Automation of NLO computations using the FKS subtraction method

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in collaboration with

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Contents

- The FKS subtraction
- Automated in MadFKS
- Some results for MadFKS standalone (i.e. without virtual corrections)
- Results in collaboration with BlackHat and Rocket: e^+e^- -> jets at NLO

Real and Virtual corrections

$$\sigma^{\rm NLO} = \int_{m+1} d^{(d)}\sigma^R + \int_m d^{(d)}\sigma^V + \int_m d^{(4)}\sigma^B$$

- Contributions to a NLO computation
  - Real emission -> IR divergent
  - (UV-renormalized) virtual corrections -> IR divergent
  - Born contribution (finite)
- After integration, the sum of all contributions is finite (for infrared-safe observables)

Subtraction terms

$$\sigma^{\text{NLO}} = \int_{m+1} \left[d^{(4)}\sigma^{R} - d^{(4)}\sigma^{A} \right] + \int_{m} \left[d^{(4)}\sigma^{B} + \int_{\text{loop}} d^{(d)}\sigma^{V} + \int_{1} d^{(d)}\sigma^{A} \right]_{\epsilon=0}$$

- Include subtraction terms to make real emission contributions and virtual contributions separately finite
- Both contributions can be integrated numerically

FKS subtraction

- FKS subtraction: Frixione, Kunszt & Signer. Standard subtraction method in MC@NLO and POWHEG, but can also be used for ‘normal’ NLO computations
- Also known as “residue subtraction”
- Based on using plus-distributions to regulate the infrared divergences of the real emission matrix elements

FKS for beginners

- Easiest to understand by starting from real emission:

$$d\sigma^R = |M^{n+1}|^2 d\phi_{n+1}$$

- $|M^{n+1}|^2$ blows up like $\dfrac{1}{\xi_i^2}\dfrac{1}{1-y_{ij}}$ with $\begin{array}{l}\xi_i = E_i/\sqrt{\hat{s}} \\ y_{ij} = \cos\theta_{ij}\end{array}$

- Partition the phase space in such a way that each partition has at most one soft and one collinear singularity

$$d\sigma^R = \sum_{ij} S_{ij}|M^{n+1}|^2 d\phi_{n+1} \qquad\qquad \sum_{ij} S_{ij} = 1$$

- Use plus distributions to regulate the singularities

$$d\tilde{\sigma}^R = \sum_{ij}\left(\frac{1}{\xi_i}\right)_+\left(\frac{1}{1-y_{ij}}\right)_+ \xi_i(1-y_{ij})S_{ij}|M^{n+1}|^2 d\phi_{n+1}$$

FKS for beginners

$$d\tilde{\sigma}^R = \sum_{ij} \left(\frac{1}{\xi_i}\right)_+ \left(\frac{1}{1-y_{ij}}\right)_+ \xi_i(1-y_{ij}) S_{ij} |M^{n+1}|^2 d\phi_{n+1}$$

- Definition plus distribution

$$\int d\xi \left(\frac{1}{\xi}\right)_+ f(\xi) = \int d\xi \, \frac{f(\xi) - f(0)}{\xi}$$

- One event has maximally three counter events:
  - Soft: $\xi_i \to 0$
  - Collinear: $y_{ij} \to 1$
  - Soft-collinear: $\xi_i \to 0 \quad y_{ij} \to 1$

FKS FOR BEGINNERS

$$d\tilde{\sigma}^R = \sum_{ij} \left(\frac{1}{\xi_i}\right)_{\xi_{cut}} \left(\frac{1}{1-y_{ij}}\right)_{\delta_O} \xi_i (1-y_{ij}) S_{ij} |M^{n+1}|^2 d\phi_{n+1}$$

- Definition plus distribution

$$\int d\xi \left(\frac{1}{\xi}\right)_{\xi_{cut}} f(\xi) = \int d\xi \, \frac{f(\xi) - f(0)\Theta(\xi_{cut} - \xi)}{\xi}$$

- One event has maximally three counter events:
  - Soft: $\xi_i \to 0$
  - Collinear: $y_{ij} \to 1$
  - Soft-collinear: $\xi_i \to 0 \qquad y_{ij} \to 1$

Subtraction terms

$$\sigma^{\rm NLO} = \int_{m+1} \left[d^{(4)}\sigma^R - d^{(4)}\sigma^A \right] + \int_m \left[d^{(4)}\sigma^B + \int_{\rm loop} d^{(d)}\sigma^V + \int_1 d^{(d)}\sigma^A \right]_{\epsilon=0}$$

- This defines the subtraction terms for the reals
- They need to be integrated over the one parton (analytically) and added to the virtual corrections
- All formulae can be found in the MadFKS paper, arXiv:0908.4247

MadFKS

- Automatic FKS subtraction within the MadGraph/MadEvent framework
- Given the (n+1) process, it generates the real, all the subtraction terms and the Born processes
- For a NLO computation, only the finite parts of the virtual corrections needed from the user
- Phase-space integration integrates (n) and (n+1) body processes at the same time
- So far, only implemented for e^+e^- collisions, but no difficulties foreseen in hadronic initial states

Optimization

- Each phase space partition can be run completely independent of all the others -> genuine parallelization
- MadFKS uses the symmetry of the matrix elements to reduce the number of phase space partitions:
  - adding multiple gluons does not increase the complexity of the subtraction structure
- Within each phase space partition: usual MadGraph 'Single diagram enhanced multi-channel' phase space integration, but using the Born diagrams
- Born amplitudes are computed only once for each event, and used for the Born and collinear, soft and soft-collinear counter events

