

NLO QCD corrections to WW +jet production including leptonic W decays at hadron colliders

Stefan Kallweit¹

in collaboration with S. Dittmaier and P. Uwer
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arXiv:0908.4124 [hep-ph] (to appear in *Nucl. Phys. B*)

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- 1 Introduction
- 2 Calculation of NLO QCD corrections via dipole subtraction
 - Virtual corrections
 - Real corrections
- 3 Implementation of W decays
 - Improved narrow-width approximation
- 4 Numerical results for LHC and Tevatron
 - Comparison of decay descriptions at LO
 - Scale dependence of NLO cross sections
 - Differential NLO cross sections
- 5 Conclusions & Outlook

Relevance of $pp/p\bar{p} \rightarrow WW + \text{jet} + X$

- **Important background process** at the LHC (and for Tevatron Higgs searches):
 - background to Higgs searches: $H + \text{jet}$,
 - background to new-physics searches: $\text{leptons} + \cancel{E}_T + \text{jet}$.
- ↔ $VV + \text{jet}$ is on the top of the “Les Houches experimenter’s wishlist '05”

Progress on $VV + \text{jet}$ so far:

- $pp \rightarrow WW + \text{jet}$ Dittmaier/S.K./Uwer '07, '09
Campbell/Ellis/Zanderighi '07
Binoth/Guillet/Karg/Kauer/Sanguinetti (in progress)
- $pp \rightarrow W\gamma + \text{jet}$ Campanario/Englert/Spannowsky/Zeppenfeld '09

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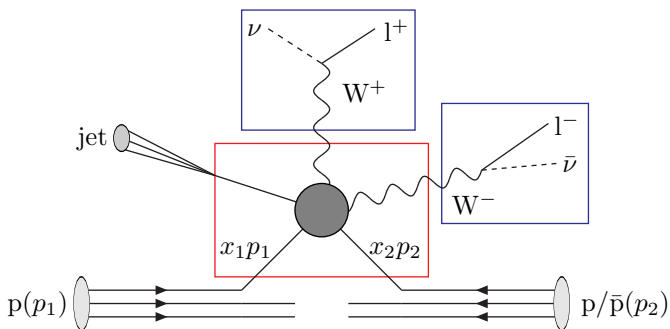
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- $WW + \text{jet}$ delivers the **real-virtual contribution to WW production at NNLO QCD**. (two-loop results known [Chachamis, Czakon, Eiras '08])
- Process was an **important test ground** before approaching more complicated many-particle processes at NLO.

Hadronic cross section

Schematic illustration of the hadronic process $pp/p\bar{p} \rightarrow W^+W^- + \text{jet} + X$:



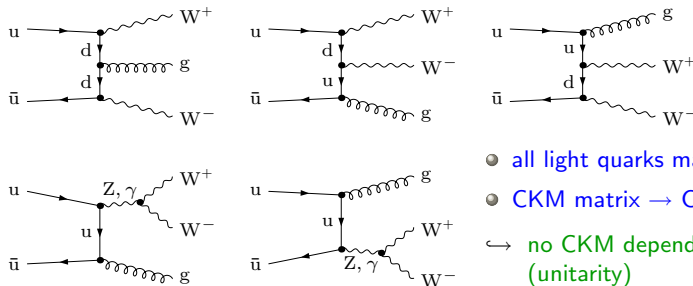
$$\sigma^{pp/p\bar{p}}(p_1, p_2) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 \underbrace{f_{a(p)}(x_1, \mu_F) f_{b(p/\bar{p})}(x_2, \mu_F)}_{\text{PDF's of parton } a/b \text{ in } p/\bar{p}} \underbrace{\hat{\sigma}^{ab}(x_1 p_1, x_2 p_2)}_{\text{partonic cross section}}$$

Subprocesses contributing at leading order

6 partonic channels in LO (12 flavour channels for 2 generations, b quarks omitted):

$$\begin{aligned}
 u\bar{u} &\rightarrow W^+W^-g, & ug &\rightarrow W^+W^-u, & g\bar{u} &\rightarrow W^+W^-\bar{u}, \\
 d\bar{d} &\rightarrow W^+W^-g, & dg &\rightarrow W^+W^-d, & g\bar{d} &\rightarrow W^+W^-\bar{d}
 \end{aligned}$$

Diagrams for $u\bar{u}$ initial state:



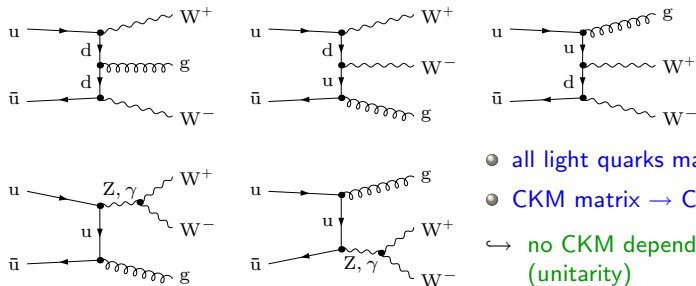
- all light quarks massless
- CKM matrix \rightarrow Cabibbo matrix
- \hookrightarrow no CKM dependence in LO (unitarity)

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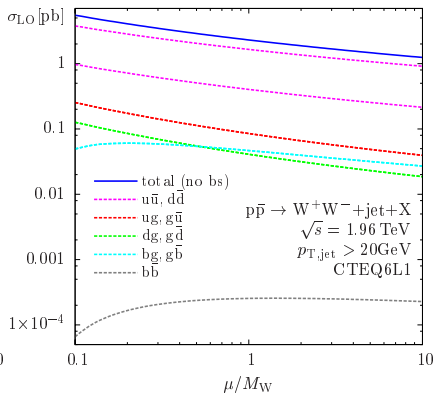
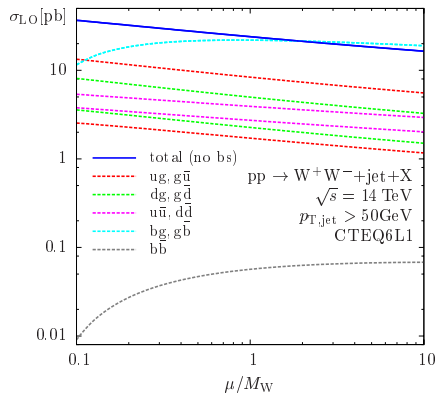


- all light quarks massless
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The amplitudes for all other channels are generated by crossing the gluon into the initial state and by $SU(2)$ symmetry ($u \leftrightarrow d, W^+ \leftrightarrow W^-$).

