$b \to s\ell^+\ell^-$ in the high q^2 region at two-loops

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Outline

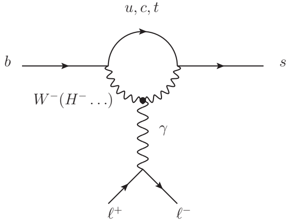
Framework and status of the calculation

NNLO calculation in the high q^2 region

Numerical issues

Some features about $b \to s\ell^+\ell^-$

- Induced by flavour changing neutral current
  $\Rightarrow$ loop-induced in the SM and sensitive to new physics



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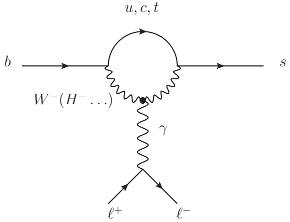

- Three body decay
  $\Rightarrow$ many kinematic observables can be measured like invariant mass spectrum of $\ell^+\ell^-$ and forward-backward asymmetry

Theoretical treatment of the decay mode

- Expansion in $1/m_b$ by means of operator product expansion (OPE)

$$\Gamma(B \to X_s \ell^+ \ell^-) = \Gamma(b \to X_s \ell^+ \ell^-) + \mathcal{O}(\frac{\Lambda_{\rm QCD}^2}{m_b^2})$$

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- Leading power is approximated by partonic decay rate
- Power corrections start at $1/m_b^2$
- We aim an accuracy of 10% in the region where OPE is valid
- However OPE is not valid over the complete range of the invariant mass squared of $\ell^+\ell^-$

Break down of OPE for dilepton invariant mass squared q^2 at

- $c\bar{c}$ resonances (e.g. $B \to X_s J/\psi \to X_s \ell^+ \ell^-$)
  $\Rightarrow$ Precise theoretical predictions are possible by appropriate cuts:

  Low q^2: $1\text{GeV}^2 < q^2 < 6\text{GeV}^2$
  High q^2: $q^2 > 14.4\text{GeV}^2$ (Topic of the present talk)

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- the endpoint m_b^2

  For $\int_{q_0^2}^{m_b^2} dq^2 \Gamma(B \to X_s \ell^+ \ell^-)$ effective expansion in $\Lambda_{\text{QCD}}/(m_b - \sqrt{q_0^2})$ (Bauer, Ligeti, Luke '00, Neubert '00)

  Normalizing by $\int_{q_0^2}^{m_b^2} dq^2 \Gamma(B \to X_u \ell \nu)$ reduces the effect of $1/m_b^3$ corrections (Ligeti, Tackmann '07)

Effective Hamiltonian

- Decay amplitude is given by matrix elements of an effective Hamiltonian:
$$\langle s\ell^+\ell^-|\mathcal{H}_{\text{eff}}|b\rangle = \sum_i C_i \langle s\ell^+\ell^-|\mathcal{O}_i|b\rangle$$
with

$$\mathcal{O}_1 = (\bar{s}_L\gamma_\mu T^a c_L)(\bar{c}_L\gamma^\mu T^a b_L) \qquad \mathcal{O}_2 = (\bar{s}_L\gamma_\mu c_L)(\bar{c}_L\gamma^\mu b_L)$$
$$\mathcal{O}_3 = (\bar{s}_L\gamma_\mu b_L)\sum_q(\bar{q}\gamma^\mu q) \qquad \mathcal{O}_4 = (\bar{s}_L\gamma_\mu T^a b_L)\sum_q(\bar{q}\gamma^\mu T^a q)$$
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$$\mathcal{O}_7 = \frac{e}{g_s^2} m_b(\bar{s}_L\sigma^{\mu\nu} b_R)F_{\mu\nu} \qquad \mathcal{O}_8 = \frac{1}{g_s} m_b(\bar{s}_L\sigma^{\mu\nu} T^a b_R)G^a_{\mu\nu}$$
$$\mathcal{O}_9 = \frac{e^2}{g_s^2}(\bar{s}_L\gamma_\mu b_L)\sum_\ell(\bar{\ell}\gamma^\mu\ell) \qquad \mathcal{O}_{10} = \frac{e^2}{g_s^2}(\bar{s}_L\gamma_\mu b_L)\sum_\ell(\bar{\ell}\gamma^\mu\gamma_5\ell)$$

