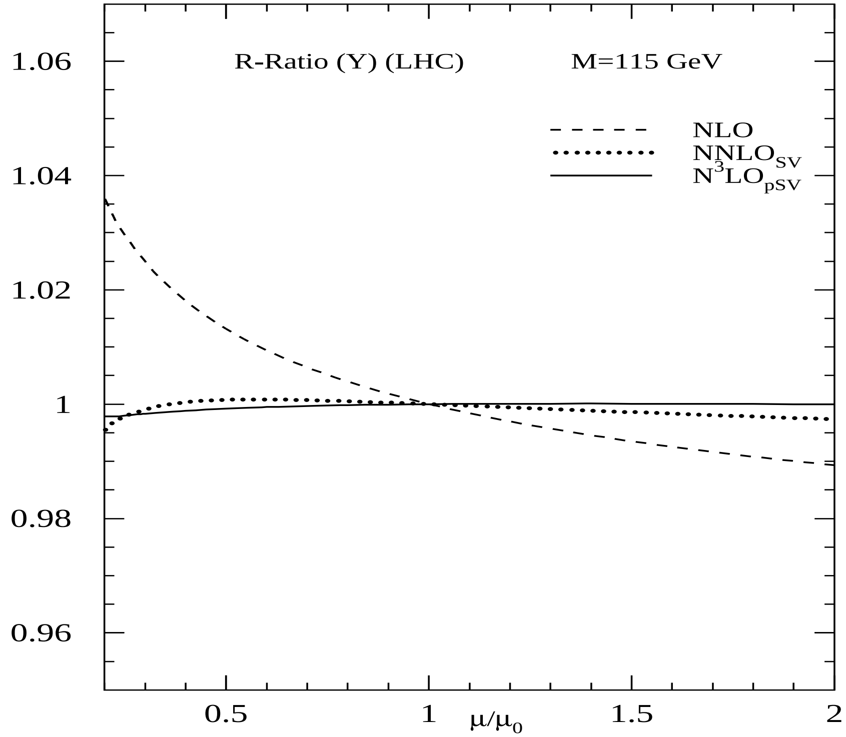

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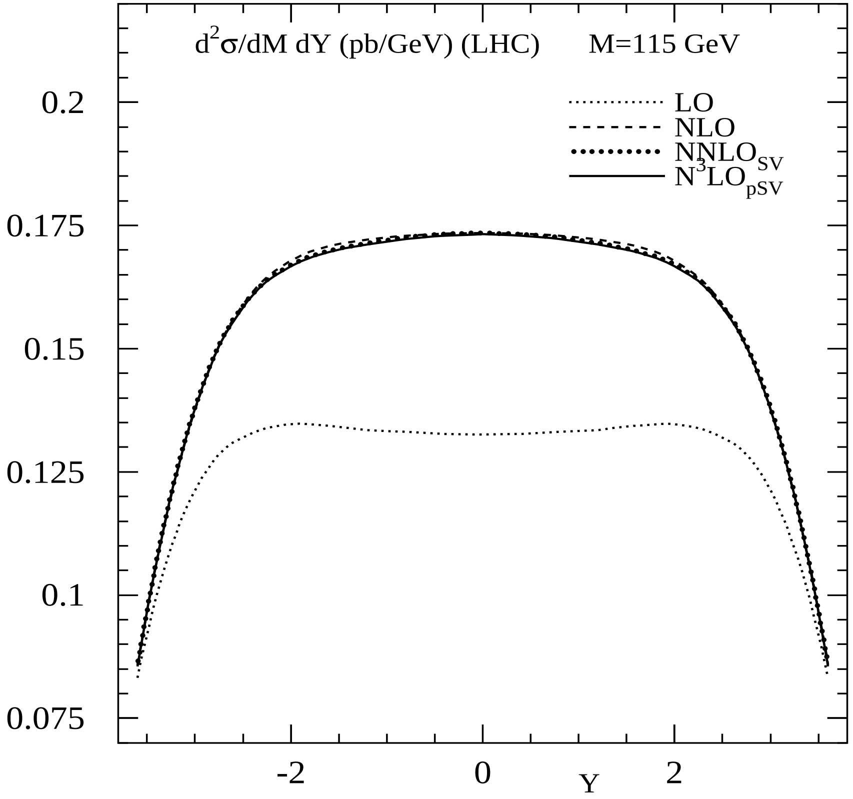

