NNLO CORRECTIONS TO SEMILEPTONIC AND HADRONIC B DECAYS

[GUIDO BELL]

based on: G. Bell, Nucl. Phys. B **795** (2008) 1
G. Bell, Nucl. Phys. B **812** (2009) 264
G. Bell, Nucl. Phys. B **822** (2009) 172
G. Bell, V. Pilipp, Phys. Rev. D **80** (2009) 054024

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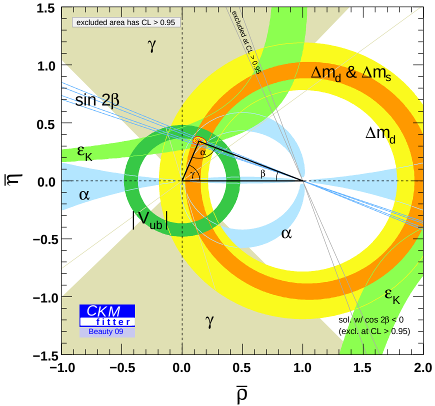
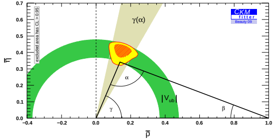
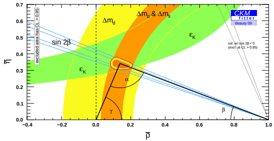
The CKM picture today



trees:



loops:



- impressive confirmation of CKM mechanism of flavour mixing and CP violation
- global CKM fit still driven by loop-induced observables that are sensitive to NP

Puzzles in Flavour Physics

But there exist observables that show interesting tensions, e.g.

$S_{\phi K_S}$, $A_{CP}(\pi K)$, $S_{\psi\phi}$, $D_s \to \ell\nu$, $(g-2)_\mu$, ... $\to$ departures from the SM?

In order to resolve these (and new) puzzles we need ...

- further progress on the experimental side (Belle, CDF, D0, LHCb, Super B)
- more precise determinations of non-perturbative parameters (mainly lattice)
- to work out subleading perturbative corrections (NNLO)

NNLO programme complete for $\mathcal{H}_{\rm eff} = \sum C_i(\mu_F)\, Q_i(\mu_F)$

$C_i(M_W)$ 2-loop / 3-loop matching corrections [Bobeth, Misiak, Urban 99; Misiak, Steinhauser 04]

$C_i(m_b)$ 3-loop / 4-loop anomalous dimensions [Gorbahn, Haisch 04; Gorbahn, Haisch, Misiak 05; Czakon, Haisch, Misiak 06]

$\to$ need hadronic matrix elements $\langle Q_i \rangle$ to same level of precision!

current studies: $B \to X_s\gamma$, $B \to X_s\ell^+\ell^-$, $B \to X_u\ell\nu$, $B \to MM$, ...

Charmless hadronic B decays

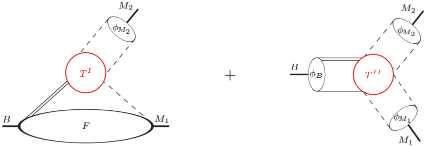
$B \to \pi\pi,\ \rho\pi,\ \rho\rho,\ K\pi,\ KK,\ K^*\pi,\ \ldots$

theoretically challenging, but phenomenolog. important with $\mathcal{O}(100)$ observables

Hadronic matrix elements factorize in heavy quark limit $m_b \to \infty$

[Beneke, Buchalla, Neubert, Sachrajda 99,01]

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \simeq F^{BM_1}(0) \int du\, T_i^I(u)\, \phi_{M_2}(u) + \int d\omega\, du\, dv\, T_i^{II}(\omega,u,v)\, \phi_B(\omega)\, \phi_{M_1}(v)\, \phi_{M_2}(u)$$



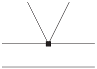
vertex corrections $T^I = 1 + \mathcal{O}(\alpha_s)$ spectator-scattering $T^{II} = \mathcal{O}(\alpha_s)$

- no strong phases at LO, i.e. NLO is leading contribution for direct CP asymmetries

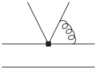
$\rightarrow$ NNLO calculation particularly important for direct CP asymmetries!

