Exclusive Monte Carlo modelling of NLO DGLAP evolution

The KRKMC Project

S. JADACH
in collaboration with

A. Kusina, M. Skrzypek and M. Sławińska

IFJ-PAN, Kraków and CERN-EP/TH, Geneva

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More in http://jadach.web.cern.ch/

The aim of KRKMC project

Can we construct NLO Parton Shower Monte Carlo for QCD Initial State Radiation:

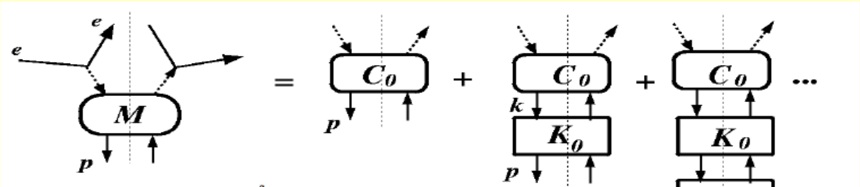
- based firmly on Feynman Diagrams (ME) and LIPS,
- based rigorously on the collinear factorization (EGMPR, CSS,...),
- implementing *exactly* NLO $\overline{MS}$ DGLAP evolution,
- for fully unintegrated exclusive PDFs (ePDFs);
- with NLO evolution done by the MC itself, using new Exclusive NLO kernels.

We are going to show that **YES!** We can do it!
And report on the first Monte Carlo implementation
– the proof of the concept for non-singlet NLO DGLAP.

Scheme of collinear factorization of EGMPR (78) used by CFP (79)

EGMPR = Ellis+Georgi+Machacek+Politzer+Ross

"Raw" factorization of the IR collinear singularities



$$M = M_{ij} = \int \mathrm{dLips} \sum_{\mathrm{spin}} \mathcal{M}_i \mathcal{M}_j^*$$

- Cut vertex M: spin sums and Lips integrations over all lines cut across
- C_0 and K_0 and are 2-particle irreducible (2PI)
- C_0 is IR finite, while K_0 encapsulates all IR collinear singularities
- Use of the axial gauge essential for the proof
- Formal proof given in EGMPR NP B152 (1979) 285
- Notation next slide

$$M = C_0\left(1 + K_0 + K_0^2 + \cdots\right) = C_0 \frac{1}{1 - K_0} \equiv C_0 \Gamma_0$$

EGMPR scheme customized to $\overline{MS}$ by Furmanski and Petronzio (80):

$$F = C_0 \cdot \frac{1}{1-K_0} = C\left(\alpha, \frac{Q^2}{\mu^2}\right) \otimes \Gamma\left(\alpha, \frac{1}{\epsilon}\right),$$

$$= \left\{ C_0 \cdot \frac{1}{1-(1-\mathbb{P})\cdot K_0} \right\} \otimes \left\{ \frac{1}{1-\left(\mathbb{P} K_0 \cdot \frac{1}{1-(1-\mathbb{P})\cdot K_0}\right)} \right\}_{\otimes},$$

$$\Gamma\left(\alpha, \frac{1}{\epsilon}\right) \equiv \left(\frac{1}{1-K}\right)_{\otimes} = 1 + K + K\otimes K + K\otimes K\otimes K + ...,$$

$$K = \mathbb{P} K_0 \cdot \frac{1}{1-(1-\mathbb{P})\cdot K_0}, \quad C = C_0 \cdot \frac{1}{1-(1-\mathbb{P})\cdot K_0}.$$

Ladder part Γ corresponds to MC parton shower

C is the hard process part

$\mathbb{P}$ is the projection operator: $\mathbb{P} = P_{spin}\, P_{kin}\, PP$

Projection operator of Curci-Furmanski-Petronzio (CFP)

$$\mathbb{P} = P_{spin}\; P_{kin}\; PP$$

consists of:

- the kinematic (on-shell) proj. operator P_{kin},
- spin proj. operator P_{spin}
- and the pole part PP extracting $\frac{1}{\epsilon^k_{IR}}$ part.

Multiplication symbol $\cdot$ means full phase space integration $d^n k$ while convolution $\otimes$ only the integration over the 1-dim. lightcone variable.

GENERAL IMPORTANT REMARKS:

- **Monte Carlo has to be in FOUR dimensions $d = 4$!**
- We'll emphasis on resummation of single collinear logs; in practice (MC) problems will often come from Sudakov double logs!

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For MC we use right now brute force interpretation of collinear ε-poles:

$$\frac{1}{\varepsilon} = \int_0^{\mu_F} \frac{dk^T}{k^T} \left(\frac{k^T}{\mu_F}\right)^{\varepsilon}.$$

CFP (1980) factorization scheme

$$F = C_0 \cdot \frac{1}{1-K_0} = C_0 \cdot \frac{1}{1-(1-\mathbb{P})\cdot K_0} \otimes \Gamma, \quad \Gamma = \frac{1}{1-\left(\mathbb{P} K_0 \cdot \frac{1}{1-(1-\mathbb{P})\cdot K_0}\right)},$$

introduces enormous oversubtractions/cancellations. At LO we have:

$$\Gamma \simeq \frac{1}{1-\left(1-e^{-\frac{1}{\varepsilon}}\right)} = 1 + \left(1-e^{-\frac{1}{\varepsilon}}\right) + \left(1-e^{-\frac{1}{\varepsilon}}\right)^2 + \ldots$$

while from RGE and explicit LO calculation give us directly

$$\Gamma = e^{+\frac{1}{\varepsilon}} = 1 + \frac{1}{\varepsilon} + \frac{1}{2!}\frac{1}{\varepsilon^2} + \ldots$$

We want this exponent directly from the Feynman diagrams!!!

