Electroweak corrections to hadronic event shapes

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Contents

1 Introduction

Jet event-shape observables at e^+e^- colliders

- characterize the topology of hadronic events without referring to details of the hadronic particle content
- allow for crucial tests of jet dynamics predicted by QCD
- sensitive to $\alpha_s(Q)$ at variable scale $Q = \sqrt{s}$

α_s from hadronic event shapes — status 2006

Bethke '06

Q[GeV]	$\alpha_s(M_Z)$	Δ_{EXP}	Δ_{TH}
14.0	$0.120\,^{+0.010}_{-0.008}$	0.002	$^{+0.009}_{-0.008}$
22.0	$0.118\,^{+0.009}_{-0.008}$	0.003	$^{+0.009}_{-0.007}$
35.0	$0.123\,^{+0.008}_{-0.006}$	0.002	$^{+0.008}_{-0.005}$
44.0	$0.123\,^{+0.008}_{-0.006}$	0.003	$^{+0.007}_{-0.005}$
58.0	0.123 ± 0.007	0.003	0.007

Q[GeV]	$\alpha_s(M_Z)$	Δ_{EXP}	Δ_{TH}
91.2	0.121 ± 0.006	0.001	0.006
133	0.120 ± 0.007	0.003	0.006
161	0.118 ± 0.008	0.005	0.006
172	0.114 ± 0.008	0.005	0.006
183	0.121 ± 0.006	0.002	0.005
189	0.121 ± 0.005	0.001	0.005
195	0.122 ± 0.006	0.001	0.006
201	0.124 ± 0.006	0.002	0.006
206	0.124 ± 0.006	0.001	0.006

Note: $\Delta_{TH} > \Delta_{EXP}$
with TH = QCD NLO$\oplus$resummations

$\hookrightarrow$ high motivation for recent QCD calculations at NNLO and NNLL
Gehrmann et al. '07–'09; Weinzierl '08,'09; Becher, Schwartz '08

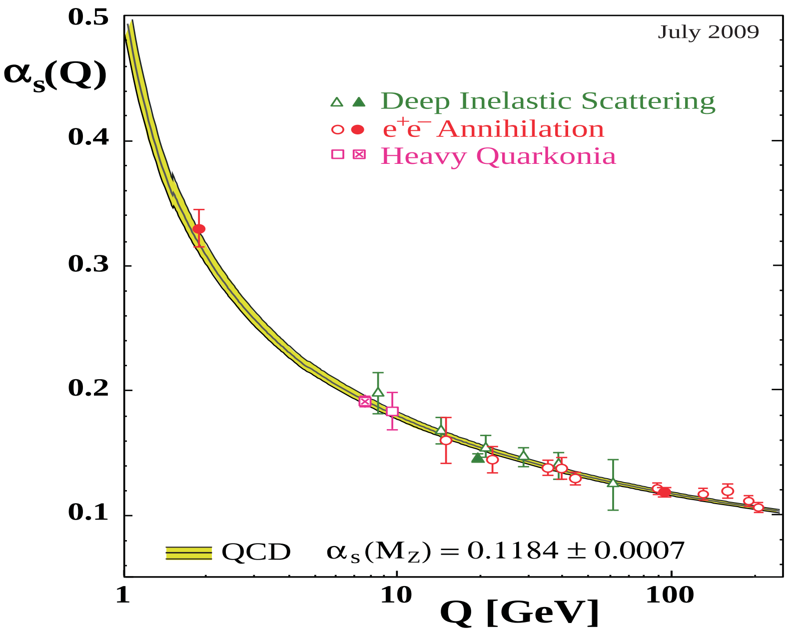
Jet event shapes and α_s determination in 2009

Bethke '09

Process	Q[GeV]	$\alpha_s(Q)$	$\alpha_s(M_Z)$	excl. mean $\alpha_s(M_Z)$	std. dev.
τ-decays	1.78	0.330 ± 0.014	0.1197 ± 0.0016	0.11818 ± 0.00070	0.9
DIS [F_2]	2–15	–	0.1142 ± 0.0023	0.11876 ± 0.00123	1.7
DIS [ep → jets]	6–100	–	0.1198 ± 0.0032	0.11836 ± 0.00069	0.4
$Q\overline{Q}$ states	7.5	0.1923 ± 0.0024	0.1183 ± 0.0008	0.11862 ± 0.00114	0.2
Υ decays	9.46	$0.184^{+0.015}_{-0.014}$	$0.119^{+0.006}_{-0.005}$	0.11841 ± 0.00070	0.1
e^+e^- [jets&shps]	14–44	–	0.1172 ± 0.0051	0.11844 ± 0.00076	0.2
e^+e^- [ew]	91.2	0.1193 ± 0.0028	0.1193 ± 0.0028	0.11837 ± 0.00076	0.3
e^+e^- [jets&shps]	91–208	–	0.1224 ± 0.0039	0.11831 ± 0.00091	1.0

World average:

$$\alpha_s(M_Z) = 0.1184 \pm 0.0007$$



Definition of jet event-shape observables

Step 1: jet definition by jet algorithm

$\hookrightarrow$ cluster partons (particles in exp.) that are close to each other, e.g.

Durham alg.: successively recombine partons until all parton pairs i, j obey

$$y_{ij,\mathrm{D}} = \frac{2\min\{E_i^2, E_j^2\}\,(1-\cos\theta_{ij})}{s} > y_{\mathrm{cut}}$$

Note: different schemes for recombination $(i,j) \to ij$ possible (E, E_0, P, P_0)

Step 2: calculate n-jet cross sections $\sigma_{n-\mathrm{jet}}$ and event-shape observables y

$\hookrightarrow$ confront theory with data

- hadronic cross section: $\sigma_{\mathrm{had}} = \sum_{n=1}^{\infty} \sigma_{n-\mathrm{jet}}$
- n-jet rates: $R_n(y_{\mathrm{cut}}, \sqrt{s}) = \sigma_{n-\mathrm{jet}}/\sigma_{\mathrm{had}}$
- normalized event-shape distributions: $\frac{1}{\sigma_{\mathrm{had}}}\frac{\mathrm{d}\sigma(y)}{\mathrm{d}y}$

Frequently used event-shape observables y

- Thrust: $T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$
- Normalized heavy jet mass: $\rho = \max\{M_1^2, M_2^2\}/s$

  ($M_i =$ inv. mass flowing into hemispheres H_i defined by plane perpendicular to thrust axis)
- Wide / total jet broadenings:

  $$B_{\mathrm{W}} = \max\{B_1, B_2\}, \quad B_{\mathrm{T}} = B_1 + B_2, \qquad B_i = \frac{\sum_{j\in H_i} |\vec{p}_j \times \vec{n}|}{2\sum_k |\vec{p}_k|}$$
- C parameter:

  $$C = 3(\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1), \quad \{\lambda_i\} = \text{eigenvalues of } \Theta = \frac{1}{\sum_i |\vec{p}_i|} \sum_j \frac{\vec{p}_j \otimes \vec{p}_j}{|\vec{p}_j|}$$
- Jet transition variable:

  $Y_3 =$ value of y_{cut} at which the event turns from 3-jet to 2-jet type

Note: 2-jet configuration appears at an endpoint of $\frac{\mathrm{d}\sigma(y)}{\mathrm{d}y}$ (e.g. at $T \to 1$)

$\hookrightarrow$ shapes of distributions sensitive to 3 and more jets, and thus to α_{s}

Theory prediction for jet event shapes ($\mathrm{e^+e^-} \to n\,\mathrm{jets},\ n \geq 3$)

$$\frac{1}{\sigma_{\mathrm{had}}}\frac{\mathrm{d}\sigma(y)}{\mathrm{d}y} = \underbrace{\alpha_s C^{\mathrm{QCD}}_{\mathrm{LO}} + \alpha_s^2 C^{\mathrm{QCD}}_{\mathrm{NLO}}} + \underbrace{\alpha_s^3 C^{\mathrm{QCD}}_{\mathrm{NNLO}}}$$

R.K.Ellis, Ross, Terrano '81; Kunszt '81
Vermaseren, Gaemers, Oldham '81
Giele, Glover '92; Catani, Seymour '96

Gehrmann-DeRidder, Gehrmann, Glover, Heinrich '07–'09; Weinzierl '08,'09

$+$ NLL resummation $+$ NLL/NNLO matching ($+$ NNLL resummation for T)

NLL resummation: Catani, Turnock, Webber, Trentadue '91,'93

NLL/NNLO matching: Gehrmann, Luisoni, Stenzel '08

NNLL resummation for T: Becher, Schwartz '08

$+$ non-perturbative hadronization effects

Korchemsky, Sterman '95; Dokshitzer, Webber '95,'97
Dokshitzer, Lucenti, Marchesini, Salam '98

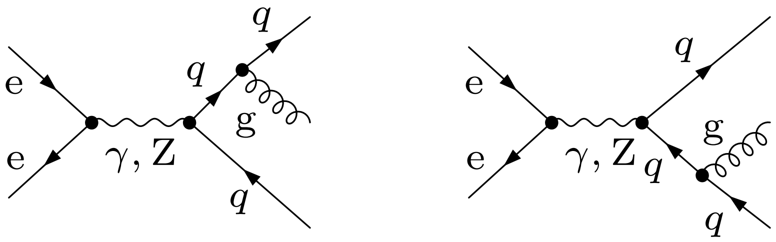
$$+ \underbrace{\alpha\, C^{\mathrm{EW}}_{\mathrm{LO}} + \alpha\alpha_s C^{\mathrm{EW}}_{\mathrm{NLO}} + \alpha^2\alpha_s C^{\mathrm{ISR}}_{\mathrm{LL}}}$$

subject of our calculation
partial results from Maina, Moretti, Ross '03
Carloni-Calame, Moretti, Piccinini, Ross '08

- Recent NNLO QCD results already included in α_s fit to event shapes
  Gehrmann, Luisoni, Stenzel '08; Dissertori et al. '08; Bethke et al. '08; Davison, Webber '08
- NLO EW corrections potentially of same size as NNLO QCD, since $\mathcal{O}(\alpha) \sim \mathcal{O}(\alpha_s^2)$

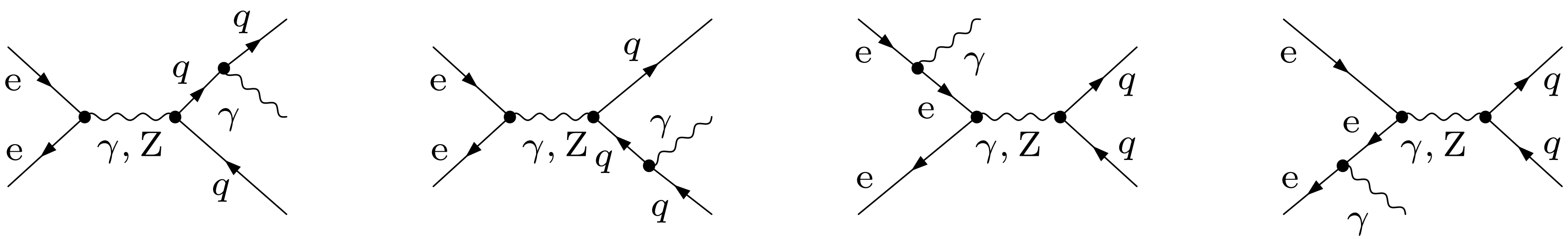
2 Calculation of NLO corrections

LO diagrams for $\mathrm{e}^+\mathrm{e}^- \to q\bar{q}\mathrm{g}$: $\mathcal{O}(\alpha^2\alpha_\mathrm{s})$