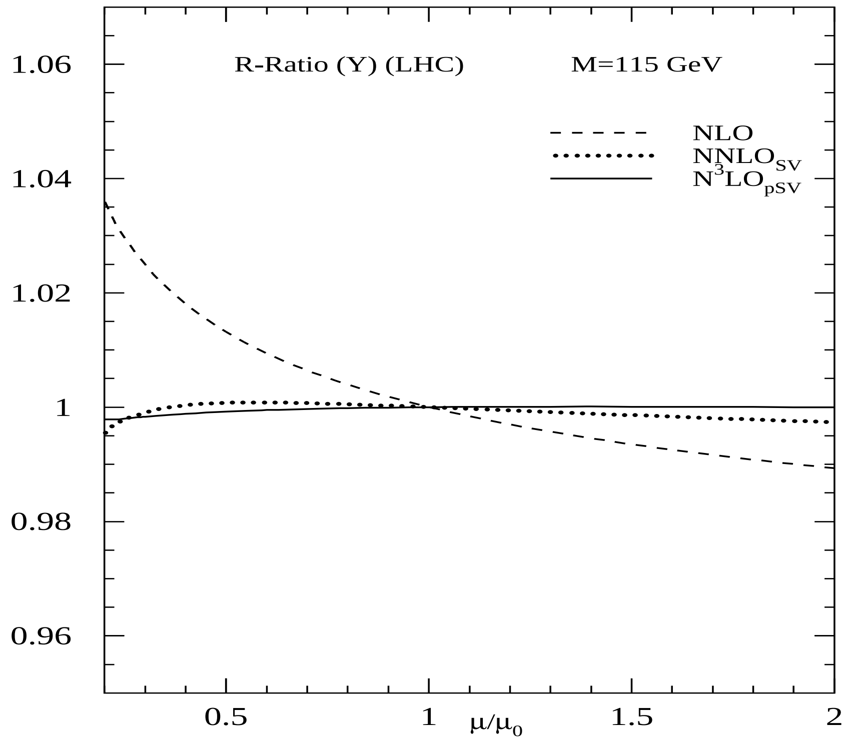

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Soft gluons beyond leading $1 - x$ order

$Laenen,\ Magnea,\ Moch, Vogt,\ Grunberg, VR$

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Consider DIS structure function:

$$F_2(x,Q^2)/x \quad = \quad \sum_j \mathcal{C}_{2,j}(x,Q^2,\mu_F^2) \otimes f_{a_j}(x,\mu_F^2)$$

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$$\begin{aligned} d_1(x) &= c_1(x) \\ d_2(x) &= 2c_2(x) - c_1^{\otimes 2}(x) \\ d_3(x) &= 3c_3(x) - 3c_2(x) \otimes c_1(x) + c_1^{\otimes 3}(x)\ , \end{aligned}$$

where $C_{2,j} = \delta(1-x) + \sum_{k=1} a_s^k c_k(x)$ $\qquad a_s \equiv a_s(Q^2),$

Ansatz beyond leading $1 - x$ order

$Grunberg, VR$

Ansatz beyond leading $1 - x$ order

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Ansatz with $\bar{\mathcal{J}}_0$ and $\mathcal{J}_0$:

$$K(x,Q^2) \quad = \quad K_{resum}(x,Q^2) + \left[\bar{\mathcal{J}}_0\left(W^2\right)\ln(1-x) + \mathcal{J}_0\left(W^2\right)\right] + \mathcal{O}\left(r\ln^2 r\right).$$