δ_O	$a_S = b_S$	$\xi_{cut} = \xi_{\max}$	$\xi_{cut} = 0.3$	$\xi_{cut} = 0.1$	$\xi_{cut} = 0.01$
		`useenergy=.true.`			
	1.0	3.5988 ± 0.0146	3.6173 ± 0.0122	3.6190 ± 0.0140	3.6126 ± 0.0141
2	1.5	3.6085 ± 0.0126	3.5942 ± 0.0143	3.5956 ± 0.0115	3.5989 ± 0.0133
	2.0	3.6127 ± 0.0121	3.6122 ± 0.0158	3.6020 ± 0.0147	3.5956 ± 0.0144
	1.0	3.6196 ± 0.0142	3.6012 ± 0.0139	3.5888 ± 0.0142	3.5833 ± 0.0130
0.6	1.5	3.5941 ± 0.0123	3.6012 ± 0.0139	3.6009 ± 0.0138	3.6047 ± 0.0114
	2.0	3.6066 ± 0.0120	3.6111 ± 0.0117	3.6053 ± 0.0110	3.5950 ± 0.0150
	1.0	3.6350 ± 0.0151	3.5927 ± 0.0145	3.5813 ± 0.0128	3.5811 ± 0.0146
0.2	1.5	3.6020 ± 0.0119	3.6086 ± 0.0133	3.6104 ± 0.0127	3.5993 ± 0.0119
	2.0	3.5815 ± 0.0140	3.5966 ± 0.0136	3.5938 ± 0.0121	3.6079 ± 0.0125
	1.0	3.6053 ± 0.0202	3.5998 ± 0.0181	3.5988 ± 0.0122	3.6088 ± 0.0165
0.06	1.5	3.6144 ± 0.0161	3.5986 ± 0.0140	3.5847 ± 0.0119	3.5884 ± 0.0126
	2.0	3.5990 ± 0.0166	3.6016 ± 0.0158	3.6014 ± 0.0147	3.6191 ± 0.0133
		`useenergy=.false.`			
	1.0	3.6078 ± 0.0164	3.6149 ± 0.0162	3.6145 ± 0.0158	3.6085 ± 0.0140
2	1.5	3.5695 ± 0.0156	3.5841 ± 0.0180	3.5975 ± 0.0165	3.5986 ± 0.0142
	2.0	3.5921 ± 0.0125	3.6260 ± 0.0211	3.6034 ± 0.0134	3.6007 ± 0.0149
	1.0	3.5891 ± 0.0199	3.5786 ± 0.0164	3.6084 ± 0.0232	3.5956 ± 0.0151
0.6	1.5	3.6083 ± 0.0152	3.5944 ± 0.0136	3.6040 ± 0.0123	3.6018 ± 0.0147
	2.0	3.5838 ± 0.0141	3.5633 ± 0.0154	3.5964 ± 0.0129	3.5920 ± 0.0158
	1.0	3.5976 ± 0.0171	3.5790 ± 0.0166	3.5702 ± 0.0155	3.6155 ± 0.0132
0.2	1.5	3.5804 ± 0.0163	3.5925 ± 0.0136	3.6012 ± 0.0137	3.6091 ± 0.0138
	2.0	3.5978 ± 0.0148	3.5749 ± 0.0144	3.5825 ± 0.0128	3.5902 ± 0.0145
	1.0	3.6122 ± 0.0170	3.5942 ± 0.0158	3.5743 ± 0.0146	3.5962 ± 0.0167
0.06	1.5	3.6064 ± 0.0198	3.5977 ± 0.0136	3.6047 ± 0.0115	3.5886 ± 0.0123
	2.0	3.5971 ± 0.0169	3.6018 ± 0.0136	3.5991 ± 0.0148	3.6040 ± 0.0148

Table 1: Cross section (in pb) and Monte Carlo integration errors for the $(n+1)$-body process $e^+e^- \to Z \to u\bar{u}ggg$. See the text for details.

- Our 'benchmark process': e^+e^- -> Z -> uubar ggg
- Result is independent of internal (non-physical) parameters
- Also the integration uncertainty is independent of the choice for the internal parameters
- run-time: 1-4 minutes for each integration channel

Six-fold increase of the statistics:

δ_O	$a_S = b_S$	$\xi_{cut} = \xi_{\max}$	$\xi_{cut} = 0.3$	$\xi_{cut} = 0.1$	$\xi_{cut} = 0.01$
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	1.0	3.5988 ± 0.0146	3.6173 ± 0.0122	3.6190 ± 0.0140	3.6126 ± 0.0141
2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	1.0	3.6196 ± 0.0142	3.6012 ± 0.0139	3.5888 ± 0.0142	3.5833 ± 0.0130
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	2.0	[illegible]	3.6111 ± 0.0117	3.6053 ± 0.0110	3.5950 ± 0.0150
	1.0	3.6350 ± 0.0151	[illegible]$.5927 \pm 0.0145$	3.5813 ± 0.0128	3.5811 ± 0.0146
0.2	1.5	[illegible]	$3.6086 \pm$ [illegible]	[illegible]	3.5993 ± 0.0119
	2.0	3.5815 ± 0.0140	[illegible]	[illegible]	[illegible]$.6079 \pm 0.0125$
	1.0	3.6053 ± 0.0202	3.5998 [illegible]	[illegible]	[illegible]$.6088 \pm 0.0165$
0.06	1.5	3.6144 ± 0.0161	$3.5986 \pm$ [illegible]	[illegible]	3.5884 ± 0.0126
	2.0	3.5990 ± 0.0166	3.6016 ± 0.0158	3.6014 ± 0.0147	3.6191 ± 0.0133
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	1.0	3.6078 ± 0.0164	3.6149 ± 0.0162	3.6145 ± 0.0158	3.6085 ± 0.0140
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0.6	1.5	3.6083 ± 0.0152	[illegible]	3.6040 ± 0.0123	3.6018 ± 0.0147
	2.0	3.5838 ± 0.014[illegible]	3.5633 ± 0.0154	[illegible]$.5964 \pm 0.0129$	3.5920 ± 0.0158
	1.0	3.5976 ± 0.0171	[illegible]	[illegible]	[illegible]
0.2	1.5	3.5804 ± 0.0163	3.5925 ± 0.0136	3.6012 [illegible]	[illegible]
	2.0	3.5978 ± 0.0148	3.5749 ± 0.0144	3.5825 [illegible]	[illegible]
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0.06	1.5	3.6064 ± 0.0198	3.5977 ± 0.0136	3.6047 ± 0.0115	3.5886 ± 0.0123
	2.0	3.5971 ± 0.0169	3.6018 ± 0.0136	3.5991 ± 0.0148	3.6040 ± 0.0148