Structure of perturbative expansion

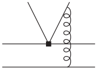
LO: "naive" factorization $\langle M_1 M_2 | Q_i | \bar{B} \rangle = F^{BM_1}(0)\; f_{M_2}$



NLO: 1-loop vertex corrections

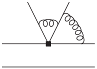


tree-level spectator scattering

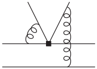


[Beneke, Buchalla, Neubert, Sachrajda 01]

NNLO: 2-loop vertex corrections

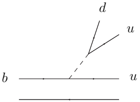


1-loop spectator scattering

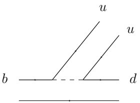


Status of NNLO calculation

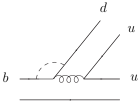
Topological amplitudes



colour-allowed tree α_1



colour-suppressed tree α_2



QCD penguins α_4

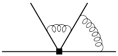
	2-loop vertex corrections	1-loop spectator scattering
Trees	[GB 07, 09] [Beneke, Huber, Li in preparation]	[Beneke, Jäger 05] [Kivel 06] [Pilipp 07]
Penguins	in progress	[Beneke, Jäger 06] [Jain, Rothstein, Stewart 07]

- first NNLO results for tree-dominated decays [GB, Pilipp 09]
- direct CP asymmetries not yet available at NNLO

2-loop vertex corrections

Characterization

- 2-loop calculation with 4 on-shell legs (1 massive)
- $\sim$ 75 diagrams, 2 scales (except for charm loops)



Automatized reduction to Master Integrals

- integration-by-parts techniques + Laporta-algorithm [Chetyrkin, Tkachov 81; Laporta 00]

$\rightarrow$ $\mathcal{O}(6.000)$ scalar integrals reduced to 36 Master Integrals

Calculation of Master Integrals (analytical, $1/\varepsilon_{IR}^4$)

- method of differential equations [Kotikov 91; Remiddi 97]
- Harmonic Polylogarithms [Remiddi, Vermaseren 00]
- Mellin-Barnes techniques (for boundary conditions) [Smirnov 99; Tausk 99]
- method of sector decomposition (for numerical cross-check) [Binoth, Heinrich 04]

$\rightarrow$ several independent calculations in agreement

[GB 07; Bonciani, Ferrogila 08; Asatrian, Greub, Pecjak 08; Beneke, Huber, Li 08]

IR-subtractions

UV-renormalized matrix elements are IR-divergent

$$\underbrace{\langle Q_i \rangle^{(2)}_{ren}}_{\sim \frac{1}{\varepsilon_{IR}^2}} = F^{(0)} \, \underbrace{T_i^{(2)}}_{\text{finite!}} * \phi^{(0)} + \underbrace{F^{(1)}}_{\sim \frac{1}{\varepsilon_{IR}^2}} \, T_i^{(1)} * \phi^{(0)} + F^{(0)} \, T_i^{(1)} * \underbrace{\phi^{(1)}}_{\sim \frac{1}{\varepsilon_{IR}}} + \ldots$$

→ have to show that IR-divergences can be absorbed into non-perturbative objects (form factor F and light-cone distribution amplitude ϕ)

→ involves subtle finite subtraction terms from evanescent four-quark operators (particularly involved for α_2 due to evanescent operator at tree level)

New calculation by Beneke/Huber/Li [talk by T. Huber at QCD conference, Berlin, October 09]

- confirms result from [GB 07, 09]
- obtains the charm mass dependence analytically

NNLO result for tree amplitudes

Input parameters for default scenario

μ_h	μ_{hc}	$F_+^{B\pi}(0)$	$\lambda_B(1\text{ GeV})$	$a_2^\pi(1\text{ GeV})$	$10^3\,\vert V_{ub}\vert$	γ
$4.8^{+4.8}_{-2.4}$	$1.5^{+1.5}_{-0.7}$	0.26 ± 0.04	0.40 ± 0.15	0.25 ± 0.15	3.95 ± 0.35	$(70 \pm 20)^\circ$

