Measuring the running top-quark mass

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in collaboration with **U. Langenfeld** and **P. Uwer** on arXiv:0906.5273

– 9th International Symposium on Radiative Corrections *RADCOR 2009*, Ascona, Oct 27, 2009 –

Proton colliders

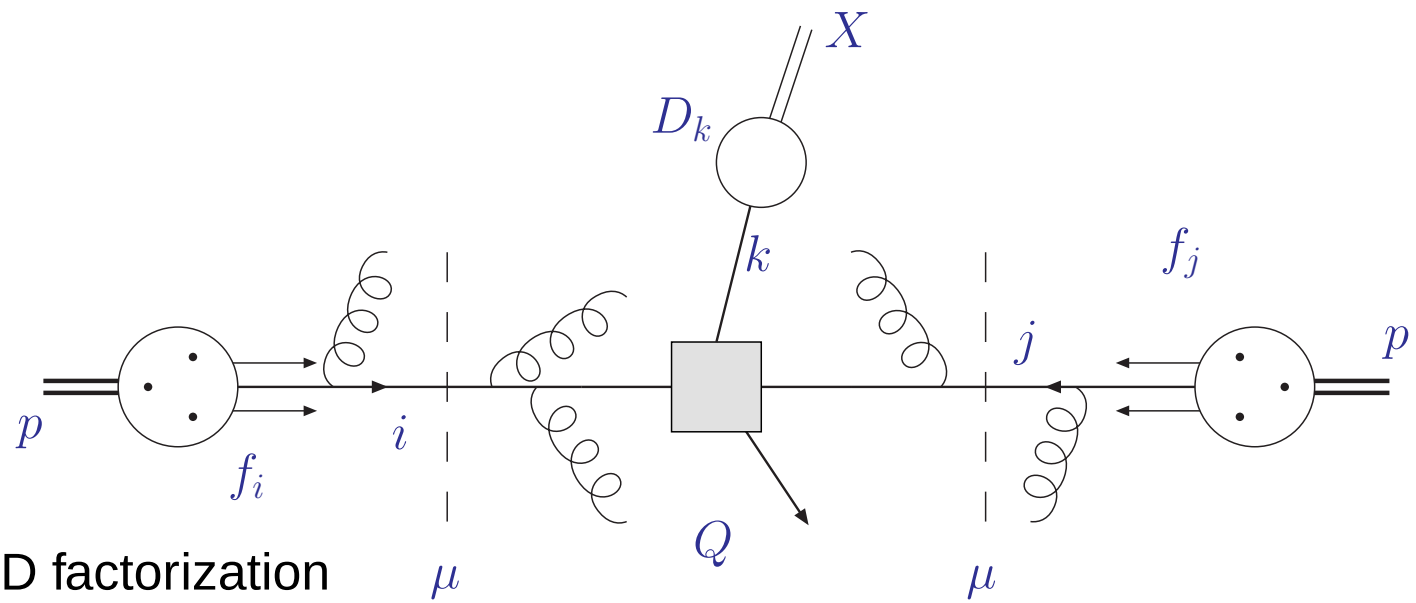
- Tevatron: energy frontier at $\sqrt{S} = 1.96$TeV
  top quark discovery
- LHC: in comissioning phase
  Higgs boson search at highest energies: $\sqrt{S} = 7 \ldots 14$TeV

Abundant production of top-quarks

- Orders for top-quarks from www.particlezoo.com

Perturbative QCD at colliders

- Hard hadron-hadron scattering
  - constituent partons from each incoming hadron interact at short



- QCD factorization
  - separate sensitivity to dynamics from different scales

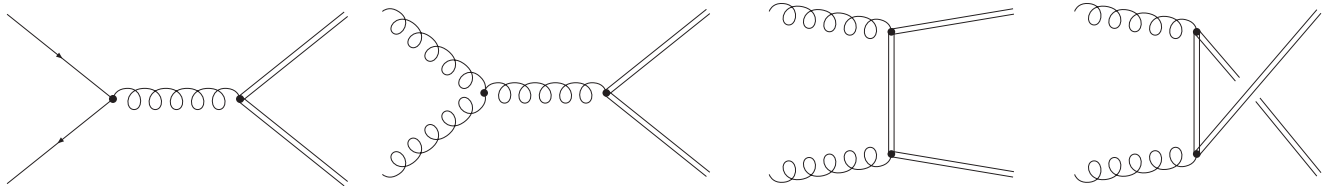
$$\sigma_{pp\to X} = \sum_{ijk} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to k}\left(\alpha_s(\mu^2), Q^2, \mu^2\right) \otimes D_{k\to X}(\mu^2)$$

  - factorization scale μ, subprocess cross section $\hat{\sigma}_{ij\to k}$ for parton types i, j and hadronic final state X

Top quark production

- Leading order Feynman diagrams

$$\mathrm{q} + \bar{\mathrm{q}} \longrightarrow \mathrm{Q} + \bar{\mathrm{Q}}$$
$$\mathrm{g} + \mathrm{g} \longrightarrow \mathrm{Q} + \bar{\mathrm{Q}}$$

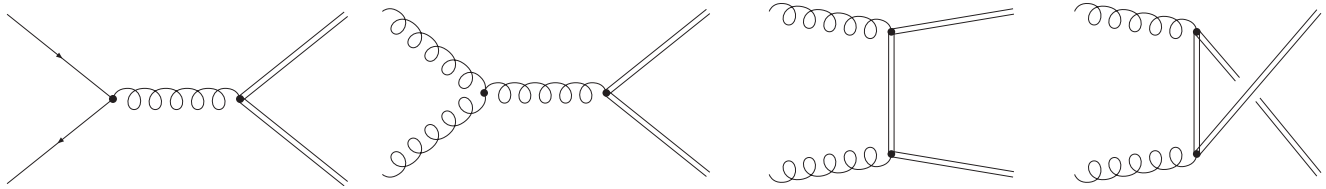


- NLO in QCD Nason, Dawson, Ellis '88; Beenakker, Smith, van Neerven '89; Mangano, Nason, Ridolfi '92; Bernreuther, Brandenburg, Si, Uwer '04; Mitov, Czakon '08; ...
  - accurate to $\mathcal{O}(15\%)$ at LHC

Top quark production

- Leading order Feynman diagrams

$$\begin{aligned} \mathrm{q} + \bar{\mathrm{q}} &\longrightarrow \mathrm{Q} + \bar{\mathrm{Q}} \\ \mathrm{g} + \mathrm{g} &\longrightarrow \mathrm{Q} + \bar{\mathrm{Q}} \end{aligned}$$



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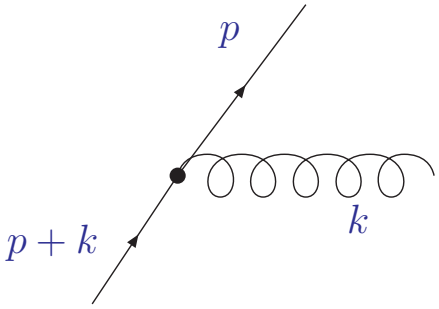
  - accurate to $\mathcal{O}(15\%)$ at LHC
- First steps towards higher orders in QCD: explore limits
- Study of massive QCD amplitudes in limit $m \to 0$
  - exploit relation of massive to massless amplitudes Mitov, S.M. '06
- Partonic threshold $s \simeq 4m^2$
  - Sudakov logarithms $\ln \beta$ (velocity of heavy quark $\beta = \sqrt{1 - 4m^2/s}$)
  - theory status and phenomenology S.M., Uwer '08