3.6007 ± 0.0053

3.6086 ± 0.0051

Table 1: Cross section (in pb) and Monte Carlo integration errors for the $(n+1)$-body process $e^+e^- \to Z \to u\bar{u}ggg$. See the text for details.

- Our 'benchmark process': e^+e^- -> Z -> uubar ggg
- Result is independent of internal (non-physical) parameters
- Also the integration uncertainty is independent of the choice for the internal parameters
- run-time: 1-4 minutes for each integration channel

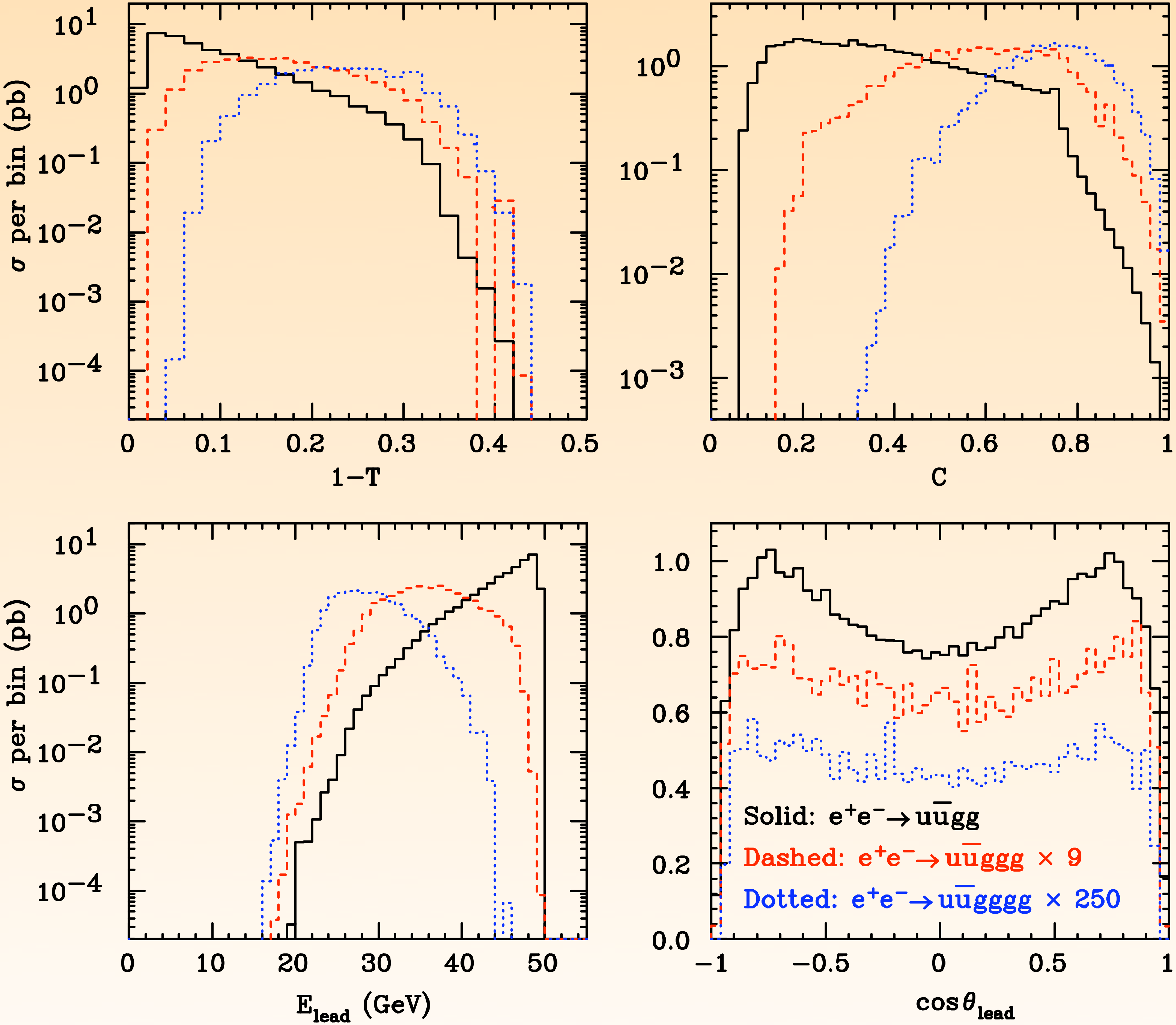
$(n+1)$-body process	cross section	$\overline{N}_{\text{FKS}}$	iterations $\times$ points	N_{ch}	ϵ
$e^+e^- \to Z \to u\bar{u}gg$	$(0.4144 \pm 0.0006\ (0.15\%))\times 10^{2}$	3	10×50k	6	0.536
$e^+e^- \to Z \to u\bar{u}ggg$	$(0.3601 \pm 0.0014\ (0.38\%))\times 10^{1}$	3	10×50k	18	0.167
$e^+e^- \to Z \to u\bar{u}gggg$	$(0.8869 \pm 0.0054\ (0.61\%))\times 10^{-1}$	3	10×350k	52	0.031
$e^+e^- \to \gamma^*/Z \to jjjj$	$(0.1801 \pm 0.0002\ (0.12\%))\times 10^{3}$	14	10×50k	56	0.520
$e^+e^- \to \gamma^*/Z \to jjjjj$	$(0.1529 \pm 0.0004\ (0.26\%))\times 10^{2}$	30	10×50k	328	0.171
$e^+e^- \to \gamma^*/Z \to jjjjjj$	$(0.3954 \pm 0.0015\ (0.38\%))\times 10^{0}$	55	10×350k	2450	0.033
$e^+e^- \to Z \to t\bar{t}gg$	$(0.1219 \pm 0.0003\ (0.24\%))\times 10^{-1}$	3	10×10k	6	0.899
$e^+e^- \to Z \to t\bar{t}ggg$	$(0.1521 \pm 0.0013\ (0.83\%))\times 10^{-2}$	3	10×10k	18	0.708
$e^+e^- \to Z \to t\bar{t}gggg$	$(0.1108 \pm 0.0031\ (2.76\%))\times 10^{-3}$	3	10×20k	52	0.427
$e^+e^- \to Z \to t\bar{t}b\bar{b}g$	$(0.1972 \pm 0.0024\ (1.23\%))\times 10^{-4}$	4	10×10k	16	1.000
$e^+e^- \to Z \to t\bar{t}b\bar{b}gg$	$(0.2157 \pm 0.0029\ (1.34\%))\times 10^{-4}$	5	10×10k	120	0.824
$e^+e^- \to Z \to \tilde{t}_1\bar{\tilde{t}}_1ggg$	$(0.3712 \pm 0.0037\ (1.00\%))\times 10^{-8}$	3	10×10k	18	0.764
$e^+e^- \to Z \to \tilde{g}\tilde{g}ggg$	$(0.1584 \pm 0.0020\ (1.23\ \%))\times 10^{-1}$	2	10×10k	9	0.753
$\mu^+\mu^- \to H \to gggg$	$(0.1404 \pm 0.0005\ (0.34\ \%))\times 10^{-7}$	1	10×50k	2	0.559
$\mu^+\mu^- \to H \to ggggg$	$(0.2575 \pm 0.0018\ (0.69\ \%))\times 10^{-8}$	1	10×50k	4	0.165
$\mu^+\mu^- \to H \to gggggg$	$(0.1186 \pm 0.0008\ (0.70\ \%))\times 10^{-9}$	1	10×350k	9	0.031

