Recent developments in PHOTOS MC $\gamma* \rightarrow \pi^-\pi^+(\gamma)$ and $K^\pm \rightarrow \pi^+\pi^-e^\pm\nu_e(\gamma)$

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G. Nanava, Q. Xu and Z. Was, **arXiv:0906.4052**

RADCOR2009, Ascona, Switzerland

Outline

- Brief Introduction to PHOTOS
- $\gamma^* \longrightarrow \pi^+\pi^-(\gamma)$
- $K^\pm \longrightarrow \pi^+\pi^- e^\pm \nu_e(\gamma)$
- Summary and Outlook

Brief Introduction to PHOTOS

- PHOTOS Monte Carlo is used to simulate effect of QED in decays
- It can be combined with other main processes, generators
- Factorization of Amplitude
  - $|M_{\rm virt}|^2 = |M_{\rm Born}|^2 \delta_{\rm virt}$ IR divergent
  - $|M_{\rm soft}|^2 = |M_{\rm Born}|^2 \delta_{\rm soft}$ IR divergent
  - In soft and collinear limits
    $|M_{\rm real}|^2 \to |M_{\rm Born}|^2 \times$ a factor $= |M_{\rm kernel}|^2$
  - Amplitude of hard photon emission is expressed approximately as a process independent kernel
- Phase space factorization is done similarly, but it is exact and full phase space is covered
- $d\sigma \propto |M_{\rm Born}|^2 \left(1 + \underbrace{\delta_{\rm virt} + \delta_{\rm soft}}_{\text{IR finite}}\right) d\Omega_{\rm Born} + |M_{\rm kernel}|^2 d\Omega_{\rm Born} d\Omega_\gamma$

Brief Introduction to PHOTOS

- In PHOTOS, processes can also be simulated using exact matrix element by implementing a weight $wt = \frac{|M_{\mathrm{exact}}|^2}{|M_{\mathrm{kernel}}|^2}$
- Results were compared process after process. We found the kernel in PHOTOS is a very good approximation
- QED, scalar QED decaying processes in PHOTOS
  - $Z \ \ (\gamma^*, H) \to \mu^+\mu^-(\gamma)$
  - $B^0 \to K^+K^-, \pi^+\pi^-, K^+\pi^-(\gamma)$, $B^\pm \to K^\pm K^0, \pi^\pm\pi^0(\gamma)$
  - $W^\pm \to l^\pm \nu_l(\gamma)$
  - $\gamma^* \to \pi^-\pi^+(\gamma)$
  - $K^\pm \to \pi^+\pi^- e^\pm \nu_e(\gamma)$ at work
- Multi-photon emission can be simulated

$\gamma^* \to \pi^+\pi^-(\gamma)$

In PHOTOS, kernel for scalars is obtained from B decays
$B_0(P) \to \pi^\pm(q_1)K^\mp(q_2)\gamma(k,\epsilon)$

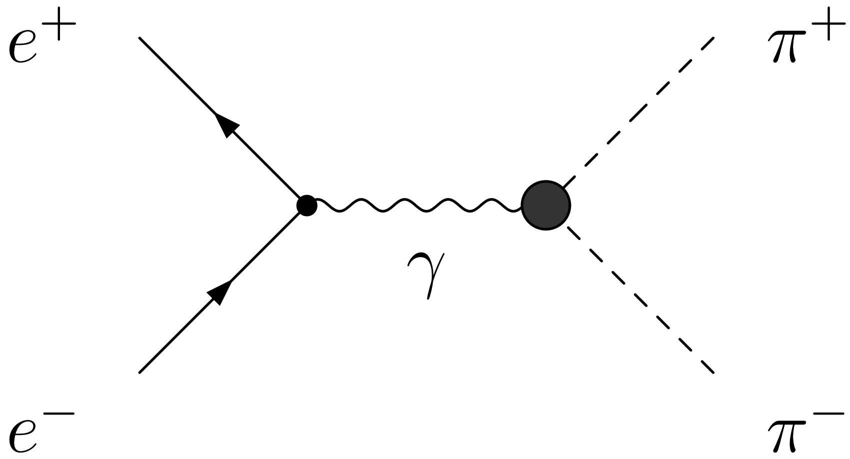
$$|M|^2_{\mathrm{PHOTOS}} = 4\pi\alpha|M_{Born}|^2 \left(Q_1\frac{q_1\cdot\epsilon}{q_1\cdot k} - Q_2\frac{q_2\cdot\epsilon}{q_2\cdot k}\right)^2$$

$Q_1,\ Q_2$ are the charges of final particle

Since spin structure of $e^+e^- \to \gamma^* \to \pi^+\pi^-\gamma$ is different from B decays, $\sum_{\lambda,\epsilon}|M|^2(\gamma^* \to \pi^+\pi^-\gamma)$ is different from kernel used in standard PHOTOS for scalars!