Recent theory activities

- Amplitudes
  - two-loop virtual corrections to $\mathrm{q\bar{q}} \to \mathrm{t\bar{t}}$ and $\mathrm{gg} \to \mathrm{t\bar{t}}$
    - small-mass limit $m^2 \ll s, t, u$ Czakon, Mitov, S.M. '07
    - complete IR singularities Ferroglia, Neubert, Pecjak, Yang '09
  - one-loop squared terms (NLO $\times$ NLO) Anastasiou, Mert Aybat '08; Kniehl, Merebashvili, Körner, Rogal '08
  - two-loop virtual corrections for $\mathrm{q\bar{q}} \to \mathrm{t\bar{t}}$ (analytic, n_f-terms) Bonciani, Ferroglia, Gehrmann, Maitre, Studerus '08; (numerical result) Czakon '08
- $\mathrm{t\bar{t}}$+jet production at NLO Dittmaier, Uwer, Weinzierl '07-'08
- Threshold resummation
  - updates of cross section predictions based on resummation S.M., Uwer '08; Cacciari, Frixione, Mangano, Nason, Ridolfi '08; Kidonakis, Vogt '08
  - coulomb corrections Hagiwara, Sumino, Yokoya '08; Kiyo, Kühn, S.M., Steinhauser, Uwer '08
- Parton luminosity
  - correlation of cross section at NLO with gluon PDFs Nadolsky, Lai, Cao, Huston, Pumplin, Stump, Tung, Yuan '08
- Definition of mass parameter Hoang, Jain, Scimemi, Stewart '08

Sudakov logarithms

- Recall perturbative QCD:
  - calculation of observables as series in $\alpha_s \ll 1$
  - but: large logarithmic corrections, $\ln(\dots) \gg 1$
    double logarithms (Sudakov)
- Soft/Collinear regions of phase space
  - double logarithms from singular regions in Feynman diagrams
  - propagator vanishes for: $E_g = 0$, soft $\theta_{qg} = 0$ collinear



$$\frac{1}{(p+k)^2} = \frac{1}{2p\cdot k} = \frac{1}{2E_qE_g(1-\cos\theta_{qg})}$$

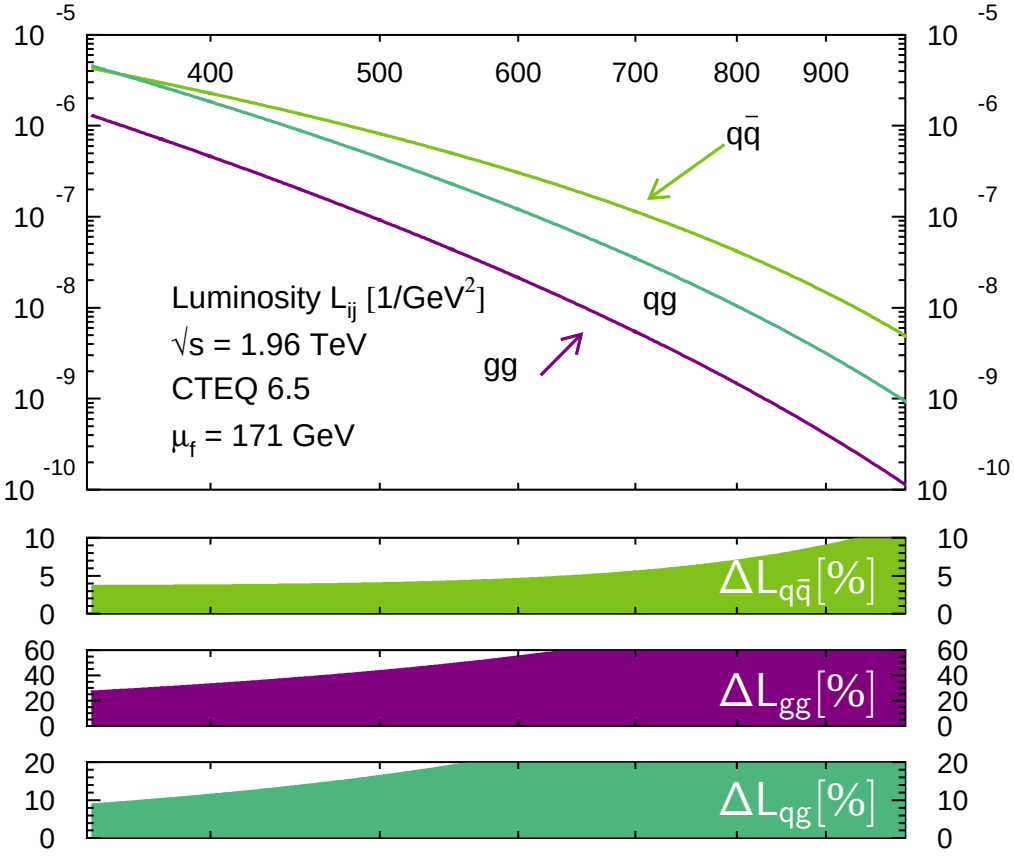
$$\alpha_s \int d^4k \frac{1}{(p+k)^2} \longrightarrow \alpha_s \int dE_g\, d\theta_{qg} \frac{1}{2E_qE_g(1-\cos\theta_{qg})}$$

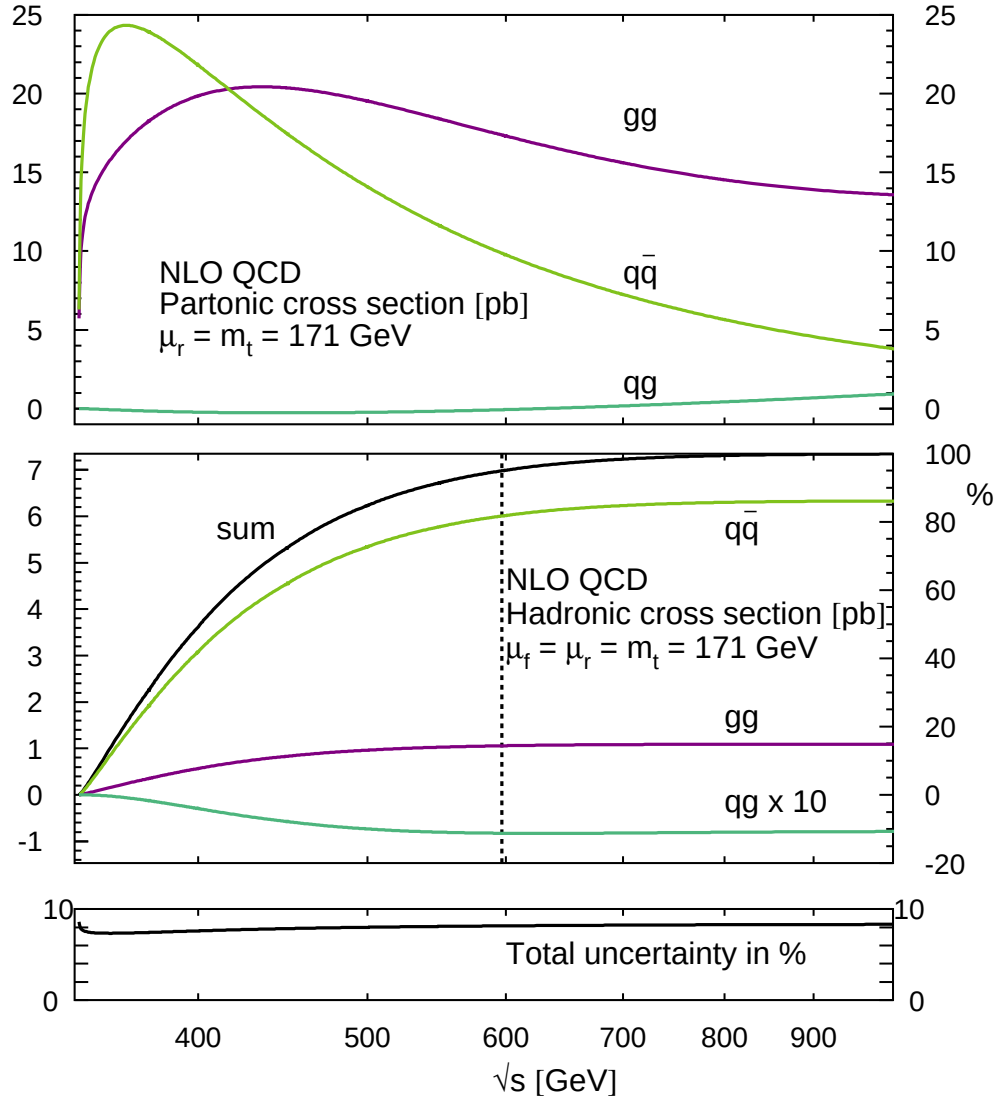
$$\longrightarrow \alpha_s \ln^2(\dots)$$

- Improved perturbation theory: resum logarithms to all orders
  - long history of resummation Kidonakis, Sterman '97; Bonciani, Catani, Mangano, Nason '98; Kidonakis, Laenen, S.M., Vogt '01; ...