Tree amplitudes

$$\begin{aligned}
\alpha_1(\pi\pi) &= [\,1.008\,]_{V_0} + [\,0.022 + 0.009\,i\,]_{V_1} + [\,0.024 + 0.026\,i\,]_{V_2} \\
&\quad - [\,0.014\,]_{S_1} - [\,0.016 + 0.012\,i\,]_{S_2} - [\,0.008\,]_{1/m_b} \\
&= 1.015^{+0.020}_{-0.029} + (0.023^{+0.015}_{-0.015})\,i \\
\alpha_2(\pi\pi) &= [\,0.224\,]_{V_0} - [\,0.174 + 0.075\,i\,]_{V_1} - [\,0.029 + 0.046\,i\,]_{V_2} \\
&\quad + [\,0.084\,]_{S_1} + [\,0.037 + 0.022\,i\,]_{S_2} + [\,0.052\,]_{1/m_b} \\
&= 0.194^{+0.130}_{-0.095} - (0.099^{+0.057}_{-0.056})\,i
\end{aligned}$$

V_0 : naive factorization
V_1 : 1-loop vertex corrections
V_2 : 2-loop vertex corrections

S_1 : tree level spectator scattering
S_2 : 1-loop spectator scattering
$1/m_b$: power corrections (estimate)

$B \to \pi\pi/\pi\rho/\rho_L\rho_L$ branching ratios

[GB, Pilipp 09]

Mode	Theory	CKM	had	μ	pow	Experiment
$B^- \to \pi^-\pi^0$	$6.22^{+2.37}_{-2.01}$	$^{+1.14}_{-1.05}$	$^{+2.03}_{-1.65}$	$^{+0.16}_{-0.18}$	$^{+0.43}_{-0.42}$	$5.59^{+0.41}_{-0.40}$
$B^- \to \rho_L^-\rho_L^0$	$21.0^{+8.5}_{-7.3}$	$^{+3.9}_{-3.5}$	$^{+7.4}_{-6.1}$	$^{+0.5}_{-0.7}$	$^{+1.5}_{-1.4}$	$22.5^{+1.9}_{-1.9}$
$B^- \to \pi^-\rho^0$	$9.34^{+4.00}_{-3.23}$	$^{+2.00}_{-1.81}$	$^{+3.22}_{-2.51}$	$^{+0.31}_{-0.34}$	$^{+1.24}_{-0.84}$	$8.3^{+1.2}_{-1.3}$
$B^- \to \pi^0\rho^-$	$15.1^{+5.7}_{-5.0}$	$^{+2.9}_{-2.8}$	$^{+4.8}_{-4.1}$	$^{+0.3}_{-0.4}$	$^{+1.0}_{-0.7}$	$10.9^{+1.4}_{-1.5}$
$\bar{B}^0 \to \pi^+\pi^-$	$8.96^{+3.78}_{-3.32}$	$^{+1.87}_{-1.91}$	$^{+3.02}_{-2.62}$	$^{+0.16}_{-0.20}$	$^{+1.28}_{-0.71}$	$5.16^{+0.22}_{-0.22}$
$\bar{B}^0 \to \pi^0\pi^0$	$0.35^{+0.37}_{-0.21}$	$^{+0.16}_{-0.14}$	$^{+0.20}_{-0.09}$	$^{+0.03}_{-0.03}$	$^{+0.26}_{-0.11}$	$1.55^{+0.19}_{-0.19}$
$\bar{B}^0 \to \pi^+\rho^-$	$22.8^{+9.1}_{-8.0}$	$^{+4.2}_{-4.0}$	$^{+7.8}_{-6.8}$	$^{+0.4}_{-0.5}$	$^{+1.9}_{-1.4}$	$15.7^{+1.8}_{-1.8}$
$\bar{B}^0 \to \pi^-\rho^+$	$11.5^{+5.1}_{-4.3}$	$^{+2.3}_{-2.1}$	$^{+4.2}_{-3.6}$	$^{+0.2}_{-0.2}$	$^{+1.8}_{-1.0}$	$7.3^{+1.2}_{-1.2}$
$\bar{B}^0 \to \pi^\pm\rho^\mp$	$34.3^{+11.5}_{-10.0}$	$^{+6.3}_{-5.7}$	$^{+8.9}_{-7.8}$	$^{+0.6}_{-0.7}$	$^{+3.7}_{-2.4}$	$23.0^{+2.3}_{-2.3}$
$\bar{B}^0 \to \pi^0\rho^0$	$0.52^{+0.76}_{-0.42}$	$^{+0.10}_{-0.09}$	$^{+0.62}_{-0.21}$	$^{+0.10}_{-0.10}$	$^{+0.41}_{-0.34}$	$2.0^{+0.5}_{-0.5}$
$\bar{B}^0 \to \rho_L^+\rho_L^-$	$30.3^{+12.9}_{-11.2}$	$^{+5.6}_{-5.3}$	$^{+11.2}_{-9.6}$	$^{+0.6}_{-0.7}$	$^{+2.9}_{-2.3}$	$23.6^{+3.2}_{-3.2}$
$\bar{B}^0 \to \rho_L^0\rho_L^0$	$0.44^{+0.66}_{-0.37}$	$^{+0.10}_{-0.09}$	$^{+0.50}_{-0.18}$	$^{+0.10}_{-0.09}$	$^{+0.40}_{-0.30}$	$0.69^{+0.30}_{-0.30}$