$\gamma^* \to \pi^+\pi^-(\gamma)$

$$e^+(p_1,\lambda_1)e^-(p_2,\lambda_2) \to \gamma^* \to \pi^+(q_1)\pi^-(q_2)$$



$$M_{born} = V_\mu H_0^\mu$$
$$V_\mu = \imath e\bar{u}(p_1,\lambda_1)\gamma_\mu v(p_2,\lambda_2)$$
$$H_0^\mu = \frac{eF_{2\pi}(S)}{S}(q_1-q_2)^\mu$$

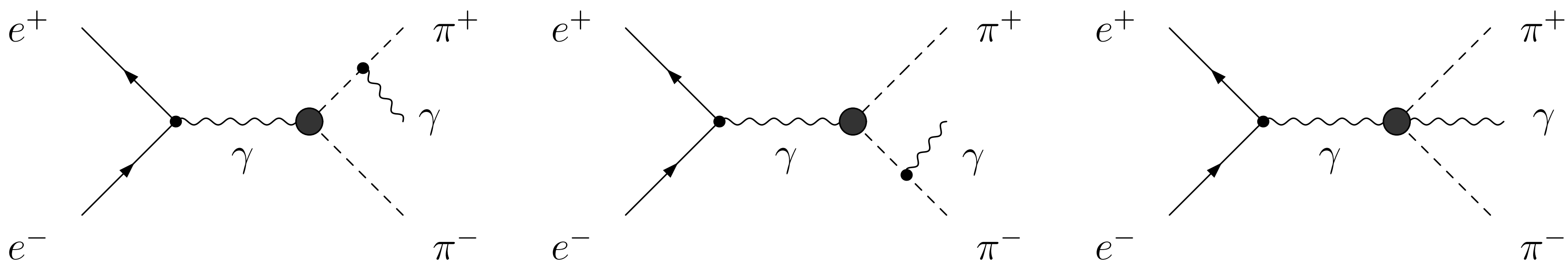
$$\sum_\lambda |M_{Born}|^2(S,T,U) = \frac{8(4\pi\alpha)^2 F_{2\pi}^2(S)}{S^2}\left(TU - m_\pi^2 S\right) \propto q^2 \sin^2\theta_B$$

$$S = 2p_1\cdot p_2, \quad T = 2p_1\cdot q_1, \quad U = 2p_1\cdot q_2$$

q is the length of $\vec{q_1}$, $\theta_B = \angle p_1 q_1$

$\gamma^* \to \pi^+\pi^-\gamma$

$$e^+(p_1,\lambda_1)e^-(p_2,\lambda_2) \to \gamma^* \to \pi^+(q_1)\pi^-(q_2)\gamma(k,\epsilon)$$



$$M = V_\mu H^\mu \ \ , \ \ V_\mu = \imath e \bar{u}(p_1,\lambda_1)\gamma_\mu v(p_2,\lambda_2)$$

$$H^\mu = \frac{e^2 F_{2\pi}(S)}{S}\left\{(q_1+k-q_2)^\mu \frac{q_1\cdot\epsilon}{q_1\cdot k} + (q_2+k-q_1)^\mu \frac{q_2\cdot\epsilon}{q_2\cdot k} - 2\epsilon^\mu\right\}$$

Rewrite H^μ into two gauge invariant parts $H^\mu = H_I^\mu + H_{II}^\mu$

$$H_I^\mu = \frac{e^2 F_{2\pi}(S)}{S}\left((q_1-q_2)^\mu + k^\mu \frac{q_2\cdot k - q_1\cdot k}{q_2\cdot k + q_1\cdot k}\right)\left(\frac{q_1\cdot\epsilon}{q_1\cdot k} - \frac{q_2\cdot\epsilon}{q_2\cdot k}\right)$$

$H_I^\mu \to \sqrt{4\pi\alpha}H_0^\mu \left(\frac{q_1\cdot\epsilon}{q_1\cdot k} - \frac{q_2\cdot\epsilon}{q_2\cdot k}\right)$ for soft and collinear limits

$\gamma^* \to \pi^+\pi^-(\gamma)$

$$H^\mu_{II} = \frac{e^2 F_{2\pi}(S)}{S}\left(k^\mu\left(\frac{q_1\cdot\epsilon}{q_1\cdot k}+\frac{q_2\cdot\epsilon}{q_2\cdot k}\right) - 2\epsilon^\mu - k^\mu \frac{q_2\cdot k - q_1\cdot k}{q_2\cdot k + q_1\cdot k}\left(\frac{q_1\cdot\epsilon}{q_1\cdot k}-\frac{q_2\cdot\epsilon}{q_2\cdot k}\right)\right)$$
$$= \frac{2e^2 F_{2\pi}(S)}{S}\left(\frac{k^\mu}{q_2\cdot k + q_1\cdot k}(q_1\cdot\epsilon + q_2\cdot\epsilon) - \epsilon^{*\mu}\right)$$

free of soft and collinear and singularities!

Similar factor like $k^\mu \frac{q_2\cdot k - q_1\cdot k}{q_2\cdot k + q_1\cdot k}$ in QCD amplitude

(see A. van Hameren and Z. Was, Eur.Phys.J.C61:33-49,2009)

$$A_I = \sum_{\lambda,\epsilon} |V_\mu H^\mu_I|^2 \propto |\vec{q}_1 - \vec{q}_2 + \vec{k}\frac{q_2\cdot k - q_1\cdot k}{q_2\cdot k + q_1\cdot k}|^2 \sin^2\theta$$

$$A'_I = A_I \frac{S - 4m_\pi^2}{|\vec{q}_1 - \vec{q}_2 + \vec{k}\frac{q_2\cdot k - q_1\cdot k}{q_2\cdot k + q_1\cdot k}|^2} = 4\pi\alpha \sum_\lambda |M_{Born}|^2 \sum_\epsilon \left(\frac{q_1\cdot\epsilon}{q_1\cdot k} - \frac{q_2\cdot\epsilon}{q_2\cdot k}\right)^2$$