Total cross section at Tevatron

$$\sigma_{pp\to t\bar{t}} = \sum_{ij} f_i \otimes f_j \otimes \hat{\sigma}_{ij\to t\bar{t}}$$





Two-loop results

- NNLO cross section for heavy-quark hadro-production near threshold (all powers of $\ln \beta$ and Coulomb corrections) S.M., Uwer '08; Langenfeld, S.M., Uwer '09
  - e.g. gg-fusion for $n_f = 5$ light flavors at $\mu = m_t$

$$\hat{\sigma}^{(1)}_{gg\to t\bar{t}} = \hat{\sigma}^{(0)}_{gg\to t\bar{t}} \left\{ 96 \ln^2 \beta - 9.5165 \ln \beta + 35.322 + 5.1698 \frac{1}{\beta} \right\}$$

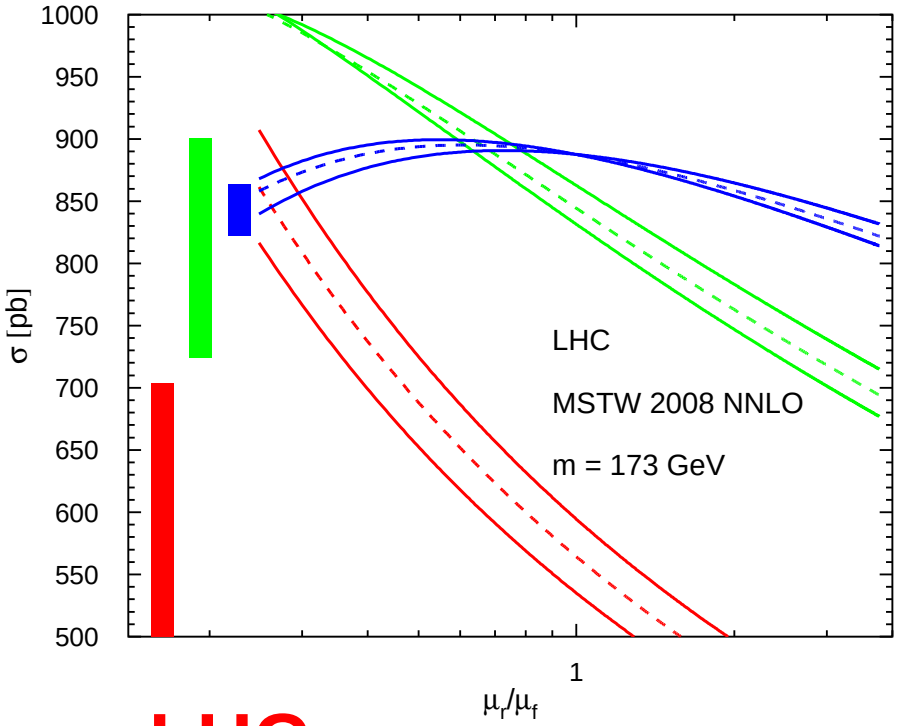
$$\hat{\sigma}^{(2)}_{gg\to t\bar{t}} = \hat{\sigma}^{(0)}_{gg\to t\bar{t}} \left\{ 4608 \ln^4 \beta - 1894.9 \ln^3 \beta + \left(-912.35 + 496.30 \frac{1}{\beta} \right) \ln^2 \beta \right.$$
$$\left. + \left(2747.5 + 321.14 \frac{1}{\beta} \right) \ln \beta + 68.547 \frac{1}{\beta^2} - 196.93 \frac{1}{\beta} + C^{(2)}_{gg} \right\}$$

Upshot

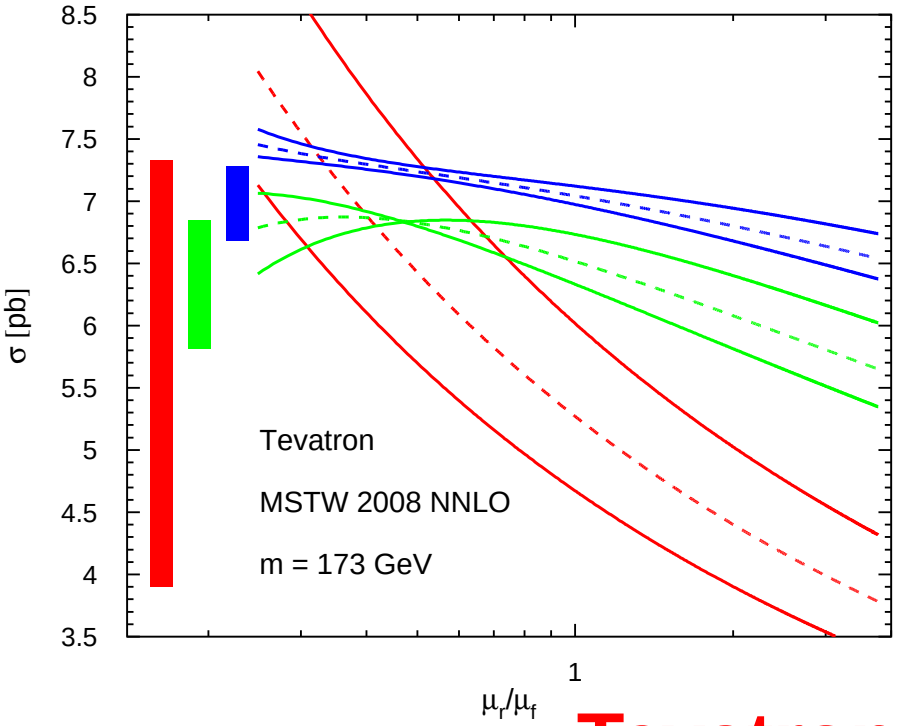
- Best present approximation to complete NNLO
- Similar results for new massive colored particles (4th generation quarks, squarks, gluinos, . . .)
  S.M., Uwer '08; S.M., Langenfeld '08

Scale dependence

- Theoretical uncertainty from variation of scales μ_R, μ_F
  - plot with PDF set MSTW 2008 (but largely independent on PDFs)
  - mass $m_t = 173$ GeV (from $m_t = 173.1 \pm 1.3$ GeV Tevatron winter '09)
  - stable predictions in range $\mu_R, \mu_F \in [m_t/2, 2m_t]$
    - $-3\% \leq \Delta\sigma \leq +1\%$ at LHC
    - $-5\% \leq \Delta\sigma \leq +3\%$ at Tevatron