This is what we actually implement in the present MC!

$$F = \frac{1}{1-K_0} = C_0 \cdot \overleftarrow{\mathbb{R}}_\mu[K_0] \cdot \exp_{TO}\left(\overleftarrow{\mathbb{P}}' \left\{ {}^s K_0 \cdot \overleftarrow{\mathbb{R}}_s[K_0] \right\}\right)(\mu)$$

$$\overleftarrow{\mathbb{R}}_\mu(K_0) = \overleftarrow{\mathbb{B}}_\mu \left[\frac{1}{1-K_0}\right] \equiv 1 + \overleftarrow{\mathbb{B}}_\mu[K_0] + \overleftarrow{\mathbb{B}}_\mu[K_0 \cdot K_0] + \overleftarrow{\mathbb{B}}_\mu[K_0 \cdot K_0 \cdot K_0] + \dots$$

Explaining the notation/meaning step by step:

- $\exp_{TO}$ means time ordered exponential in the time evolution variable = log of factorization scale, next slide.
- Operator $\overleftarrow{\mathbb{B}}$ is defined recursively (similarly as β-functions in Yennie-Frautschi-Suura 1961 subtraction scheme):

$$\overleftarrow{\mathbb{B}}_\mu[K_0] = K_0 - \mathbb{P}'_\mu\{K_0\},$$

$$\overleftarrow{\mathbb{B}}_\mu[K_0 \cdot K_0] = K_0 \cdot K_0 - \mathbb{P}'_\mu\{{}^{s_2}K_0\} \cdot \mathbb{P}'_{s_2}\{{}^{s_1}K_0\} - \mathbb{P}'_\mu\{{}^{s_2}K_0 \cdot \overleftarrow{\mathbb{B}}_{s_2}[K_0]\} - \overleftarrow{\mathbb{B}}_\mu[K_0] \cdot \mathbb{P}'_\mu\{K_0\},$$

$$\overleftarrow{\mathbb{B}}_\mu[K_0 \cdot K_0 \cdot K_0] = K_0 \cdot K_0 \cdot K_0 - \mathbb{P}'_\mu\{{}^{s_3}K_0\} \cdot \mathbb{P}'_{s_3}\{{}^{s_2}K_0\} \cdot \mathbb{P}'_{s_2}\{{}^{s_1}K_0\} - \dots$$

- The key point is the definition of new $\mathbb{P}'$ projection operator.

New factorization formula = algebraic structure for MC

$$F = \frac{1}{1-K_0} = C_0 \cdot \overleftarrow{\mathbb{R}}_\mu[K_0] \cdot \exp_{TO}\left(\overleftarrow{\mathbb{P}}'\left\{{}^sK_0 \cdot \overleftarrow{\mathbb{R}}_s[K_0]\right\}\right)(\mu)$$

Modified projection operator $\overleftarrow{\mathbb{P}}'$:

- Does spin projection as in $\mathbb{P}$ of CFP
- sets its own (cut) real momenta on-shell to its left
- acts on integrand, leaves intact Lorentz invar.ph.sp. (LIPS)
- sets upper limit μ on the phase space for all its own real (cut) partons, eg. $\mu > s(k_1, .., k_n) = \max(k_i^T)$,
- our preferred choice is rapidity ordering choice: $s(k_1, .., k_n) = a(k_1, .., k_n) = \max(k_i^T/\alpha_i)$, $\alpha_i = k_i^+/E$
- $\overleftarrow{\mathbb{P}}(A)$ acts on A which is at most single-log (col.) divergent and extracts this singularity from the LIPS integrand, (for instance by rescaling all $k_i^T \to \lambda k_i^T$ and taking coefficient in front of $1/\lambda$)
- $\overleftarrow{\mathbb{P}}'(K_0)$ is OK. because K_0 is single-log divergent.
- Nesting like $\overleftarrow{\mathbb{P}}[K_0 \cdot (1 - \overleftarrow{\mathbb{P}}(K_0))]$ is allowed, as long as its argument is at most single-log divergent.

New factorization formula = algebraic structure for MC

$$F = \frac{1}{1-K_0} = C_0 \cdot \overleftarrow{\mathbb{R}}_\mu[K_0] \cdot \exp_{TO}\left(\overleftarrow{\mathbb{P}}'\left\{{}^{s}K_0 \cdot \overleftarrow{\mathbb{R}}_s[K_0]\right\}\right)(\mu)$$

Time ordered exponential:

$$\exp_{TO}\left(\mathbb{P}'_\mu\{A\}\right)(\mu) = 1 + \mathbb{P}'_\mu\{A\} + \mathbb{P}'_\mu\{{}^{s_2}A\} \cdot \mathbb{P}'_{s_2}\{{}^{s_1}A\} + \mathbb{P}'_\mu\{{}^{s_3}A\} \cdot \mathbb{P}'_{s_3}\{{}^{s_2}A\} \cdot \mathbb{P}'_{s_2}\{{}^{s_1}A\} + \dots$$

How to understand that?
For $A = \int dLips(k_1, k_2, ..., k_n)\ f(k_1, ..., k_n)$,
where k_i are on-shell cut lines (real emitted partons)
the notation $\{{}^{s_3}A\}$ defines $s_3 = a(a_1, ..., a_n) = \max(a_1, ..., a_n)$.
From above definition and def. of $\mathbb{P}'$ follows that term like

$$\mathbb{P}'_\mu\{{}^{s_3}A\} \cdot \mathbb{P}'_{s_3}\{{}^{s_2}A\} \cdot \mathbb{P}'_{s_2}\{{}^{s_1}A\}$$

has its entire integrand multiplied by $\theta_{\mu > s_3 > s_2 > s_1}$,
where μ is constant and s_i are integration variables dependent.