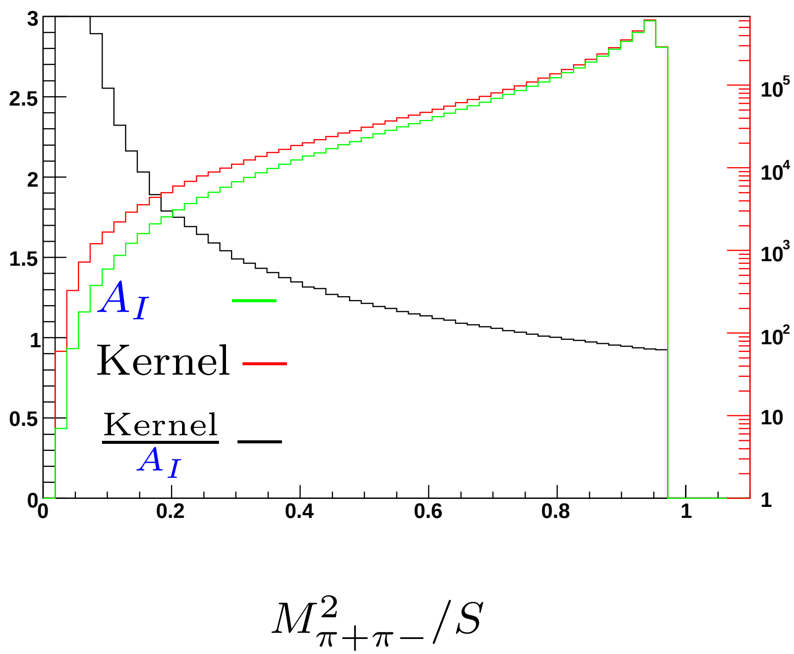
$$\sum_{\lambda,\epsilon} |M|^2 = A'_I + A_{\text{remain}}$$

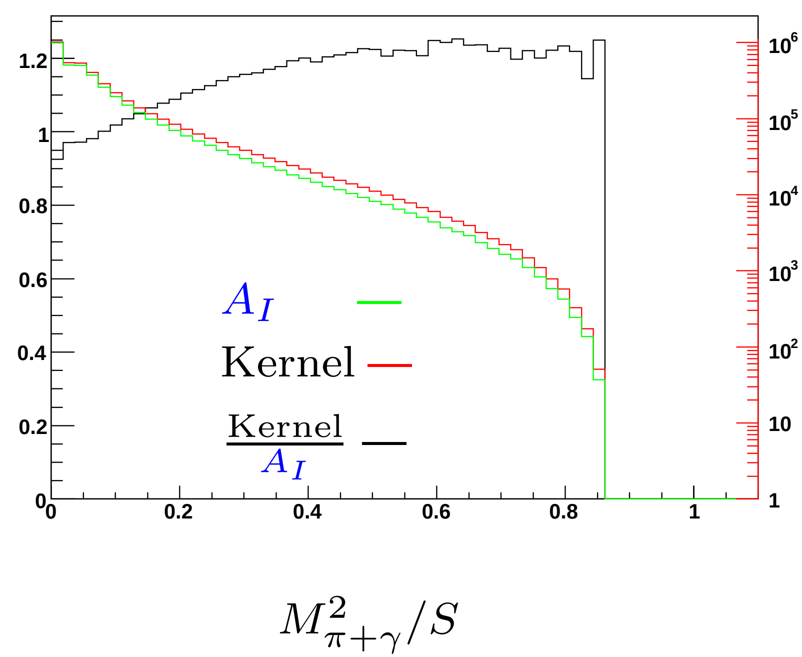
$\gamma^* \to \pi^+\pi^-(\gamma)$ Numerical Results

Comparison of A_I with kernel for scalars in PHOTOS at $\sqrt{S} = 2$GeV

Fraction of events with hard photons ($E_\gamma > 50MeV$):