LHC



Tevatron

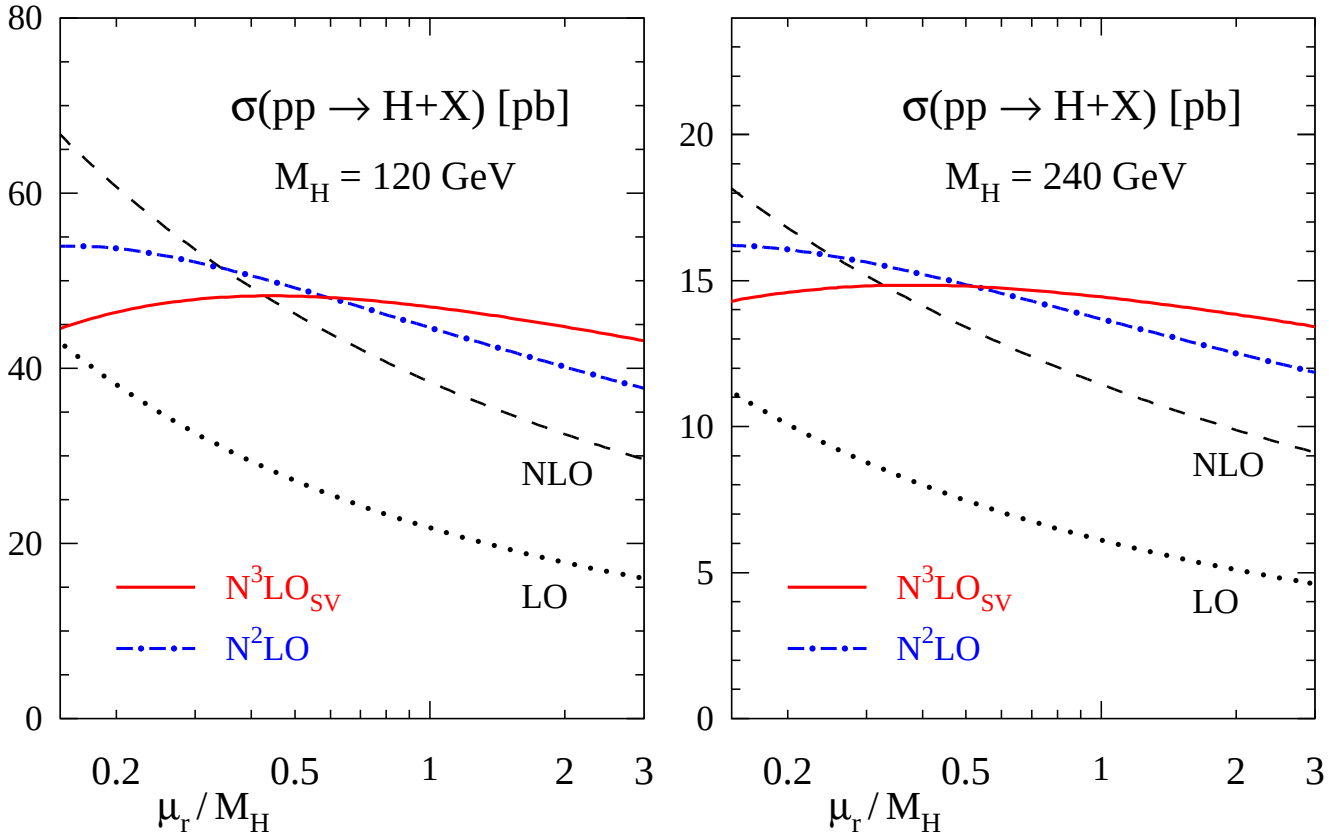
The total cross section

- pole mass $m_t = 173$ GeV
  - different values of α_s and light quark PDFs at large x for CTEQ6.6 and MSTW 2008

$$\sigma_{\rm LHC} = 874\ {\rm pb}\ {}^{+9}_{-33}\ {\rm pb\ (scale)}\ {}^{+28}_{-28}\ {\rm pb\ (CTEQ6.6)}$$
$$\sigma_{\rm Tev} = 7.34\,{\rm pb}\ {}^{+0.24}_{-0.38}\,{\rm pb\ (scale)}\ {}^{+0.41}_{-0.41}\,{\rm pb\ (CTEQ6.6)}$$

$$\sigma_{\rm LHC} = 887\ {\rm pb}\ {}^{+9}_{-33}\ {\rm pb\ (scale)}\ {}^{+15}_{-15}\ {\rm pb\ (MSTW2008)}$$
$$\sigma_{\rm Tev} = 7.04\,{\rm pb}\ {}^{+0.24}_{-0.36}\,{\rm pb\ (scale)}\ {}^{+0.14}_{-0.14}\,{\rm pb\ (MSTW2008)}$$

Cross section for Higgs production



- Variation of cross section at LHC with renormalizaion scale for different Higgs masses: $M_H = 120$GeV (left) and $M_H = 240$GeV (right)
  - NNLO corrections
    Harlander, Kilgore '02; Anastasiou, Melnikov '02; Ravindran, Smith, van Neerven '03
  - complete soft N^{3}LO corrections (dominant part at three loops) S.M., Vogt '05

Quality control

- Total cross section with $m_t = 173$ GeV at scale $\mu = \mu_R = \mu_F$
  - numbers from arXiv:0804.1476, arXiv:0807.2794

	CTEQ6.6	MSTW08
$\sigma_{\rm NNLO}[\rm pb]$	872	

LHC

CTEQ6.6	MSTW08
7.34	

Tevatron

Quality control

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LHC

CTEQ6.6	MSTW08
7.34+0.0	

Tevatron

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Tevatron

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7.34	7.04

Tevatron

- Check of systematics: soft gluons
  - compare exact results against (at NLO and for NNLO scale dep.)

$$\hat{\sigma}^{(n)}_{\rm N\ldots LL} \quad = \quad \hat{\sigma}^{(0)} \left\{ \# \ln^{2n} \beta + \cdots + \# \ln \beta + \cdots + \frac{\#}{\beta} \right\}$$

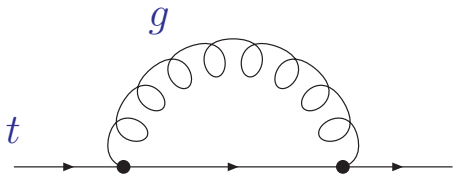
- Check of systematics: hard radiation
  - $\rm t\bar{t}$+jet production (large rates at LHC)
  - NLO for $\rm t\bar{t}$+jet is important part of NNLO for inclusive $\rm t\bar{t}$
  - corrections are almost zero at scale $\mu_R = \mu_F = m_t$
- Estimate systematic uncertainty of total cross section $\Delta\sigma \sim \mathcal{O}(2\%)$
  - $\Delta\sigma \sim \mathcal{O}(10 - 15)\,\rm pb$ at LHC
  - $\Delta\sigma \sim \mathcal{O}(0.15 - 0.2)\,\rm pb$ at Tevatron

Top-quark mass definition

Pole mass scheme

- Based on (unphysical) concept of top-quark being a free parton

$$\not{p} - m_t - \Sigma(p, m_t)\Big|_{p^2 = m_t^2}$$



  - heavy-quark self-energy $\Sigma(p, m_t)$ receives contributions from regions of all loop momenta – also from momenta of $\mathcal{O}(\Lambda_{\rm QCD})$

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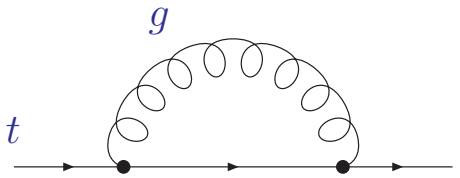
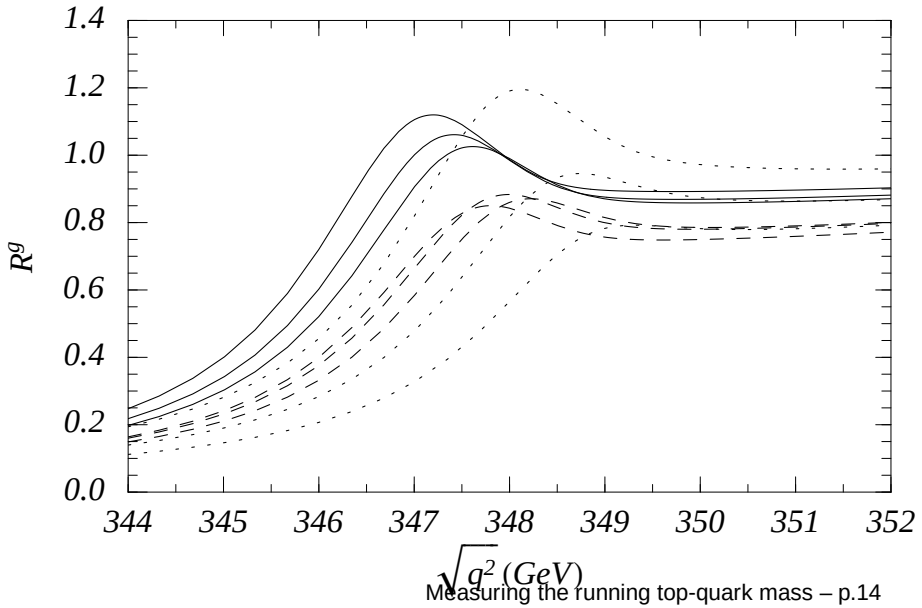


