Soft and Coulomb gluon resummation in squark-antisquark production at the LHC

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IPPP Durham

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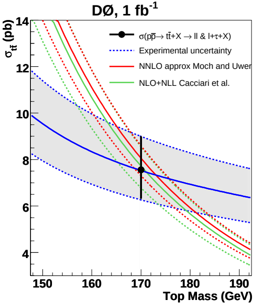
Based on **M. Beneke, PF, C. Schwinn**, **[arXiv:0907.1443[hep-ph]]**
and work in preparation

Motivation

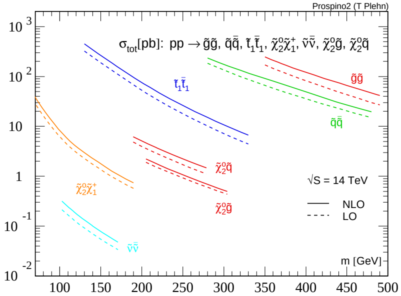
IN THIS TALK: Pair production of **coloured heavy particles** at LHC

$$p_i + p_j \longrightarrow HH' + X \qquad H, H' = \text{top, } \textbf{squarks}\text{, gluinos...}$$

Accurate theoretical prediction of the cross section **phenomenologically important** (sensitivity to **mass parameters**, **exclusion bounds**, **model discrimination**...)



[D0 Collaboration ’09]



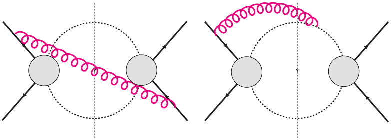
[Plehn, Prospino]

+ theoretically interesting due to **non-trivial colour exchange**

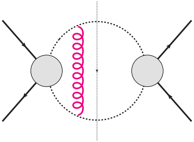
Soft-gluon and Coulomb corrections

Partonic cross section enhanced in the **threshold region**, $\beta \equiv \sqrt{1-(m_H+m_{H'})^2/\hat{s}} \to 0$:

- **Threshold logarithms**: $\sim \alpha_s^n \ln^m \beta$
  $\Leftrightarrow$ soft-gluon exchange between **initial-initial**, **initial-final** ($\alpha_s \log^{2,1} \beta$) and **final-final** state particles ($\alpha_s \log \beta$)



- **Coulomb corrections**: $\sim (\alpha_s/\beta)^n$
  $\Leftrightarrow$ **static interaction** of slowly-moving particles



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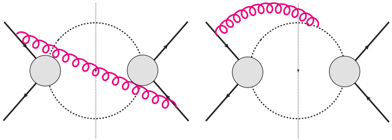

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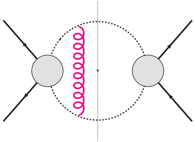
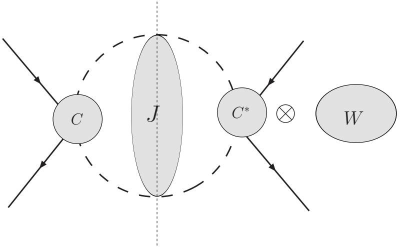


Enhanced terms can spoil convergence of perturbative series $\Rightarrow$ **RESUMMATION**

- **Normalisation** of the cross section (really important only for $m_H \gtrsim 1$ TeV...)
- Generally observed to reduce dependence on the **factorisation-scale** (even for small masses...)
- Allows to predict classes of **higher-order corrections** ($\Rightarrow$ see **M. Beneke**'s and **M. Czakon**'s talks)

Factorisation of pair production near threshold

Effective-theory description of pair production near threshold [Beneke, PF, Schwinn '09]:
SCET (collinear/soft modes) +**(P)NRQCD** (heavy non-relativistic fields)
⇒ arbitrary colour representations + factorisation of Coulomb corrections



$$\hat{\sigma}_{pp'}(\hat{s},\mu) = \sum_i H_i(M,\mu) \int d\omega \sum_{R_\alpha} J_{R_\alpha}(E - \frac{\omega}{2})\, W_i^{R_\alpha}(\omega,\mu)$$

- Hard function H_i depends on the precise nature of the physics model
- Process-independent soft function $W_i^{R_\alpha}$ ($\sim \alpha_s^n \log^m \beta$)
- Potential function J_{R_α} encodes Coulomb effects($\sim \alpha_s^n/\beta^n$)

Formula valid up to corrections of $O(\alpha_s^2 \log \beta)$ [Beneke, PF, Schwinn '09; Ferroglia et al. '09]