$3.8329 \pm 0.0020\%$, $4.2279 \pm 0.0021\%$



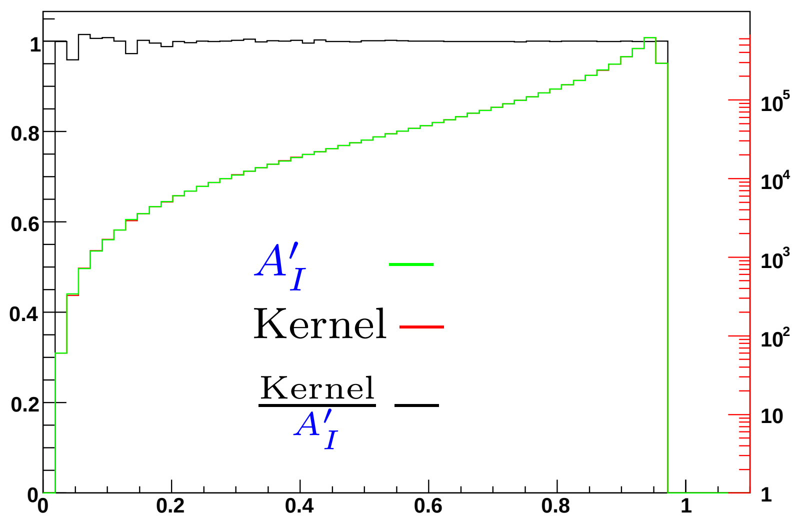


$\gamma^* \to \pi^+\pi^-(\gamma)$ Numerical Results

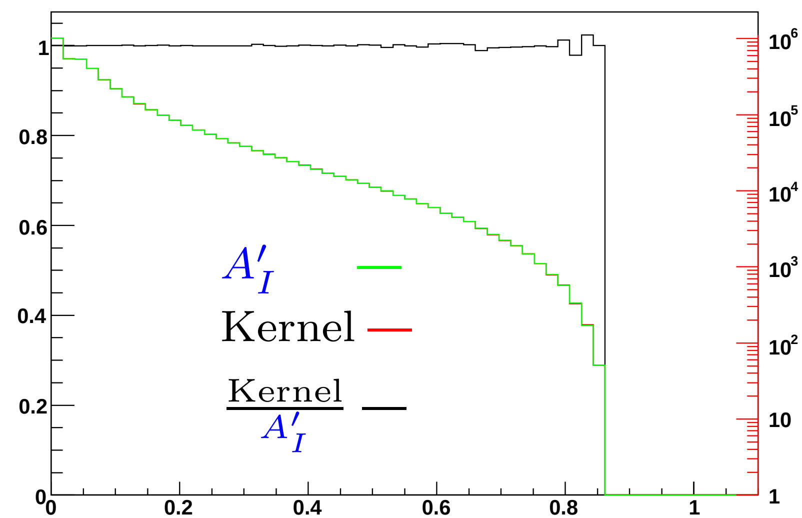
Comparison of A_I' with kernel for scalars in PHOTOS at $\sqrt{S} = 2$GeV

Fraction of events with hard photons ($E_\gamma > 50MeV$):

$4.2278 \pm 0.0021\%$, $4.2279 \pm 0.0021\%$. Very good agreement



$M^2_{\pi+\pi-}/S$



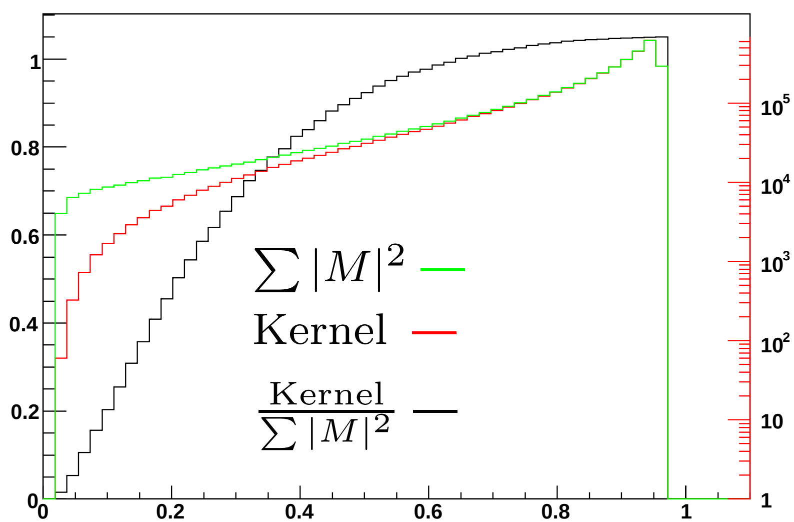
$M^2_{\pi+\gamma}/S$

$\gamma^* \to \pi^+\pi^-(\gamma)$ Numerical Results

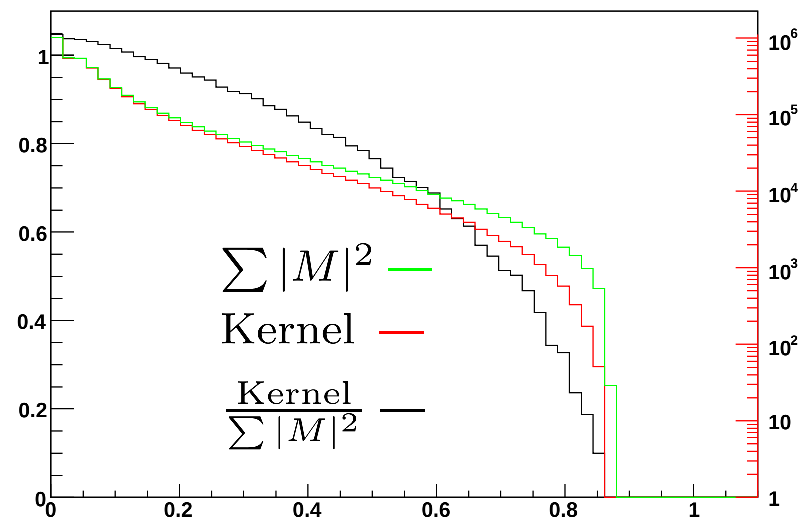
Comparison of complete amplitude with kernel for scalars in PHOTOS at $\sqrt{S} = 2$GeV

Fraction of events with hard photons ($E_\gamma > 50 MeV$):