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Renormalisation group invariance:

$$\mathcal{J}\left(W^2\right) = j_1 a_s + a_s^2 \left(-j_1 \beta_0 \ln\left(\frac{W^2}{Q^2}\right) + j_2 \right)$$

$$\mathcal{J}_0\left(W^2\right) = j_{02} a_s^2 + a_s^3 \left(-2 j_{02} \beta_0 \ln\left(\frac{W^2}{Q^2}\right) + j_{03} \right) + \ldots .$$

where

$$W^2 \equiv \frac{1-x}{x} Q^2 .$$

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$$W^2 \equiv \frac{1-x}{x}\,Q^2\,.$$

Explicit result on $C_2(x,Q^2)$ upto two loops fixes:

$$j_1 \quad = \quad A_1 \qquad\qquad j_2 = A_2 + 3C_F\beta_0$$

$$j_{02} \quad = \quad A_1 B_1^\delta - 11 C_F\beta_0 \qquad\qquad \bar{j}_{02} = A_1^2$$

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Does not predict anything at two loops! But it predictions at "three loops".

Order a_s^3 prediction from the Ansatz

$Grunberg, VR$

Order a_s^3 prediction from the Ansatz

Grunberg, VR

$$K^{(3)}(x,Q^2)|_{ansatz} \sim a_s^3 \Big[-2\bar{j}_{02}\beta_0 \ln^2(1-x) + (2\beta_0\,(j_1\beta_0 - j_{02}) + \bar{j}_{03})\ln(1-x)$$
$$+(-j_1\beta_1 - 2\beta_0\, j_2 + j_{03})\Big] + \ldots$$

Order a_s^3 prediction from the Ansatz

Grunberg, VR

$$K^{(3)}(x,Q^2)|_{ansatz} \sim a_s^3\Big[-2\bar{j}_{02}\beta_0\ln^2(1-x)+(2\beta_0(j_1\beta_0-j_{02})+\bar{j}_{03})\ln(1-x)$$
$$+(-j_1\beta_1-2\beta_0 j_2+j_{03})\Big]+\dots$$

The ansatz thus predicts

1. that the leading $\mathcal{O}(r^0)$ logarithm at $\mathcal{O}(a_s^3)$ in $K(x,Q^2)$ should be a double logarithm, and
2. that its coefficient should be $-2\bar{j}_{02}\beta_0=-2C_2\beta_0=-2A_1^2\beta_0$, which can be compared to the $\mathcal{O}(a_s^3)$ exact result.

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Grunberg, VR

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$\mathcal{O}(a_s^3)$ exact result:

$$\begin{aligned} K(x,Q^2) &= a_s\,P_0(x)+a_s^2[P_1(x)-\beta_0\,c_1(x)] \\ &+ a_s^3[P_2(x)-\beta_1\,c_1(x)-\beta_0\,d_2(x)]+\ldots. \end{aligned}$$

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- we find that ansatz predicts the correct logarithmic structure but with wrong numerical coefficients.
- it predicts $24C_F^2\,\log^2(1-x)$ instead of the correct $32C_F^2\,\log^2(1-x)$.

Order a_s^3 prediction from the Ansatz

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Order a_s^3 prediction from the Ansatz

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	C_F^2		$C_F \beta_0$	
$\frac{\ln^3(1-x)}{1-x}$	8	8	0	0
$\ln^3(1-x)$	−8	−8	0	0
$\frac{\ln^2(1-x)}{1-x}$	−18	−18	−2	−2
$\ln^2(1-x)$	60	64	2	2