$$q = \mathrm{u, d, s, c, b}$$

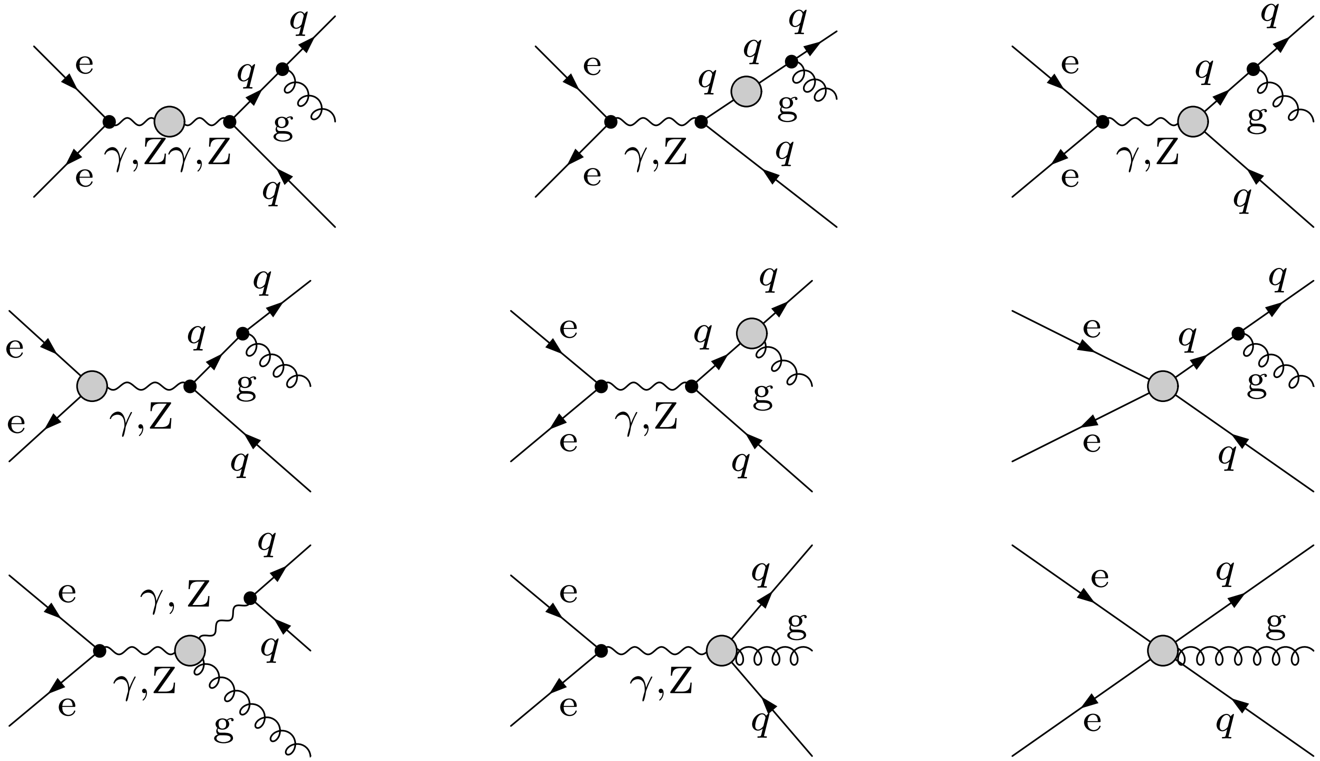
LO diagrams for $\mathrm{e}^+\mathrm{e}^- \to q\bar{q}\gamma$: $\mathcal{O}(\alpha^3)$



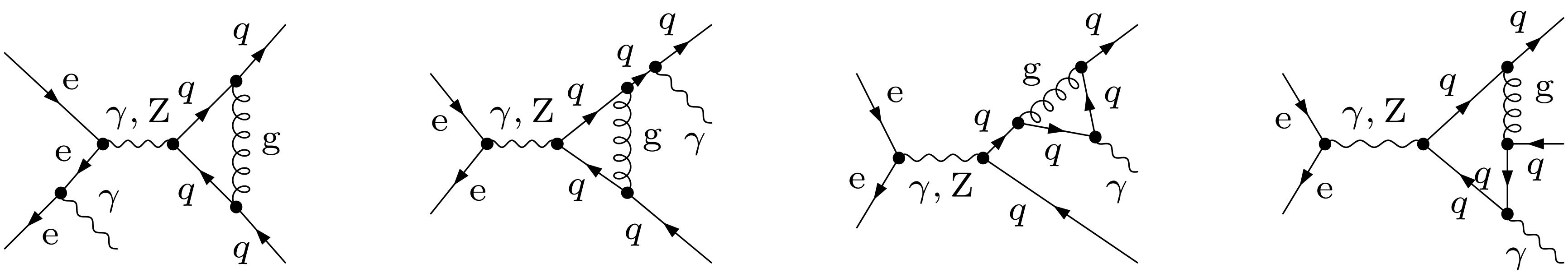
Comments:

- $q\bar{q}\gamma$ final states in LO deliver contributions if γ is merged with $q/\bar{q}$, i.e. near 2-jet configurations
- focus of our calculation: $\mathcal{O}(\alpha^3\alpha_\mathrm{s})$ = NLO EW correction to $q\bar{q}\mathrm{g}$ production
  = NLO QCD correction to $q\bar{q}\gamma$ production

1PI loop insertions in EW one-loop corrections to $\mathrm{e}^+\mathrm{e}^- \to q\bar{q}\mathrm{g}$ $\quad\mathcal{O}(200)$ diagrams

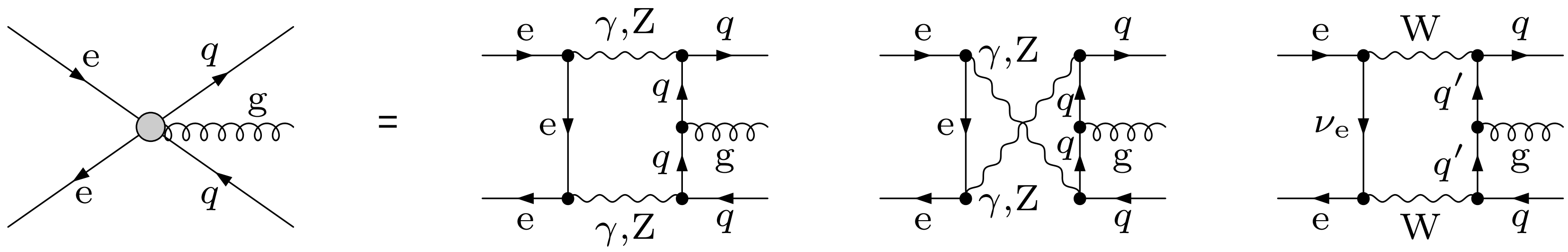


Sample QCD one-loop diagrams for $\mathrm{e}^+\mathrm{e}^- \to q\bar{q}\gamma$



Most complicated parts of the loop calculation:

- pentagon graphs:



$\hookrightarrow$ stable reduction to box integrals without inverse Gram determinants

Denner, S.D. ’02,’05 (similar to Binoth et al. ’05)

- numerical instabilities in Passarino–Veltman reduction of tensor integrals

$\hookrightarrow$ expansion about exceptional points

Denner, S.D. ’05 (similar to Giele et al. ’04; R.K.Ellis et al. ’05)

- gauge-invariant treatment of Z resonance

$\hookrightarrow$ “complex-mass scheme” Denner, S.D., Roth, Wieders ’05

The complex-mass scheme at NLO Denner, S.D., Roth, Wieders '05

Basic idea: mass^2 = location of propagator pole in complex p^2 plane

$\hookrightarrow$ consistent use of complex masses everywhere !

Application to gauge-boson resonances:

- replace $M_\mathrm{W}^2 \to \mu_\mathrm{W}^2 = M_\mathrm{W}^2 - \mathrm{i} M_\mathrm{W}\Gamma_\mathrm{W}, \qquad M_\mathrm{Z}^2 \to \mu_\mathrm{Z}^2 = M_\mathrm{Z}^2 - \mathrm{i} M_\mathrm{Z}\Gamma_\mathrm{Z}$

  and define (complex) weak mixing angle via $c_\mathrm{W}^2 = 1 - s_\mathrm{W}^2 = \frac{\mu_\mathrm{W}^2}{\mu_\mathrm{Z}^2}$

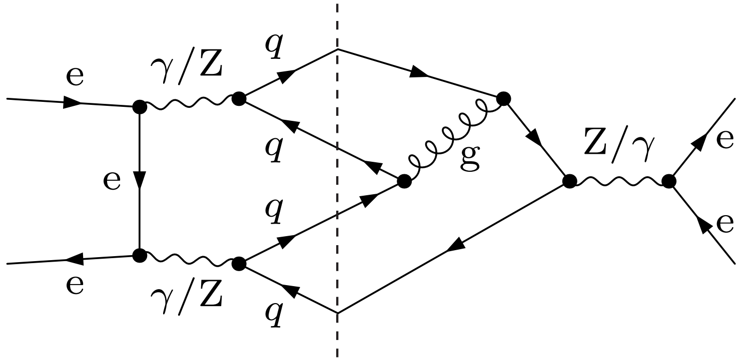
- virtues:
  - ◇ gauge-invariant result (Slavnov–Taylor identities, gauge-parameter independence)
    $\hookrightarrow$ unitarity cancellations respected !
  - ◇ perturbative calculations as usual (loops and counterterms)
  - ◇ no double counting of contributions (bare Lagrangian unchanged !)

- drawbacks:
  - ◇ unitarity-violating spurious terms of $\mathcal{O}(\alpha^2)$ $\to$ but beyond NLO accuracy !
    (from t-channel/off-shell propagators and complex mixing angle)
  - ◇ complex gauge-boson masses also in loop integrals

Real emission corrections at $\mathcal{O}(\alpha^3\alpha_s)$ and beyond

- $e^+e^- \to q\bar{q}g\gamma$ = photon bremsstrahlung to $q\bar{q}g$ production
  = gluon bremsstrahlung to $q\bar{q}\gamma$ production

- QCD–EW interferences for $e^+e^- \to q\bar{q}q\bar{q}$



$\hookrightarrow$ non-singular contributions of $\mathcal{O}(\alpha^3\alpha_s)$ = same order as NLO EW

Interferences included in our calculation

$\hookrightarrow$ effect phenomenologically negligible ($< 0.1\%$)

- higher-order photonic ISR included via leading-log structure functions up to order LO $\times\, \mathcal{O}(\alpha^3)$

Further technical details and checks

- Two completely independent calculations of all ingredients
  $\hookrightarrow$ results in mutual agreement

- Combination of virtual and real corrections:
  dipole subtraction for QCD and photonic IR (soft and collinear) singularities
  Catani, Seymour '96; S.D. '99; S.D., Kabelschacht, Kasprzik '08
  $\hookrightarrow$ checked against one-cutoff and two-cutoff phase-space slicing
  Giele, Glover '92 Baur, Keller, Wackeroth '98, etc.
  GG + Kosower '93

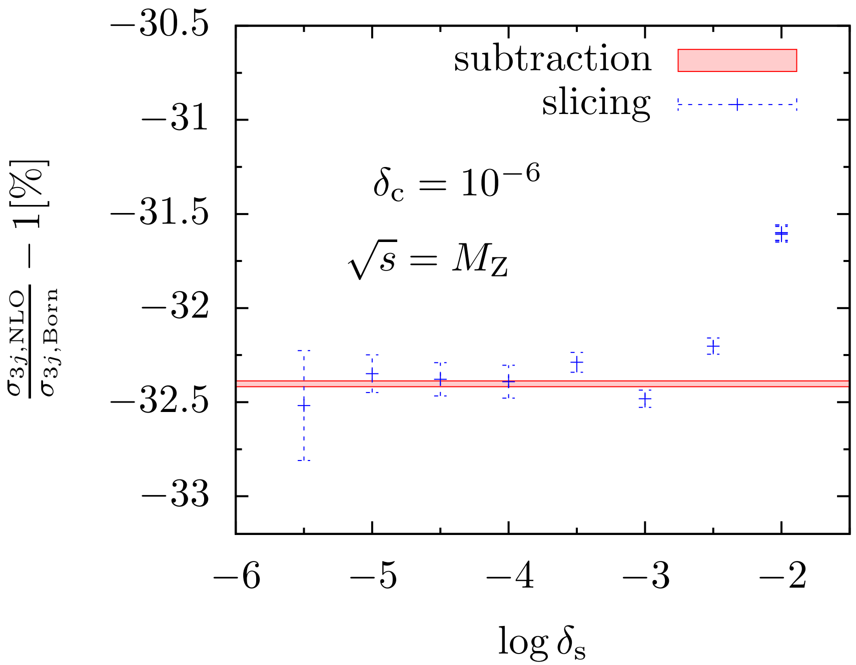
- Loop amplitude generation with FEYNARTS 1 or FEYNARTS 3
  Böhm, Denner, Küblbeck '90 Hahn '00