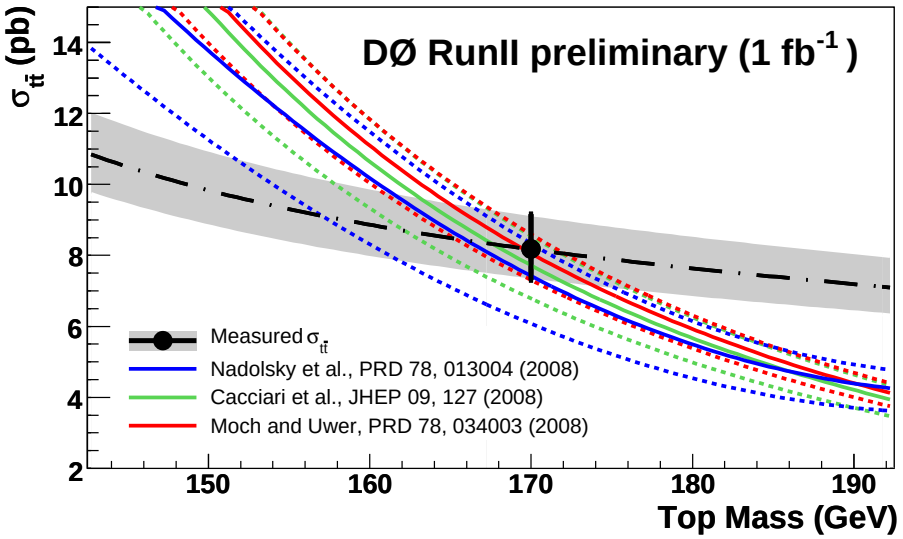
  - heavy-quark self-energy $\Sigma(p, m_t)$ receives contributions from regions of all loop momenta – also from momenta of $\mathcal{O}(\Lambda_{\rm QCD})$

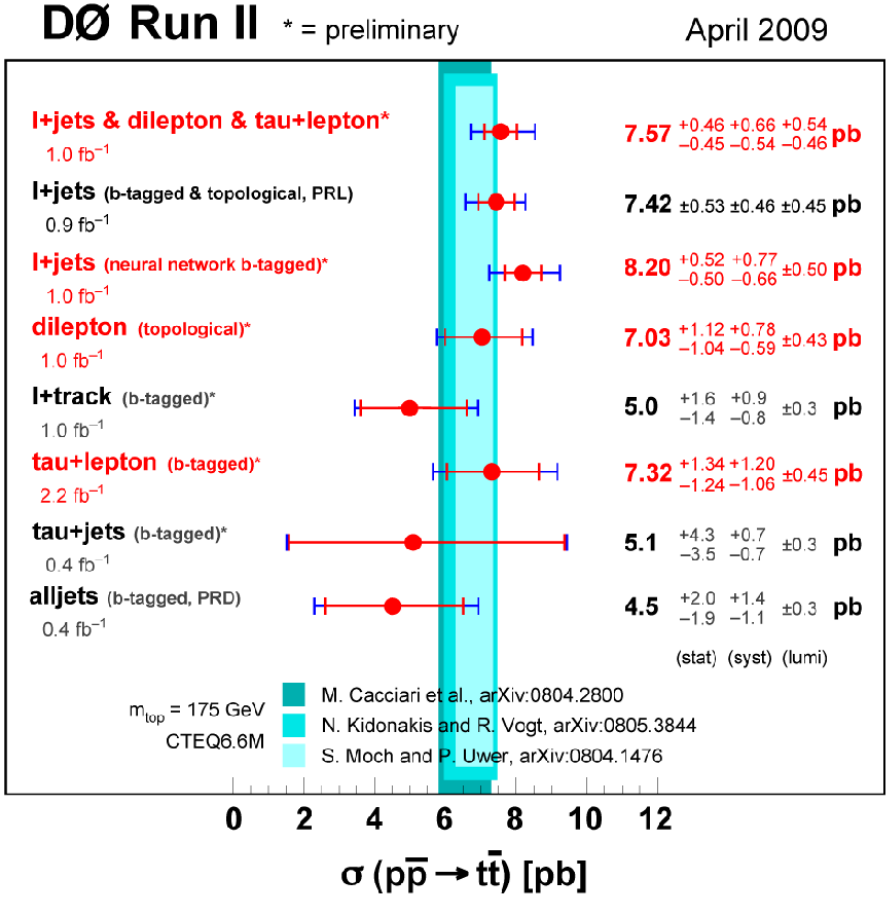
- Pole mass measurements are strongly order-dependent
  - e.g. threshold scan of cross section in e^+e^- collision
    Beneke, Signer, Smirnov ‘99;
    Hoang, Teubner ‘99;
    Melnikov, Yelkhovsky ‘98;
    Penin, Pivovarov ‘99;
    Yakovlev ‘99
  - LO (dotted), NLO (dashed), NNLO (solid)



Tevatron analyses

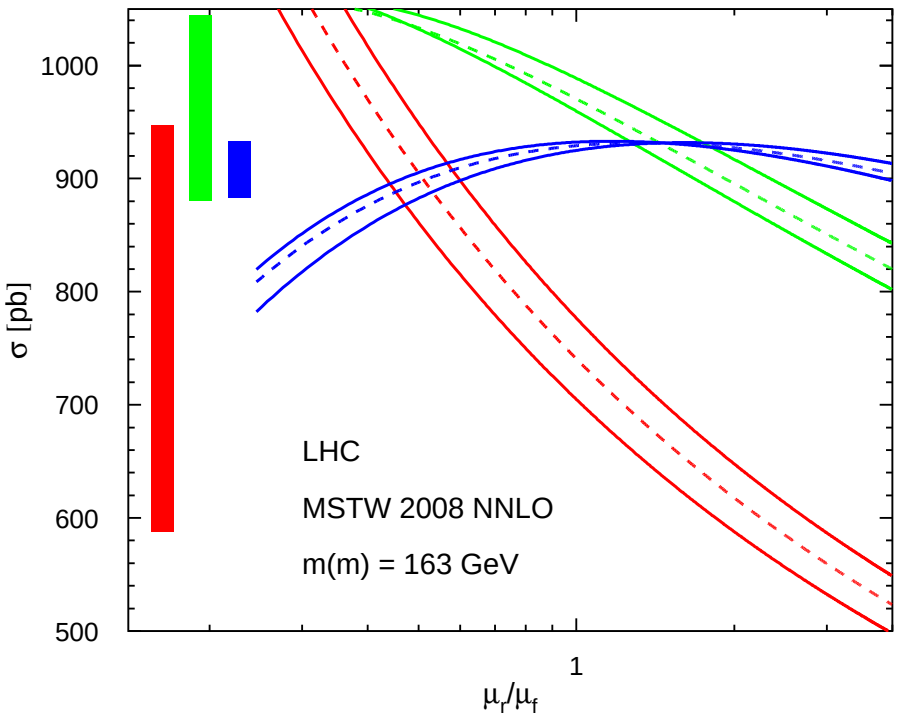
- Total cross section and different channels of Tevatron analyses (theory uncertainty band from scale variation)
- Determination of m_t from total cross section (slope $d\sigma/dm_t$)
  - e.g. DZero '09: NLO $m_t = 165.5^{+6.1}_{-5.9}$; NNLO $m_t = 169.1^{+5.9}_{-5.2}$; ...