Resummation of logs in momentum space

Factorisation-scale independence of the total cross section translates into RG evolution equations for the soft function $W_i^{R_\alpha}$ and the hard function $H_i^{R_\alpha}$
(generalisation of analogous DY result [Becher, Neubert, Xu '07])

$$
\begin{aligned}
\frac{d}{d\log\mu_f} W_i^{R_\alpha}(\omega,\mu_f) &= -2\left[\left(\Gamma^r_{\text{cusp}}+\Gamma^{r'}_{\text{cusp}}\right)\log\left(\frac{\omega}{\mu}\right)+2\gamma^{R_\alpha}_{H,s}+2\gamma^r_s+\gamma^{r'}_s\right]W_i^{R_\alpha}(\omega,\mu_f)\\
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and similar for hard function $H_i(M,\mu_f)$

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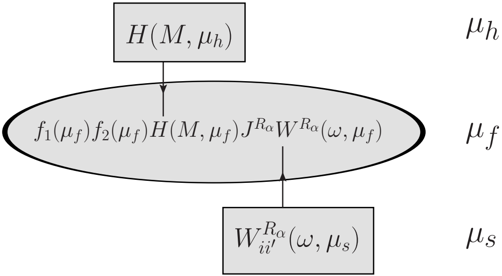
Resummation strategy

- Solve evolution equation in momentum space
- Evolve the function H_i from the hard scale μ_h to μ_f
- Evolve soft function $W_i^{R_\alpha}$ from a low scale μ_s to μ_f.



Resummed soft function and hard matching coefficient

Solutions to the RG evolutions equations

[Neubert, Becher, Xu '07; Beneke, PF, Schwinn, in preparation]

$$H_i^{\text{res}}(M,\mu) = \exp[4S(\mu_h,\mu) - 2a_i^V(\mu_h,\mu)] \left(-\frac{M^2}{\mu_h^2}\right)^{-2a_\Gamma(\mu_h,\mu)} H_i(M,\mu_h)$$

$$W_i^{R_\alpha,\text{res}}(\omega,\mu) = \exp[-4S(\mu_s,\mu) + 2a_{W,i}^{R_\alpha}(\mu_s,\mu)]\tilde{s}_i^{R_\alpha}(\partial_\eta,\mu_s)\frac{1}{\omega}\left(\frac{\omega}{\mu_s}\right)^{2\eta}\theta(\omega)\frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}$$

$$S(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha_s \frac{\Gamma^r_{\text{cusp}}(\alpha_s) + \Gamma^{r'}_{\text{cusp}}(\alpha_s)}{2\beta(\alpha_s)} \int_{\alpha_s(\nu)}^{\alpha_s} \frac{d\alpha_s'}{\beta(\alpha_s')}$$

$$a_\Gamma(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha_s \frac{\Gamma^r_{\text{cusp}}(\alpha_s) + \Gamma^{r'}_{\text{cusp}}(\alpha_s)}{2\beta(\alpha_s)}$$

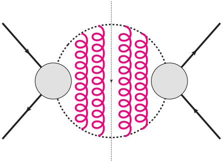
$$a_i^X(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha_s \frac{\gamma_i^X(\alpha_s)}{\beta(\alpha_s)}$$

- Resummation controlled by **cusp and soft anomalous dimensions**: $\Gamma^r_{\text{cusp}}, \gamma_i^V, \gamma^r, \gamma_{H,s}^{R_\alpha}$
- **Hard and soft scales** chosen to minimise higher-order terms in fixed-order expansions of $H_i(M,\mu_h)$ and $\tilde{s}_i^{R_\alpha}(L,\mu_s) \overset{\text{Laplace tr.}}{\longleftrightarrow} W_i^{R_\alpha}(\omega,\mu_s)$

Resummation of Coulomb corrections

Near threshold exchange of Coulomb gluons between the pair H, H' is also kinematically enhanced: $\Delta\sigma^{\mathrm{Coul},(1)}/\sigma^{\mathrm{tree}} \sim \alpha_s/\beta \sim 1$

$\Rightarrow$ **Leading Coulomb corrections must be resummed to all orders as well**

$J_{R_\alpha}(E) \Leftrightarrow$ 

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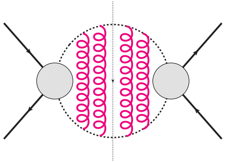
$J_{R_\alpha}(E) \Leftrightarrow$ 

Resummation of Coulomb effects well understood from **PNRQCD** and quarkonia physics.