- Compared to the Born the error is only 1.9-4.5 times larger with the same statistics*

* 2 exceptions; ttbbg: 7 & ttgggg: 9

Further optimization (not yet used)

- The results presented here do not use possible optimization related to
  - running the important integration channels with higher statistics
  - using the Monte Carlo to sum over the helicities of the external particles
- Diagram information is only used for defining the integration channels: use recursive relations for the rest?
- More improvements possible for treatment of massive quarks: under investigation



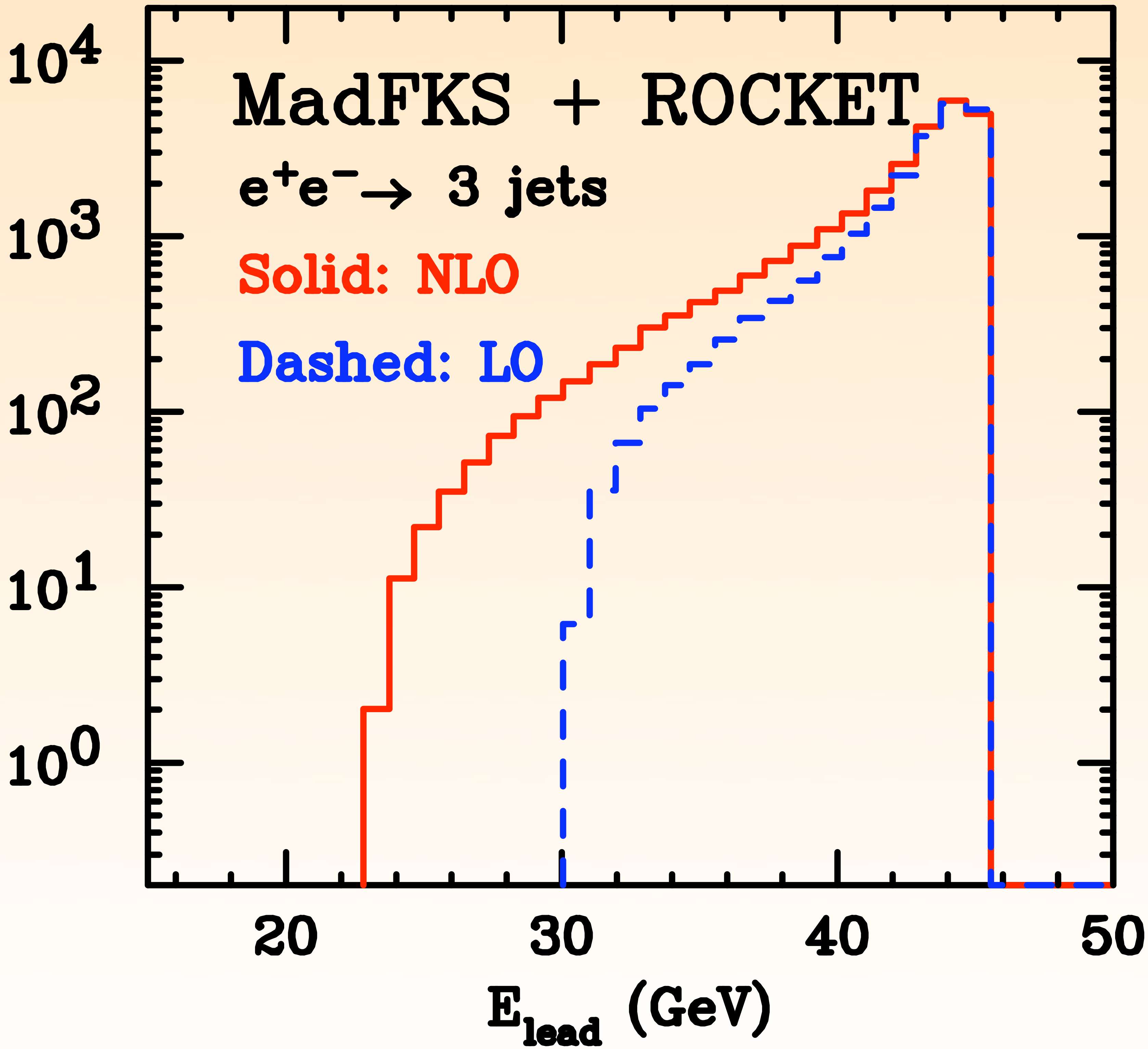
- Sqrt(s)=100 GeV
- ren. & fac. scales equal to Z mass
- kt jet clustering with $Y_{cut}=(10\ \mathrm{GeV})^2$
- **Finite part of virtual correction not included**