Two scale Ansatz and Order a_s^4 prediction

$Grunberg, VR$

Two scale Ansatz and Order a_s^4 prediction

$Grunberg, VR$

$$K(x,Q^2) = K_{resum}(x,Q^2) + \left[\left(\bar{\mathcal{J}}_0\left(W^2\right) - \bar{\mathcal{S}}_0\left(\widetilde{W}^2\right)\right) \ln(1-x) + \mathcal{J}_0\left(W^2\right) - \mathcal{S}_0\left(\widetilde{W}^2\right)\right] + \cdots$$

where $\widetilde{W}^2 = (1-x)^2 Q^2$,

Two scale Ansatz and Order a_s^4 prediction

Grunberg, VR

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where $\widetilde{W}^2 = (1-x)^2 Q^2$,

$$\begin{aligned}\left(\bar{\mathcal{J}}_0\left(W^2\right) - \bar{\mathcal{S}}_0\left(\widetilde{W}^2\right)\right)\ln r &= a_s^2\left(\bar{j}_{02} - \bar{s}_{02}\right)\ln(1-x) \\ &+ a_s^3\left[-2\beta_0\left(\bar{j}_{02} - 2\bar{s}_{02}\right)\ln^2(1-x) + \ldots\right] \\ &+ a_s^4\left[3\beta_0^2\left(\bar{j}_{02} - 4\bar{s}_{02}\right)\ln^3(1-x) + \ldots\right] + \ldots\end{aligned}$$

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Using available results to order a_s^3, we find

$$\begin{aligned}\bar{j}_{02} - \bar{s}_{02} &= 16C_F^2 \\ 2(\bar{j}_{02} - 2\bar{s}_{02}) &= 24C_F^2 \\ 3(\bar{j}_{02} - 4\bar{s}_{02}) &= \frac{88}{3}C_F^2\end{aligned}$$

Third equation is inconsistent with the first two implying the failure of Two- Scale Ansatz.

Two scale Ansatz and Order a_s^4 prediction

Grunberg, VR

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Order a_s^4 prediction from the Two-Scale Ansatz

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- $d_3(x)$ contains less power of logarithms for a given color factor then $c_3(x)$. The resulting necessary cancellations in $d_3(x)$ allow to constrain $c_3(x)$ given $c_1(x), c_2(x)$.
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	$C_F^2 n_f$		$C_F \beta_0^2$	
$\frac{\ln^5(1-x)}{1-x}$	0	0	0	0
$\ln^5(1-x)$	0	0	0	0
$\frac{\ln^4(1-x)}{1-x}$	$\frac{40}{9}$	$\frac{40}{9}$	0	0
$\ln^4(1-x)$	$-\frac{40}{9}$	$-\frac{40}{9}$	0	0
$\frac{\ln^3(1-x)}{1-x}$	$-\frac{280}{9}$	$-\frac{280}{9}$	$\frac{4}{3}$	$\frac{4}{3}$
$\ln^3(1-x)$	$\frac{1832}{27}$	64	$-\frac{4}{3}$	$-\frac{4}{3}$

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$\ln^3(1-x)$	$\frac{1832}{27}$	64	$-\frac{4}{3}$	$-\frac{4}{3}$

	C_F^3		$C_F^2 C_A$	
$\frac{\ln^5(1-x)}{1-x}$	8	8	0	0
$\ln^5(1-x)$	-8	-8	0	0
$\frac{\ln^4(1-x)}{1-x}$	-30	-30	$-\frac{220}{9}$	$-\frac{220}{9}$
$\ln^4(1-x)$	92	92	$\frac{220}{9}$	$\frac{220}{9}$
$\frac{\ln^3(1-x)}{1-x}$	$-96\zeta_2 - 36$	$-96\zeta_2 - 36$	$-32\zeta_2 + \frac{1732}{9}$	$-32\zeta_2 + \frac{1732}{9}$
$\ln^3(1-x)$	$32\zeta_2 - 38$	$32\zeta_2 - 38$	$64\zeta_2 - \frac{10976}{27}$	$64\zeta_2 - \frac{1156}{3}$

Conclusions

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Conclusions

- Using Sudakov resummation of soft and collinear gluons we obtain N^3LO soft gluon corrections to Higgs and Drell-Yan productions.
- Resummed rapidity distributions can be obtained at N^3LO level in the soft and collinear region.
- Structure of single pole terms in the form factors is completely understood.
- Few attempts to resum sub-leading logarithms in the threshold regions.