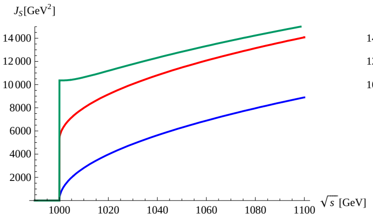
For HH' system in **irreducible representation** R_α:

$$J_{R_\alpha}(E) = -\frac{(2m_{\text{red}})^2}{2\pi}\text{Im}\left\{\sqrt{-\frac{E}{2m_{\text{red}}}} + \alpha_s(-C_{R_\alpha})\left[\frac{1}{2}\ln\left(-\frac{8\,m_{\text{red}}E}{\mu_f^2}\right) - \frac{1}{2} + \gamma_E + \psi\left(1 - \frac{\alpha_s(-C_{R_\alpha})}{2\sqrt{-E/(2m_{\text{red}})}}\right)\right]\right\} \qquad E \equiv \sqrt{s} - M$$

$C_S = -C_F,\ C_8 = C_A/2 - C_F$

Resummed green functions and bound-state contributions

$J^{(0)}_{R_\alpha}$, $J^{(1)}_{R_\alpha}$ and full resummed green function J_{R_α} for $m_H = m_{H'} = 500\,\mathrm{GeV}$



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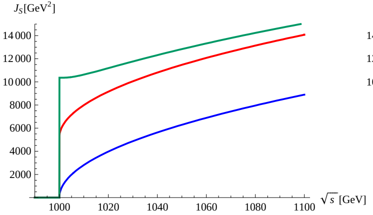
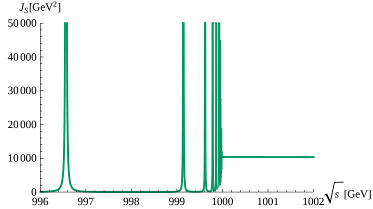


Below threshold contributions from singlet squark-antisquark bound states

For $\Gamma_{\tilde{q}} \to 0$:

$$J_S = 2 \sum_{n=1}^{\infty} \left(\frac{m_{\tilde{q}} \alpha_s C_F}{2n} \right)^3 \delta\left(E - E_n\right)$$

$$E_n = -m_{\tilde{q}} \left(\frac{\alpha_s C_f}{2n} \right)^2$$



See also [Fadin, Khoze '87; Kiyo et al. '09; Hagiwara, Yokoya '09]

Squark-antisquark production at the LHC

In the rest of this talk:

$$PP \longrightarrow \tilde{q}\bar{\tilde{q}} + X$$

Apply **NLL soft resummation** and **Coulomb resummation** to total cross section for squark-antisquark production

$$\hat{\sigma}^{\text{Res}}_{pp'}(\hat{s},\mu) = \sum_i H_i^{\text{NLL}}(m_{\tilde{q}},\mu) \int d\omega \sum_{R_\alpha} J_{R_\alpha}(E - \frac{\omega}{2}) \; W_i^{R_\alpha,\text{NLL}}(\omega,\mu)$$

Resummed cross section is matched onto the full NLO result!
[Zerwas et al., '96; Langenfeld, Moch '09]

$$\hat{\sigma}^{\text{match}}_{pp'}(\hat{s},\mu_f) = \left[\hat{\sigma}^{\text{Res}}_{pp'}(\hat{s},\mu_f) - \hat{\sigma}^{\text{Res}}_{pp'}(\hat{s},\mu_f)\big|_{\text{NLO}}\right] + \hat{\sigma}^{\text{NLO}}_{pp'}(\hat{s},\mu_f)$$