$4.4320 \pm 0.0021\%$, $4.2279 \pm 0.0021\%$



$M^2_{\pi^+\pi^-}/S$



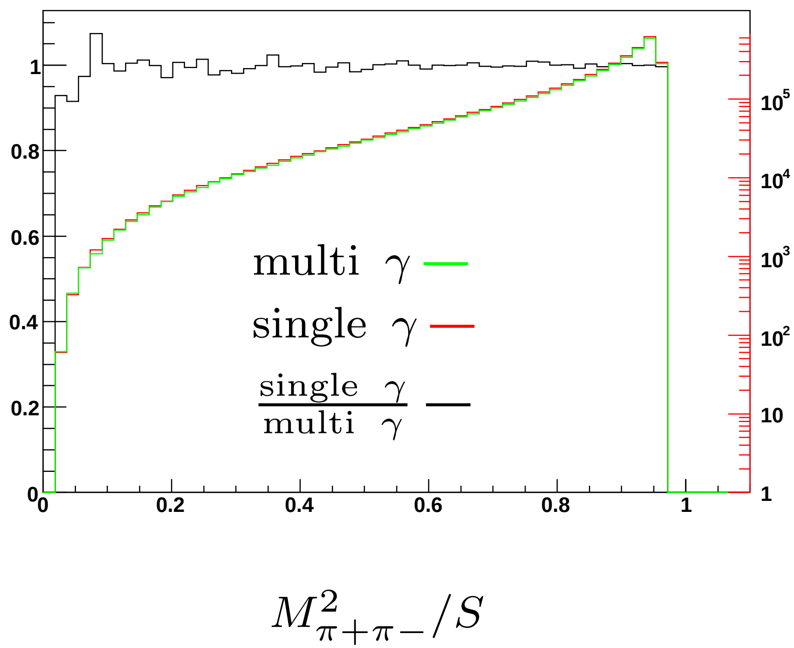
$M^2_{\pi^+\gamma}/S$

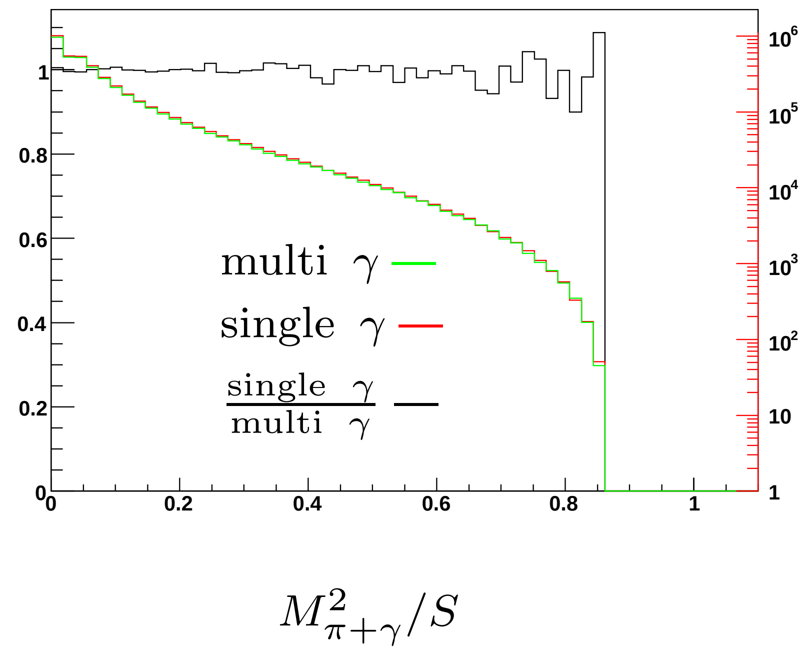
$\gamma^* \to \pi^+\pi^-(\gamma)$ Numerical Results

Comparison of kernel for scalars in the case of one photon emission with the case where multi-photon are emitted at $\sqrt{S} = 2$GeV

Fraction of events with hard photons ($E_\gamma > 50 MeV$):

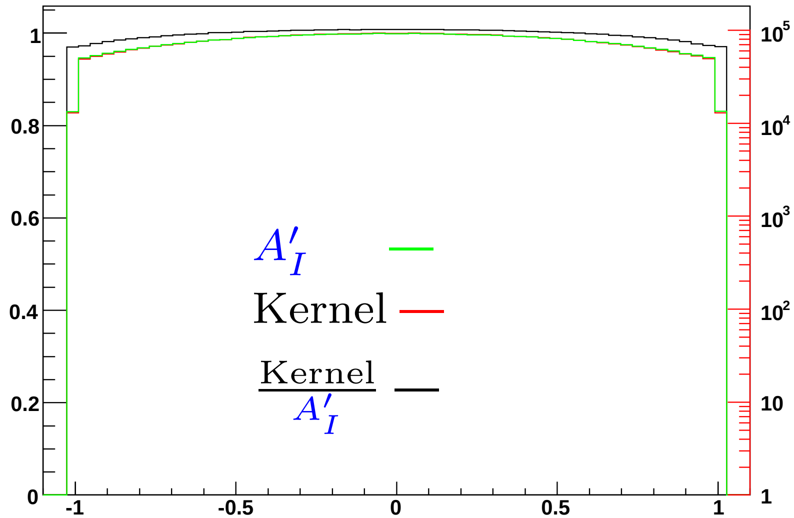
$4.2279 \pm 0.0021\%$, $4.1377 \pm 0.0020\%$