Making the whole story short...
... more details in the folowing explicit examples

In the factoriz. formula $F(Q) = C(Q,\mu) \cdot D(\mu)$, where $C(Q,\mu) = C_0 \cdot \overleftarrow{\mathbb{R}}_\mu[K_0]$
our main interest for today is in the the exclusive PDF (integral over ePDF):

$$D(\mu) = \exp_{TO}\left(\overleftarrow{\mathbb{P}}'\left\{{}^sK_0 \cdot \overleftarrow{\mathbb{R}}_s[K_0]\right\}\right)(\mu) = \exp_{TO}(K),$$

where LO and NLO truncations of the evolution kernel K_μ are:

$$K_\mu^{LO} = \overleftarrow{\mathbb{P}}'_\mu\left\{{}^sK_0\right\}, \quad \text{taken at } \mathcal{O}(\alpha^1),$$

$$K_\mu^{NLO} = \overleftarrow{\mathbb{P}}'_\mu\left\{{}^sK_0 + K_0 \cdot (1 - \overleftarrow{\mathbb{P}}') \cdot K_0\right\}, \text{ truncated at } \mathcal{O}(\alpha^2).$$

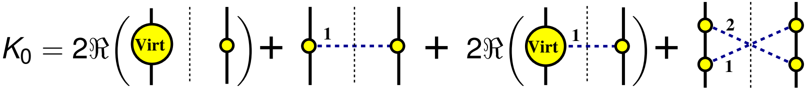
NB. The x-dependent $D(\mu, x)$ obeys ordinary evolution equation
$\boxed{\partial_\mu D(\mu, x) = \mathcal{P} \otimes D(\mu)(x)}$ with the inclusive DGLAP kernel

$$\mathcal{P}(x) = \frac{\partial}{\partial \ln(\mu)} \int d\text{Lips}\ \delta(x = \dots)\ K_\mu$$

$$= \int d\text{Lips}\ \delta\left(x = \frac{\sum k_i^+}{E_0}\right)\delta\left(1 - \frac{s}{\mu}\right)\overleftarrow{\mathbb{P}}'_\mu\left\{{}^sK_0 \cdot \overleftarrow{\mathbb{R}}_s[K_0]\right\}.$$

NB2. Our factorization formula defines hard process part $C(Q,\mu)$ and also encodes the precise method of combining the two.

The 2PI kernel K_0 of CFP scheme at LO+NLO is:

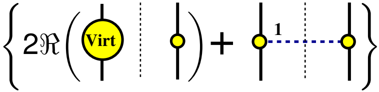


where dashed lines are gluons, blobs marked “Virt” may include several (one loop) subgraphs.

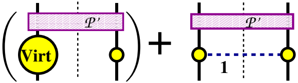
Take Only two LO terms in the time ordered exponential:

$$D(Q) = \exp_{T.O.}\left(\mathbb{P}'_Q\{K\}\right) \simeq 1 + \mathbb{P}'_Q\{K_0\} =$$

$= 1 + \mathbb{P}'_Q$

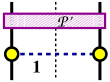


$= 1 + 2\Re$



Explicit LO expression for single real emission,
where we also introduced cutoff q_0 in the $a \to 0$ limit

Familiar LO/LL bremsstrahlung distribution:

$$\bar{D}_{B1r}(Q) =$$ 

$$= \int \frac{d^3 k_1}{2k_1^0} \Pi(k_1)\, \bar{\rho}_{B1r}(k_1)$$

$$= \int \frac{d\alpha_1}{\alpha_1} d^2\mathbf{k}_1 d\phi_1\, \Pi(k_1)\, \bar{\rho}_{B1r}(k_1),$$

$$\bar{\rho}_{B1r}(k_1) = \frac{2C_F\alpha_s}{4\pi^2}\, \frac{1+(1-\alpha_1)^2}{2}\, \frac{1}{\mathbf{k}_1^2}\theta_{\alpha_1>\delta}\ ,$$

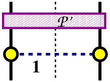
$$\Pi(Q, q_0|k_1) = \theta_{Q>a_1>q_0}.$$

$$\mathbf{a}_i \equiv \mathbf{k}_1/\alpha_1,\quad a_1 = |\mathbf{a}_1|,$$

$$a_1 = \exp(\textit{rapidity of particle } \mathbf{1}).$$

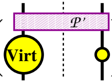
Trivial phase space integration

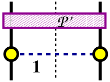
Trivial phase space integration gives Sudakov double log:

$\bar{D}_{B1r}(Q) =$ 

$$= \frac{2C_F\alpha_s}{\pi} \ln\frac{Q}{q_0} \left(\ln\frac{1}{\delta} - \frac{3}{4}\right) = S_{\rm ISR}. \quad (1)$$

Seemingly trivial results:

$\bar{D}_{B1}(Q) = 1 + \mathbb{P}'_Q\{K_0\} = 1 + 2\Re\Big($ 

$\Big) +$ 

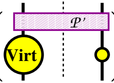
$= 1 + S_{\rm ISR} - S_{\rm ISR} = 1,$

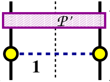
BUT... the insertions of δ-function defining x exposes inclusive LO kernel...