Anatomy of soft and Coulomb-gluon resummation

$$\hat{\sigma}^{\text{Res}}_{pp'} = \sum_i \hat{\sigma}^{i,(0)}_{pp'} \Bigg\{ U_i^{R_\alpha} \left(\frac{Ee^{-\gamma_E}}{m_{\tilde{q}}}\right)^{2\eta} \Bigg[\underbrace{\frac{\sqrt{\pi}}{2\Gamma(2\eta+\frac{3}{2})}}_{\text{NLL soft corrections}} - \underbrace{\frac{C_{R_\alpha}\alpha_s(\mu_C)}{\Gamma(2\eta+1)}\sqrt{\frac{m_{\tilde{q}}}{E}}}_{\text{NLL soft}\times\text{ first Coulomb}} \Bigg]$$

$$- \underbrace{C_{R_\alpha}\alpha_s(\mu_C)\sqrt{\frac{m_{\tilde{q}}}{E}}\,\text{Im}\left[\psi\left(1+\frac{iC_{R_\alpha}\alpha_s(\mu_C)}{2}\sqrt{\frac{m_{\tilde{q}}}{E}}\right)\right]}_{\text{Higher-order Coulomb}} + \underbrace{\cdots}_{\text{NLL Soft}\times\text{ HO Coulomb}} \Bigg\}$$

$$U_i^{R_\alpha} = \exp[4S(\mu_h,\mu_s) - 2a_i^V(\mu_h,\mu_s) + 4a^{\phi,r}(\mu_s,\mu_f)] \left(\frac{4m_{\tilde{q}}^2}{\mu_h^2}\right)^{-2a_\Gamma(\mu_h,\mu_s)}$$

Interference of soft-gluon resummation and higher-order Coulomb expected to be negligible
⇒ **Here consider only interference of first-Coulomb exchange with all-order soft emission**

Scale choice for μ_s, μ_h and μ_C

What is a good choice for μ_s and μ_h (and μ_C)?

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- Choose μ_s such that one-loop soft corrections to the **hadronic cross section** are minimised [Becher, Neubert, Xu '07]

$$\frac{\partial}{\partial \bar{\mu}_s} \int dx_1 d_2 f(x_1, \bar{\mu}_s) f(x_2, \bar{\mu}_s) \Delta\hat{\sigma}^{S,(1)}(\hat{s}, \bar{\mu}_s) = 0$$

This choice guarantees well-behaved perturbative expansion at the low scale $\bar{\mu}_s$

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- In the following identify **hard scale and factorisation scale**: $\mu_h = \mu_f \sim m_{\tilde{q}}$
  $\rightarrow$ **No large logs of the hard scale** ($\log(\mu_h/\mu_f) \sim 0$)

$$H_i^{\text{NLL}}(m_{\tilde{q}}, \mu) \stackrel{\mu_h=\mu_f}{\longrightarrow} H_i^{\text{Tree}}(m_{\tilde{q}}, \mu)$$

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Freedom to choose μ_f independently for J_{R_α} (scale dependence cancelled by HO corrections in PNRQCD...)

Scale μ_C for Coulomb interactions set by typical **virtuality of a Coulomb gluons** $\sqrt{|q^2|} \sim m_{\tilde{q}}\beta \sim m_{\tilde{q}}\alpha_s$

$$\Rightarrow \mu_C = \max\{2m_{\tilde{q}}\beta, C_F m_{\tilde{q}}\alpha_s(\mu_C)\}$$

$\hookrightarrow$ twice inverse Bohr radius of first bound state

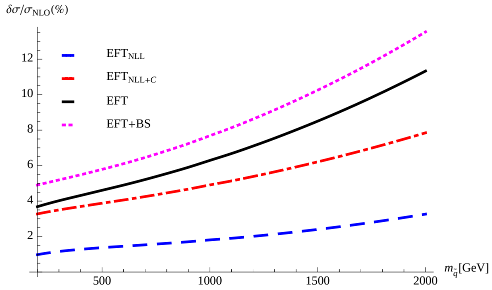
Squark-antisquark resummed cross section

Beneke, PF, Schwinn, PRELIMINARY

- $\mathbf{EFT_{NLL}}$: NLL soft resummation, no Coulomb resummation
- $\mathbf{EFT_{NLL+C}}$: NLL soft resummation **AND** Coulomb resummation (above threshold). No soft/Coulomb interference
- **EFT**: NLL soft resummation + Coulomb resummation (above threshold) + soft/1st Coulomb interference
- $\mathbf{EFT + BS}$: **EFT**+ Bound-state effects

Setup:

- PP@ 14 TeV
- MSTW2008 PDFs
- equal squark masses
- no stops
- $m_{\tilde{g}} = 1.25 m_{\tilde{q}}$
- $\mu_f = m_{\tilde{q}}$