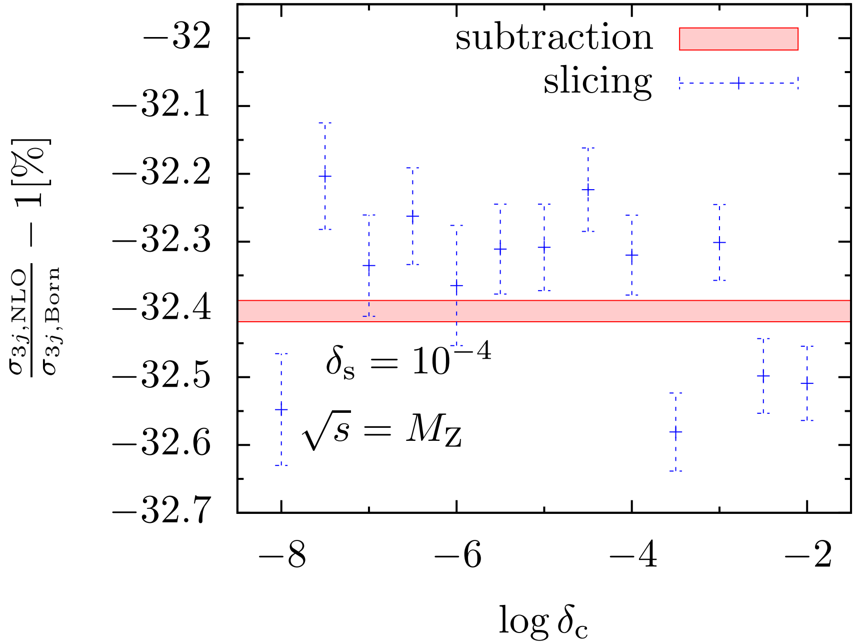
- Algebraic reduction of loops to standard form with FORMCALC Hahn '98–'09
  or inhouse MATHEMATICA routines

- Further checks:
  - ◇ UV finiteness
  - ◇ IR structure of loops (cancellation if soft/coll. singularities in loops+subtraction endpoint)
  - ◇ soft/coll. singularities treated in dimensional and mass regularization

EW corrections to the three-jet rate — dipole subtraction vs. two-cutoff slicing

$(y_{\rm cut} = 0.0006)$





(δ_s = soft cut, δ_c = collinear cut)

Here #(slicing events) $=$ $10 \times$ #(subtraction events)

$\hookrightarrow$ Dipole subtraction shows much better efficiency than phase-space slicing.

3 Definition of jet observables

Event selection: (closely following the procedure employed by ALEPH)

1. Discard particles too close to the beams, i.e. if $|\cos\theta_i| > \cos\theta_{\rm cut} = 0.965$.
2. Reject event if $M_{\rm visible} < 0.9E_{\rm CM}$.
3. Boost to CM system of observed final-state particles.
4. Apply Durham jet algorithm with E recombination and $y_{\rm cut} = 0.002$ to $q, \bar{q}, {\rm g}, \gamma$
   $\hookrightarrow$ photons appear inside jets
5. Reject "photonic events" where photon energy fraction $z > z_{\rm cut} = 0.9$ in a jet.

Subtleties arising at NLO EW level:

- Step 3 minimizes boost effects from collinear ISR photons
  (otherwise two-jet configurations do not always appear at event-shape endpoints)

  But: at LEP two-jet events were shifted to endpoints "by hand"
  $\hookrightarrow$ renders confrontation between theory and LEP results difficult

- Step 5 is not collinear safe
  $\hookrightarrow$ perturbative result is plagued by quark-mass singularities $\propto \alpha \ln m_q$
  Solution: include photon fragmentation function with non-perturbative input

Photon–jet separation via photon fragmentation function $D_{q\to\gamma}$

Why does a naive γ–jet separation by a jet algorithm not work ?

- collinear quarks and photons have to be recombined $\to$ $(q\gamma) =$ jet
  otherwise corrections $\propto \ln(m_q^2/Q^2)$ $\to$ perturbative “IR instability”
- quark and gluon jets cannot be distinguished event by event
  $\hookrightarrow$ common recombination required for quarks/gluons with photons

$$\Rightarrow \underbrace{(\mathrm{g}_{\mathrm{hard}}+\gamma_{\mathrm{soft}})}_{\text{EW corr. to 3 jets}} \text{ and } \underbrace{(\mathrm{g}_{\mathrm{soft}}+\gamma_{\mathrm{hard}})}_{\text{QCD corr. to 2 jets} + \gamma} \quad \text{both appear as 3 jets}$$

Solution:

- exclude events with photon energy fraction $z_\gamma = \dfrac{E_\gamma}{E_{\mathrm{jet}}+E_\gamma} > z_0$ for $(\mathrm{jet}+\gamma)$ quasiparticles
- subtract convolution of LO cross section with

$$D_{q\to\gamma}^{\overline{\mathrm{MS}}}(z_\gamma,\mu_{\mathrm{fact}})\Big|_{\mathrm{mass.reg.}} = P_{q\to\gamma}(z_\gamma)\left[\ln\frac{m_q^2}{\mu_{\mathrm{fact}}^2}+2\ln z_\gamma+1\right] \quad \leftarrow \text{cancels coll. singularities}$$

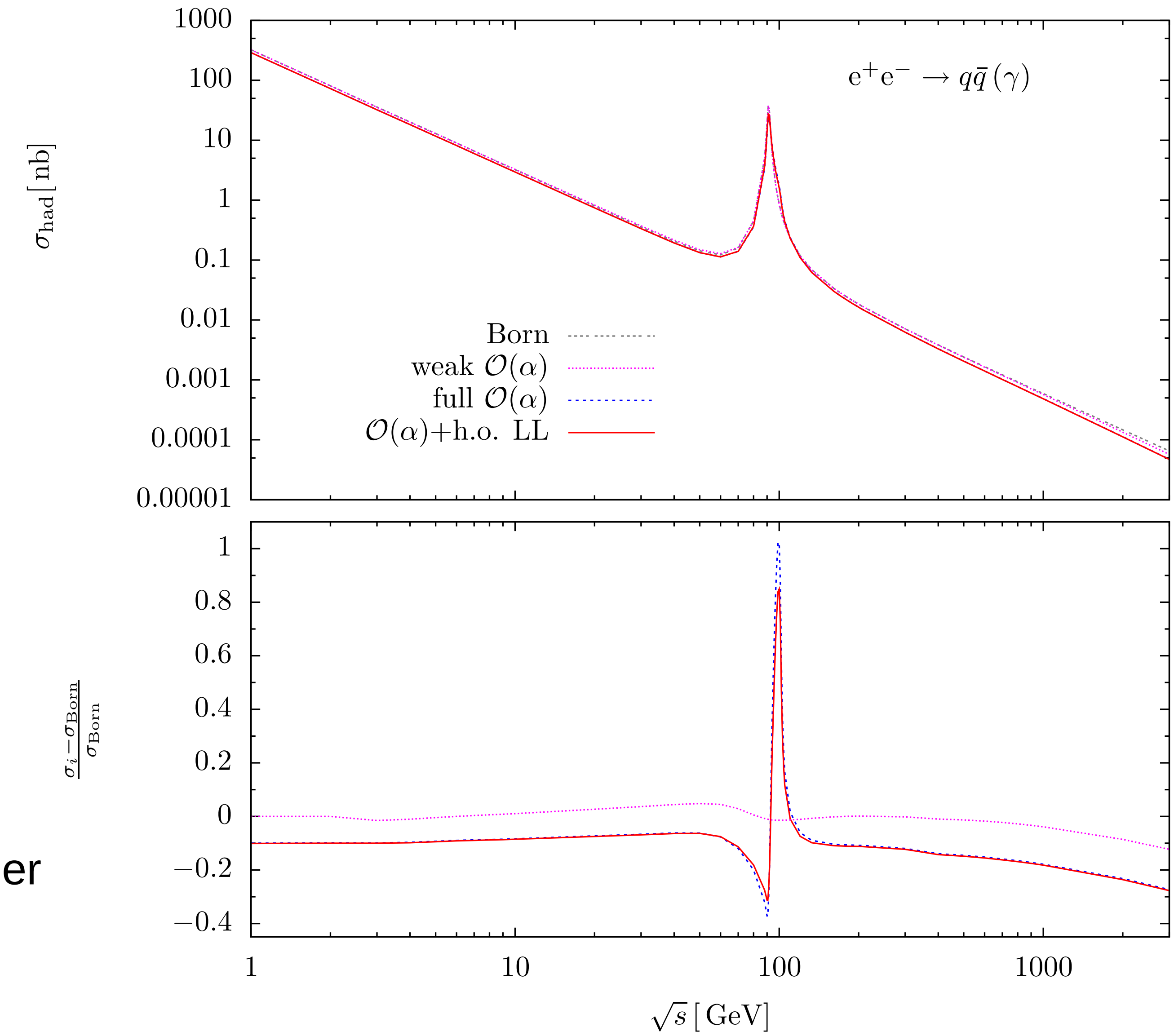
$$+\ D_{q\to\gamma}^{\mathrm{ALEPH}}(z_\gamma,\mu_{\mathrm{fact}}) \quad \leftarrow \text{non-perturbative part fitted to ALEPH data on } \mathrm{e}^+\mathrm{e}^- \to \mathrm{jet}+\gamma$$

where $P_{q\to\gamma}(z_\gamma) = \frac{1+(1-z_\gamma)^2}{z_\gamma}$ = quark-to-photon splitting function

4 Numerical results

Total hadronic cross section

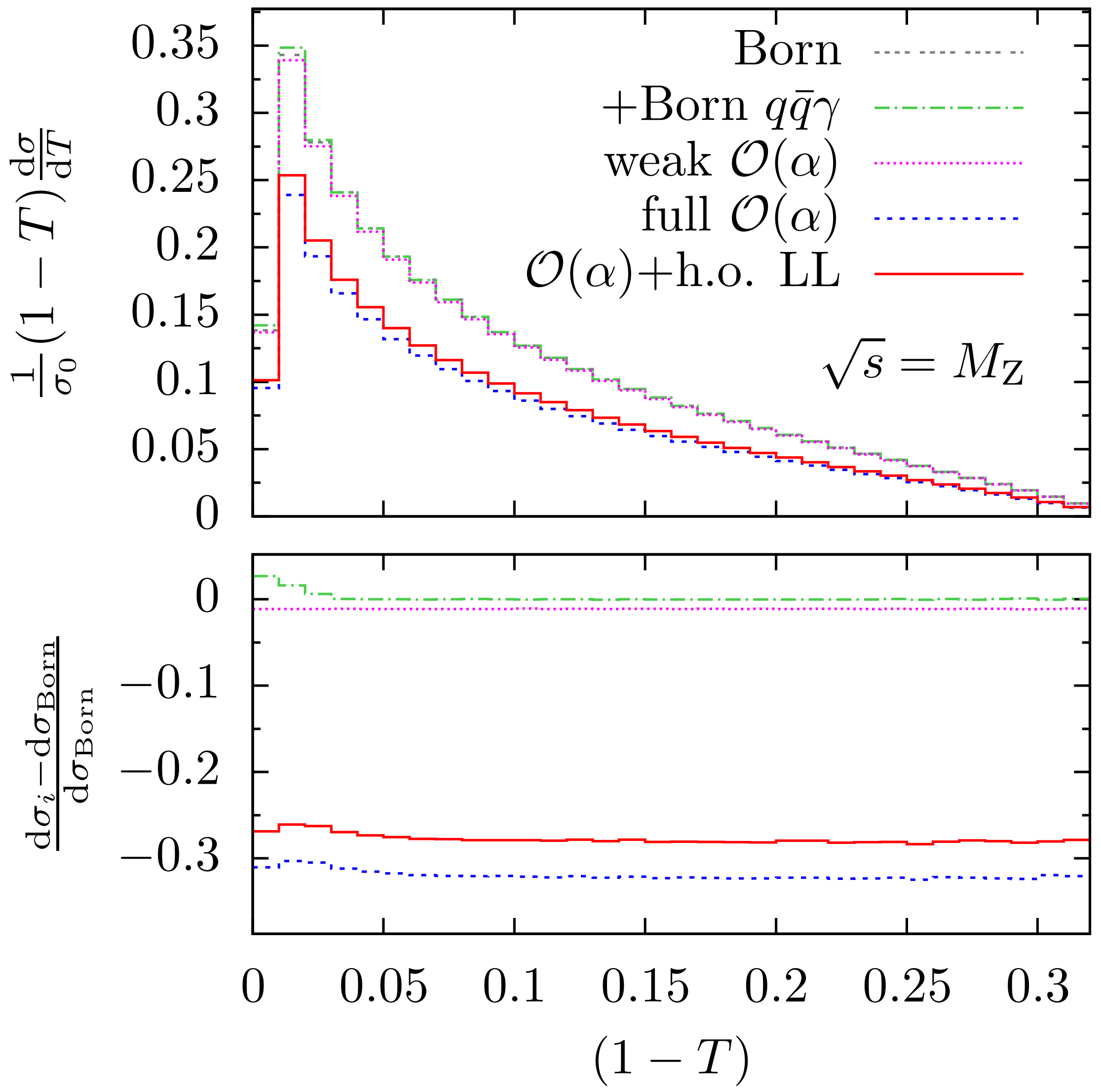
- largest EW corrections due to ISR
  (radiative return cut off by cut $M_{\mathrm{visible}} < 0.9\sqrt{s}$)
- ISR beyond one loop relevant (some $\%$) for $\sqrt{s} \sim M_{\mathrm{Z}}$
- weak corrs. of $\mathcal{O}(5\%)$, increasingly negative for large $\sqrt{s}$
- Note: σ_{had} calculated to same perturbative order as $\mathrm{d}\sigma/\mathrm{d}y$ to obtain a proper normalization