Leading-order prediction

Scale dependence of LO cross section (with $\mu = \mu_{\text{fact}} = \mu_{\text{ren}}$):



LHC: Cross section changes by 12% (30%) when scaling μ by a factor of 2 (5).

Tevatron: Cross section changes by 25% (75%) when scaling μ by a factor of 2 (5).

↔ For precise predictions the calculation of NLO QCD corrections is required.

NLO cross section with the dipole subtraction formalism

Schematic formula for the NLO cross section in the situation of two initial-state hadrons (LHC and Tevatron):

$$\sigma^{\text{NLO}} = \underbrace{\int_{m+1} d\sigma^R}_{\text{real corrections}} + \underbrace{\int_m d\sigma^V}_{\text{virtual corrections}} + \underbrace{\int_0^1 dx \int_m d\sigma^C}_{\text{collinear-subtraction counterterm}}$$

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$$d\sigma^A = \sum_{\text{dipoles}} d\sigma^B \otimes dV_{\text{dipole}}$$

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 &= \int_{m+1} \left[d\sigma^R - d\sigma^A \right]_{\epsilon=0} \quad \Rightarrow \text{R} - \text{A} \\
 &\quad + \int_m \left[d\sigma^V + \sum_{\text{dipoles}} d\sigma^B \otimes V_{\text{dipole}}(1) \right]_{\epsilon=0} \quad \Rightarrow \text{V} + \text{A} \\
 &\quad + \int_0^1 dx \int_m \left[d\sigma^C + \sum_{\text{dipoles}} \int_1^x d\sigma^B(x) \otimes [dV_{\text{dipole}}(x)]_+ \right]_{\epsilon=0} \quad \Rightarrow \text{C} + \text{A} \\
 &\quad dV_{\text{dipole}}(x) = [dV_{\text{dipole}}(x)]_+ + dV_{\text{dipole}}(1)\delta(1-x)
 \end{aligned}$$

Virtual corrections

For each channel $\mathcal{O}(100)$ 1-loop diagrams contribute, which can be classified as

- “bosonic” corrections (exchange of an additional gluon)
- “fermionic” corrections (closed quark loops)

Renormalization leads to counterterm diagrams contributing on 1-loop level.

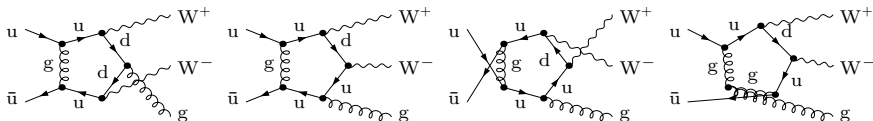
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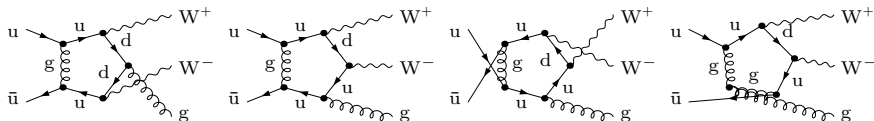
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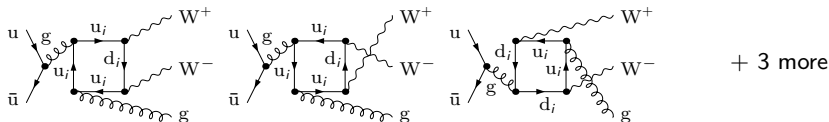
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Renormalization leads to **counterterm diagrams** contributing on 1-loop level.

- **Bosonic corrections:** e. g. **pentagon contributions** (5-point functions)



- **Fermionic corrections:** e. g. **box contributions** (4-point functions)



Virtual corrections

Two different strategies for evaluation of loop amplitudes
(realized in two independent calculations!)

- Analogous to NLO calculation for

$pp \rightarrow t\bar{t}H$ [Beenakker et al. '02] and $pp \rightarrow t\bar{t} + \text{jet}$ [Dittmaier, Uwer, Weinzierl '07]

- diagrams generated with FEYNARTS 1.0 [Küblbeck, Böhm, Denner '90] and reduced with in-house MATHEMATICA routines \rightarrow FORTRAN
- analytical extraction of IR singularities [Beenakker et al. '02, Dittmaier '03]
- reduction of 5-point to 4-point integrals according to [Denner, Dittmaier '02], numerical stabilization for exceptional momentum regions via expansions [DD '05]

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- Alternative calculation with available tools

- diagrams generated with FEYNARTS 3.4 [Hahn '00]
- algebraic reduction/numerical evaluation with FORMCALC 6.0/LOOPTOOLS: [Hahn, Perez-Victoria '98]
 - reduction of 5-point integrals à la [Denner, Dittmaier '02]
 - regular scalar integrals with FF [v.Oldenborgh '91]
- dimensionally regularized singular integrals added to LOOPTOOLS:
 - box integrals checked against result of [Bern, Dixon, Kosower '93]

Real corrections

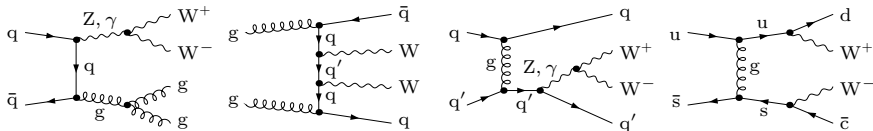
Contributing subprocesses are generated by two types of generic amplitudes,

- $0 \rightarrow W^+W^-q\bar{q}gg$,
- $0 \rightarrow W^+W^-q\bar{q}q'\bar{q}'$,

and crossing any two partons into the initial state.

↪ Large number of contributions (136 flavour channels for 2 generations)!

Sample of real-correction diagrams:



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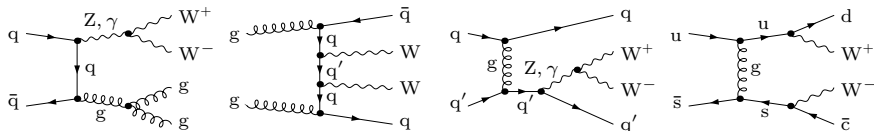
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Two evaluations of helicity amplitudes:

- application of (4-dimensional) spinor techniques
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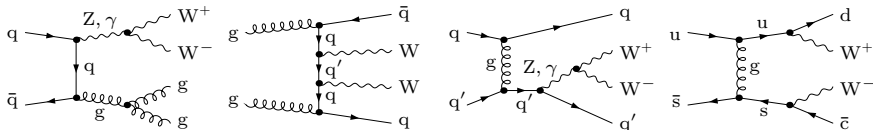
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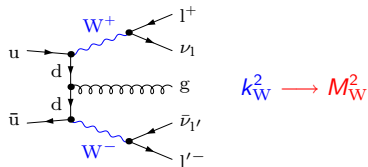
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Two versions of Monte Carlo integrators:

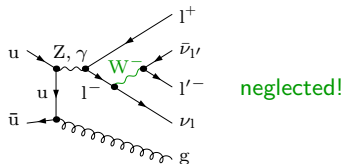
- based on multi-channel MC technique [Berends, Pittau, Kleis '94, Kleis, Pittau '94]
- second version based on simple mapping (phase space by sequential splitting)

W decays via improved NWA

Double-resonant diagrams:

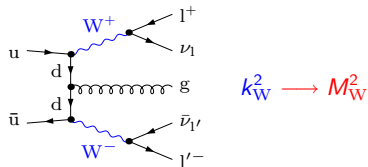


Single-resonant diagrams:

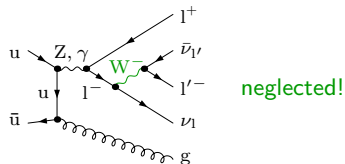


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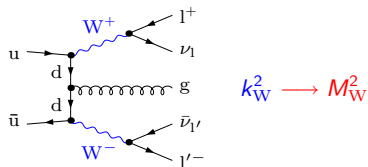
- on-shell approximation for resonant matrix elements:

$$\mathcal{M}_{\text{resonant}} \sim \sum_{\lambda_W} \frac{\mathcal{M}_{\text{prod}}^W(k_W^2 \rightarrow M_W^2, \lambda_W) \times \mathcal{M}_{\text{decay}}^W(k_W^2 \rightarrow M_W^2, \lambda_W)}{k_W^2 - M_W^2 + i\Gamma_W M_W}$$

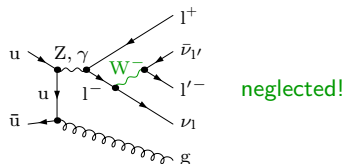
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Spin correlations respected!

- replacement of Breit–Wigner-propagator by a δ -function approximation:

$$\int_{\min}^{\max} dk_W^2 \int d\Phi_{\text{prod}}^W(k_W^2) \frac{1}{(k_W^2 - M_W^2)^2 + \Gamma_W^2 M_W^2} \int d\Phi_{\text{decay}}^W(k_W^2)$$

$$\longrightarrow \int_{-\infty}^{\infty} dk_W^2 \int d\Phi_{\text{prod}}^W(M_W^2) \frac{\pi}{\Gamma_W M_W} \delta(k_W^2 - M_W^2) \int d\Phi_{\text{decay}}^W(M_W^2)$$

W decays via improved NWA

Two different (equivalent) implementation strategies

$$\hat{k} = \hat{k}_1 + \hat{k}_2, \quad \hat{k}^2 = M_V^2$$

- Decay correlation matrix

$$\sigma_{ab \rightarrow VX \rightarrow f\bar{f}'X} \underset{\Gamma_V \rightarrow 0}{\widetilde{}} \frac{1}{2s_{ab}} \int d\Phi_{VX}(\hat{k}) \sum_{\lambda, \lambda' = 0, \pm 1} \mathcal{M}_{ab \rightarrow VX}(\lambda')^* \mathcal{M}_{ab \rightarrow VX}(\lambda) \\ \times \text{BR}_{V \rightarrow f\bar{f}'} \int \frac{d\Phi_{f\bar{f}'}(\hat{k}_1, \hat{k}_2)}{\Phi_{f\bar{f}'}} \underbrace{\Delta_{\lambda'\lambda}^{V \rightarrow f\bar{f}'}(\hat{k}_1, \hat{k}_2)}_{\text{decay correlation matrix}},$$

$$\Delta_{\lambda'\lambda}^{V \rightarrow f\bar{f}'}(\hat{k}_1, \hat{k}_2) = \frac{\Phi_{f\bar{f}'}}{2M_V \Gamma_{V \rightarrow f\bar{f}'}} \mathcal{M}_{V \rightarrow f\bar{f}'}(\lambda')^* \mathcal{M}_{V \rightarrow f\bar{f}'}(\lambda).$$

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- Replacement of polarization vectors by leptonic currents in the amplitude

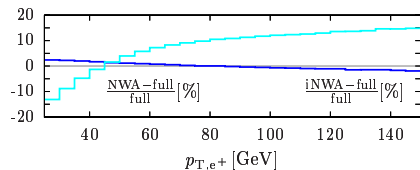
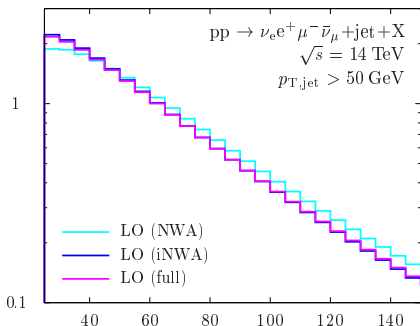
$$\sigma_{ab \rightarrow VX \rightarrow f\bar{f}'X} \quad \widetilde{\Gamma_{V \rightarrow 0}} = \frac{1}{2s_{ab}} \int d\Phi_{VX}(\hat{k}) \int \frac{d\Phi_{f\bar{f}'}(\hat{k}_1, \hat{k}_2)}{2M_V \Gamma_V} \sum_{\sigma = \pm 1} |\tilde{\mathcal{M}}_{ab \rightarrow V(\rightarrow f\bar{f}')X}(\sigma)|^2,$$

$$\tilde{\mathcal{M}}_{ab \rightarrow V(\rightarrow f\bar{f}')X}(\sigma) = \mathcal{M}_{ab \rightarrow VX}(\lambda) \text{ with}$$

$$\varepsilon_V^{*\mu}(\lambda) \longrightarrow \tilde{\varepsilon}_{V \rightarrow f\bar{f}'}^{*\mu}(\sigma) = e g_{Vff'}^\sigma \bar{u}_f(\hat{k}_1) \gamma^\mu \omega_\sigma v_{\bar{f}'}(\hat{k}_2).$$