The insertions of δ-function defining x inside phases space (histograming in the MC) exposes standard inclusive LO kernel $\mathcal{P}_{qq}(x)$ (we incluse $2C_F\alpha/\pi$ in the kernel):

$$D_{B1}(Q,x) = \delta(1-x) + \mathbb{P}'_Q\{K_0\}_x$$

$$= \delta_{x=1} + \; 2\Re\Big(\;\; \Big)_x + \Big(\;\; \Big)_x$$





$$= \delta_{x=1} - S_{\rm ISR}\,\delta_{x=1} + \ln\frac{Q}{q_0}\,\frac{2C_F\alpha_s}{\pi}\,\frac{1+x^2}{2(1-x)}\theta_{1-x>\delta}$$

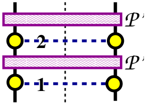
$$= \delta_{x=1} + \ln\frac{Q}{q_0}\,\frac{2C_F\alpha_s}{\pi}\left(\frac{1+x^2}{2(1-x)}\right)_+ = \delta_{x=1} + \ln\frac{Q}{q_0}\,\mathcal{P}_{qq}(x),$$

Next term in the LO time-ordered exponential in next slide...

Simlar relation will hold for LO+NLO excl./incl. kernels

Double emission term in the LO time-ord. exponent

$$\mathbb{P}'_Q\{^{s_2}K_0\}\cdot\mathbb{P}'_{s_2}\{^{s_1}K_0\} =$$



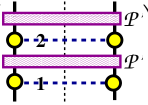
$$= \int \frac{d^3k_1}{2k_1^0}\frac{d^3k_2}{2k_2^0}\Pi(k_1)\Pi(k_2)\,\frac{1}{2}\bar\rho_{B2r}(k_2,k_1)\theta_{a_2>a_1} = \frac{1}{2}(S_{\rm ISR})^2.$$

$$\bar\rho_{B2r}(k_2,k_1) = \left(\frac{2C_F\alpha_s}{4\pi^2}\right)^2 \frac{1+z_2^2}{2\mathbf{k}_2^2}\theta_{1-z_1>\delta}\,\frac{1+z_1^2}{2\mathbf{k}_1^2}\theta_{1-z_2>\delta},$$

$$z_1 = 1-\alpha_1,\qquad z_2 = \frac{1-\alpha_1-\alpha_2}{1-\alpha_1},\qquad \mathbf{a}_i = \frac{\mathbf{k}_i}{\alpha_i},\; a_i = |\mathbf{a}_i|,\; \alpha_i = \frac{k_i^+}{E_0}$$

And the x-dependent version:

$$\mathbb{P}'_Q\{^{s_2}K_0\}\cdot\mathbb{P}'_{s_2}\{^{s_1}K_0\}_x = \Big(\quad \Big)_x$$



$$= \frac{1}{2}\ln^2\frac{Q}{q_0}\;\;[\mathcal{P}^\theta_{qq}\otimes\mathcal{P}^\theta_{qq}](x),$$

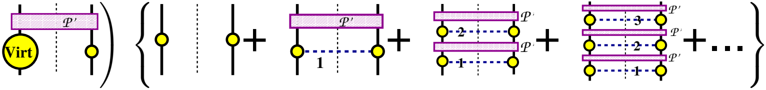
$$4[\mathcal{P}^\theta_{qq}\otimes\mathcal{P}^\theta_{qq}](z) = \left(\frac{2C_F\alpha_s}{\pi}\right)^2\left[\frac{1+z^2}{1-z}\left(4\ln\frac{1}{\delta}+4\ln(1-z)\right)+(1+z)\ln z-2(1-z)\right].$$

LO parton shower MC starts to unfold!

After tedious but simple resumming of the virtual $(-S_{\rm ISR})$:

$$D_B(Q) =$$

$$= 1 + \mathbb{P}'_Q\{K_0\} + \mathbb{P}'_Q\{^{s_2}K_0\}\cdot\mathbb{P}'_{s_2}\{^{s_1}K_0\} + \mathbb{P}'_Q\{^{s_3}K_0\}\cdot\mathbb{P}'_{s_3}\{^{s_2}K_0\}\cdot\mathbb{P}'_{s_2}\{^{s_1}K_0\} + \dots$$

$= \exp$ ($\Re$ 

$$= e^{-S_{\rm ISR}}\left\{1 + \sum_{n=1}^{\infty}\prod_{j=1}^{n}\int \frac{d^3k_j}{2k_j^0}\,\Pi(k_j)\,\bar\rho_{B1}^{LO}(k_j)\theta_{a_j>a_{j-1}}\right\} = e^{-S_{\rm ISR}}\,\left\{e^{+S_{\rm ISR}}\right\} = 1,$$

and the x-dependent version (histogramming in Markovian MC) follows:

$$D_B(x,Q) = e^{-S_{\rm ISR}}\left\{\delta_{x=1} + \sum_{n=1}^{\infty}\left(\prod_{j=1}^{n}\int_{q_{j-1}}^{Q}\frac{da_j}{a_j}\int_0^{1-\delta}dz_j\int_0^{2\pi}\frac{d\varphi_j}{2\pi}\,\mathcal{P}^{\theta}_{qq}(z_j)\right)\delta_{x=1-\prod_i z_i}\right\}.$$

THE END of LO warm-up introduction.