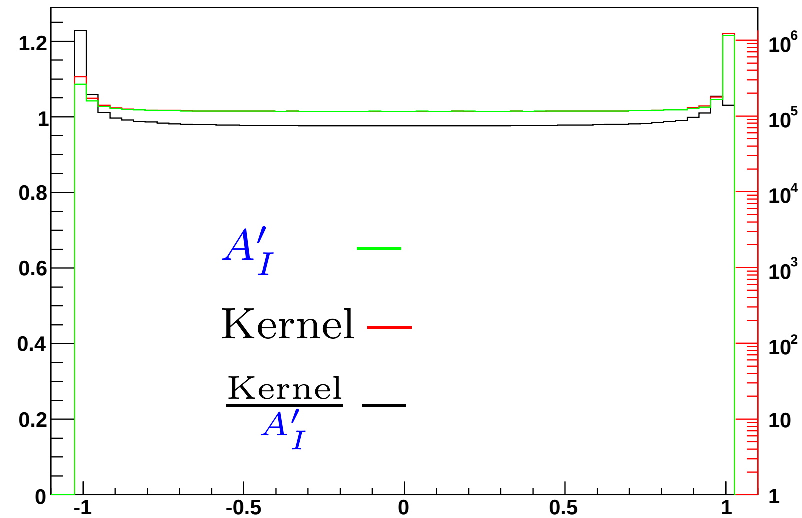


$\gamma^* \rightarrow \pi^+\pi^-(\gamma)$ Numerical Results

Comparison of A'_I with kernel for scalars in PHOTOS at $\sqrt{S} = 2$GeV



θ_γ distribution



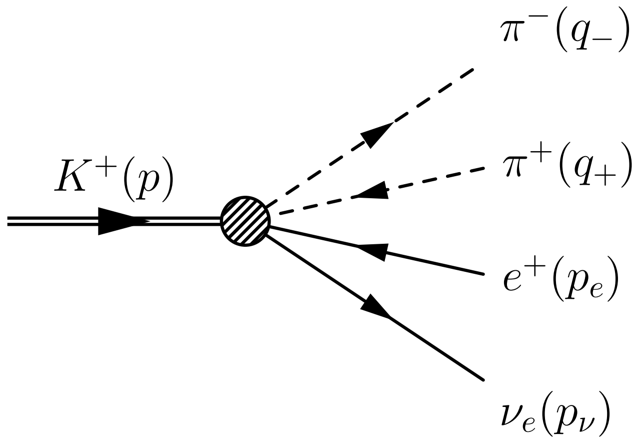
θ_{π^+} distribution

A little difference for θ_γ and θ_{π^+} distribution

angles are respect to the beam direction

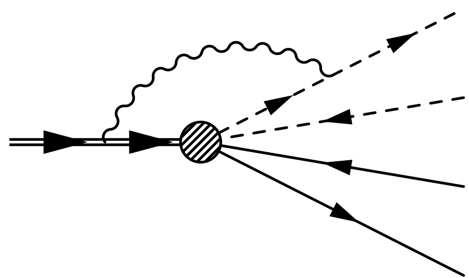
$K^{\pm} \to \pi^{+}\pi^{-}e^{\pm}\nu_e(\gamma)$

Born

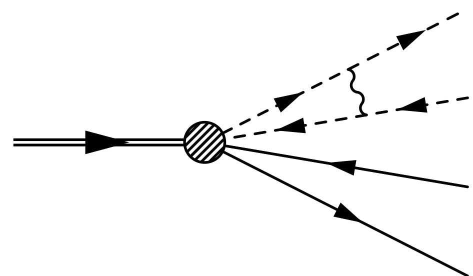


$$s_\pi = (q_+ + q_-)^2,$$
$$s_e = (p_e + p_\nu)^2,$$
$$\phi, \theta_\pi, \theta_e,$$

Virtual



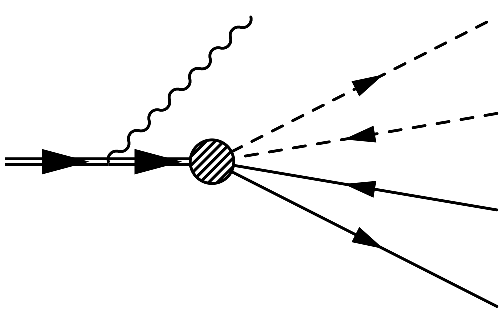
$a.)$



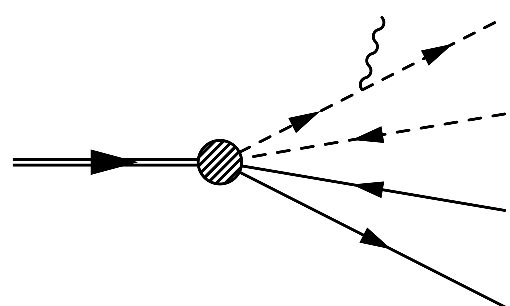
$b.)$

$\cdots + $ CTs

Real



$a.)$



$b.)$

$\cdots$

$K^\pm \to \pi^+\pi^- e^\pm \nu_e(\gamma)$

Analytical Results

$$\frac{d\Gamma_{\rm virt}}{d\Gamma_{\rm Born}} = \frac{\alpha}{\pi}\left[\ln\frac{m}{\lambda}\left(4+\frac{1}{\beta_-}L_- - \frac{1}{\beta_+}L_+ - 2\rho - \frac{1+\beta^2}{\beta}L_\beta + 2\ln\frac{p_e\cdot q_+}{p_e\cdot q_-}\right)\right.$$
$$\left. + \quad \pi^2\frac{1+\beta^2}{2\beta} + \rho^2 + \frac{1}{2}\rho + 2\rho\ln\frac{m}{2E_e} + K_v\right]$$

m is charged pion mass, $\rho = \ln\frac{2E_e}{m_e}$

$\beta = \sqrt{1-\frac{4m^2}{s_\pi}}$, $L_\beta = \ln\frac{1+\beta}{1-\beta}$

$\beta_\pm = \sqrt{1-\frac{m^2}{E_\pm^2}}$, $L_\pm = \ln\frac{1+\beta_\pm}{1-\beta_\pm}$ K_v depends on masses, kinematics

$$\frac{d\Gamma^{soft}}{d\Gamma_{\rm Born}} = \frac{\alpha}{\pi}\left[\ln\left(\frac{\lambda}{2\Delta\epsilon}\right)\left(4+\frac{1}{\beta_-}L_- - \frac{1}{\beta_+}L_+ - 2\rho - \frac{1+\beta^2}{\beta}L_\beta + 2\ln\frac{2p_e\cdot q_+}{2p_e\cdot q_-}\right)\right.$$
$$\left. + \quad \rho - \rho^2 + K_s\right]$$

$\Delta\epsilon$ is the energy cut of soft photon, K_s depends on masses, kinematics