Thrust distribution — normalized to LO hadronic cross section σ_0

$\sqrt{s} = M_Z$

- large ISR effects as for $\sigma_{\rm had}$
- h.o. ISR $\sim$ some $\%$
- genuine weak effects of few $\%$, but flat
- $q\bar{q}\gamma$ final states visible for $T \to 1$



Thrust distribution — normalized to full hadronic cross section σ_{had}

$\sqrt{s} = M_{\mathrm{Z}}$

- large ISR effects in $\frac{\mathrm{d}\sigma}{\mathrm{d}T}$ cancel against σ_{had}

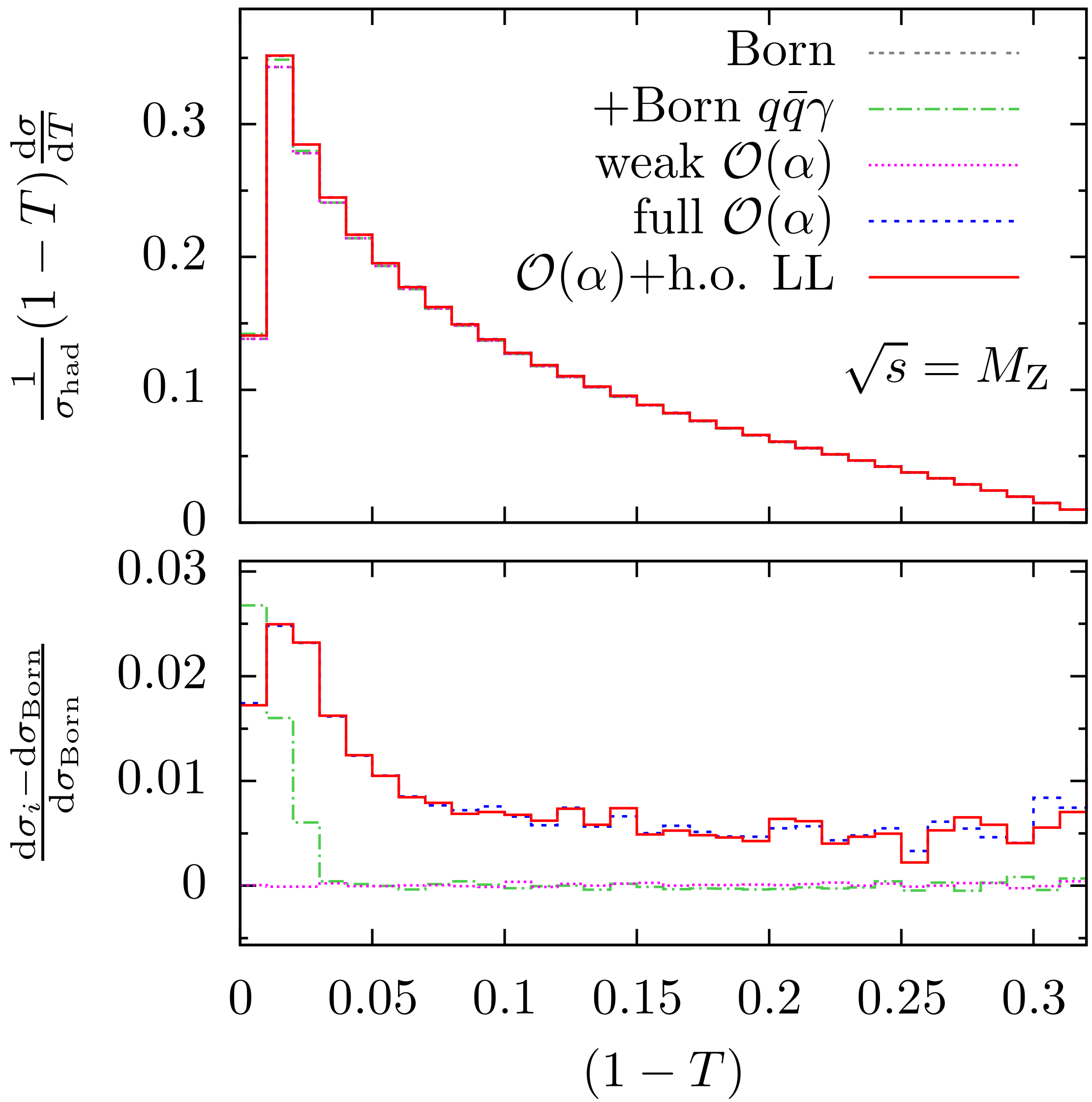
  $\hookrightarrow$ % effects

- h.o. ISR irrelevant

  (proper normalization important)

- genuine weak effects below 0.1%

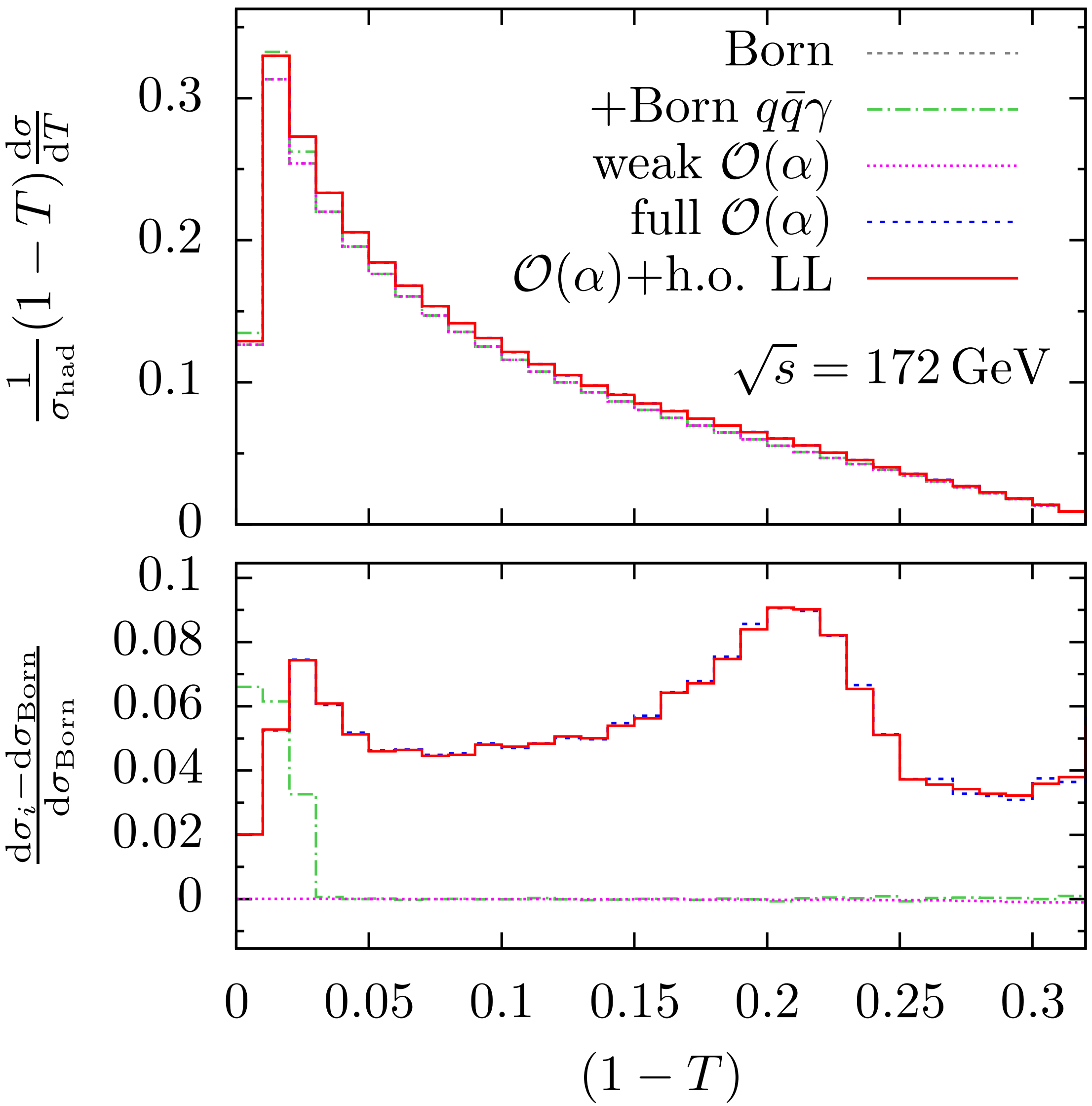
- $q\bar{q}\gamma$ final states $\sim \mathcal{O}(2\%)$ for $T \gtrsim 0.97$



Normalized thrust distribution — increasing CM energy

$\sqrt{s} = 172\,\mathrm{GeV}$

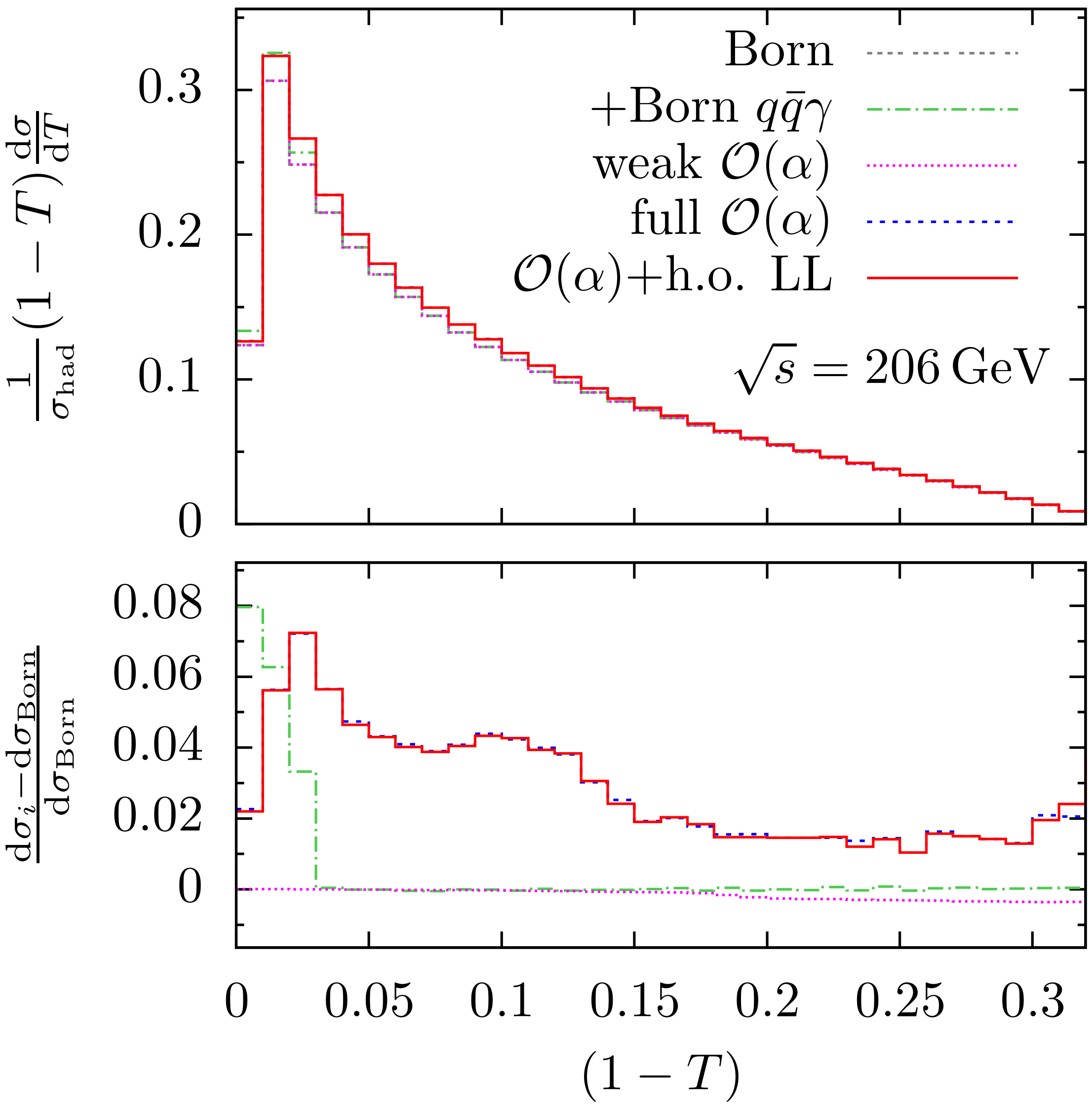
- ISR develops structures that depend on y_{cut} and on event selection
- h.o. ISR irrelevant
- genuine weak effects remain small (∼ 1% at $\sqrt{s} = 500\,\mathrm{GeV}$)
- $q\bar{q}\gamma$ final states ∼ some % for $T \gtrsim 0.97$