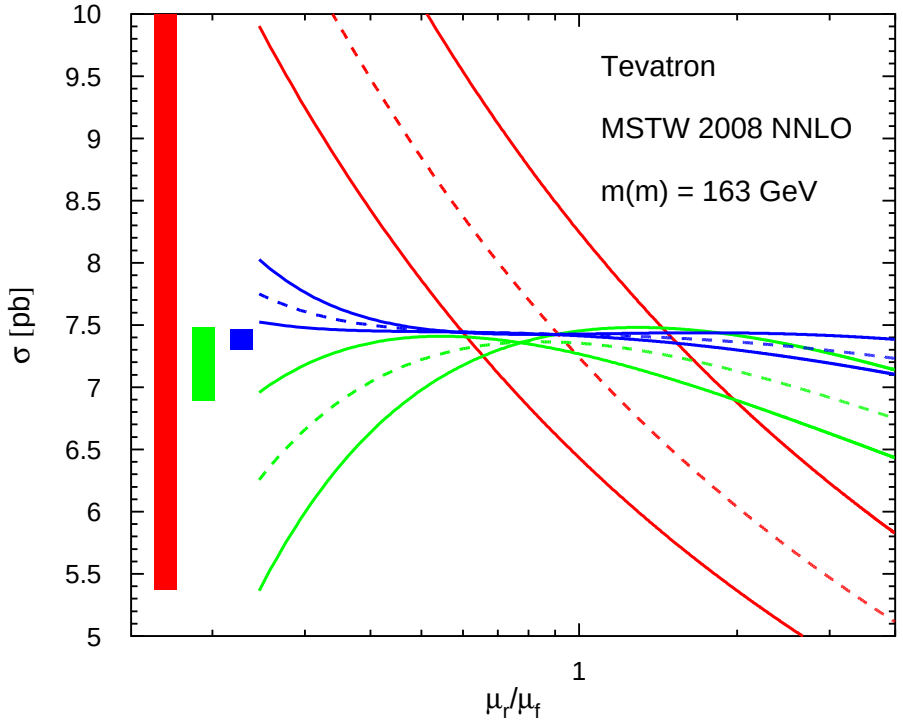




The running top-quark mass

- $\overline{MS}$ mass definition $m(\mu_R)$ realizes running mass (scale dependence)
  - short distance mass probes at scale of hard scattering
  - conversion between pole mass and $\overline{MS}$ mass definition in perturbation theory: $m_t = m(\mu_R)\left(1 + a_s(\mu_R)d^{(1)} + a_s(\mu_R)^2 d^{(2)}\right)$
- Scale dependence greatly reduced





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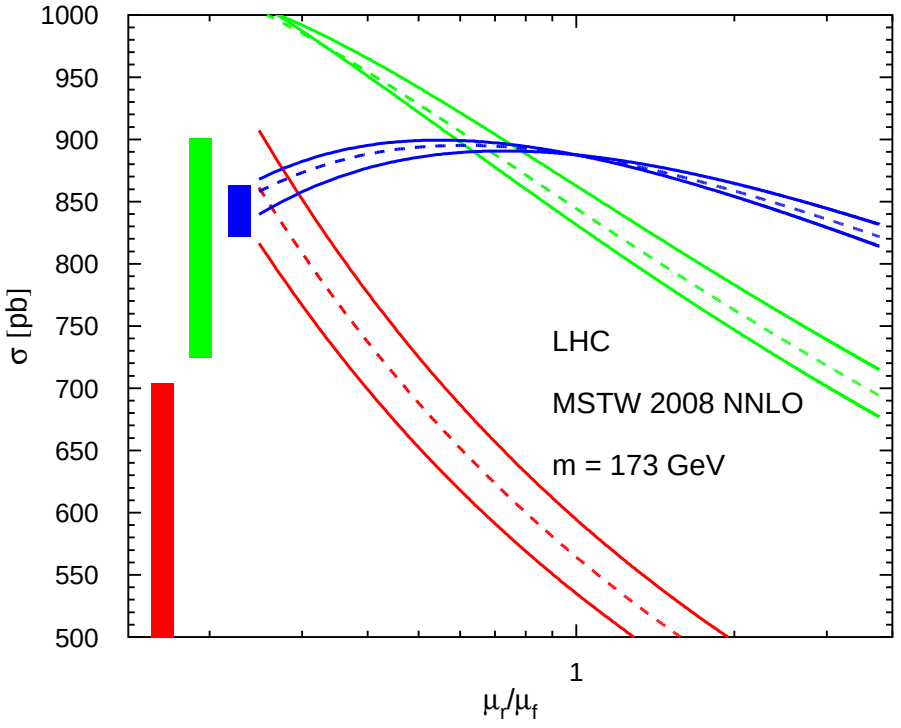
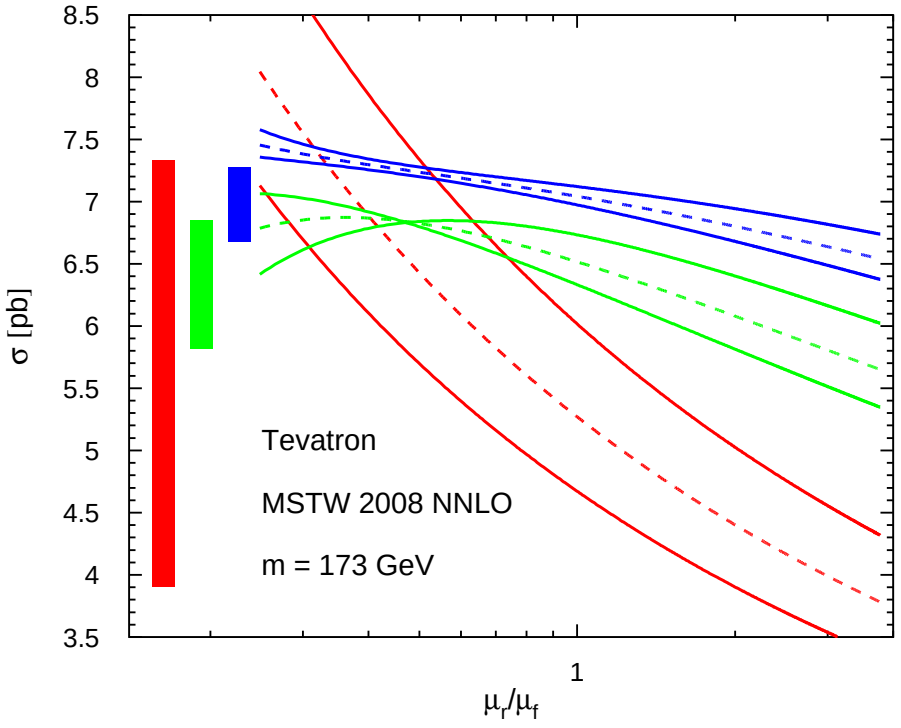
- Pole mass scheme for comparison



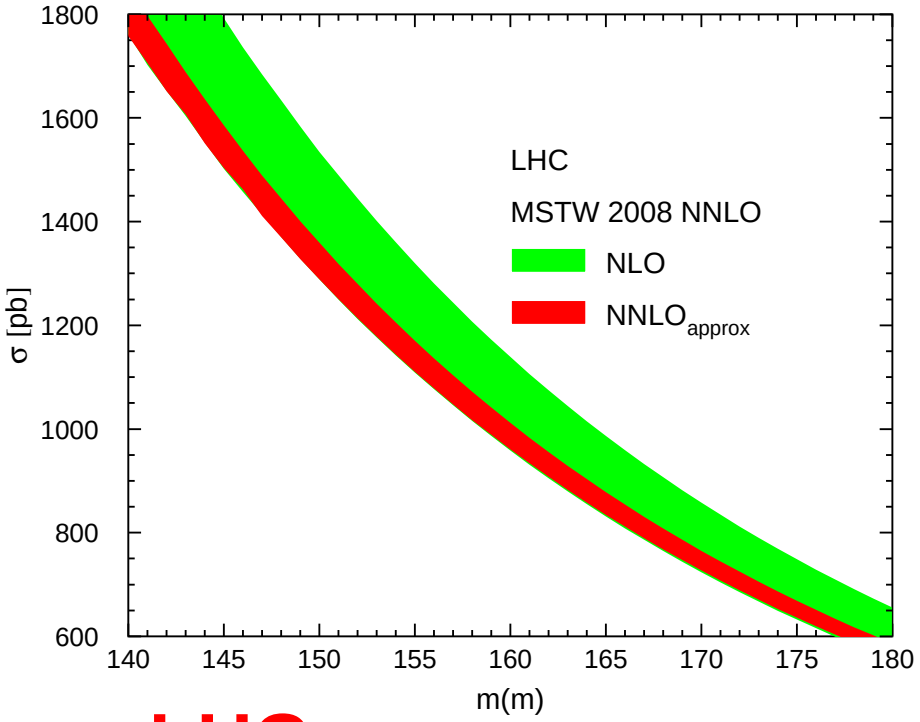


Mass dependence in $\overline{MS}$ mass scheme

- Total top-quark cross section as function of $\overline{m}$

  Langenfeld, S.M., Uwer '09

  - theoretical uncertainity (band) due to variation of $\mu_R \in [\overline{m}/2, 2\overline{m}]$ for fixed set $\mu_F \in \overline{m}/2, \overline{m}, 2\overline{m}$