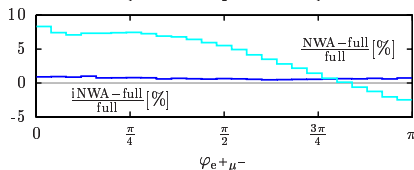
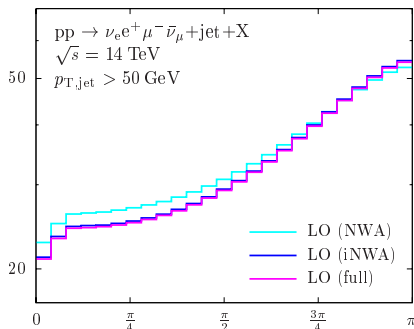
Full calculation vs. sNWA vs. iNWA at LO at LHC

$$\frac{d\sigma}{dp_{T,e^+}} \left[\frac{\text{fb}}{\text{GeV}} \right]$$



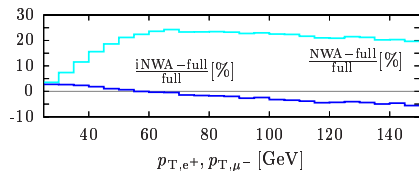
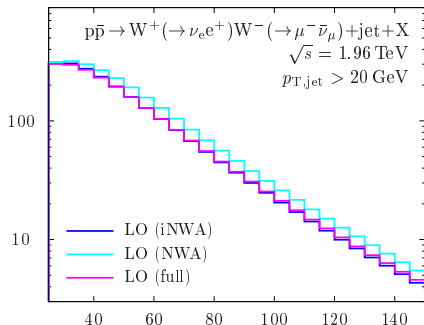
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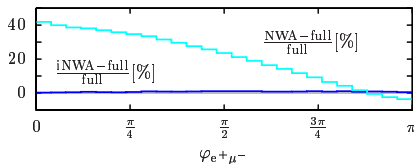
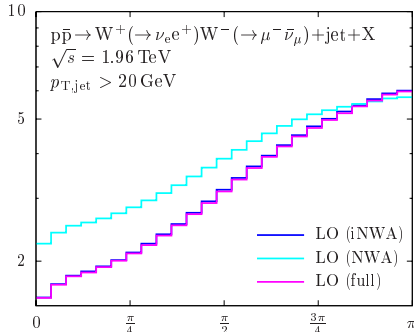
Full calculation vs. sNWA vs. iNWA at LO at Tevatron

$$\frac{d\sigma}{dp_{T,e^+}}, \frac{d\sigma}{dp_{T,\mu^-}} \left[\frac{\text{ab}}{\text{GeV}} \right]$$



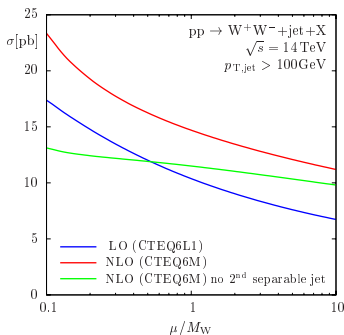
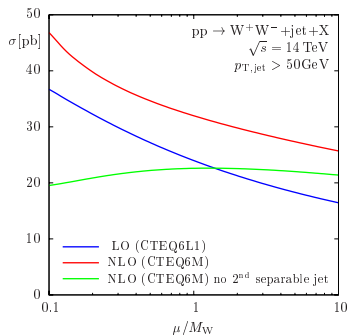
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Scale dependence of cross sections at the LHC

LO versus NLO cross sections with stable W bosons:



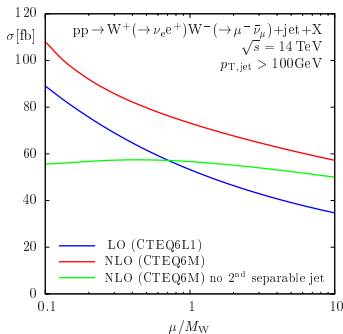
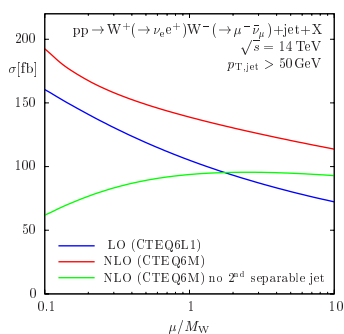
$$\mu = \mu_{\text{fact}} = \mu_{\text{ren}}$$

Jet definition: successive combination algorithm (with $R = 1$) [S.D.Ellis, Soper '93]

- Scale dependence stabilizes at NLO for genuine $WW + \text{jet}$ production.
- But: Significant scale dependence is introduced by $WW + 2\text{jets}$ events.

Scale dependence of cross sections at the LHC

LO versus NLO cross sections including leptonic W decays via improved NWA:



$$\mu = \mu_{\text{fact}} = \mu_{\text{ren}}$$

Additional cuts:

$$|\eta_{\text{jet}}| < 4.5$$

$$p_{T,\text{lep}} > 25 \text{ GeV}$$

$$|\eta_{\text{lep}}| < 2.5$$

$$p_{T,\text{miss}} > 25 \text{ GeV}$$

$$R_{\text{jet,lep}} > 0.4$$

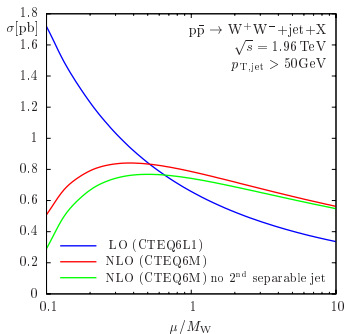
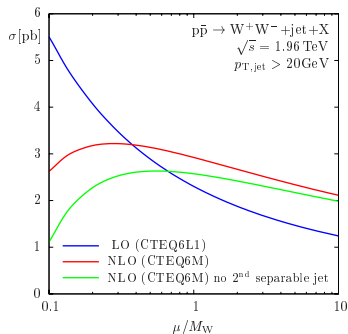
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Scale dependence of cross sections at the Tevatron

LO versus NLO cross sections with stable W bosons:

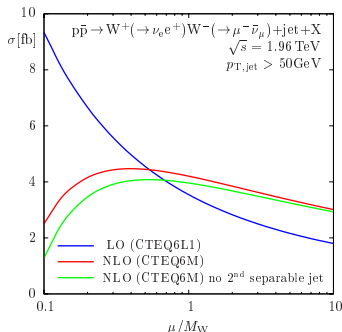
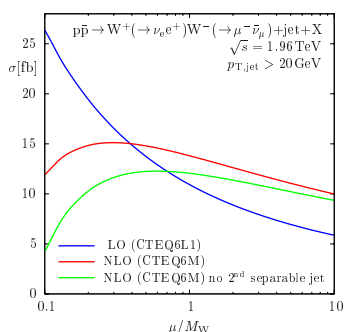


Jet definition: successive combination algorithm (with $R = 1$) [S.D.Ellis, Soper '93]

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- Only small influence from WW+2jets production.