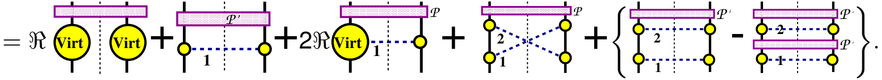
NLO bremsstrahlung ingredients

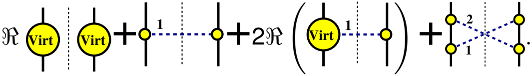
Again the same T.O. exponent:

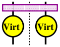
$$D_B(Q) = 1 + \mathbb{P}'_Q\{K_0\} + \mathbb{P}'_Q\{{}^{s_2}K_0\}\cdot\mathbb{P}'_{s_2}\{{}^{s_1}K_0\} + \mathbb{P}'_Q\{{}^{s_3}K_0\}\cdot\mathbb{P}'_{s_3}\{{}^{s_2}K_0\}\cdot\mathbb{P}'_{s_2}\{{}^{s_1}K_0\} + \dots$$

but the exclusive NLO kernel is now:

$$\mathbb{P}'_Q\{K^{NLO}\} = \mathbb{P}'_Q\{{}^{s}K_0\} + \mathbb{P}'_Q\{{}^{s}K_0\cdot K_0 - K_0\mathbb{P}'_s\{K_0\}\} =$$

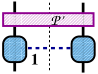


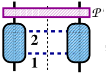
where 2PI kernel is $K_0 = $ 

Zero-emission part 

(wave function renormalization up to second order) exponentiates/factorizes as in the LO.

For the remaining 1-emission and 2-emission parts we introduce separate

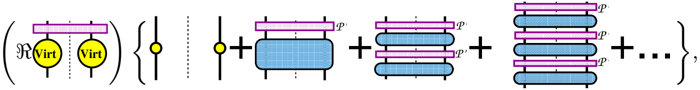
graphical notation: 

and 

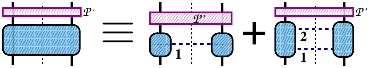
, see next slide.

$$D_B^{[1]}(Q) = \exp(-S_{ISR}^{[1]})\Big(1 + \mathbb{P}'_Q\{K_0^r\} + \mathbb{P}'_Q\{^{a_2}K_0^r\}\cdot\mathbb{P}'_{a_2}\{^{a_1}K_0^r\} +$$

$$+ \mathbb{P}'_Q\{^{a_3}K_0^r\}\cdot\mathbb{P}'_{a_3}\{^{a_2}K_0^r\}\cdot\mathbb{P}'_{a_2}\{^{a_1}K_0^r\} + \dots\Big)$$

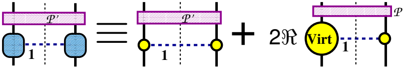


We define: $K_0^r =$



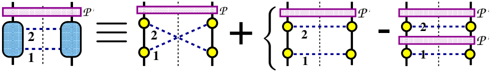
where LO dressed with virt. corrs is

$K_0^{1r} =$



and pure 2-real gluon part is

$K_0^{2r} =$

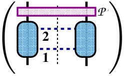


Still one difficult step (BE symmetrization) on the way to MC.

Let's have a closer look into the last 2-emission part, before MC.

Are we really in $\overline{MS}$ scheme of CFP paper?

For the dressed LO part K_0^{1r} we follow dully CFP $\overline{MS}$, when calculating virtual corrections in the dimensional regularization.
For 2-real part K_0^{2r} in the x-dependent version, we integrate analytically over 2-gluon phase space in d=4:



$$\Big(\;\cdots\;\Big)_x = \int \frac{d^3k_2}{2k_2^0} \int \frac{d^3k_1}{2k_1^0} \theta_{Q>\max(a_2,a_1)>q_0} \beta_{1B}(k_2,k_1)\delta_{x=\alpha_2+\alpha_1} = \ln\frac{Q}{q_0}\mathcal{P}^{(1r)}_{qq}(x),$$

where

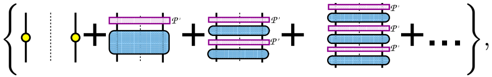
$$\mathcal{P}^{(1r)}(x) = \frac{2C_F^2\alpha}{\pi}\left(\frac{1+3x^2}{16(1-x)}\ln^2(x) + \frac{2-x}{4}\ln(x) + \frac{3}{8}(1-x)\right).$$

This agrees with the corresponding part of the NLO kernel in CFP paper.
However, we possibly miss term due to ε term in the γ-trace
$= \frac{2C_F^2\alpha}{\pi}(1-x)\ln(x)$.
Moreover, we use different ordering variable than k_i^T, then the above result in $d=4$ changes, while CFP is completely independent of this choice.
For instance for angular ordering our result gets extra term:

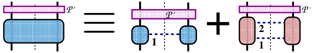
$$\Delta\mathcal{P}^{(1r)}(x) = \frac{2C_F^2\alpha}{\pi}\left(-\frac{1+x}{2}\ln^2(x) + (1-x)\ln(x)\right).$$

The above differences with $\overline{MS}$ are controlled, accounted for, and full compatibility/agreement with $\overline{MS}$ of CFP is kept in our MC.

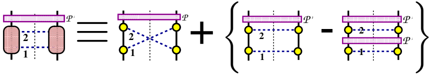
What else on the way to Monte Carlo? DEFACTORIZATION!
STEP ONE: expand kernels

$D_B^{[1]}(Q) = e^{-S_{ISR}^{[1]}}$ 

The above master NLO formula is useless for the MC. Second term in:

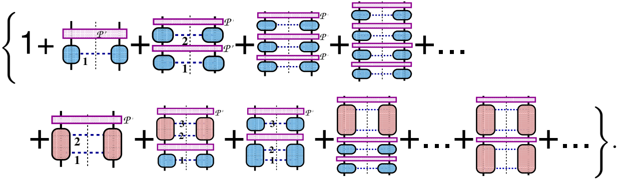
$K_0^r =$ 

which is this one:

$K_0^{2r} =$ 

is terrible! NON-positive, CANNOT be generated separately in the MC!