$K^\pm \to \pi^+\pi^- e^\pm \nu_e(\gamma)$

$$\frac{d\Gamma_{\text{Born+virt+soft}}}{d\Omega} = \frac{d\Gamma_{\text{Born}}}{d\Omega}\left(1 + \sigma P_\delta + \frac{\pi\alpha(1+\beta^2)}{2\beta} + \frac{\alpha}{\pi}K_{vs}\right)$$

$$\sigma = \tfrac{\alpha}{2\pi}(2\rho - 1), \qquad P_\delta = 2\ln\tfrac{\Delta\epsilon}{E_e} + \tfrac{3}{2}$$

K_{vs} depends on masses, kinematics and $\Delta\epsilon$

$$\frac{d\Gamma_{\text{collinear}}}{d\Omega} = \frac{d\Gamma_{\text{Born}}}{d\Omega}\left[-\sigma P_\delta + \frac{\alpha}{2\pi}\left(-P_\delta \ln\frac{\delta\theta}{2} + 3 - \frac{2}{3}\pi^2\right)\right]$$

$$E_\gamma > \Delta\epsilon, \quad 1 - \delta\theta < |\cos\theta_{e^+\gamma}| < 1$$

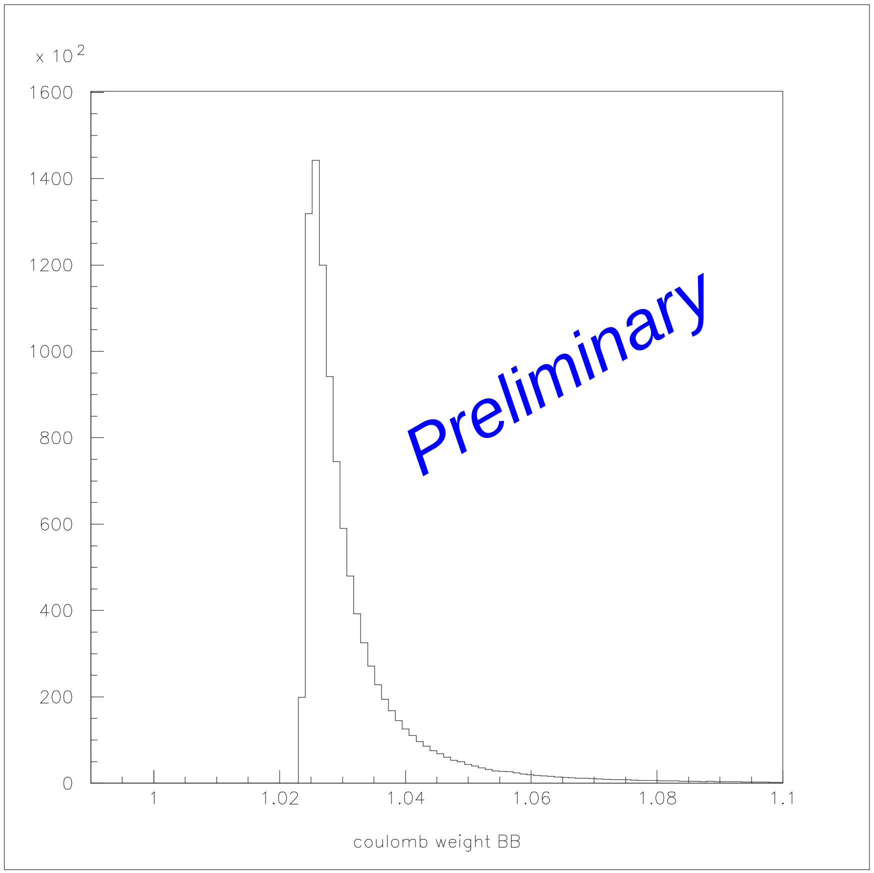
Hard Photon emission is estimated using soft and collinear approximation

$$\frac{d\Gamma_{\text{Born+virt+real}}}{d\Omega} = \frac{d\Gamma_{\text{Born}}}{d\Omega}\left(1 + \frac{\pi\alpha(1+\beta^2)}{2\beta} + \frac{\alpha}{\pi}K\right)$$

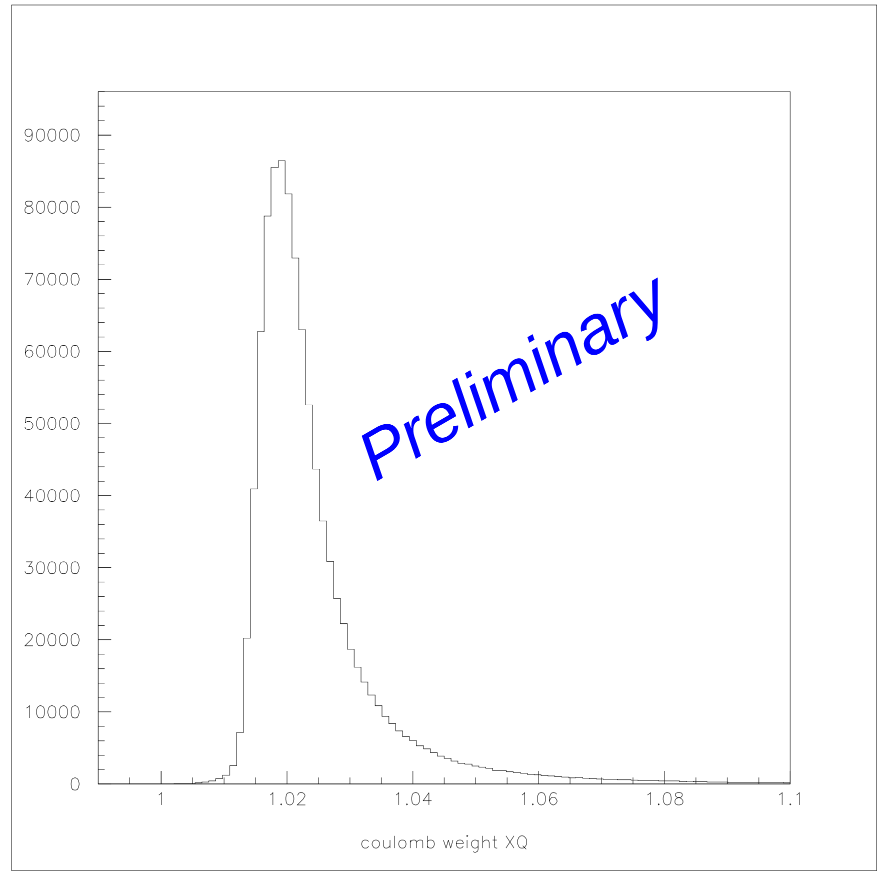
$K = K_{vs} + K_{hard}$, independent on $\Delta\epsilon$ and $\delta\theta$