Scale dependence of cross sections at the Tevatron

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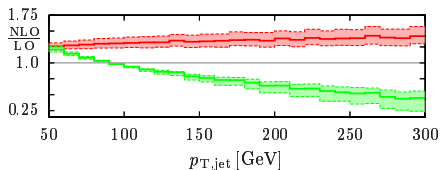
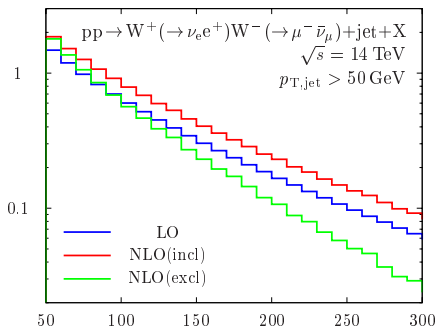
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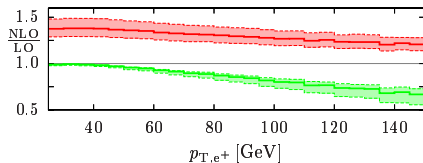
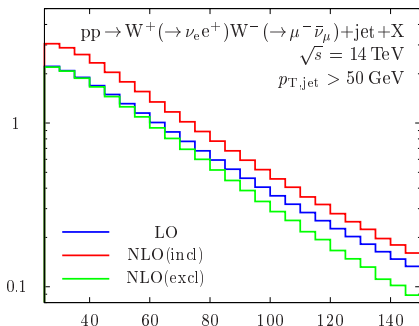
Distributions at LHC: transverse momenta

$$\frac{d\sigma}{dp_{T,\text{jet}}} \left[\frac{\text{fb}}{\text{GeV}} \right]$$



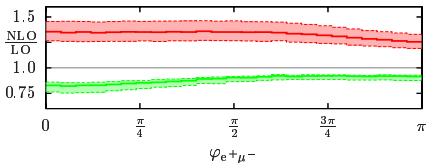
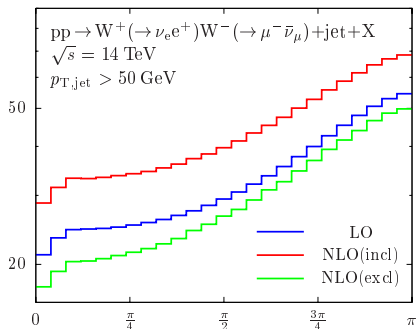
$$\frac{d\sigma}{dp_{T,e^+}} \left[\frac{\text{fb}}{\text{GeV}} \right]$$

$$\mu = \mu_{\text{fact}} = \mu_{\text{ren}} = M_W$$



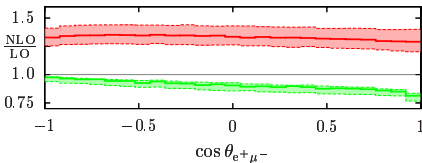
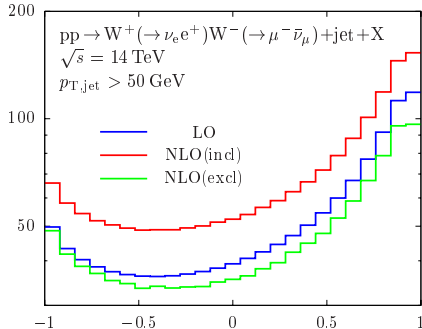
Distributions at LHC: angles between leptons

$$\frac{d\sigma}{d\varphi_{e^+\mu^-}} [\text{fb}]$$



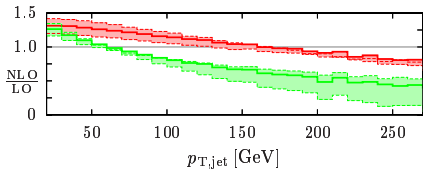
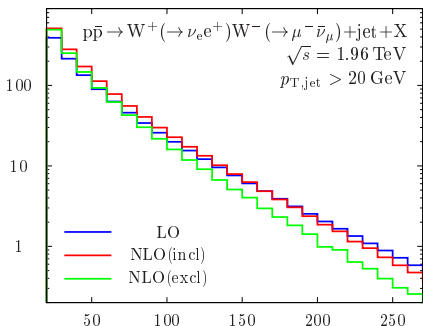
$$\frac{d\sigma}{d\cos\theta_{e^+\mu^-}} [\text{fb}]$$

$$\mu = \mu_{\text{fact}} = \mu_{\text{ren}} = M_W$$



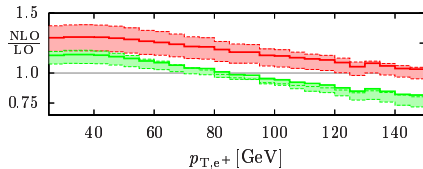
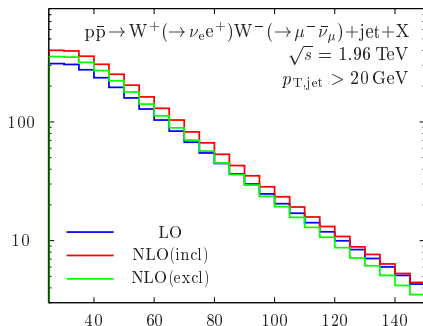
Distributions at Tevatron: transverse momenta

$$\frac{d\sigma}{dp_{T,\text{jet}}} \left[\frac{\text{ab}}{\text{GeV}} \right]$$

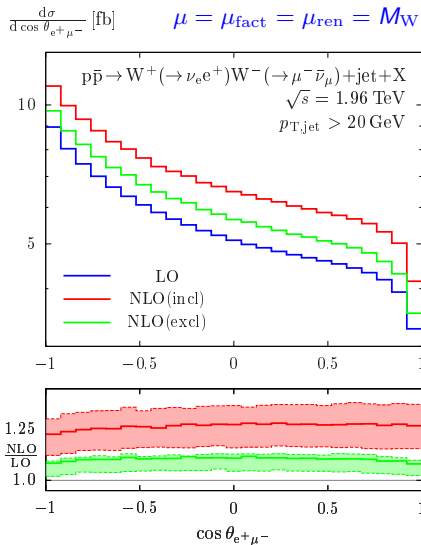
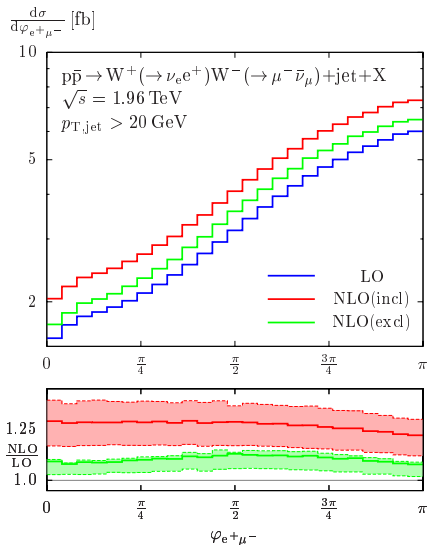