- Same runs as in the table: no ‘smoothing’ of the plots
- fine binning, and smooth results

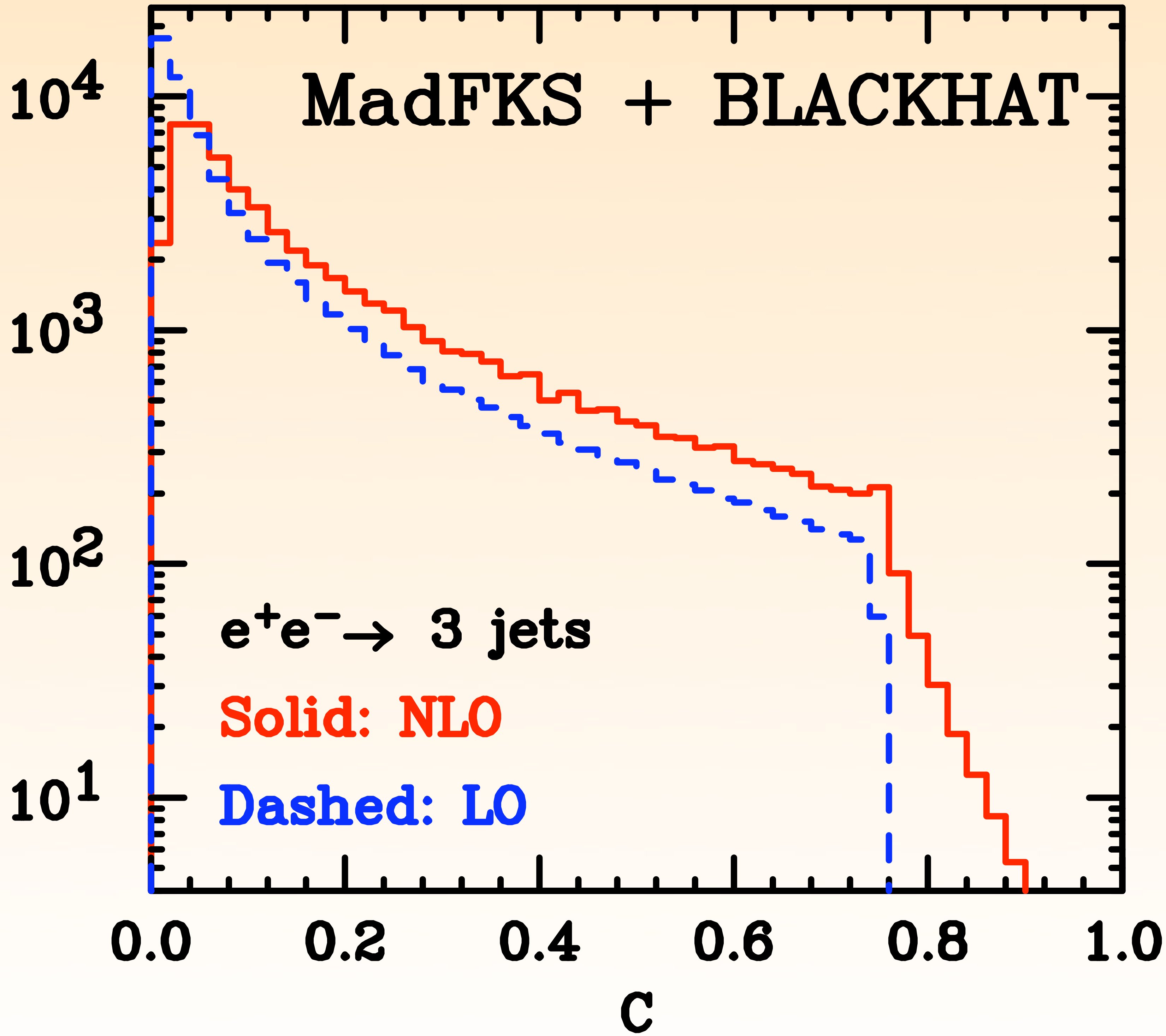
Full NLO

- Of course, to get the total NLO results the finite parts of the virtual corrections should be included as well
- Les Houches interface available
- Working interfaces to BLACKHAT and ROCKET for the finite part of the virtual corrections
- *Many thanks to Daniel Maitre and Giulia Zanderighi*

Some Results...