- substantial theoretical uncertainties $\sim 40\%$ mainly from form factors and $|V_{ub}|$
  $\rightarrow$ consider ratios of decay rates rather than absolute branching ratios!
- colour-suppressed modes $\pi^0\pi^0/\pi^0\rho^0/\rho_L^0\rho_L^0$ rather uncertain (λ_B and power)

$B \to \pi\pi/\pi\rho/\rho_L\rho_L$ branching ratios

[GB, Pilipp 09]

Mode	Theory	A	B	C	D	Experiment
$B^- \to \pi^-\pi^0$	$6.22^{+2.37}_{-2.01}$	5.97	5.46	6.22	5.64	$5.59^{+0.41}_{-0.40}$
$B^- \to \rho_L^-\rho_L^0$	$21.0^{+8.5}_{-7.3}$	20.2	21.3	21.0	23.1	$22.5^{+1.9}_{-1.9}$
$B^- \to \pi^-\rho^0$	$9.34^{+4.00}_{-3.23}$	11.2	10.4	10.3	11.8	$8.3^{+1.2}_{-1.3}$
$B^- \to \pi^0\rho^-$	$15.1^{+5.7}_{-5.0}$	11.9	11.9	15.8	11.8	$10.9^{+1.4}_{-1.5}$
$\bar{B}^0 \to \pi^+\pi^-$	$8.96^{+3.78}_{-3.32}$	6.20	5.21	10.2	5.53	$5.16^{+0.22}_{-0.22}$
$\bar{B}^0 \to \pi^0\pi^0$	$0.35^{+0.37}_{-0.21}$	0.66	0.63	0.59	0.68	$1.55^{+0.19}_{-0.19}$
$\bar{B}^0 \to \pi^+\rho^-$	$22.8^{+9.1}_{-8.0}$	20.0	13.2	24.6	15.7	$15.7^{+1.8}_{-1.8}$
$\bar{B}^0 \to \pi^-\rho^+$	$11.5^{+5.1}_{-4.3}$	13.0	8.41	13.3	11.7	$7.3^{+1.2}_{-1.2}$
$\bar{B}^0 \to \pi^\pm\rho^\mp$	$34.3^{+11.5}_{-10.0}$	33.1	21.6	37.9	27.3	$23.0^{+2.3}_{-2.3}$
$\bar{B}^0 \to \pi^0\rho^0$	$0.52^{+0.76}_{-0.42}$	0.44	1.64	0.34	1.02	$2.0^{+0.5}_{-0.5}$
$\bar{B}^0 \to \rho_L^+\rho_L^-$	$30.3^{+12.9}_{-11.2}$	26.8	22.3	33.2	27.2	$23.6^{+3.2}_{-3.2}$
$\bar{B}^0 \to \rho_L^0\rho_L^0$	$0.44^{+0.66}_{-0.37}$	0.58	1.33	0.24	1.03	$0.69^{+0.30}_{-0.30}$

- theoretical uncertainties are highly correlated

A: large weak phase ($\gamma = 110^\circ$)
B: large colour-suppressed amplitude ($\lambda_B = 0.2$, $F_+^{B\pi}(0) = 0.21$, $A_0^{B\rho}(0) = 0.27$)
C: large weak annihilation ($\rho_A = 1$, $\phi_A = 0$)
D: combined ($\lambda_B = 0.25$, $F_+^{B\pi}(0) = 0.23$, $\gamma = 90^\circ$)