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- Wilson coefficients C_i contain physics of the order m_t and M_W and resum large logarithms $\ln(m_b/M_W)$:
  LL: $(\alpha_s \ln \frac{m_b}{M_W})^n$, NLL: $\alpha_s(\alpha_s \ln \frac{m_b}{M_W})^n$, NNLL: $\alpha_s^2(\alpha_s \ln \frac{m_b}{M_W})^n$

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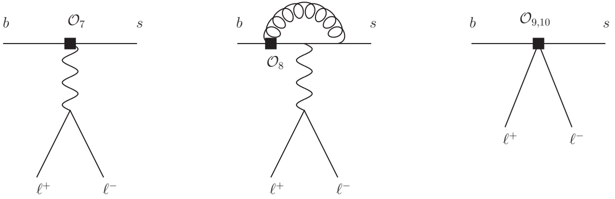
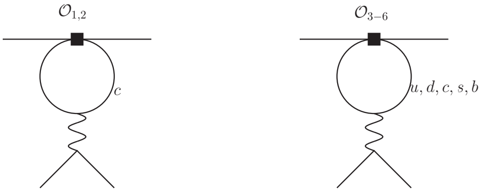
- Note extra factor $1/g_s^2$ in $\mathcal{O}_9$
$\Rightarrow$ Counting for the matrix elements: LO $\sim \alpha_s^{-1}$, NLO $\sim \alpha_s^0$, NNLO $\sim \alpha_s^1$,

Typical diagrams

- Two-quark operators



- Four-quark operators



$\Rightarrow$ lead to $c\bar{c}$ resonances that spoil OPE

Status of the calculation

- Wilson Coefficients up to NNLL

  Adel, Yao ’94; Buchalla, Buras, Lautenbacher ’96; Greub, Hurth ’97; Chetyrkin, Misiak, Münz ’97; Bobeth, Misiak, Urban ’00; Bobeth, Gambino, Gorbahn, Haisch ’04; Gorbahn, Haisch ’05

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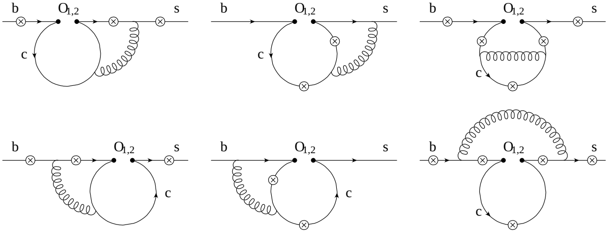
  - NNLO of $\langle \mathcal{O}_1 \rangle$ and $\langle \mathcal{O}_2 \rangle$
    - Low q^2: Expansion in m_c/m_b and q^2/m_b^2 Asatrian, Asatryan, Greub, Walker '01 '02 '02
    - High q^2:
      Numerically Ghinculov, Hurth, Isidori, Yao '04
      Analytically in an expansion in m_c/m_b Greub, V.P., Schüpbach '08

NNLO calculation in the high q^2 region

- Diagrams occurring at NNLO



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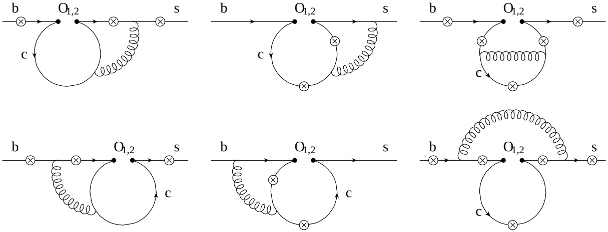

