All-order colour structure and two-loop anomalous dimension of soft radiation in heavy-particle pair production at the LHC

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Outline

- Factorization for soft gluon and Coulomb resummation
- All-order diagonal colour basis for the leading soft function
- Two-loop anomalous dimension at threshold
- Threshold expansion of the top-quark pair production at $\mathcal{O}(\alpha_s^2)$

MB, P. Falgari, C. Schwinn, 0907.1443 [hep-ph] and in preparation (factorization, colour, anomalous dimension);
MB, M. Czakon, P. Falgari, C. Schwinn, in preparation (α_s^2 log terms for $t\bar{t}$)

Pair production and resummation

Partonic cross sections $q\bar{q}, qg, gg \to HH' + X$ contain

$$\left[\alpha_s \ln^2(1-z)\right]^n, \qquad z = M_{HH'}^2/\hat{s}$$

which should be resummed, if the total hadronic cross section is dominated by the partonic threshold. ($t\bar{t}$: Catani et al., 1996; Bonciani et al, 1998; Kidonakis et al, 2001; ...; Moch, Uwer, 2008; ...; Hagiwara et.al, 2008; Kiyo et al., 2008; Sparticle pairs: Kulesza, Motyka, 2008; Langenfeld, Moch, 2009; Beenakker et al)

- Invariant mass distribution with $M_{HH'} \geq$ few $\times$ $(m_H + m_{H'})$, such that H, H' are relativistic.

  Colour exchange and s, t-dependent anomalous dimensions.

  [Formalism for $2 \to 2$ scattering processes with massless coloured particles was set up by (Kidonakis, Sterman; 1997)]

- Total cross section with $M_{HH'} = m_H + m_{H'}$, such that the threshold is dominated by non-relativistic H, H' $(1 - z = \beta^2)$.

  Colour exchange and simple anomalous dimensions. Coulomb singularities $(\alpha_s/\beta)^n$ in addition to threshold logs $(\alpha_s \ln^2 \beta)^n$.

  Combined resummation?

Not clear why the partonic threshold should be relevant at LHC energies for $t\bar{t}$. Perhaps for sparticles with masses $m_H \geq 1$ TeV. [See P. Falgari's talk]

Systematics and the factorization formula

Expansion of the partonic cross section

$$\hat{\sigma}(\beta) = \hat{\sigma}^{(0)} \sum_{k=0} \left(\frac{\alpha_s}{\beta}\right)^k \exp\Big[\underbrace{\ln\beta\, g_0(\alpha_s \ln\beta)}_{\text{(LL)}} + \underbrace{g_1(\alpha_s \ln\beta)}_{\text{(NLL)}} + \underbrace{\alpha_s g_2(\alpha_s \ln\beta)}_{\text{(NNLL)}} + \ldots\Big]$$
$$\times \left\{1 \text{ (LL,NLL)};\ \alpha_s,\ \beta \text{ (NNLL)};\ \alpha_s^2,\ \alpha_s\beta,\ \beta^2 \text{ (NNNLL)};\ \ldots\right\},$$

In fixed order [Note: NNLL can be α_s/β[Coulomb] $\times$ β[sub-leading soft] $\times$ $\alpha_s \ln^2\beta$ – beyond the standard soft gluon approximation!]

LL	$\alpha_s \left\{\frac{1}{\beta}, \ln^2\beta\right\};\ \alpha_s^2 \left\{\frac{1}{\beta^2}, \frac{\ln^2\beta}{\beta}, \ln^4\beta\right\};\ldots,$
NLL	$\alpha_s \ln\beta;\ \alpha_s^2 \left\{\frac{\ln\beta}{\beta}, \ln^3\beta\right\};\ldots,$
NNLL	$\alpha_s \left\{1, \beta\times\ln^{2,1}\beta\right\};\ \alpha_s^2 \left\{\frac{1}{\beta}, \ln^{2,1}\beta, \beta\times\ln^{4,3}\beta\right\};\ldots,$

Factorization formula [sum over a includes sub-leading powers in β.]