Precision observables

[GB, Pilipp 09]

Mode	Theory	A	B	C	D	Experiment
$\mathcal{R}_\pi(\pi^-\pi^0)$	$0.70^{+0.12}_{-0.08}$	0.68	0.95	0.70	0.82	$0.81^{+0.14}_{-0.14}$
$\mathcal{R}_\rho(\rho_L^-\rho_L^0)$	$1.91^{+0.32}_{-0.23}$	1.83	2.38	1.91	2.09	n.a.
$\mathcal{R}_\rho(\pi^-\rho^0)$	$0.85^{+0.22}_{-0.14}$	1.01	1.16	0.93	1.07	n.a.
$\mathcal{R}_\pi(\pi^0\rho^-)$	$1.71^{+0.27}_{-0.24}$	1.35	2.07	1.79	1.71	$1.57^{+0.32}_{-0.32}$
$\mathcal{R}_\pi(\pi^+\pi^-)$	$1.09^{+0.22}_{-0.20}$	0.75	0.97	1.24	0.86	$0.80^{+0.13}_{-0.13}$
$\mathcal{R}_\pi(\pi^+\rho^-)$	$2.77^{+0.32}_{-0.31}$	2.44	2.46	2.99	2.44	$2.43^{+0.47}_{-0.47}$
$\mathcal{R}_\rho(\pi^-\rho^+)$	$1.12^{+0.20}_{-0.14}$	1.27	1.01	1.29	1.13	n.a.
$\mathcal{R}_\rho(\rho_L^+\rho_L^-)$	$2.95^{+0.37}_{-0.35}$	2.61	2.68	3.22	2.64	n.a.
$R(\rho_L^-\rho_L^0/\rho_L^+\rho_L^-)$	$0.65^{+0.16}_{-0.11}$	0.70	0.89	0.59	0.79	$0.89^{+0.14}_{-0.14}$
$R(\pi^-\pi^0/\pi^+\pi^-)$	$0.65^{+0.19}_{-0.14}$	0.90	0.98	0.57	0.95	$1.01^{+0.09}_{-0.09}$
$R(\pi^+\pi^-/\pi^0\pi^0)$	$25.7^{+26.0}_{-18.7}$	9.33	8.32	17.3	8.13	$3.33^{+0.43}_{-0.43}$

- consider ratios of decay rates

$$\mathcal{R}_{M_3}(M_1M_2) \equiv \frac{\Gamma(\bar{B}\to M_1M_2)}{d\Gamma(\bar{B}^0\to M_3^+\ell^-\bar{\nu}_l)/dq^2|_{q^2=0}} \qquad R(M_1M_2/M_3M_4) \equiv \frac{\Gamma(\bar{B}\to M_1M_2)}{\Gamma(\bar{B}'\to M_3M_4)}$$

$\rightarrow$ theoretical uncertainties reduced to $\sim 15\%$, correlations largely resolved

Precision observables

[GB, Pilipp 09]

Mode	Theory	A	B	C	D	Experiment
$\mathcal{R}_\pi(\pi^-\pi^0)$	$0.70^{+0.12}_{-0.08}$	0.68	0.95	0.70	0.82	$0.81^{+0.14}_{-0.14}$
$\mathcal{R}_\rho(\rho_L^-\rho_L^0)$	$1.91^{+0.32}_{-0.23}$	1.83	2.38	1.91	2.09	n.a.
$\mathcal{R}_\rho(\pi^-\rho^0)$	$0.85^{+0.22}_{-0.14}$	1.01	1.16	0.93	1.07	n.a.
$\mathcal{R}_\pi(\pi^0\rho^-)$	$1.71^{+0.27}_{-0.24}$	1.35	2.07	1.79	1.71	$1.57^{+0.32}_{-0.32}$
$\mathcal{R}_\pi(\pi^+\pi^-)$	$1.09^{+0.22}_{-0.20}$	0.75	0.97	1.24	0.86	$0.80^{+0.13}_{-0.13}$
$\mathcal{R}_\pi(\pi^+\rho^-)$	$2.77^{+0.32}_{-0.31}$	2.44	2.46	2.99	2.44	$2.43^{+0.47}_{-0.47}$
$\mathcal{R}_\rho(\pi^-\rho^+)$	$1.12^{+0.20}_{-0.14}$	1.27	1.01	1.29	1.13	n.a.
$\mathcal{R}_\rho(\rho_L^+\rho_L^-)$	$2.95^{+0.37}_{-0.35}$	2.61	2.68	3.22	2.64	n.a.
$R(\rho_L^-\rho_L^0/\rho_L^+\rho_L^-)$	$0.65^{+0.16}_{-0.11}$	0.70	0.89	0.59	0.79	$0.89^{+0.14}_{-0.14}$
$R(\pi^-\pi^0/\pi^+\pi^-)$	$0.65^{+0.19}_{-0.14}$	0.90	0.98	0.57	0.95	$1.01^{+0.09}_{-0.09}$
$R(\pi^+\pi^-/\pi^0\pi^0)$	$25.7^{+26.0}_{-18.7}$	9.33	8.32	17.3	8.13	$3.33^{+0.43}_{-0.43}$