Normalized thrust distribution — increasing CM energy

$\sqrt{s} = 206\,\mathrm{GeV}$

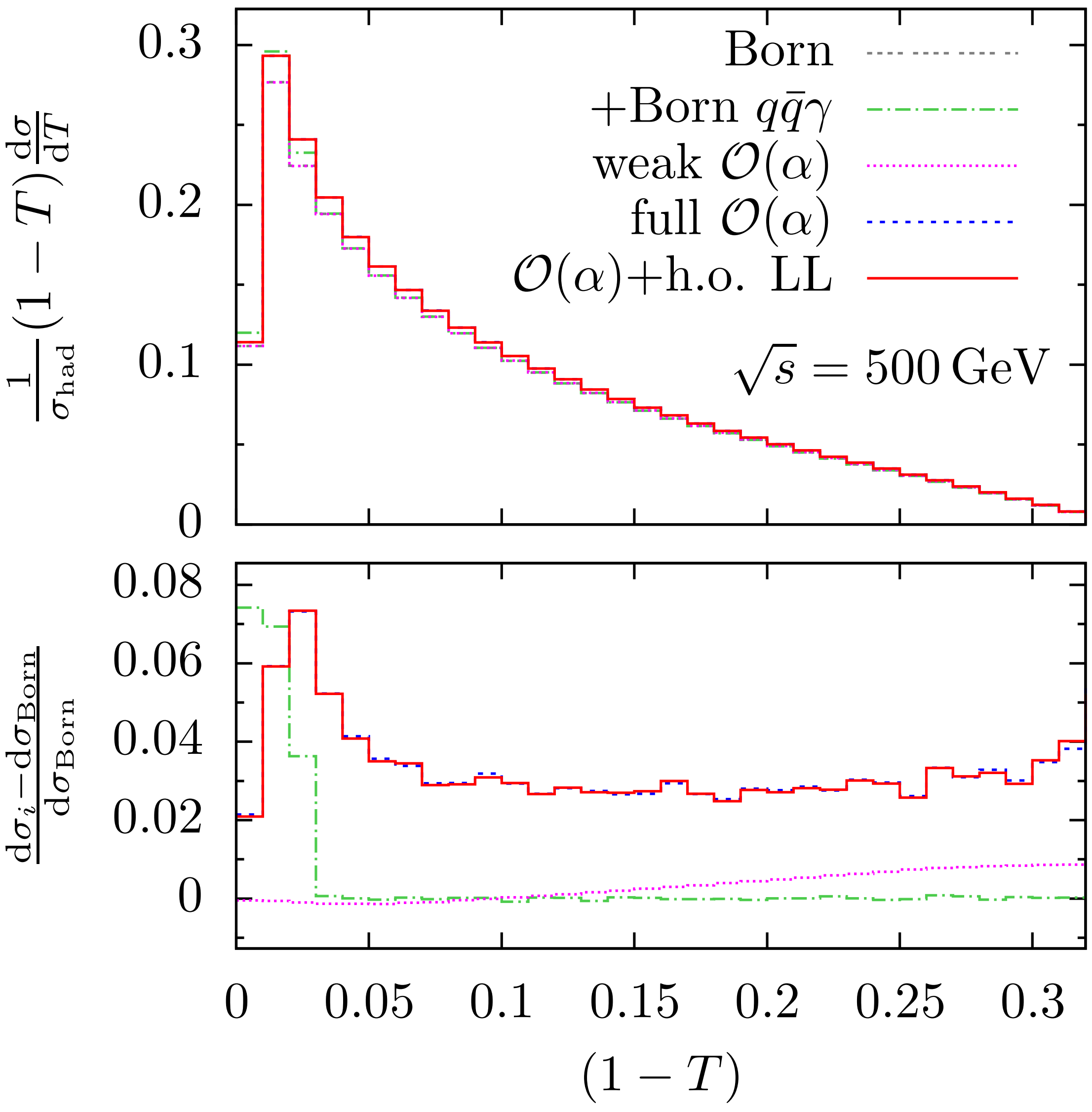
- ISR develops structures that depend on y_{cut} and on event selection
- h.o. ISR irrelevant
- genuine weak effects remain small (~ 1% at $\sqrt{s} = 500\,\mathrm{GeV}$)
- $q\bar{q}\gamma$ final states ~ some % for $T \gtrsim 0.97$



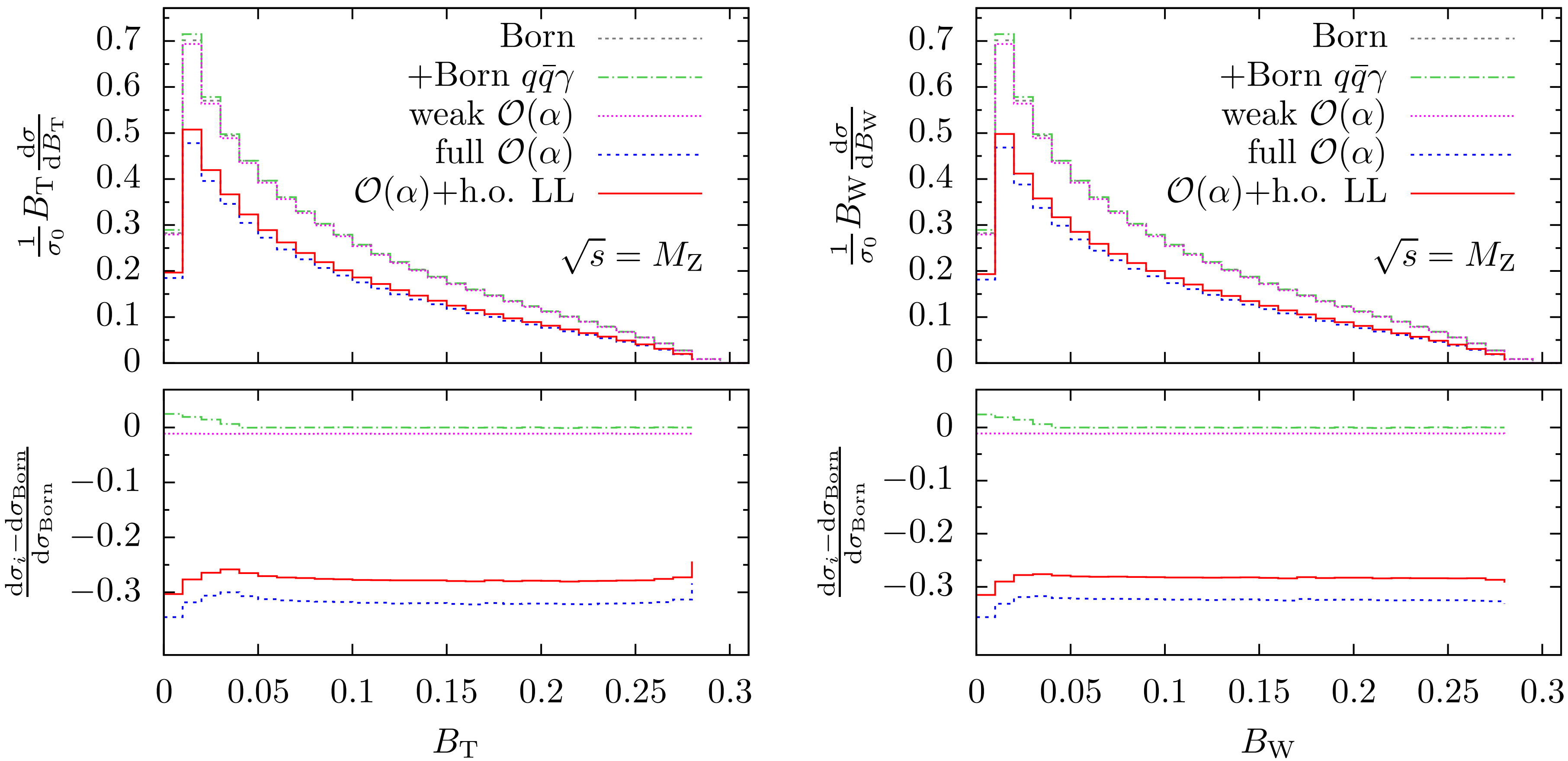
Normalized thrust distribution — increasing CM energy

$\sqrt{s} = 500\,\text{GeV}$

- ISR develops structures that depend on y_{cut} and on event selection
- h.o. ISR irrelevant
- genuine weak effects remain small ($\sim 1\%$ at $\sqrt{s} = 500\,\text{GeV}$)
- $q\bar{q}\gamma$ final states $\sim$ some $\%$ for $T \gtrsim 0.97$

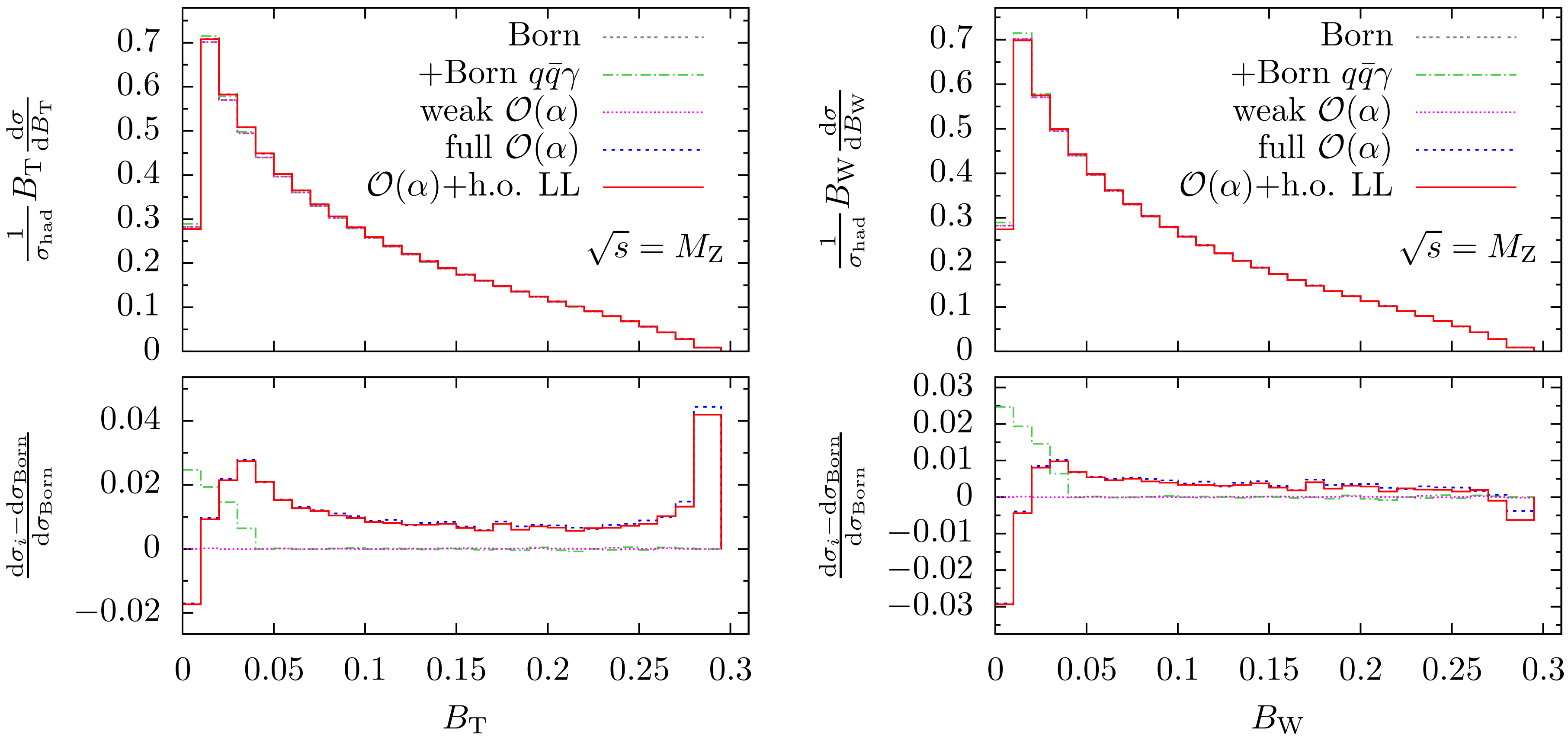


Jet broadenings — yet another example



Overall size, energy dependence, and qualitative features of EW effects similar for all event shapes.