Some Results...



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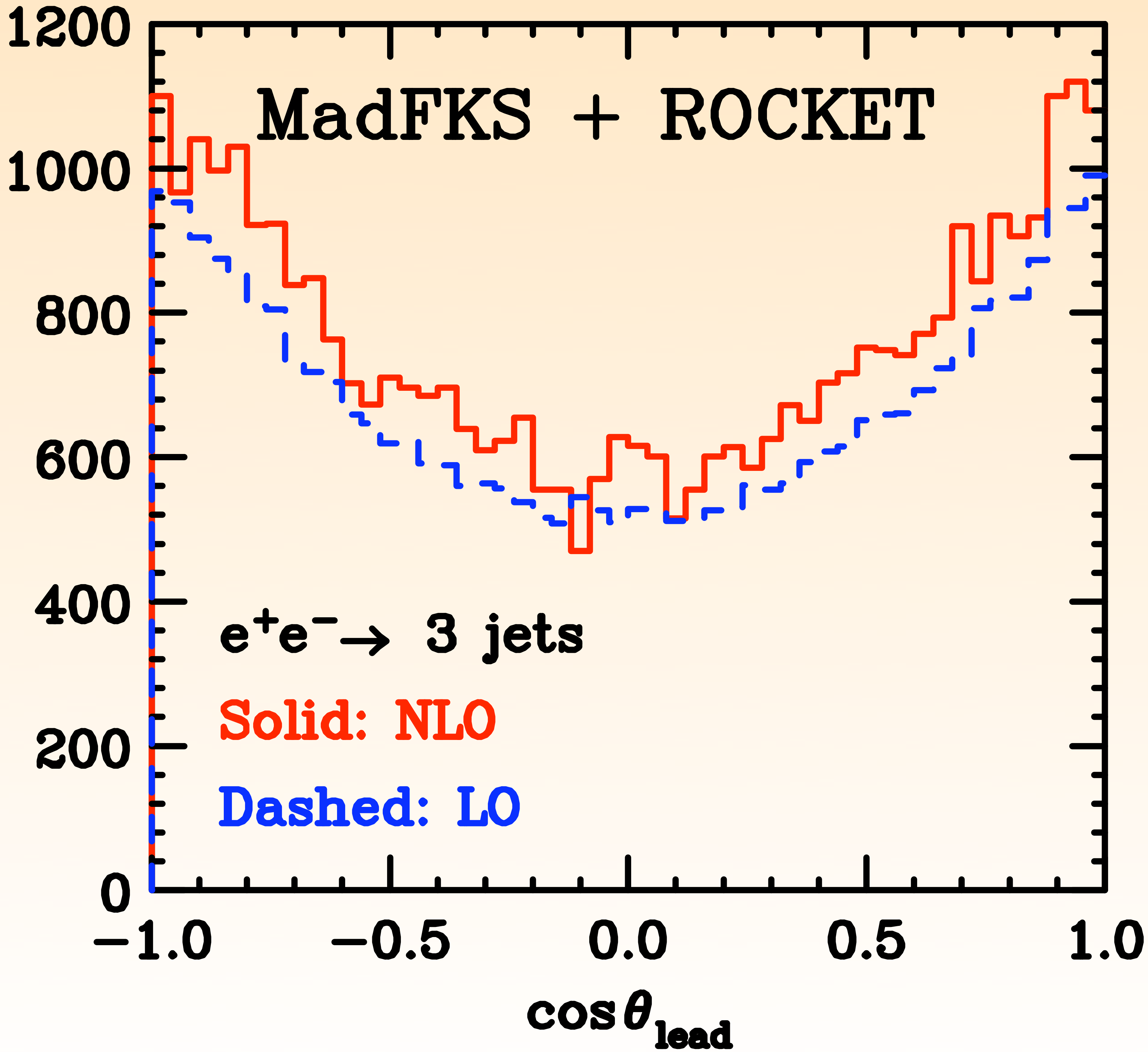