- satisfactory overall description for precision observables (preference for scenarios B/D $\longrightarrow$ hint for smaller $\lambda_B \simeq 0.25$?)
- $B \to \pi\pi$ puzzle: $R(\pi^+\pi^-/\pi^0\pi^0)$ completely off, but no precision observable

Semileptonic B decays

Determination of $|V_{ub}|$ from semileptonic decays:

$$|V_{ub}| \;=\; \begin{cases} (4.06^{+0.29}_{-0.31})\,10^{-3} & B \to X_u \ell\nu \qquad \text{(BLNP, NLO)} \\ (3.38 \pm 0.36)\,10^{-3} & B \to \pi \ell\nu \qquad \text{(lattice)} \end{cases}$$

$\rightarrow$ does the NNLO calculation bring the two values into better agreement?

Inclusive decays: $B \to X_u \ell\nu$

- experiments impose cuts to suppress background from $B \to X_c \ell\nu$
- measurements restricted to shape-function region: $E_X \sim m_b,\; m_X^2 \sim m_b \Lambda_{QCD}$
- multi-scale OPE in terms of non-local light-cone operators [Korchemsky, Sterman 94]

$$\Gamma_u \simeq \sum_{i,j} H_{ij}(n_+p) \int d\omega\; J(p_\omega^2)\; S(\omega)$$

perturbative jet function J known at 2-loop [Becher, Neubert 06]

$\rightarrow$ need 2-loop hard matching corrections H_{ij} for full NNLO analysis

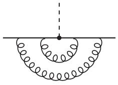
2-loop hard matching corrections

Matching heavy-to-light currents from QCD onto SCET

$$\bar{q}\,\Gamma b \;\simeq\; \sum_i \int ds\; \tilde{C}_i(s)\; [\xi W_{hc}](s n_+)\; \Gamma'_i\, h_v$$

2-loop calculation similar to the one from $B \to \pi\pi$, but ...

- 3 external lines (quark form factor)
- less diagrams, same Master Integrals
- no trouble with evanescent operators



Status of 2-loop calculation

- $\Gamma = \gamma^\mu$ $\quad (B \to X_u \ell \nu)$ [Bonciani, Ferroglia 08; Asatrian, Greub, Pecjak 08; Beneke, Huber, Li 08; GB 08]
- $\Gamma = 1, \sigma^{\mu\nu}$ $\quad (B \to X_s \ell^+ \ell^-)$ [GB, Beneke, Huber, Li in preparation]

(NDR-scheme relates currents with and without γ_5)

Wilson coefficients in NNLO

$\Gamma = \gamma^\mu \quad \rightarrow \quad C_{V,1}\,, C_{V,2}\,, C_{V,3}$

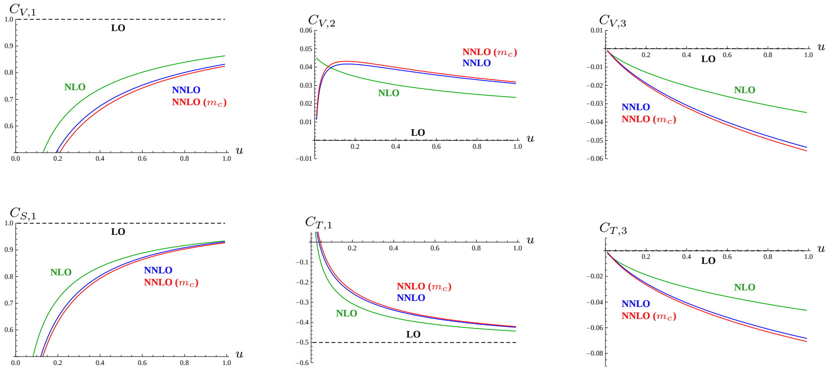
$\Gamma = 1 \quad \rightarrow \quad C_{S,1}$