Jet broadenings — yet another example



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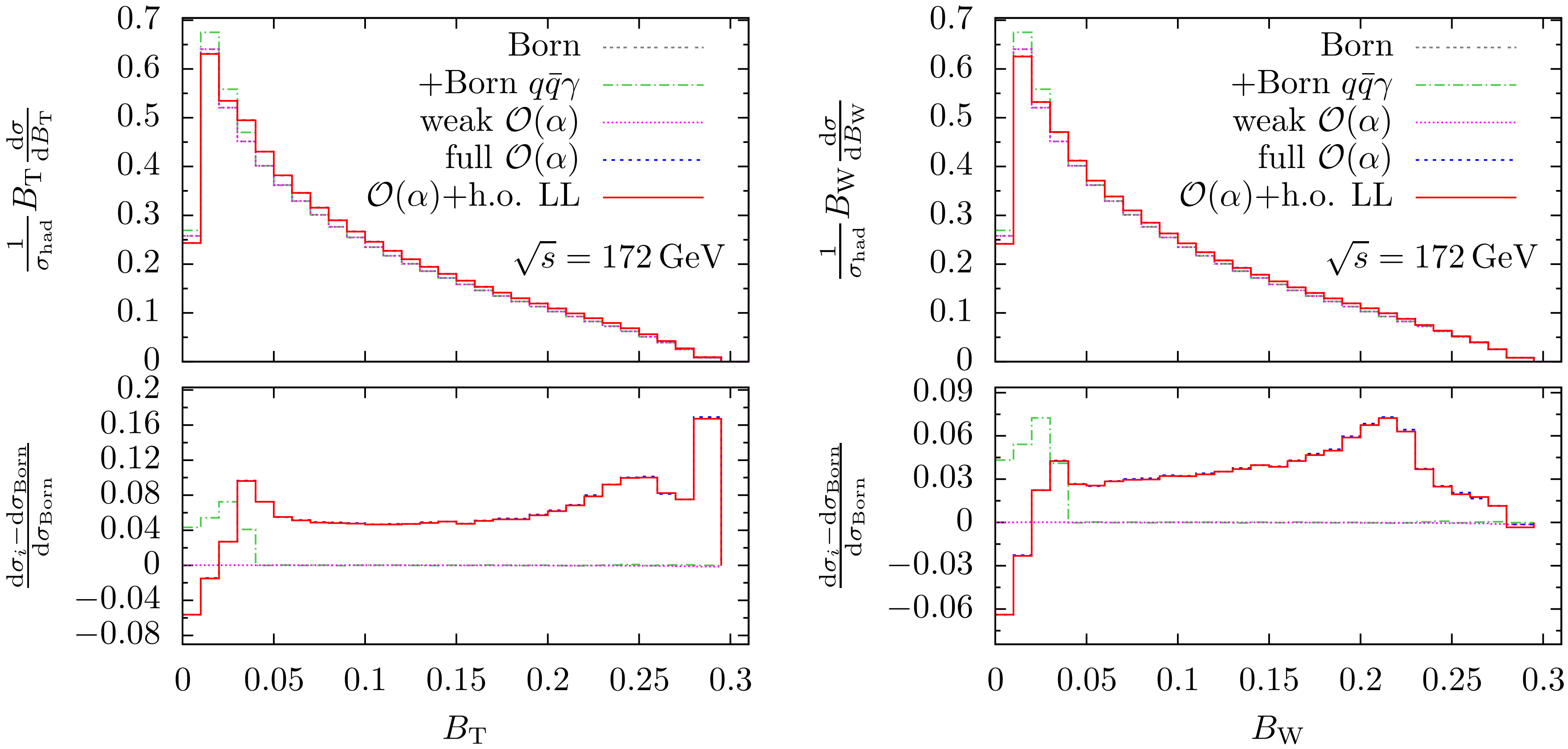

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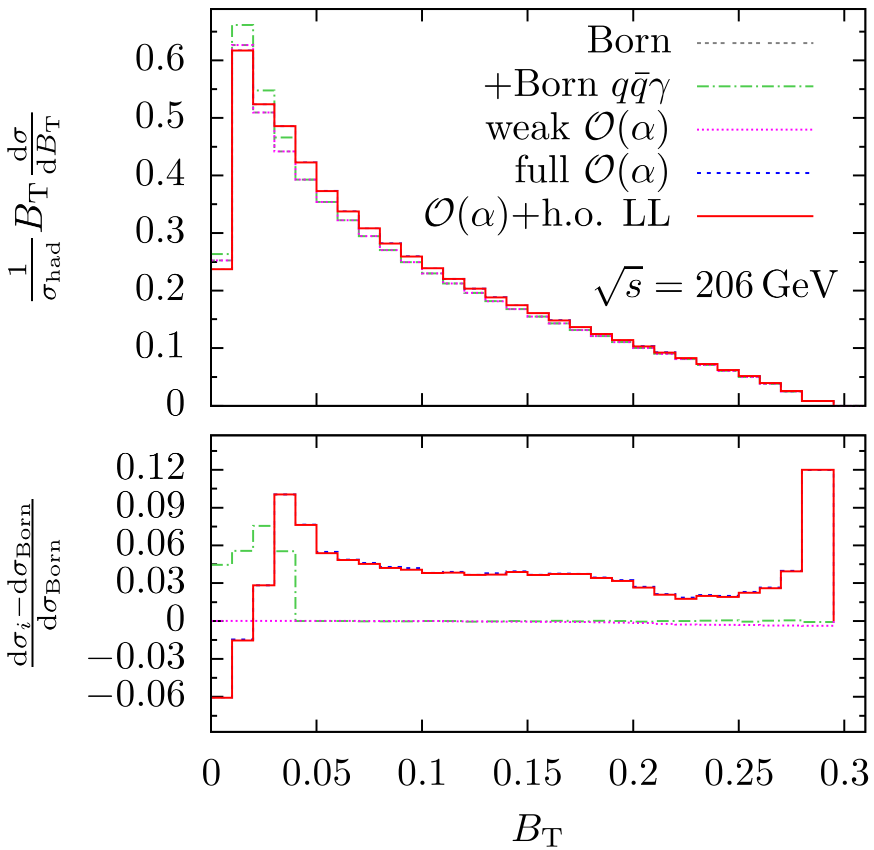

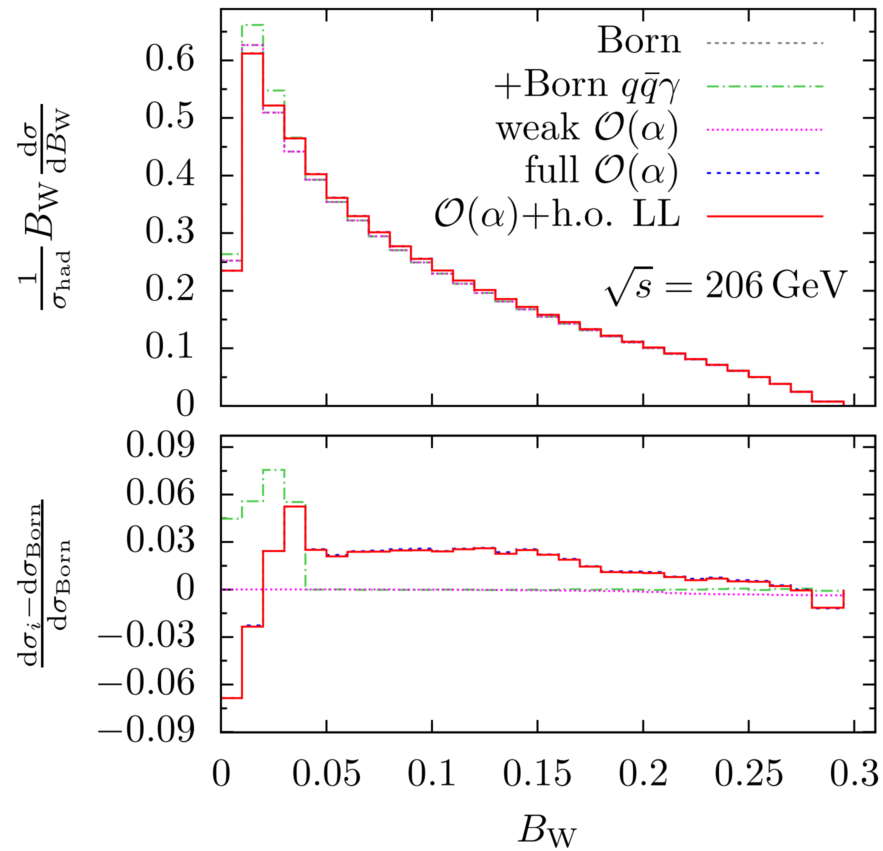

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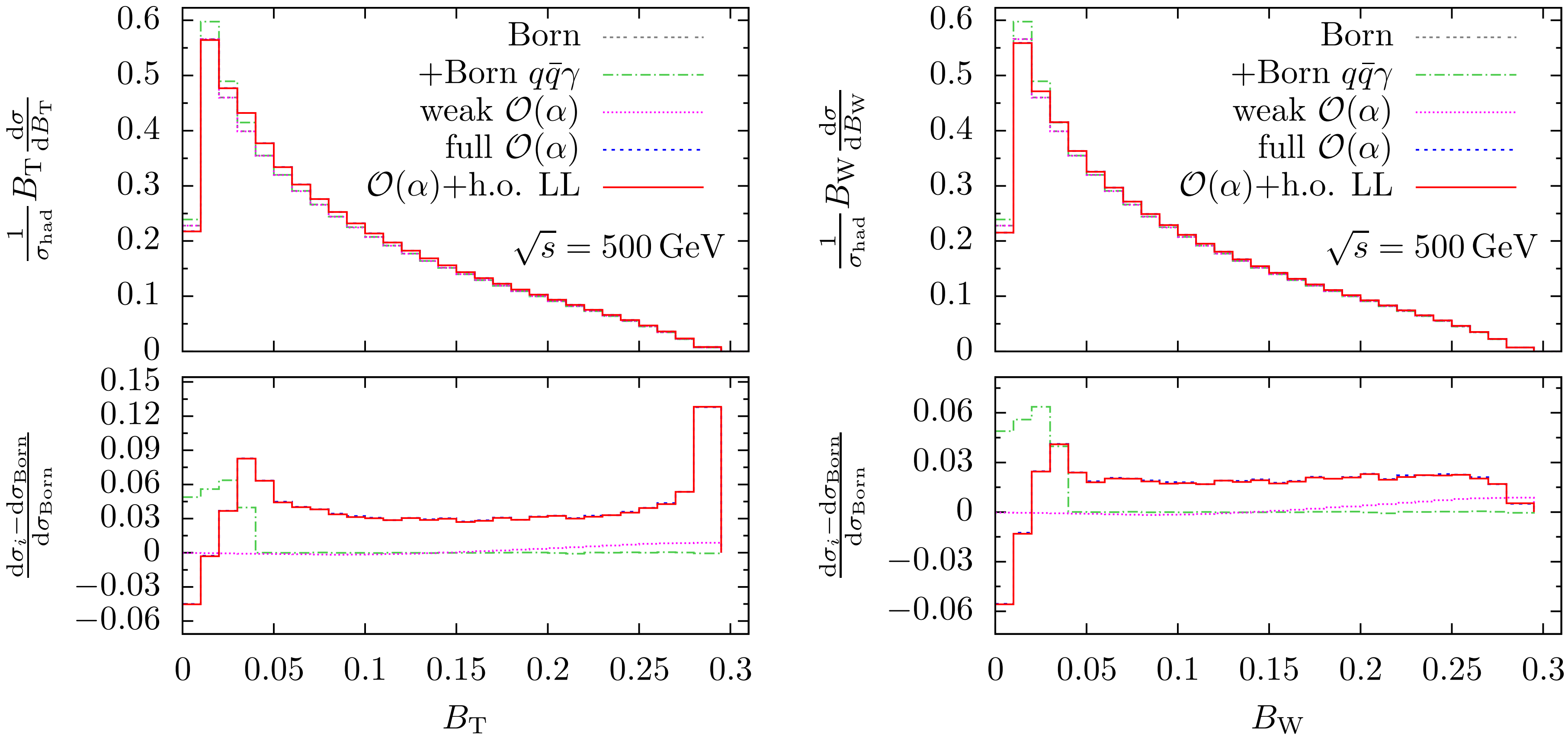
Jet broadenings — yet another example



Overall size, energy dependence, and qualitative features of EW effects similar for all event shapes.