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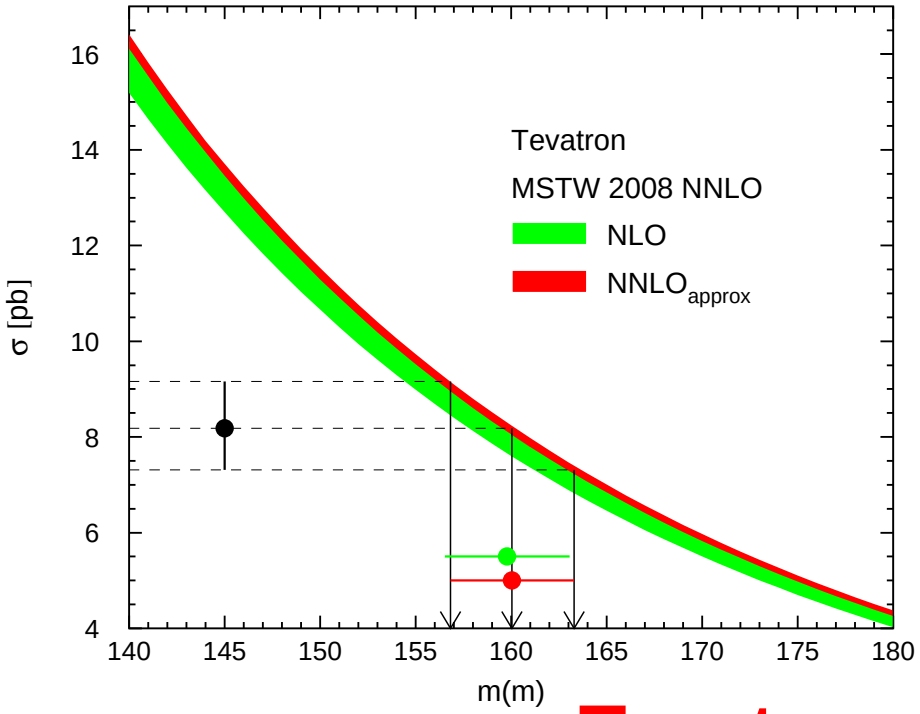

Tevatron

Analysis scheme

- Measurement of total cross section
- Determination of $\overline{MS}$ mass $\overline{m}$ from comparison of experiment to theory $\sigma(m(\mu_R), \mu_R, \mu_F)$
  - our analysis: $\overline{MS}$ mass $\overline{m} = 160.0^{+3.3}_{-3.2}\mathrm{GeV}$

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- Conversion of $\overline{MS}$ mass $\overline{m}$ to pole mass m_t within perturbation theory
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  - compatible (within errors) with world average $m_t = 173.1^{+1.3}_{-1.3}\mathrm{GeV}$

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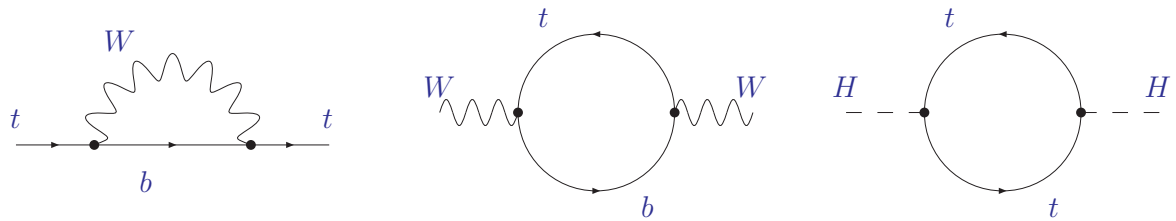
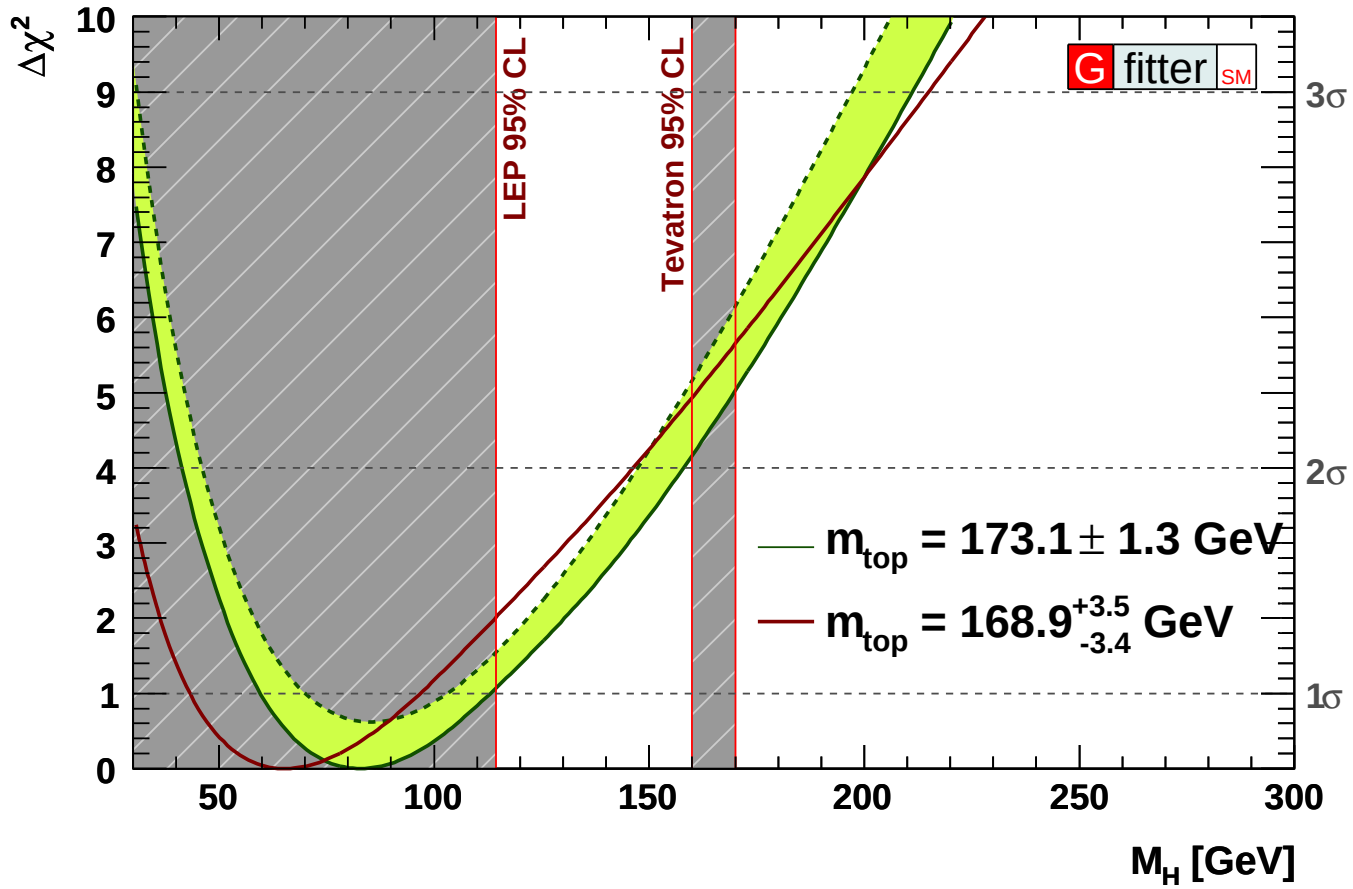
- Radiative corrections (one and two loops) provide relation between SM parameters (masses, couplings)