- Two ratios of scales: q^2/m_b^2 and m_c/m_b
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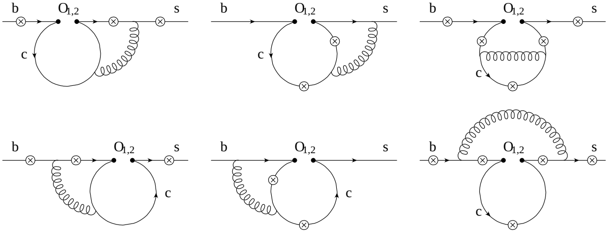

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  High q^2 region $\Rightarrow$ We keep $q^2 = \mathcal{O}(m_b^2)$ and expand in m_c/m_b
- Due to slow convergence we need powers up to $(m_c/m_b)^{20}$ to obtain an error less than 1%

Evaluation of two-loops Feynman integrals

- Reduction of tensor integrals to scalar integrals via Passarino-Veltman

$$\int d^d k_1 d^d k_2 \frac{[k_1^{\mu_1} \dots k_1^{\mu_m}][k_2^{\nu_1} \dots k_2^{\nu_n}]}{\prod D_i(k_1, k_2, p_{\text{extern}})} =$$
$$p_{\text{ext.}}^{\mu_1} \dots p_{\text{ext.}}^{\nu_n} S_1 + g^{\mu_1,\mu_2} p_{\text{ext.}}^{\mu_3} \dots p_{\text{ext.}}^{\nu_n} S_2 + \dots$$

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- Reduction of scalar integrals to a set of simpler master integrals via integration by parts identities

$$0 = \int d^d k \, p^\mu \frac{\partial}{\partial k^\mu} f(k)$$

$\Rightarrow \mathcal{O}(20)$ master integrals containing three scales m_b, m_c and q^2

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- Evaluation of master integrals in expansion in m_c/m_b

Power expansion of Feynman integrals

- Expansion of Feynman integrals in powers of $z = m_c^2/m_b^2$ by solving a set of differential equations in z

$$\frac{d}{dz} I_\alpha = \sum_\beta h_{\alpha\beta} I_\beta + g_\alpha$$

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$h_{\alpha\beta}$: rational functions in z, g_α : simpler master integrals

- Most general ansatz: Expansion of I_α in powers of z and $\ln z$

$$I_\alpha = \sum_{i,j,k \in S} I_{\alpha,i}^{(j,k)} \epsilon^i z^j \ln^k z$$

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- Set of algebraic equations

$$0 = (j+1) I_{\alpha,i}^{(j+1,k)} + (k+1) I_{\alpha,i}^{(j+1,k+1)} - \sum_\beta \sum_{i'} \sum_{j'} h_{\alpha\beta,i'}^{(j')} I_{\beta,i-i'}^{(j-j',k)} - g_{\alpha,i}^{(j,k)}$$

where $h_{\alpha\beta} = \sum_{ij} h_{\alpha,i}^{(j)} \epsilon^i z^j$ and $g_\alpha = \sum_{i,j,k} g_{\alpha,i}^{(j,k)} \epsilon^i z^j \ln^k z$

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- Evaluation of the leading power using method of regions
- Testing the correctness of our ansatz: Formalism that combines sector decomposition (Binoth, Heinrich '00) and Mellin-Barnes techniques provides a formal proof of our ansatz (V.P. '08)
- This formalism also allows for numerical evaluation of the coefficients in the expansion ⇒ additional cross-check.

A short description of this formalism

- Feynman parametrization:

$$I(z) \sim \int_0^1 d^{n-1}x \, \frac{1}{(z f_1(\vec{x}) + f_2(\vec{x}))^{n-d/2}}$$

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$$\frac{1}{(X_1 + X_2)^x} = \frac{1}{\Gamma(x)} \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} ds \, \Gamma(-s)\Gamma(s+x) X_1^s X_2^{-s-x}$$

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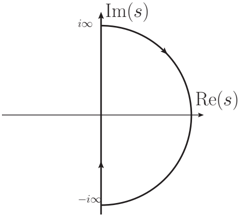
- Close integration contour to the right half