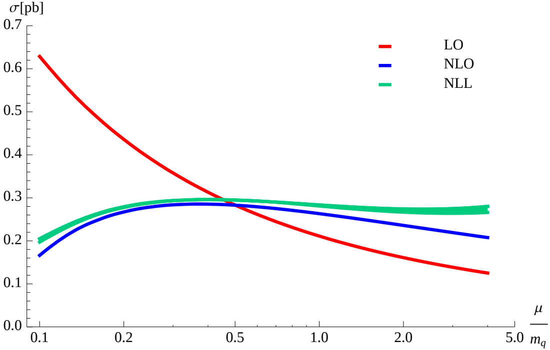


$\mathbf{EFT_{NLL}}$ result agrees well with Kulesza, Motyka '09

Factorisation-scale dependence

One of main motivations for resummation is **reduction of scale dependence** of NLO result:

$m_{\tilde{q}} = 1\text{TeV}$



$\mu_h = \mu_f$

$\bar{\mu}_s/2 < \mu_s < 2\bar{\mu}_s$

$\bar{\mu}_s \sim 365\,\text{GeV}$

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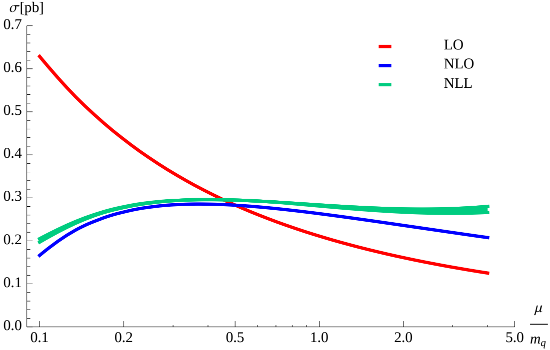

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Scale dependence only mildly reduced at small values of the factorisation scale

- $\mu_f \longrightarrow \mu_s \Rightarrow$ no large soft logs $\log(\mu_s/\mu_f) = O(1)$
- For small values of $\mu_f \ll m_{\tilde{q}}$ the choice $\mu_h = \mu_f$ is not justified
  $\hookrightarrow$ **hard scale and factorisation scale must be kept separate**

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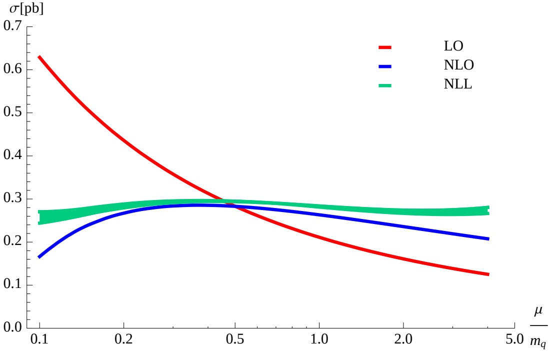

$m_{\tilde{q}}/2 < \mu_h < 2m_{\tilde{q}}$

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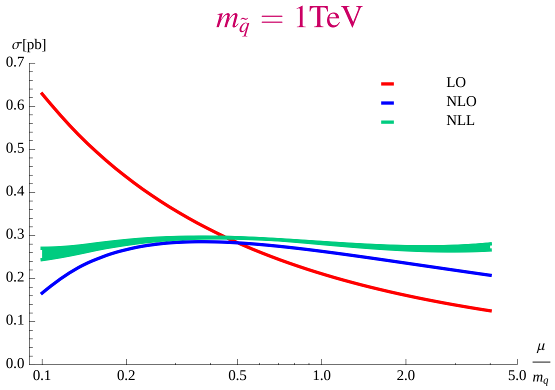

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- $\mu_f \sim m_{\tilde{q}} \Rightarrow$ resum large logs of the **soft scale** $\log(\mu_s/\mu_f)$
- $\mu_f \ll m_{\tilde{q}} \Rightarrow$ resum large logs of the **hard scale** $\log(\mu_h/\mu_f)$

Summary

- Presented results for **combined resummation of soft and Coulomb gluon effects** in **squark- antisquark** production at LHC
  - NLL resummation of threshold logarithms
  - Inclusion of all-order Coulomb corrections
  - Interference of all-order soft corrections and first Coulomb exchange
  - Bound-state effects below the production threshold
- **Corrections are sizeable** and amount to $\sim 5-13\%$ for $m_{\tilde{q}} \sim 300\text{GeV} - 2\text{TeV}$
- Observed significant **reduction of factorisation scale dependence**

Outlook

- Apply to **more processes**, ex. gluino pair production (larger colour charges...)
- Include **finite-width** effects (particularly important below threshold...)