  - use m_t in precision analysis of electroweak observables (together with M_W) for constraints on M_H

Indirect Higgs searches

- Electroweak precision data constrains M_H



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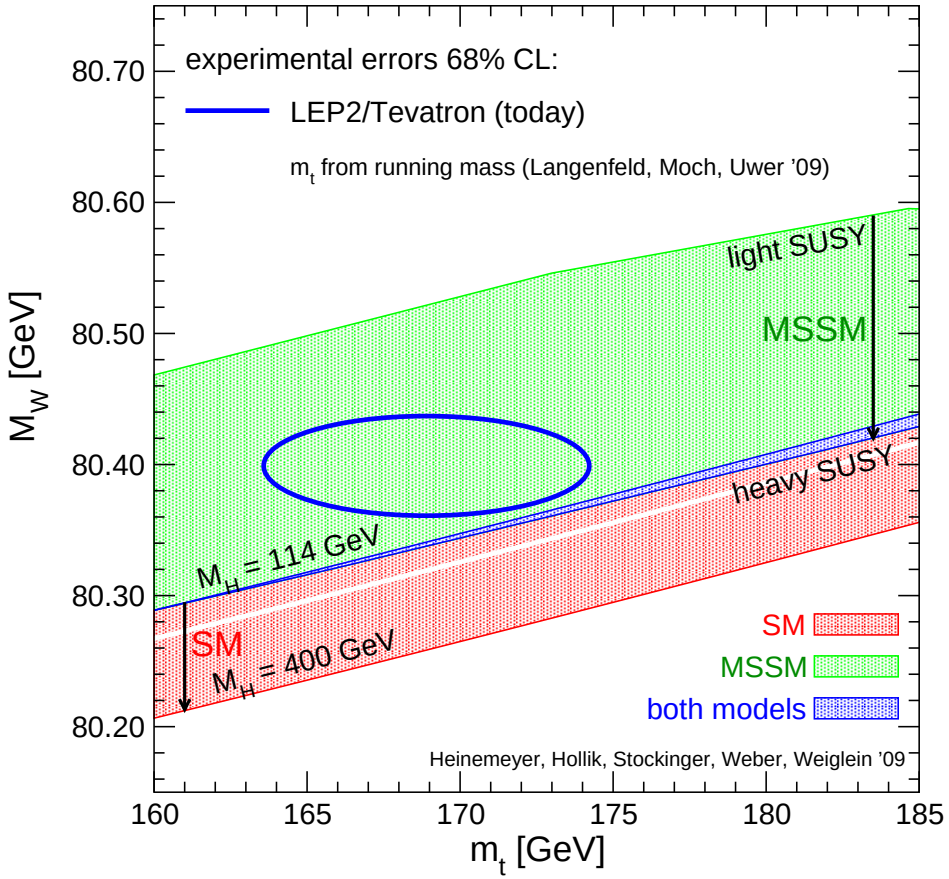
  (lighter top-quark masses disfavor SM Higgs sector)

Constraints on M_W and m_t

- Extension of electroweak precision fits to MSSM
  Heinemeyer, Hollik, Stöckinger, Weber, Weiglein ‘09
- Relations for radiative corrections known through two loops



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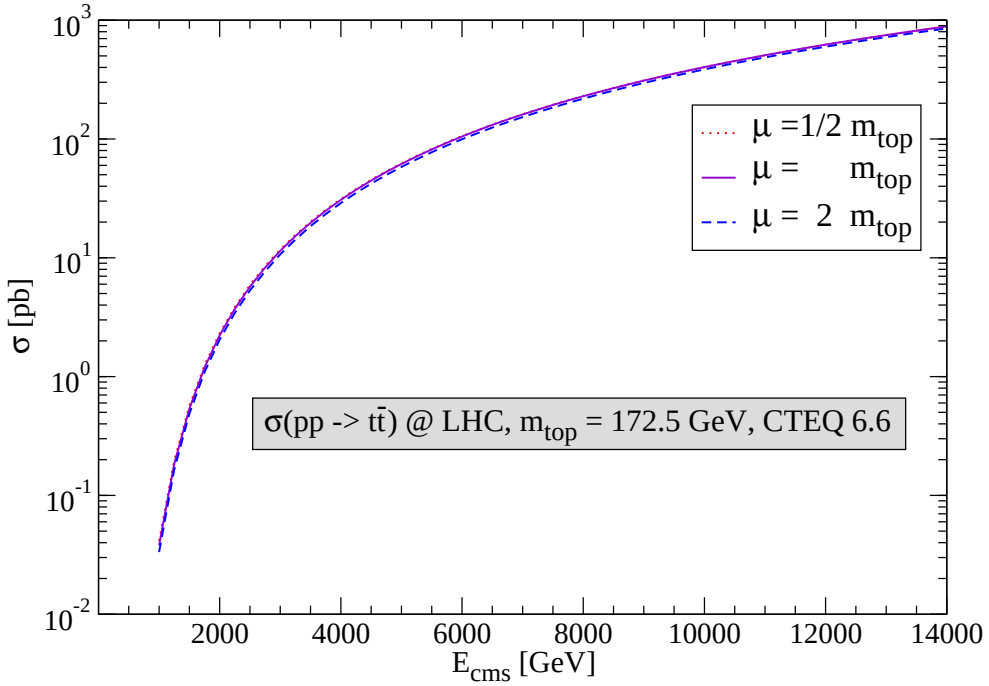
Top quarks in early phase of LHC

Energy dependence

- Early LHC phase with reduced beam energy 0.9 TeV $\leq \sqrt{s} \leq 3.5$ TeV
  - plot for $m_t = 172.5$ GeV (ATLAS default)
- Cross section at 7 TeV: $\sigma_{\mathrm{NNLO}} = 161\,\mathrm{pb}$ (MSTW08)
  - need luminosity of 10-100 pb^{-1} at 7 TeV for cross section measurement



PDF issues

- $t\bar{t}$-production cross sections at Tevatron and LHC
  - pole mass of $m_t = 173$ GeV at scale $\mu = m_t$
  - comparison of PDF dependence
    MSTW08 and Alekhin, Blümlein, Klein, Moch '09

$\sqrt{s}$ (TeV)	ABKM09	MSTW2008
1.96 $(\bar{p}p)$	6.91 ± 0.17	7.04
7 (pp)	131.3 ± 7.5	160.5
10 (pp)	343 ± 15	403
14 (pp)	780 ± 28	887

- LHC
  - correlation of cross section with gluon PDFs
    Nadolsky, Lai, Cao, Huston, Pumplin, Stump, Tung, Yuan '08
  - sensitivity to gluon PDF at large-x

Summary

- Top quark theory
  - improved understanding of theory and application of new concepts
  - resummation important for Tevatron and LHC phenomenology
- Total cross section
  - $\text{NNLO}_{\text{approx}}$ prediction with exact scale dependence $\mu_R \neq \mu_F$ ($\ln(\mu_R/m)$, $\ln(\mu_F/m)$-terms)
  - cross check on systematics with NLO correction to $t\bar{t}+$ jet
  - also: electroweak corrections; $t\bar{t}$-bound state effects; ...
- $\overline{MS}$ mass definition
  - running top-quark mass of $m(\mu = m) = 160.0^{+3.3}_{-3.2}$ GeV
  - greatly reduced scale dependence
  - much improved convergence of perturbation theory