$\Rightarrow$ Summing up residua on the positive real axis leads to power expansion in z

$\Rightarrow$ $\ln(z)$ terms originate from terms like z^ϵ/ϵ

- We have $I(z) \sim \int_{-i\infty}^{i\infty} ds\, z^s \int_0^1 d^{n-1}x\, F(\vec{x}, s)$
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  - Make sure that divergences in s come from integration over small x
  - Integral can be decomposed into terms like

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- Location of the poles can be read off

$$s_{jN} = \frac{1 + N + A_j - B_j\epsilon}{C_j} \quad N \in \mathbb{N}_0$$

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$$\int_0^1 d^{n-1}x \left(\prod_j x_j^{A_j - B_j\epsilon - C_j s} \right) \times (\text{const.} + \mathcal{O}(x))$$

- Location of the poles can be read off

$$s_{jN} = \frac{1 + N + A_j - B_j\epsilon}{C_j} \qquad N \in \mathbb{N}_0$$

- Analytical structure in z of $I(z)$ is known

$\Rightarrow$ Ansatz

$$I(z) = \sum_{i,j,k \in S} I_i^{(j,k)} \epsilon^i z^j \ln^k z$$

where the set of indices S is known

Numerical convergence of the power expansion

- Decomposition of the NNLO matrix elements

$$\langle s\ell^+\ell^-|\mathcal{O}_i|b\rangle_{\text{2-loops}} = -\left(\frac{\alpha_s}{4\pi}\right)^2 \left[F_i^{(7)}\langle\mathcal{O}_7\rangle_{\text{tree}} + F_i^{(9)}\langle\mathcal{O}_9\rangle_{\text{tree}}\right]$$

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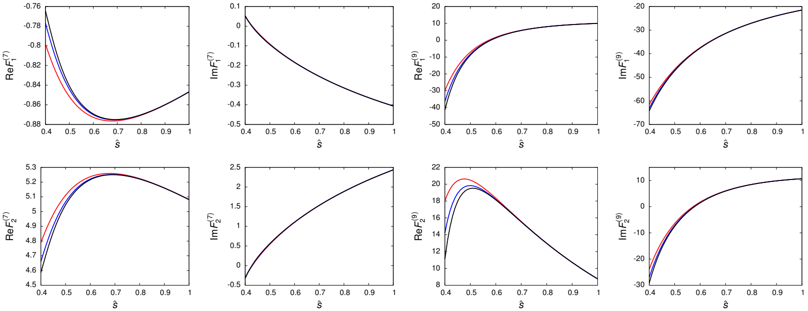

Here $z = 0.1$, $\hat{s} = q^2/m_b^2$, red curve: up to $\mathcal{O}(z^6)$, blue curve: up to $\mathcal{O}(z^8)$, black curve: up to $\mathcal{O}(z^{10})$

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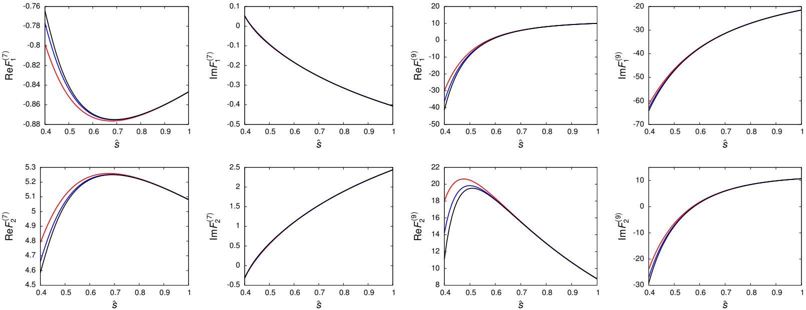