We have to expand it into LO-like part and the rest, in powers of K_0^{2r}:

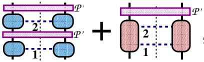
$D_B^{[1]}(Q) = e^{-S_{ISR}^{[1]}}$ 

The first raw (LO+Virt.) will go to basic MC and the rest to MC weight.

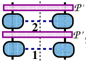
What else on the way to Monte Carlo? DEFACTORIZATION!

STEP TWO: Bose-Einstein (BE) symmetrization

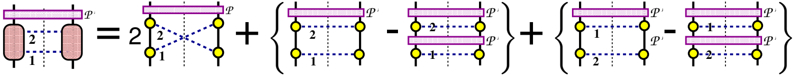
Even after kernel expansion of the previous slide, in distribution like this one:



, for 2 gluons only, we already have a problem:

It cannot be generated by MC-reweighting LO distribution ,

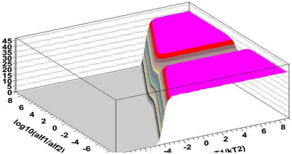
simply because it is zero outside its own simplex $Q > a_2 > a_1 > q_0$, while the target distribution is nonzero in the bigger rectangle $Q > \max(a_2, a_1) > q_0$.
Solution: include BE symmetrization in the game!
From now on we redefine:



$$= \int \frac{d^3k_2}{2k_2^0} \int \frac{d^3k_1}{2k_1^0}\; \theta_{Q>\max(a_2,a_1)>0}\; \theta_{a_2>a_1} \big(\beta_{1B}(k_2,k_1) + \beta_{1B}(k_1,k_2)\big),$$

where for purely technical reasons we include internal ordering $\theta_{a_2>a_1}$ for the already symmetric integrand.

Before BE symmetrization



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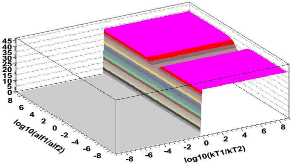

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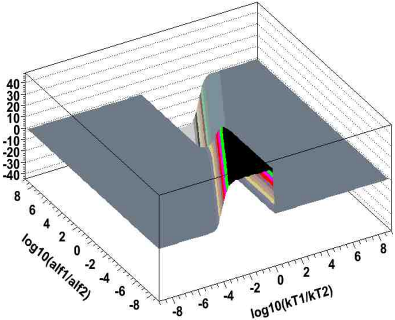
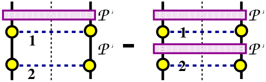
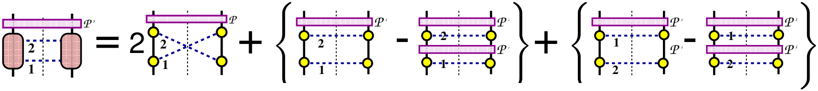
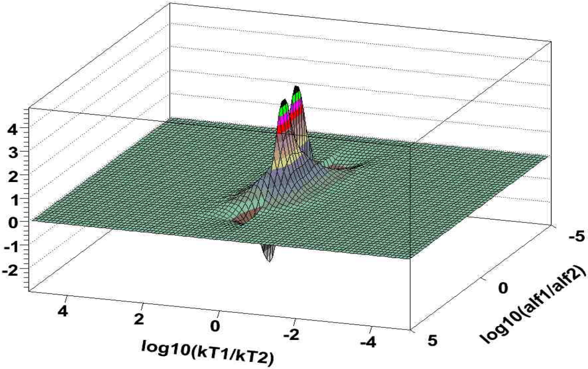


= 

Distribution in Sudakovian variables $\ln(\alpha_1/\alpha_2)$ and $\ln y = 2\ln(k_1^T/k_2^T)$. Approximate virtuality ordering? A disaster for the Monte Carlo: Huge double-log cancellations!!!

After BE symmetrization



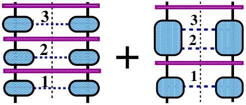


Beautiful short-range correlation type distribution, excellent for the Monte Carlo implementation.

BE symmetrization for more gluons, one NLO insertion

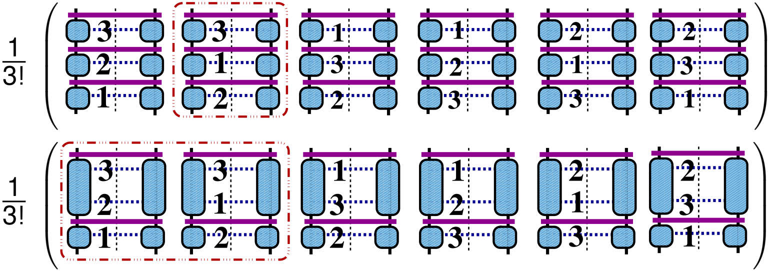
Three gluons

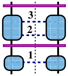
Before BE symmetrization 3-gluon distribution is:



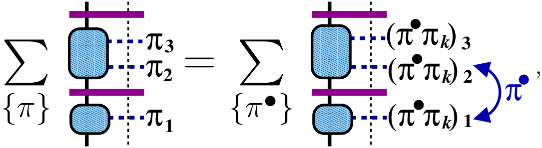
In the simplex $a_3 > a_1 > a_2$ associated with the permutation $\pi_2 = (3, 2, 1)$ the following marked terms will contribute:

$$\pi_1 = (123),\ \pi_2 = (213),\ \pi_3 = (231)\ \pi_4 = (321),\ \pi_5 = (312),\ \pi_6 = (132)$$



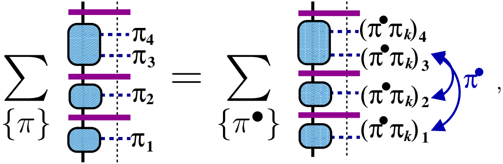
BE symmetrization over 3! permutations of 

reduces to only 2 terms:



(2)

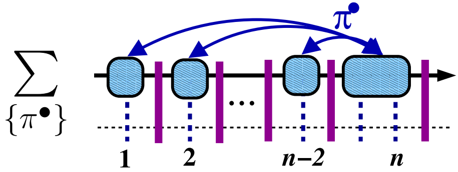
Permut. π_k does right ordering $a_{(\pi_k)_3} > a_{(\pi_k)_2} > a_{(\pi_k)_3}$ at a given phase space point.
A subset of 2 permutations $\{\pi^\bullet\} = \{(123), (213)\}$ is interchanging $(\pi_k)_2$ and $(\pi_k)_1$.
The case $n = 4$ looks quite similar:



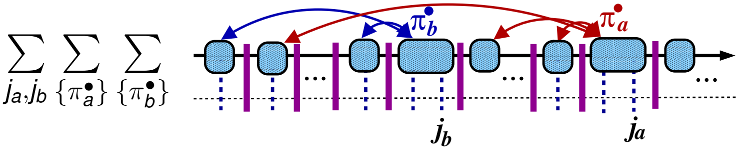
(3)

One possible swap between second gluon of NLO insertions and the two trailing spectator gluons, $\{\pi^\bullet\} = \{(1234), (1324), (3124)\}$, 3 terms out of 4! do survive.

BE symmetrization for n gluons with single NLO insertion at the end of the ladder:



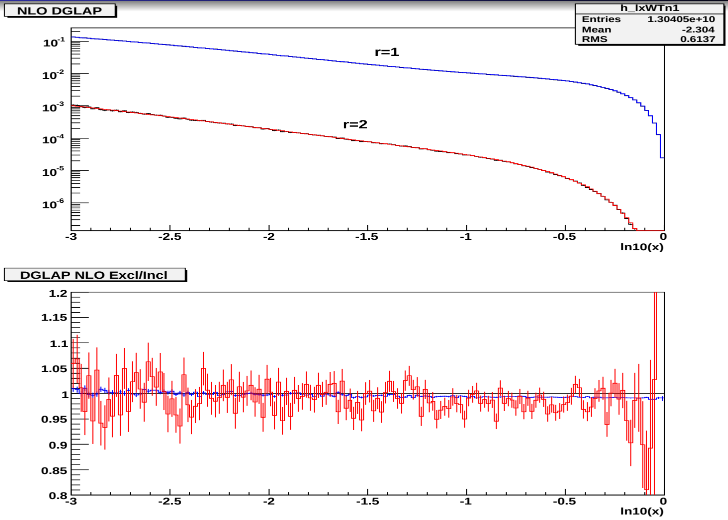
Generalization to two NLO insertions placed anywhere:



Restrictions: the same momentum k_i cannot enter as a parameter in two different NLO insertions, the same k_i cannot participate in two different *swap* permutations π_a and π_b.

Testing our idea with a prototype NLO Monte Carlo

Exclusive/Inclusive NLO MC: Slices in No. of inserts

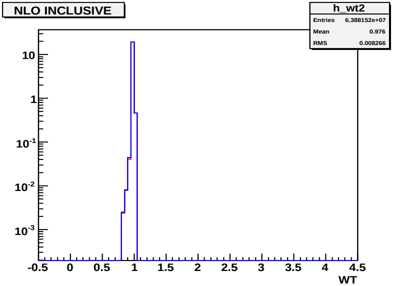


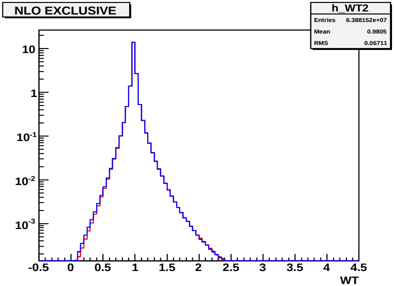
LO MC result of order ~ 1 is omitted in the plot.
NLO MC results certify our scheme with the 3-5 digits precision.

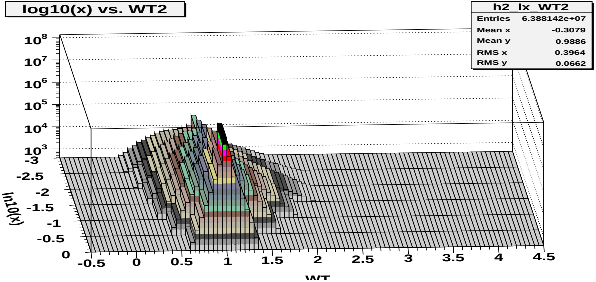
More on what is in the previous plot

- Both LO and NLO evolutions on top of the same Markovian LO MC.
  (It can be put easily on top of non-Markovian CMC.)
- MC weights positive, weight distributions very reasonable, see next slides.
- Evolution range from 10GeV to 1TeV
- LO pre-evolution staring from $\delta(1-x)$ at 1GeV to 10GeV provides initial x-distribution for the LO+NLO continuation.
- As before only C_F^2 part of gluonstrahlung.
- Non-running α_S.
- Term due to ε part of γ-traces omitted.
- NLO virtual corrections omitted.

Excellent weight distribution!