$K^{\pm} \rightarrow \pi^{+}\pi^{-}e^{\pm}\nu_e(\gamma)$

Comparison of my radiative correction with Coulomb correction from NA48



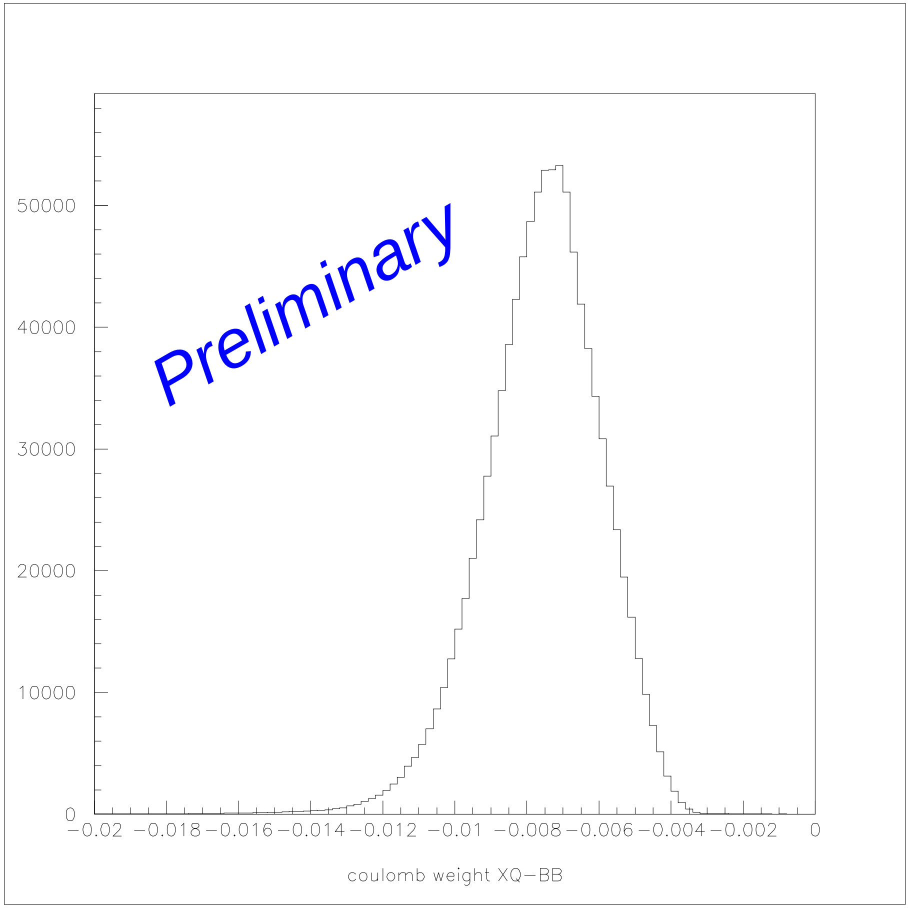
Coulomb correction from NA48



My Radiative correction

$K^{\pm} \to \pi^+\pi^-e^{\pm}\nu_e(\gamma)$

Difference between radiative correction and Coulomb correction from NA48



Next Step

- Compare hard photon emission from PHOTOS with that from approximate analytical calculation
- Implement the radiative correction for K_{e4} decay into PHOTOS Monte Carlo

Summary and Outlook

$\gamma^* \to \pi^+\pi^-(\gamma)$ process

- Its cross section can be separated into an eikonal part and a remaining part using principle of gauge invariance. The eikonal part is identical to the kernel for scalars used in standard PHOTOS
- With PHOTOS this process can be simulated using kernel for scalars and exact matrix element. Results were compared and we found the approximated, easy to use version is correct up to 0.2% level.
- Results from PHOTOS were compared with that from PHOKARA
- Analogies with QCD amplitudes are visible

Summary and Outlook

$K^{\pm} \to \pi^+\pi^- e^{\pm}\nu_e(\gamma)$ process

- It is calculated analytically
- We compare the radiative correction with Coulomb correction from NA48, find a very good agreement
- Compare hard photon emission from PHOTOS Monte Carlo with that from approximate analytical calculation
- The radiative correction for K_{e4} decay will be implemented into PHOTOS Monte Carlo
- Calculate this work in the framework of Chiral Perturbative QCD (Prof. J. Gasser)
- The radiative correction for K_{e4} decay will be used by NA48