$$\frac{d\sigma}{dp_{T,e^+}} \left[\frac{\text{ab}}{\text{GeV}} \right]$$

$$\mu = \mu_{\text{fact}} = \mu_{\text{ren}} = M_W$$



Distributions at Tevatron: angles between leptons



Conclusions & Outlook

Conclusions

$pp/p\bar{p} \rightarrow WW + \text{jet} + X$ at NLO QCD

- Important background process to Higgs and other searches at Tevatron/LHC
- Reduced scale uncertainty at the Tevatron
- Scale dependence at the LHC:
 - Significant reduction for genuine $WW + \text{jet}$ production
 - But: significant scale dependence via $WW + 2\text{jets}$ events
- Leptonic W decays included by means of an improved NWA
- Differential cross sections provided at NLO QCD

Outlook & work in progress

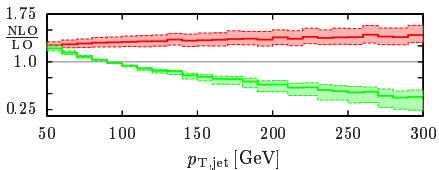
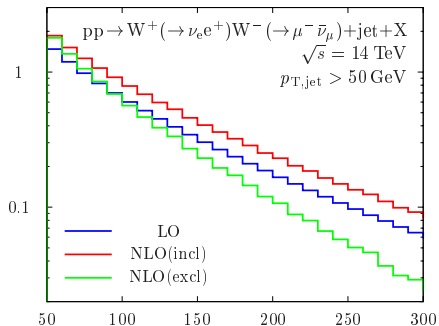
- Calculation of remaining processes of the class $pp/p\bar{p} \rightarrow VV + \text{jet} + X$
- Methods successfully applied \rightarrow more complicated applications (2 \rightarrow 4) feasible!

Backup

Backup slides

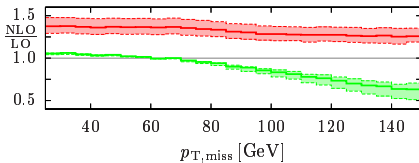
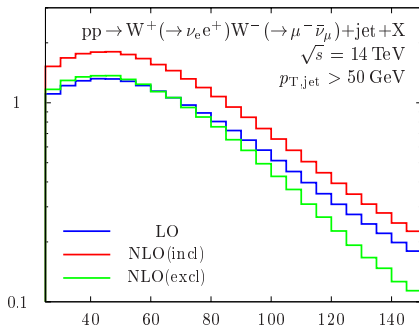
Transverse momenta at the LHC

$$\frac{d\sigma}{dp_{T,jet}} \left[\frac{\text{fb}}{\text{GeV}} \right]$$



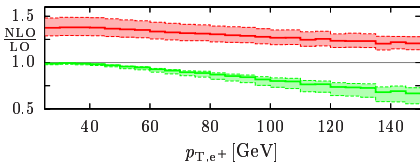
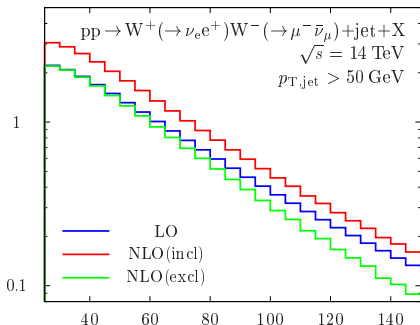
$$\frac{d\sigma}{dp_{T,miss}} \left[\frac{\text{fb}}{\text{GeV}} \right]$$

$$\mu = \mu_{\text{fact}} = \mu_{\text{ren}} = M_W$$



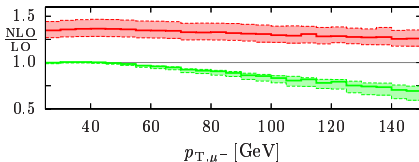
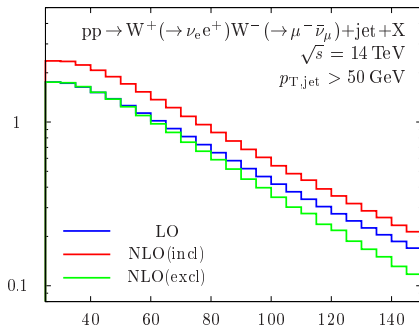
Transverse momenta of leptons at the LHC

$$\frac{d\sigma}{dp_{T,e^+}} \left[\frac{\text{fb}}{\text{GeV}} \right]$$

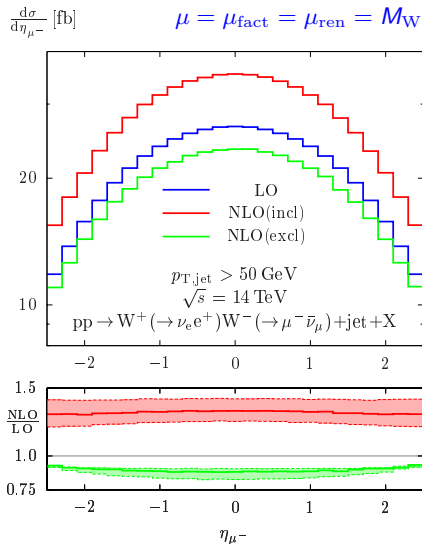
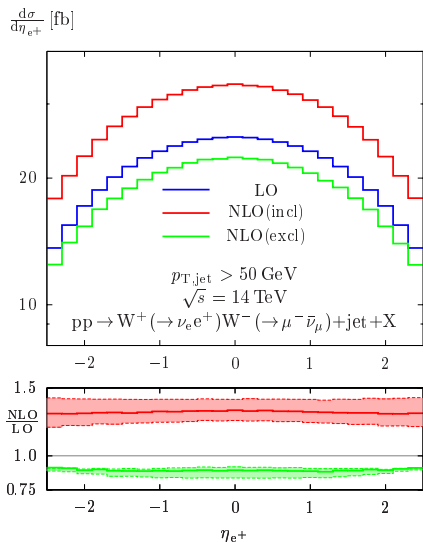