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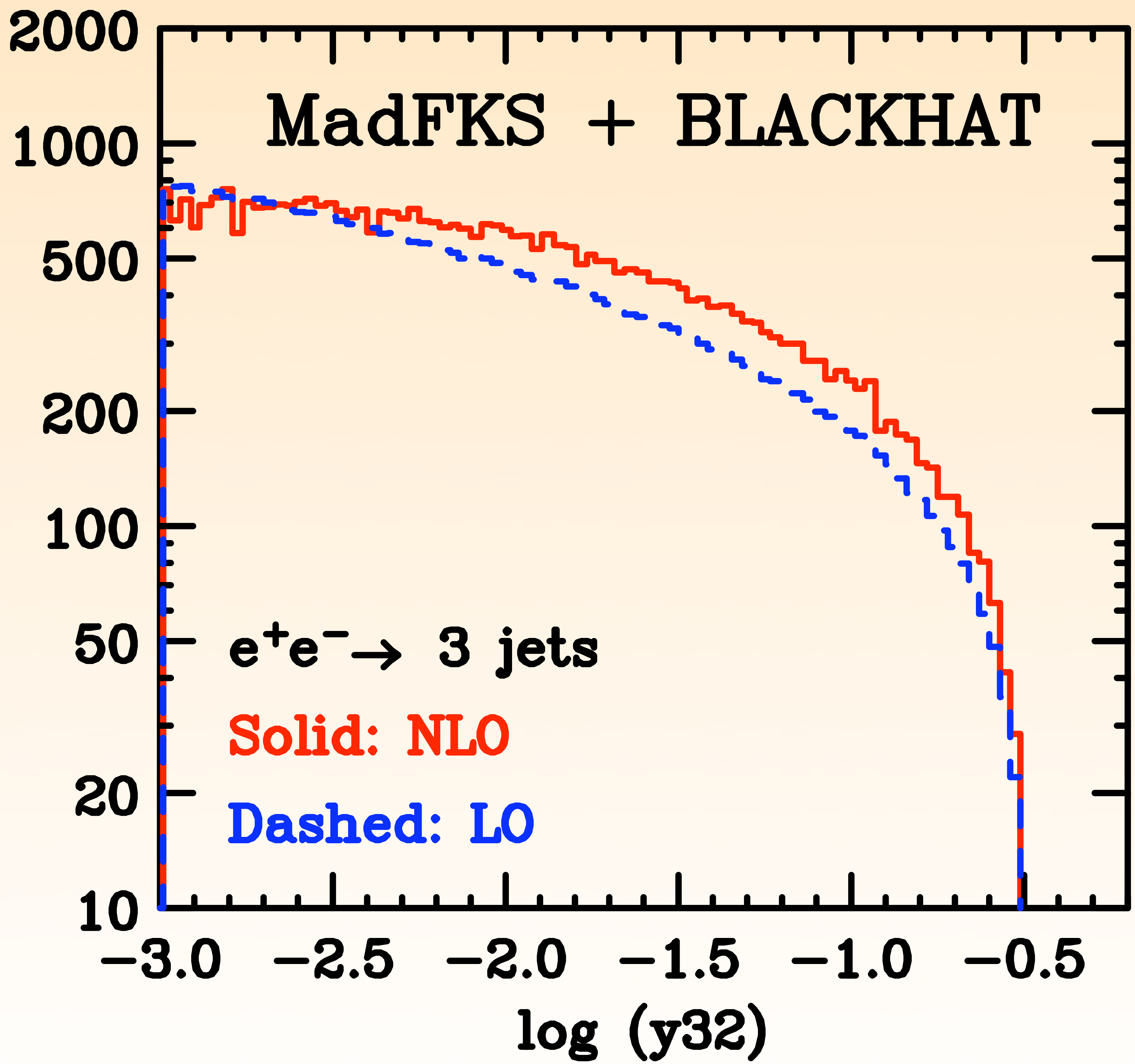