$\Gamma = i\sigma^{\mu\nu} \quad \rightarrow \quad C_{T,1}\,, C_{T,3} \quad (C_{T,2} = C_{T,4} = 0)$

momentum transfer $q^2 = (1-u)m_b^2$

$u = \frac{n_+ p}{m_b} = \frac{2E}{m_b}$

$\mu = m_b$



Impact on the determination of $|V_{ub}|$

[Greub, Neubert, Pecjak 09]

Experiments apply different cuts:

- lepton energy E_l
- hadronic invariant mass M_X
- hadronic variable $P_+ = E_X - |\vec{P}_X|$

Exp.	Method	$\Delta\mathcal{B}^{\text{exp}}$ $[10^{-4}]$	$\lvert V_{ub}\rvert$ $[10^{-3}]$ NLO	$\lvert V_{ub}\rvert$ $[10^{-3}]$ NNLO
CLEO	$E_l > 2.1\,\text{GeV}$	$3.3 \pm 0.2 \pm 0.7$	$3.56 \pm 0.40^{+0.48}_{-0.27}{}^{+0.31}_{-0.26}$	$3.81 \pm 0.43^{+0.33}_{-0.21}{}^{+0.31}_{-0.26}$
BABAR	$E_l > 2.0\,\text{GeV}$	$5.7 \pm 0.4 \pm 0.5$	$3.97 \pm 0.22^{+0.37}_{-0.23}{}^{+0.26}_{-0.25}$	$4.30 \pm 0.24^{+0.26}_{-0.20}{}^{+0.28}_{-0.27}$
BELLE	$E_l > 1.9\,\text{GeV}$	$8.5 \pm 0.4 \pm 1.5$	$4.27 \pm 0.39^{+0.32}_{-0.19}{}^{+0.25}_{-0.22}$	$4.65 \pm 0.43^{+0.27}_{-0.18}{}^{+0.27}_{-0.24}$
BELLE	$M_X < 1.7\,\text{GeV}$	$12.3 \pm 1.1 \pm 1.2$	$3.55 \pm 0.24^{+0.22}_{-0.13}{}^{+0.21}_{-0.19}$	$3.87 \pm 0.26^{+0.21}_{-0.13}{}^{+0.21}_{-0.19}$
BABAR	$M_X < 1.55\,\text{GeV}$	$11.7 \pm 0.9 \pm 0.7$	$3.67 \pm 0.18^{+0.29}_{-0.17}{}^{+0.26}_{-0.24}$	$3.96 \pm 0.19^{+0.20}_{-0.13}{}^{+0.26}_{-0.24}$
BELLE	$P_+ < 0.66\,\text{GeV}$	$11.0 \pm 1.0 \pm 1.6$	$3.56 \pm 0.31^{+0.30}_{-0.17}{}^{+0.27}_{-0.23}$	$3.84 \pm 0.33^{+0.21}_{-0.13}{}^{+0.26}_{-0.22}$
BABAR	$P_+ < 0.66\,\text{GeV}$	$9.4 \pm 1.0 \pm 0.8$	$3.30 \pm 0.23^{+0.27}_{-0.16}{}^{+0.25}_{-0.22}$	$3.55 \pm 0.24^{+0.19}_{-0.13}{}^{+0.24}_{-0.21}$

$\rightarrow$ NNLO corrections shift $|V_{ub}|_{\text{incl}}$ upwards by $\sim$ 10% compared to NLO!

recall $|V_{ub}|_{\text{excl}} = (3.38 \pm 0.36)\,10^{-3}$

Summary

Charmless hadronic $B \to MM$ decays:

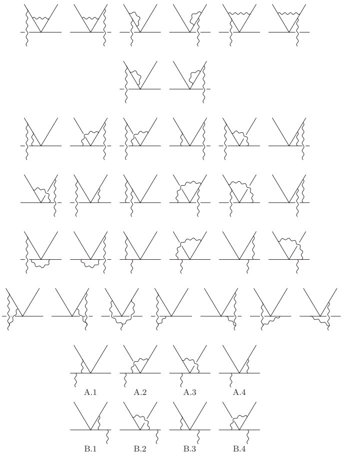
- NNLO corrections particularly important for direct CP asymmetries
- tree amplitudes available at NNLO, penguin amplitudes still incomplete
- satisfactory description of (properly normalized) $B \to \pi\pi/\pi\rho/\rho_L\rho_L$ branching ratios (colour-suppressed modes are difficult though)

Inclusive semileptonic $B \to X_u \ell\nu$ decays:

- inclusive determination of $|V_{ub}|$ has recently reached the NNLO precision
- $|V_{ub}|_{\text{incl}}$ shifted upwards, which strengthens the tension with $|V_{ub}|_{\text{excl}}$

Backup slides

1-loop spectator scattering



Characterization

scales $\mu_h \sim m_b$, $\mu_{hc} \sim \sqrt{m_b \Lambda_{QCD}}$

hard-scattering kernels factorize

$$T_i^{II} = H_i^{II}(\mu_h) \otimes J_{||}(\mu_{hc})$$

jet function $J_{||}$ known to NLO

[Becher, Hill 04, Kirilin 05, Beneke, Yang 05]

Tree amplitudes

H_i^{II} from QCD $\rightarrow$ SCET$_I$ matching

[Beneke, Jäger 05; Kivel 06]

T_i^{II} confirmed by QCD calculation

[Pilipp 07]

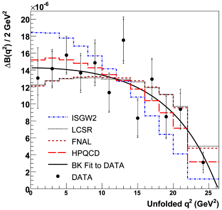
Penguin amplitudes

[Beneke, Jäger 06; Jain, Rothstein, Stewart 07]

Normalize to semileptonic decays

Differential semileptonic decay rate at maximum recoil

$$\frac{d\Gamma(\bar{B}^0 \to \pi^+ \ell^- \bar{\nu}_l)}{dq^2}\bigg|_{q^2=0} = \frac{G_F^2 (m_B^2 - m_\pi^2)^3}{192\pi^3 m_B^3} |V_{ub}|^2 |F_+^{B\pi}(0)|^2$$



$B \to \pi \ell \nu$

BaBar measurement in 12 q^2-bins (untagged)

different parameterizations to extrapolate to $q^2 = 0$

$\to \quad |V_{ub}| F_+^{B\pi}(0) = (9.1 \pm 0.6 \pm 0.3) \cdot 10^{-4}$ [Ball 07]

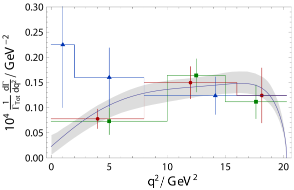
$B \to \rho \ell \nu$

BaBar / Belle / CLEO in 3-4 q^2-bins

in addition theoretical input for form factors

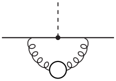
[Flynn, Nakagawa, Nieves, Toki 08]

$\to \quad |V_{ub}| A_0^{B\rho}(0) = (5.5 \pm 2.6) \cdot 10^{-4}$



Charm mass effects

charm quark enters at 2-loop through fermion bubble



$\rightarrow$ not tremendously important, but naively $\frac{m_c^2}{m_b^2} \ln \frac{m_b^2}{m_c^2} \sim 0.20 \gtrsim \frac{\alpha_s(m_b)}{\pi}$

Choose power-counting

- $m_c \sim \mu_{hc} \sim (\Lambda_{QCD} m_b)^{1/2}$

  IR-scale in hard matching ($m_c = 0$)

  m_c-dependence in jet-function

- $m_c \rightarrow \infty$, $m_b \rightarrow \infty$, m_c/m_b fixed

  m_c-dependence in hard matching

  jet function with 3 massless quarks

Adopt second scenario

$\rightarrow$ 4 new Master Integrals, modified UV- and IR-subtractions, numerical results