$$\frac{d\sigma}{dp_{T,\mu^-}} \left[\frac{\text{fb}}{\text{GeV}} \right]$$

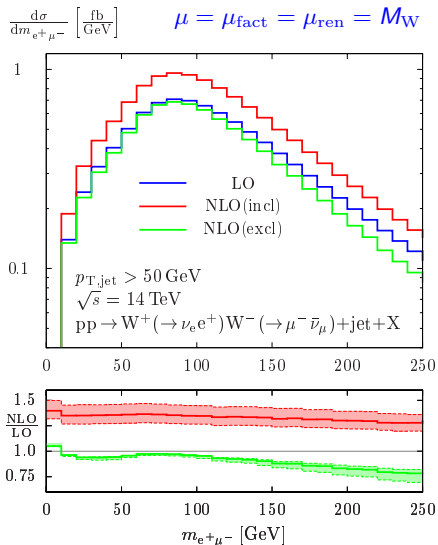
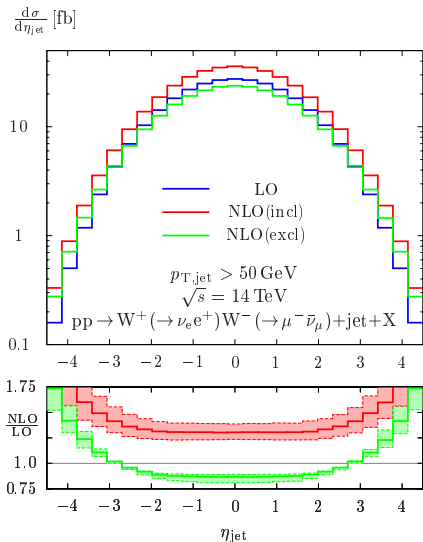
$$\mu = \mu_{\text{fact}} = \mu_{\text{ren}} = M_W$$



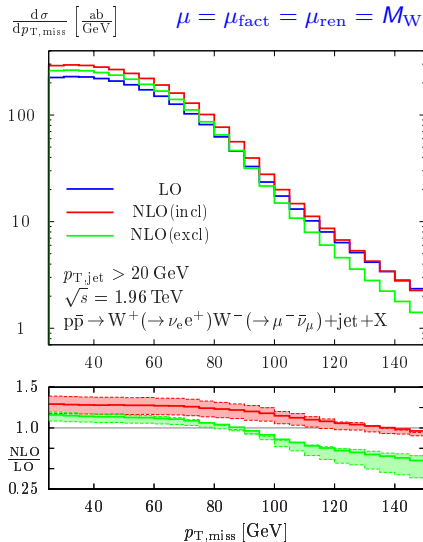
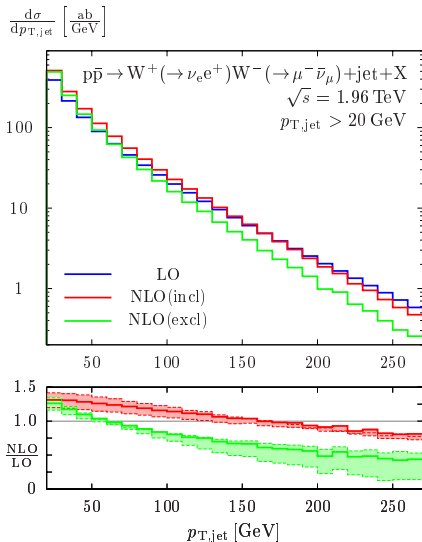
Lepton rapidities at the LHC



Jet rapidity, invariant mass of leptons at the LHC

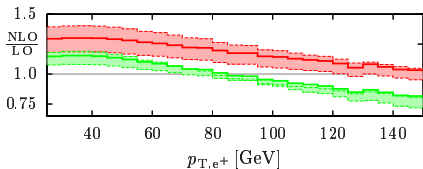
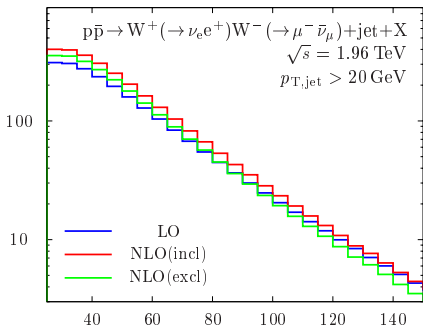


Transverse momenta at the Tevatron



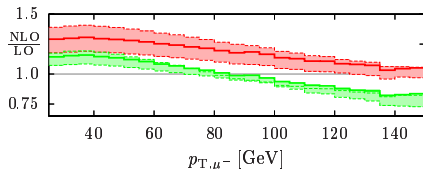
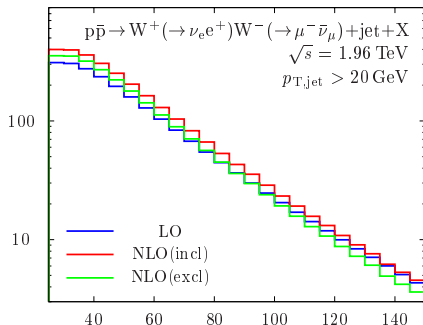
Transverse momenta of leptons at the Tevatron

$$\frac{d\sigma}{dp_{T,e^+}} \left[\frac{\text{ab}}{\text{GeV}} \right]$$



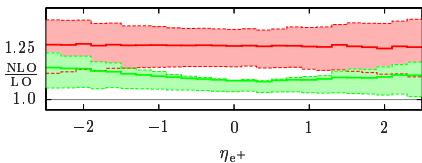
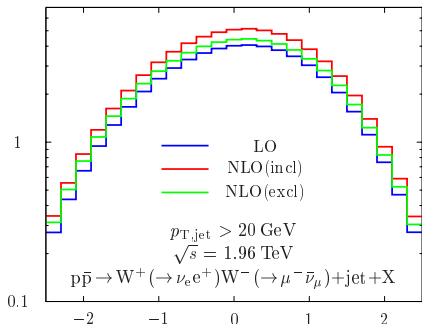
$$\frac{d\sigma}{dp_{T,\mu^-}} \left[\frac{\text{ab}}{\text{GeV}} \right]$$

$$\mu = \mu_{\text{fact}} = \mu_{\text{ren}} = M_W$$



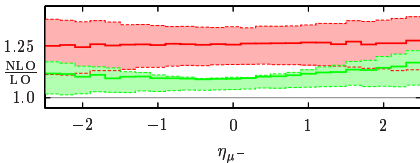
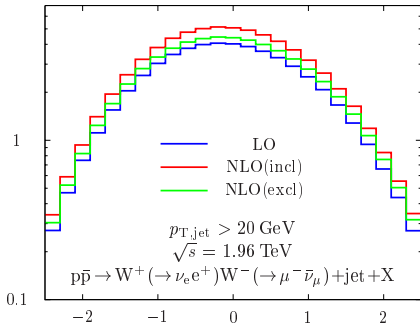
Lepton rapidities at the Tevatron