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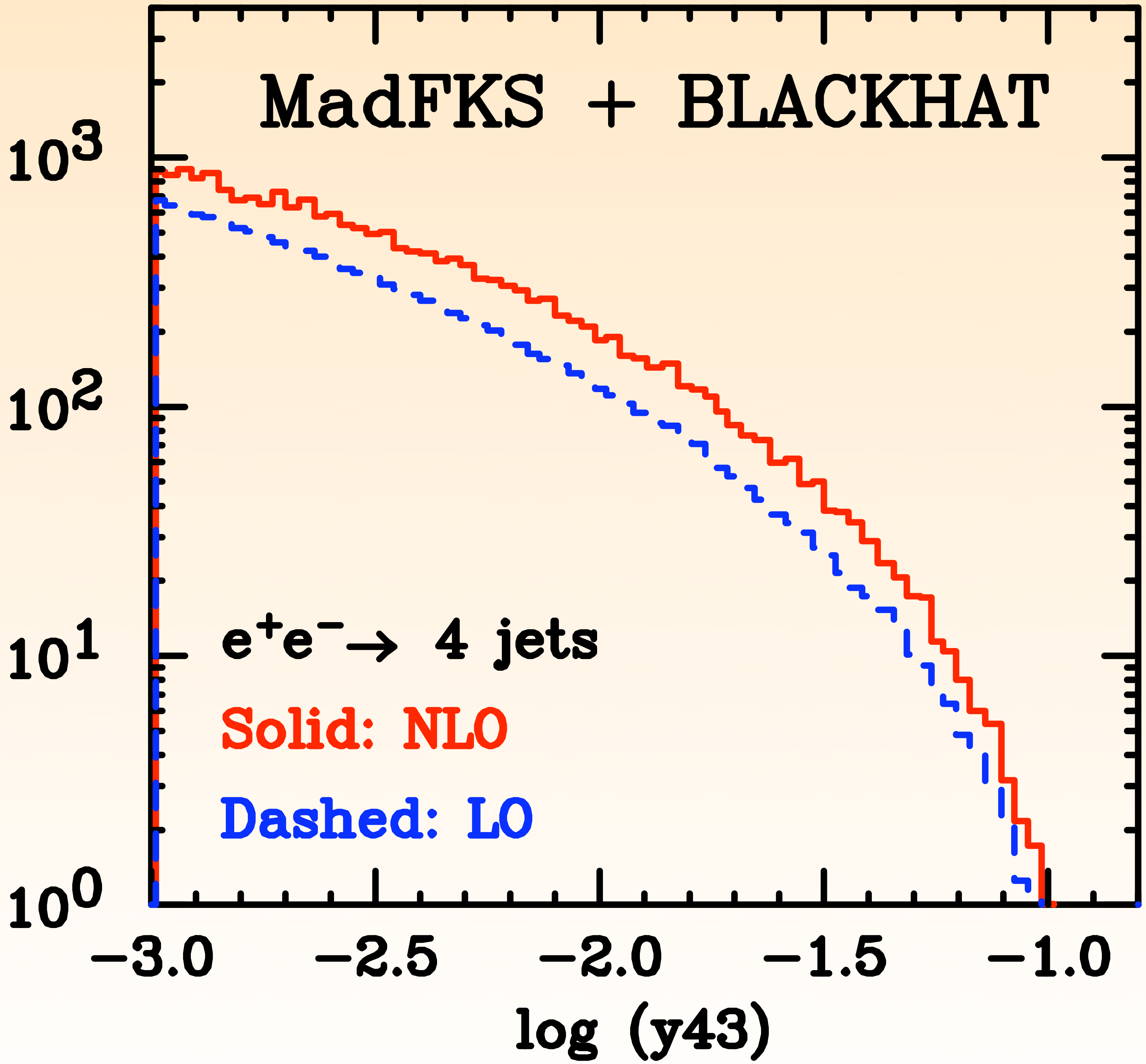
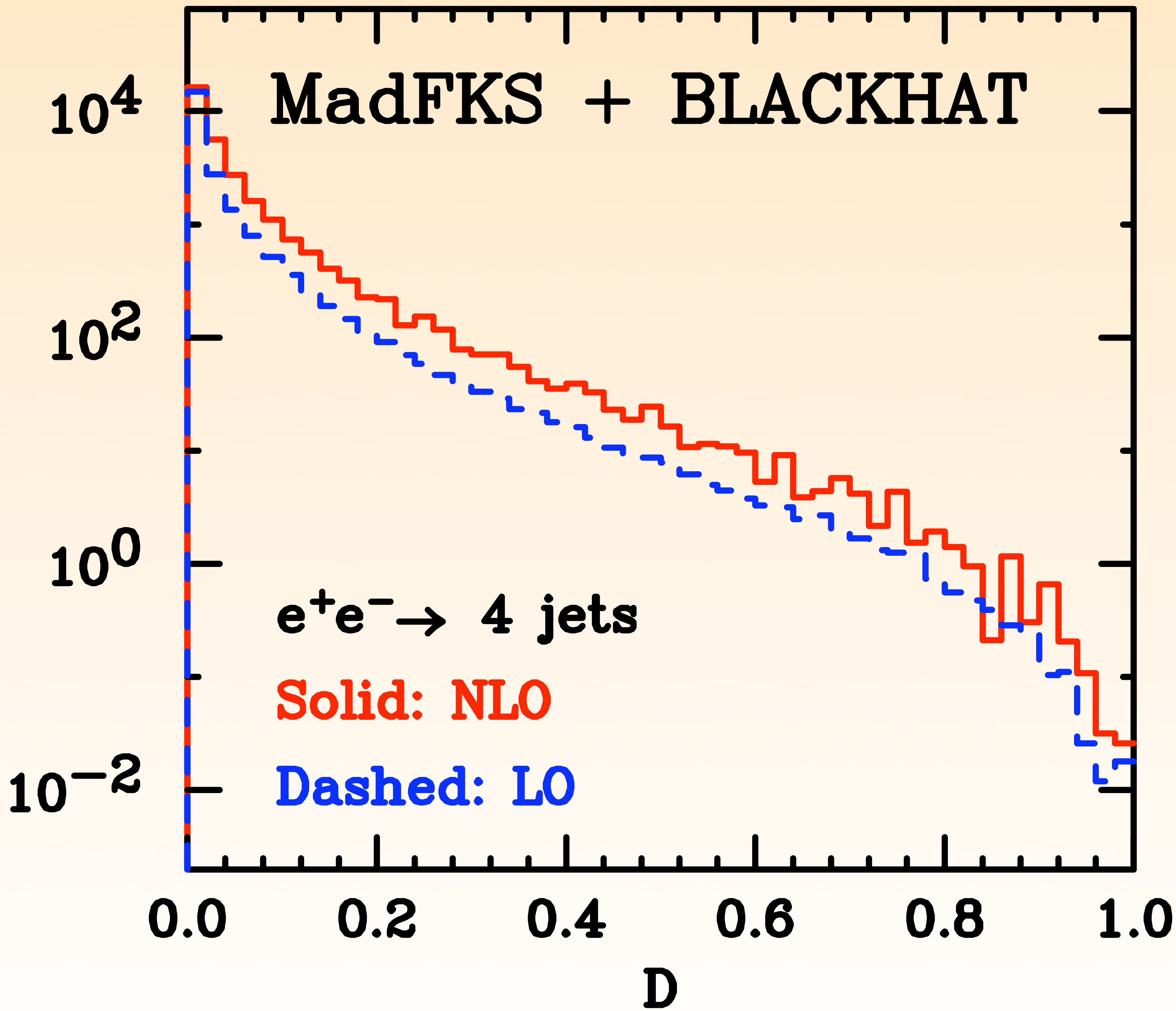


Some Results...



To conclude

- For any QCD NLO computation (SM & BSM) MadFKS takes care of:
  - Generating the Born, real emission, subtraction terms, phase-space integration and overall management of symmetry factors, subprocess combination etc.
- External program(s) needed for the (finite part of the) loop contributions (so far working with BlackHat and Rocket)
  - Other programs more than welcome!
- Next step is to include the initial state subtraction terms
- With the shower subtraction terms, interface to showers to generate automatically unweighted events at NLO is doable