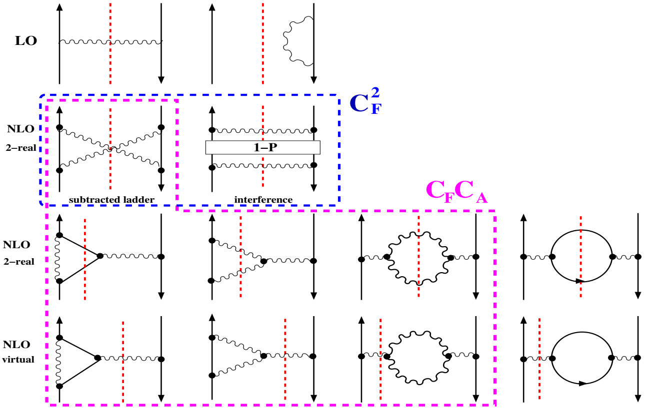






New!!! Non-singlet non-abelian $C_F C_A$ graphs

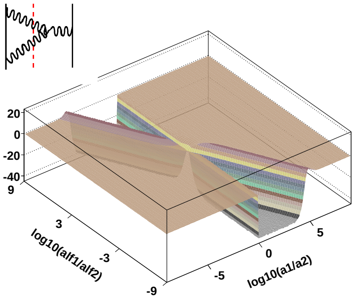
Extending NLO insertion technique. PRELIMINARY!

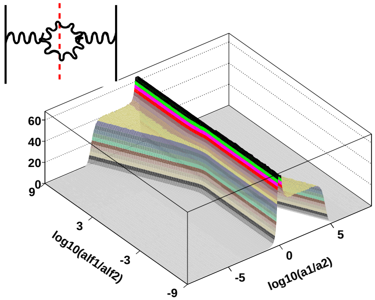


New graphs $\sim C_F C_A$ – new problems:

- Huge FSR double log cancellations between real and virtual graphs
- Need to extend NLO insertion techniques to exponentiated FSR

Non-abelian soft cancellations – colour coherence

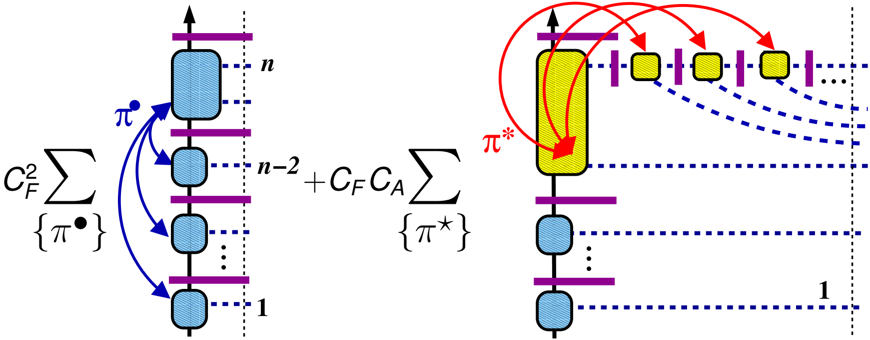




Double-log triangular structure in Sudakov plane *cancels*!
Virtuality ordering replaced by *angular ordering*!
http://arxiv.org/abs/0905.1403 by M. Slawinska and A. Kusina.

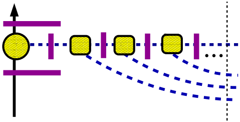
New!!! Non-singlet non-abelian $C_F C_A$ graphs

Extending NLO insertion technique. PRELIMINARY!

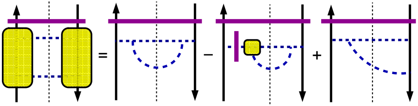


- First version already in the Monte Carlo. It works!
- New $C_F C_A$ components (FSR) explained in next slides

- Each ISR real gluon replaced by *resolved multigluon*

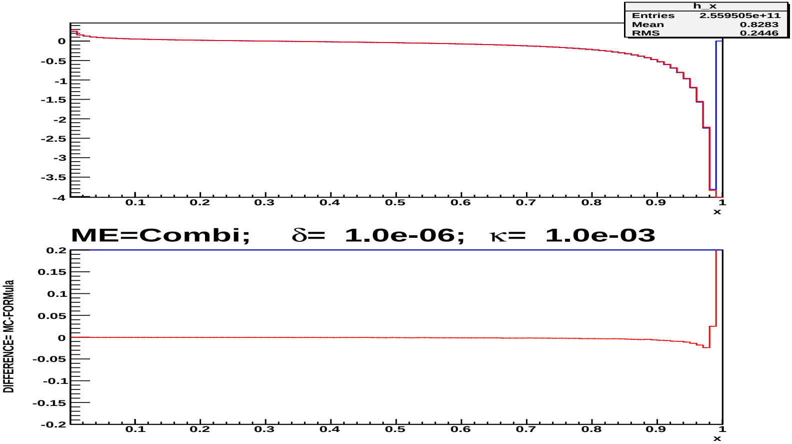


- by iterating gluon-splitting part in *LO soft counterterm*
- *soft counterterm* encapsulates all soft+collinear singularities of 2 real gluon $C_F C_A$ graphs (interferences!)
- constructing properly this *soft counterterm* is a critical step!
- LO FSR realized based on *Markovian MC algorithm with veto* using angular ordering evolution variable, as in ISR.
- NLO insertion distribution from Feynman diags.:



New!!! Non-singlet non-abelian $C_F C_A$ graphs

Three-digit numerical crosscheck of the concept



Plotted is NLO ISR+FSR insertion alone:
MC result, analytical formula and their difference.
Weight distribution is also reasonable.

Summary and Prospects

- First serious feasibility study of the true NLO exclusive MC parton shower is under construction, and well advanced...
- What next? Workplan well defined:
- Short range aim: Complete non-singlet.
- Middle range aim: Complete singlet.
- Speed up the MC weight calculation.
- Better documentation needed on what was done.
- NLO MC for W/Z production for LHC, including SANC electroweak library.
- NLO MC for DIS@HERA and more...