$$\frac{d\sigma}{d\eta_{e^+}} [\text{fb}]$$

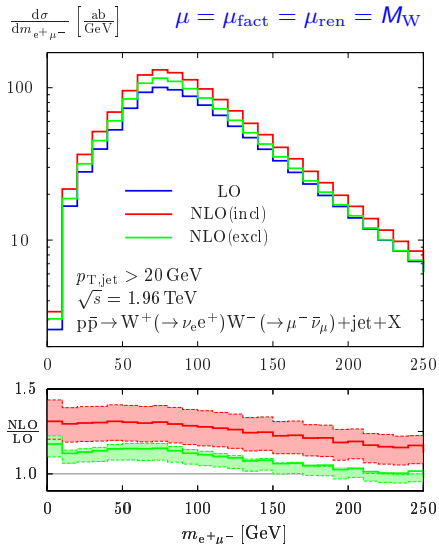
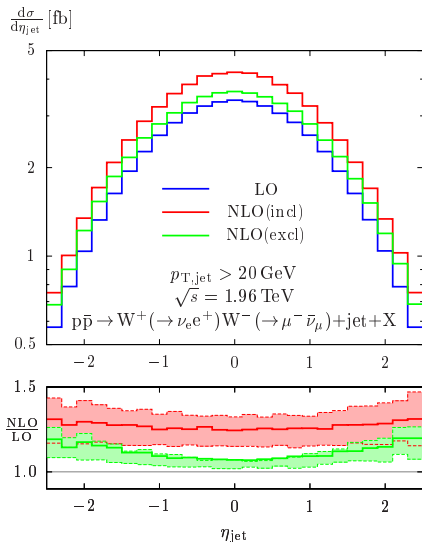


$$\frac{d\sigma}{d\eta_{\mu^-}} [\text{fb}]$$

$$\mu = \mu_{\text{fact}} = \mu_{\text{ren}} = M_W$$

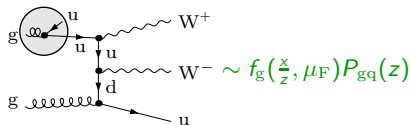
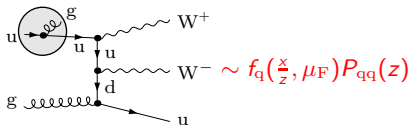


Jet rapidity, invariant mass of leptons at the Tevatron



Collinear subtraction counterterm

- **Real corrections:** divergences resulting from collinear emission off incoming partons
 \hookrightarrow no complete cancellation against virtual corrections
- **Divergences** are absorbed in redefined parton distribution functions (PDF's)



Corresponding redefinition of PDF's:

$P_{ij}(z) =$ Altarelli–Parisi splitting functions

$$\begin{aligned}
 f_q(x, \mu_F) &\rightarrow f_q(x, \mu_F) \\
 &+ \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} f_q\left(\frac{x}{z}, \mu_F\right) \left(\frac{\Gamma(1+\varepsilon)}{\varepsilon} (4\pi)^\varepsilon + \ln \frac{\mu^2}{\mu_F^2} \right) C_F [P_{qq}(z)]_+ \\
 &+ \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} f_g\left(\frac{x}{z}, \mu_F\right) \left(\frac{\Gamma(1+\varepsilon)}{\varepsilon} (4\pi)^\varepsilon + \ln \frac{\mu^2}{\mu_F^2} \right) T_R P_{gq}(z) \\
 f_g(x, \mu_F) &\rightarrow \dots \text{ analogously}
 \end{aligned}$$

Tuned comparison

Status of tuned comparison between results of [Dittmaier, S.K., Uwer '07],
[Campbell, Ellis, Zanderighi '07] and [Binoth, Guillet, Karg, Kauer, Sanguinetti (in progress)]:

↪ [NLM Les Houches report '08]

- Integrated LO results checked:

| | |
|--|----------------------------------|
| $pp \rightarrow W^+W^- + \text{jet} + X$ | $\sigma_{\text{LO}} [\text{fb}]$ |
| DKU | 10371.7(12) |
| CEZ | 10372.26(97) |
| BGKKS | 10371.7(11) |

- Results for virtual corrections checked at one phase-space point:

$$u\bar{u} \rightarrow W^+W^-g \quad |\mathcal{M}_{\text{LO}}|^2/e^4g_s^2 = 0.9963809154477200 \cdot 10^{-3}$$

$$2\text{Re}\{\mathcal{M}_V^* \cdot \mathcal{M}_{\text{LO}}\} = e^4g_s^2\Gamma(1+\varepsilon) \left(\frac{4\pi\mu^2}{M_W^2}\right)^\varepsilon \left(\frac{1}{\varepsilon^2}c_{-2} + \frac{1}{\varepsilon}c_{-1} + c_0\right)$$

All bosonic contributions:

| $u\bar{u} \rightarrow W^+W^-g$ | $c_{-2} [\text{GeV}^{-2}]$ | $c_{-1}^{\text{bos}} [\text{GeV}^{-2}]$ | $c_0^{\text{bos}} [\text{GeV}^{-2}]$ |
|--------------------------------|------------------------------------|---|--------------------------------------|
| DKU | $-1.080699305508758 \cdot 10^{-4}$ | $7.842861905263072 \cdot 10^{-4}$ | $-3.382910915425372 \cdot 10^{-3}$ |
| CEZ | $-1.080699305505865 \cdot 10^{-4}$ | $7.842861905276719 \cdot 10^{-4}$ | $-3.382910915464027 \cdot 10^{-3}$ |
| BGKKS | $-1.080699305508814 \cdot 10^{-4}$ | $7.842861905263293 \cdot 10^{-4}$ | $-3.382910915616242 \cdot 10^{-3}$ |

Fermionic contributions for 2 light generations in the loop:

| $u\bar{u} \rightarrow W^+W^-g$ | $c_{-1}^{\text{ferm}1+2} [\text{GeV}^{-2}]$ | $c_0^{\text{ferm}1+2} [\text{GeV}^{-2}]$ |
|--------------------------------|---|--|
| DKU | $2.542821895320379 \cdot 10^{-5}$ | $4.372323372044527 \cdot 10^{-7}$ |
| CEZ | $2.542821895311753 \cdot 10^{-5}$ | $4.372790378087550 \cdot 10^{-7}$ |
| BGKKS | $2.542821895314862 \cdot 10^{-5}$ | $4.372324288356448 \cdot 10^{-7}$ |