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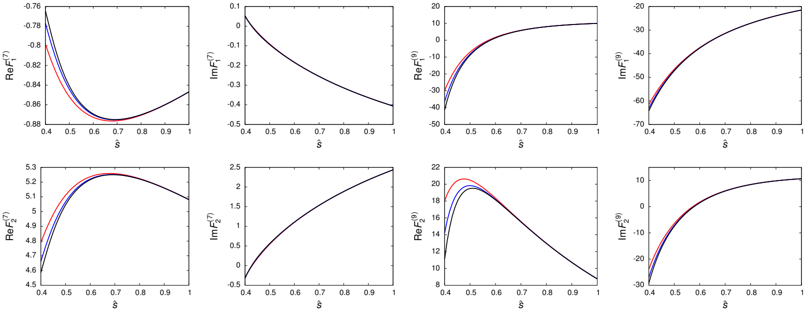

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- For $\hat{s} > 0.6$ good numerical convergence
- By comparison with numerical calculation of Ghinculov et al. we find deviation less than 1%

Numerical impact of $\langle \mathcal{O}_{1,2} \rangle_{\text{2-loops}}$ on the BRs

- Simple ratio with small dependence on $m_{b,pole}$:

$$R(\hat{s}) = \frac{1}{\Gamma(\bar{B} \to X_c e^- \bar{\nu}_e)} \frac{d\Gamma(\bar{B} \to X_s \ell^+ \ell^-)}{d\hat{s}}$$

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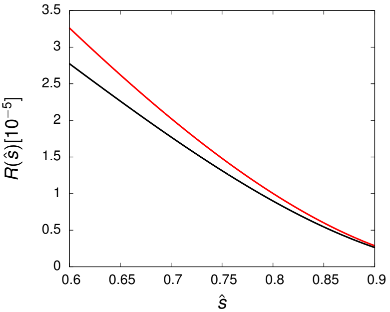
- Significant effect of 2-loops contribution on $R(\hat{s})$ of the order 10%



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not including $\langle \mathcal{O}_{1,2} \rangle_{\text{2-loops}}$
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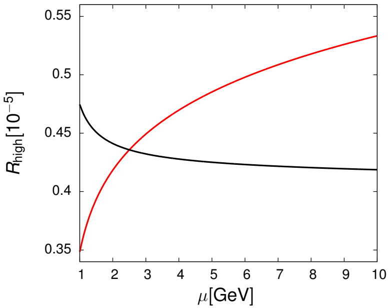
- Reduction of scale-dependence of $R_{\text{high}} = \int_{0.6}^{1} d\hat{s} R(\hat{s})$ to 2% $(2\text{GeV} \leq \mu \leq 10\text{GeV})$



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Final Analysis

- Comprehensive analysis from Huber, Hurth and Lunghi '08

$$\mathrm{Br}(\bar{B} \to X_s \ell^+ \ell^-)_{q^2 > 14.4\mathrm{GeV}^2} = \begin{cases} 2.40 \times 10^{-7}(1^{+0.29}_{-0.26}) & \ell = \mu \\ 2.09 \times 10^{-7}(1^{+0.32}_{-0.30}) & \ell = e \end{cases}$$

(Note: $\ell = \mu, e$ differ by $\ln(m_\ell / m_b)$)

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- Appropriate ratio

$$\int_{q_0^2}^{1} \frac{d\Gamma(\bar{B} \to X_s \ell^+ \ell^-)}{dq^2} \Big/ \int_{q_0^2}^{1} \frac{d\Gamma(\bar{B}^0 \to X_u \ell \nu)}{dq^2} =$$

$$\begin{cases} 2.29 \times 10^{-3}(1 \pm 0.13) & \ell = \mu \\ 1.94 \times 10^{-3}(1 \pm 0.16) & \ell = e \end{cases}$$

with $q_0^2 = 14.4\mathrm{GeV}^2$

Summary

- We did the NNLL calculation of the matrix elements of $\mathcal{O}_{1,2}$ in the high q^2 region
- Combining method of regions with differential equation techniques we obtained an expansion in m_c/m_b of the Feynman integrals
- This analytical result confirmed a former numerical calculation and is now completely published