5 Conclusions

Hadronic event shapes at e^+e^- colliders

- allow for precision measurements of α_s
- contribute to world average for $\alpha_s(M_Z)$

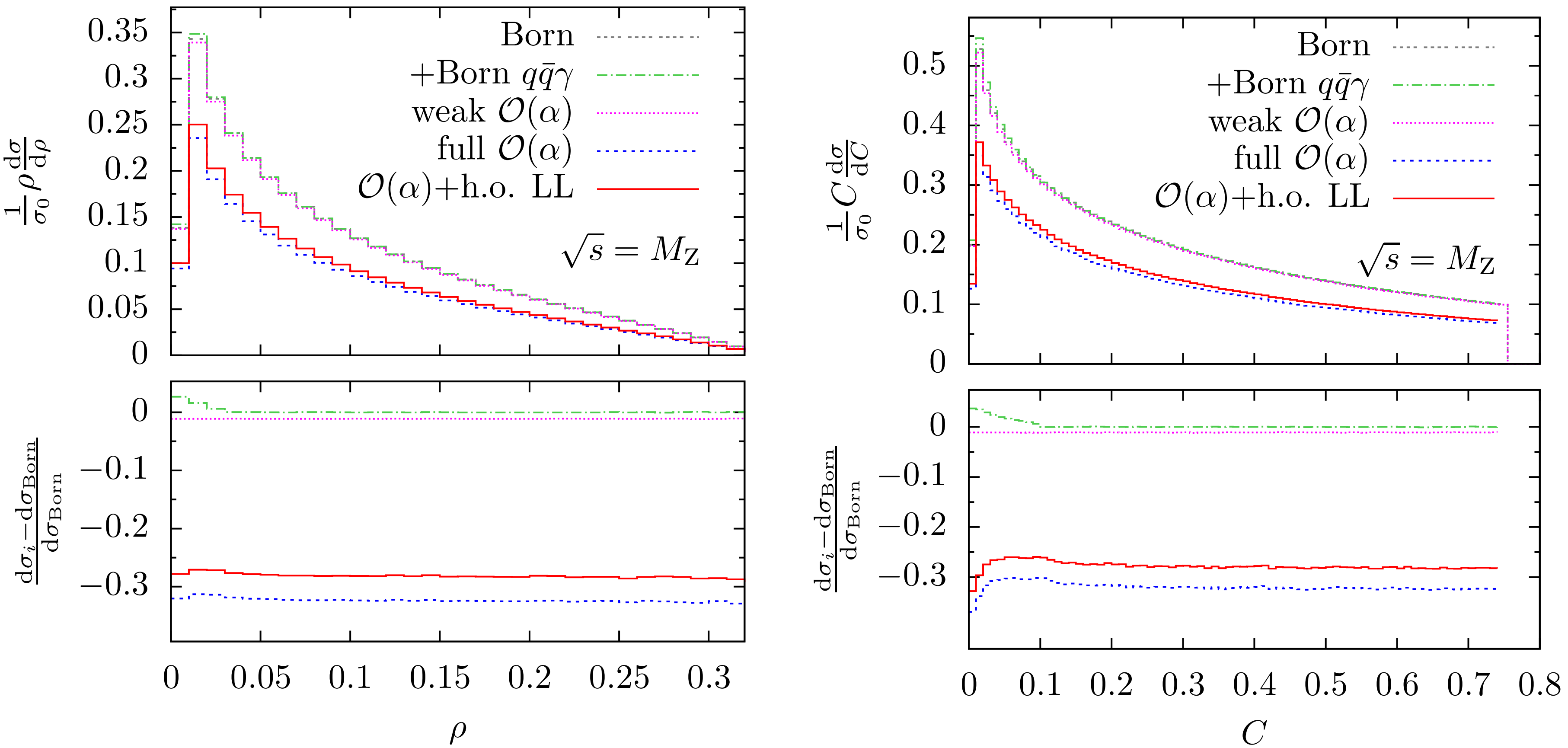
Predictions

- NNLO QCD contributions recently finished
  and matched to resummations / non-pert. effects
  $\hookrightarrow$ reduction of theoretical error in recent α_s fit
- NLO EW corrections now also available
  - ◇ largest EW corrections due to ISR,
    but reduction to $\%$ effects by normalization to σ_{had}
  - ◇ genuine weak effects negligible for LEP energies, at $\%$ level for $500\,GeV$ ILC
  - ◇ $q\bar{q}\gamma$ final states separated from 3-jet events via photon fragmentation function
    $\hookrightarrow$ $\%$ effects near 2-jet endpoints of event shapes

$\Rightarrow$ NLO EW corrections of minor relevance for JADE/LEP data,
but relevant at ILC energies

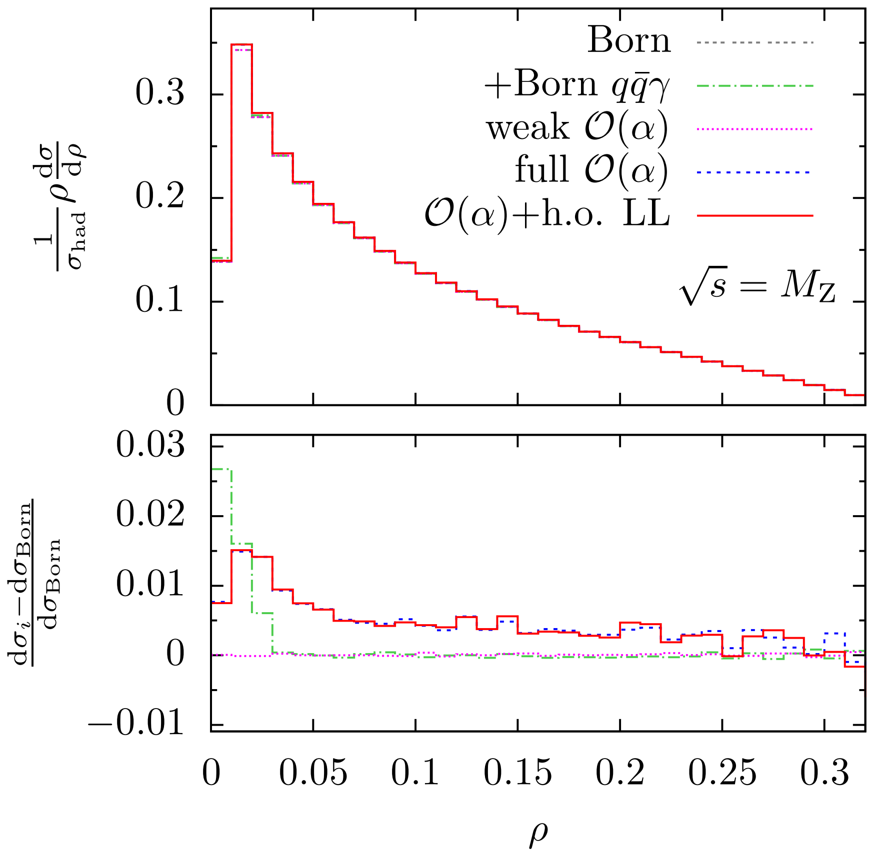
Backup slides

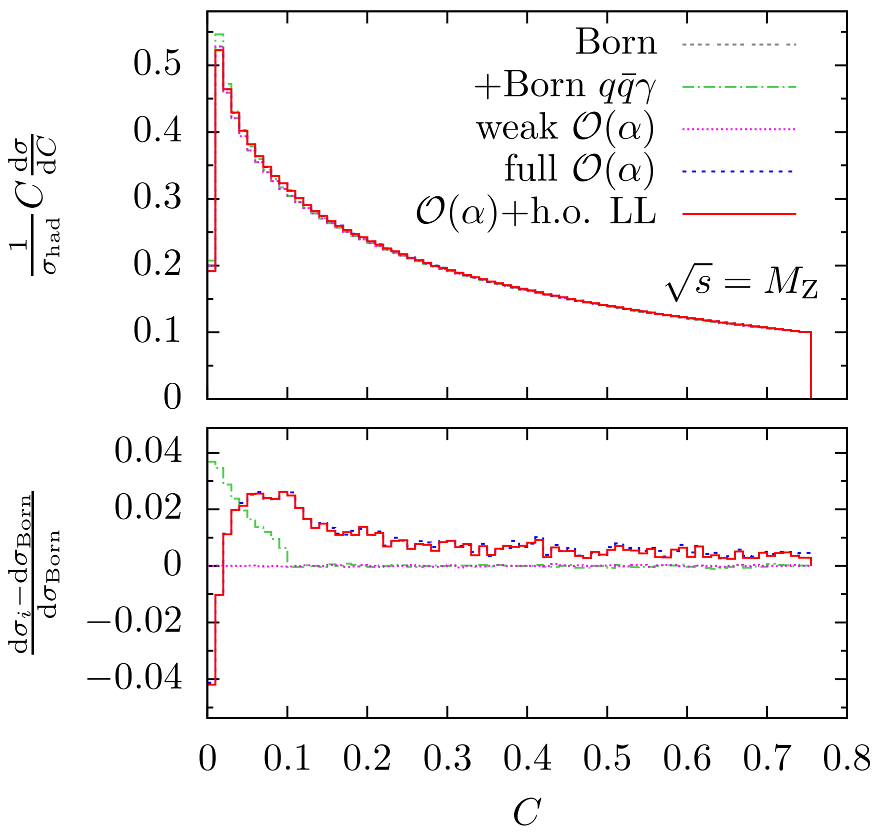
Heavy jet mass and C parameter



Overall size, energy dependence, and qualitative features of EW effects similar for all event shapes.

Heavy jet mass and C parameter





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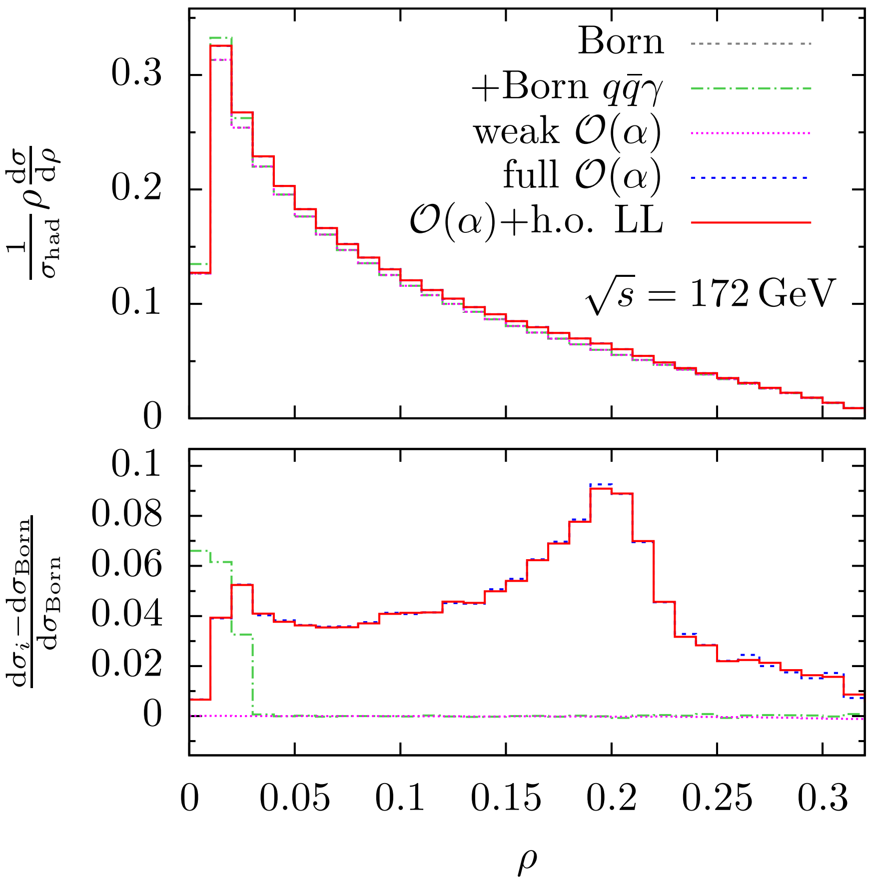
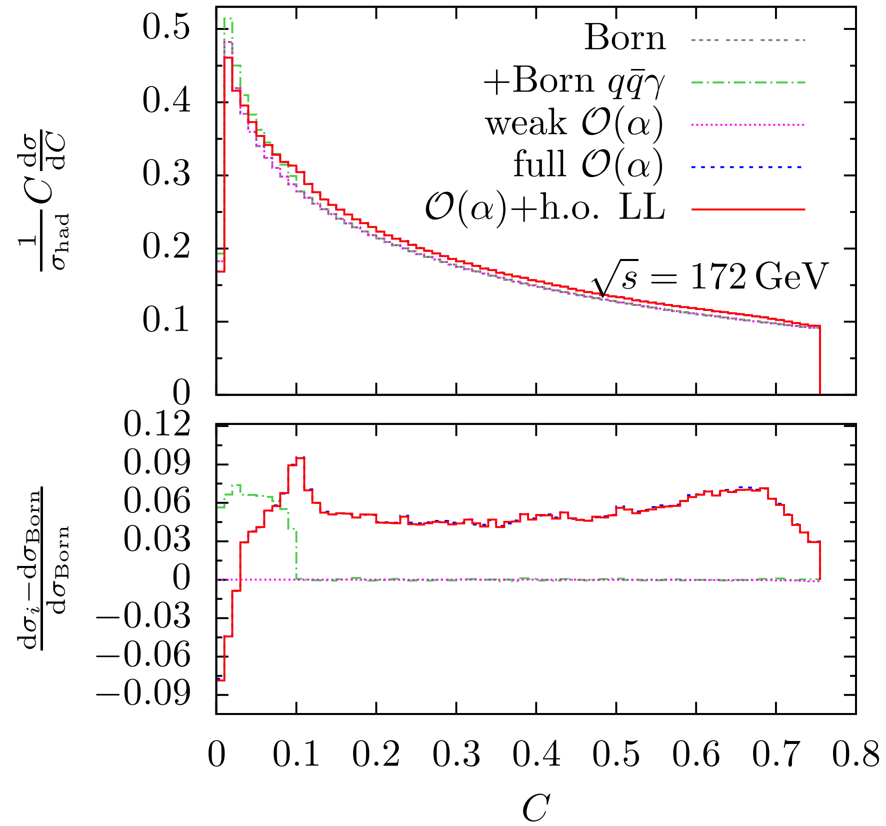
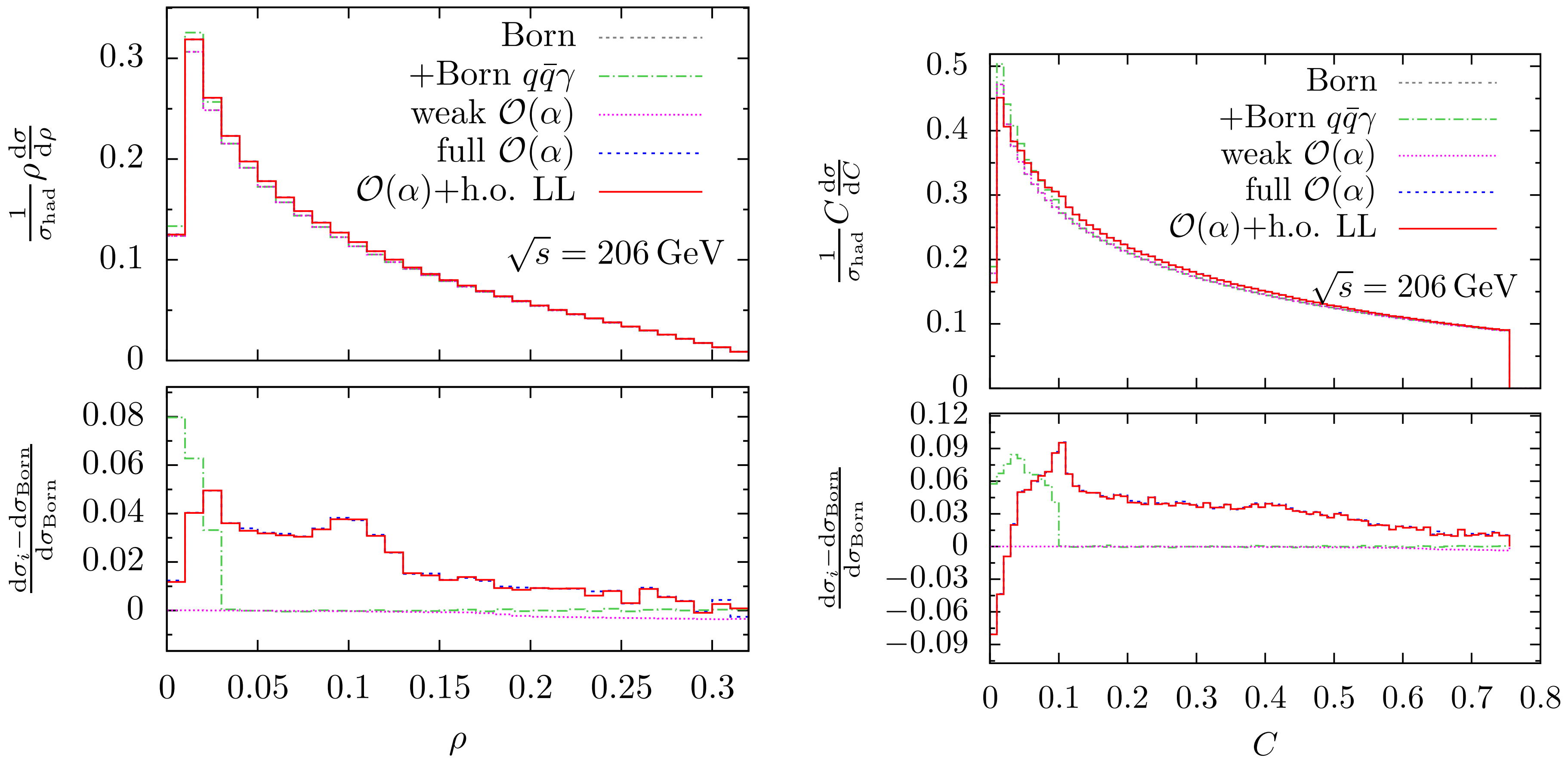
Heavy jet mass and C parameter





Overall size, energy dependence, and qualitative features of EW effects similar for all event shapes.

Heavy jet mass and C parameter



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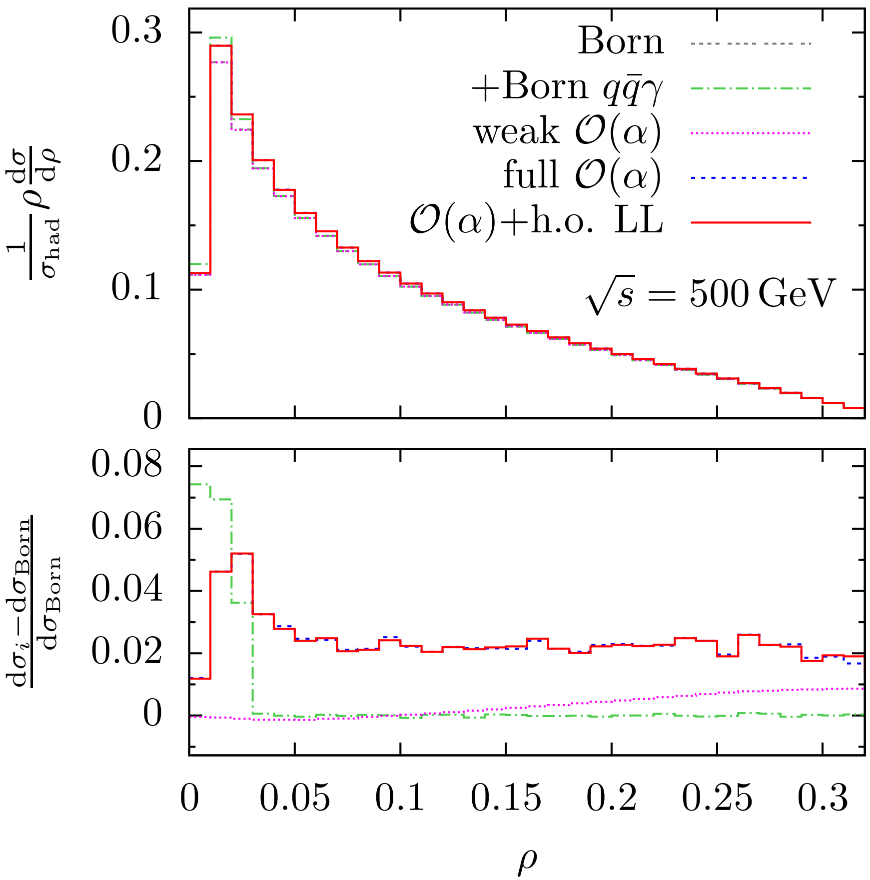
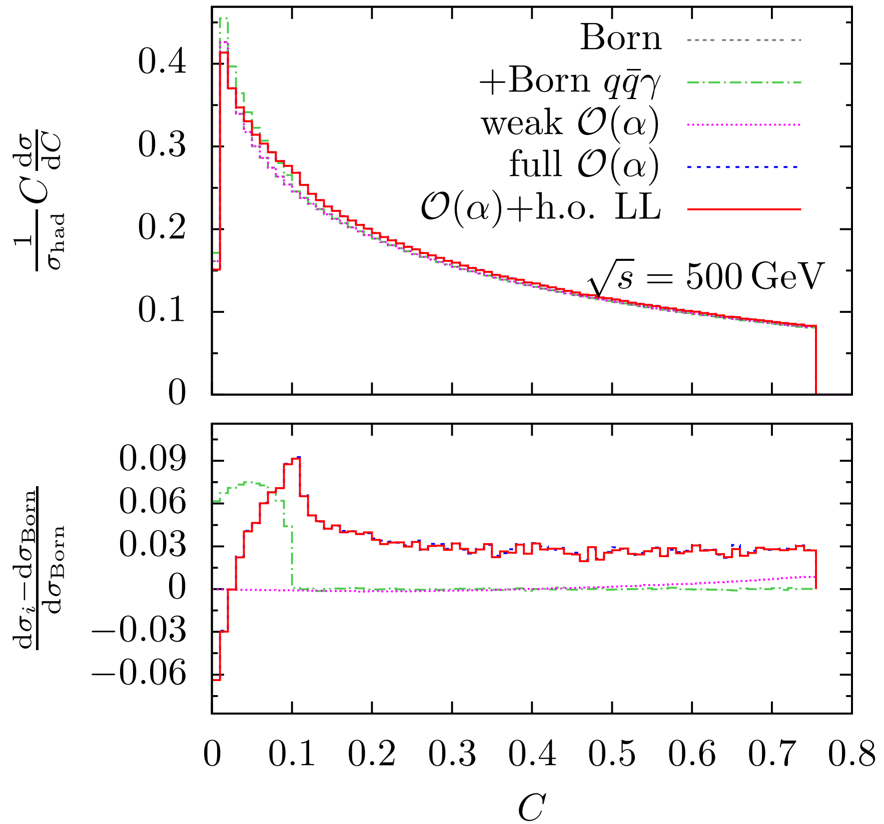
Heavy jet mass and C parameter





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