$$\hat{\sigma}(\beta,\mu) = \sum_a \sum_{i,i'} H^a_{ii'}(M,\mu) \int d\omega \sum_{R_\alpha} J^a_{R_\alpha}\left(E - \frac{\omega}{2}\right) W^{a,R_\alpha}_{ii'}(\omega,\mu).$$

Soft-gluon decoupling

From the initial state:

$$\mathcal{L}_c = \bar{\xi}_c \left(in\cdot D + i \not{D}_{\perp c} \frac{1}{i\bar{n}\cdot D_c} i\not{D}_{\perp c} \right) \frac{\not{\bar{n}}}{2}\, \xi_c - \frac{1}{2}\,\mathrm{tr}\left(F_c^{\mu\nu} F^c_{\mu\nu} \right)$$

by the SCET field redefinitions (Bauer, Pirjol, Stewart, 2001) $\xi_c(x) = S_n^{(3)}(x_-)\, \xi_c^{(0)}(x)$, $A^A_{c\mu}(x) = S_n^{(8)}(x_-)\, A^{A(0)}_{c\mu}(x)$, such that $n\cdot D \to n\cdot D_c$.

From the final state:

$$\mathcal{L}_{\mathrm{PNRQCD}} = \psi^\dagger \left(iD_s^0 + \frac{\vec{\partial}^2}{2m_H} + \frac{i\Gamma_H}{2} \right) \psi + \psi'^{\,\dagger} \left(iD_s^0 + \frac{\vec{\partial}^2}{2m_{H'}} + \frac{i\Gamma_{H'}}{2} \right) \psi' + \int d^3\vec{r}\, \left[\psi^\dagger \mathbf{T}^{(R)a} \psi\right](\vec{r}) \left(\frac{\alpha_s}{r}\right) \left[\psi'^{\,\dagger} \mathbf{T}^{(R')a} \psi'\right](0)$$

by the PNRQCD field redefinition $\psi_a(x) = S_v^{(R)}(x^0)_{ab}\, \psi_b^{(0)}(x)$, such that $D_s^0 \to \partial^0$.

[S_v drops out from the Coulomb interaction, since $S_v^{(R)\dagger}\mathbf{T}^{(R)a} S_v^{(R)\dagger} = [S^T_{\mathrm{ad}}]^{ab}\, \mathbf{T}^{(R)b}$ in any rep R; S_{ad} is real and independent of $\vec{r}$.]

Proves decoupling of soft gluon and Coulomb resummation, since soft gluons disappear from the leading-order Lagrangians for the other fields. Sub-leading interactions can be treated as perturbations in β.

Leading term

$$\hat{\sigma}(\beta,\mu) = \sum_{i,i'} H_{ii'}(M,\mu) \int d\omega \sum_{R_\alpha} J_{R_\alpha}(E - \frac{\omega}{2})\, W^{R_\alpha}_{ii'}(\omega,\mu).$$

- $H_{ii'} = C_i C^*_{i'}$ – colour-unaveraged partonic (hard) cross section directly at threshold. C_i related to matching coefficient of $\mathcal{O}^{(\ell)}_{\{a;\alpha\}}(\mu) = \left[\phi_{c;a_1}\phi_{\bar{c};a_2}\psi^\dagger_{a_3}\psi'^\dagger_{a_4}\right](\mu)$
  [No spin-separation needed at NNLL.]
- J_{R_α} – sums Coulomb-exchange to all orders for HH' in rep R_α.
  Related to a correlation function of non-relativistic fields in PNRQCD.

$$J_{R_\alpha}(E) \propto \mathrm{Im}\left[-\frac{(2m_{\mathrm{red}})^2}{4\pi}\left\{\sqrt{-\frac{E}{2m_{\mathrm{red}}}} + \alpha_s(-D_{R_\alpha})\left[\frac{1}{2}\ln\left(-\frac{8\,m_{\mathrm{red}}E}{\mu^2}\right) + \psi\left(1 - \frac{\alpha_s(-D_{R_\alpha})}{2\sqrt{-E/(2m_{\mathrm{red}})}}\right)\right]\right\}\right]$$

- $W^{R_\alpha}_{ii'}$ – generalized soft function (one for each rep R_α). Fourier transform of $\hat{W}^{R_\alpha}_{ii'}(z,\mu) = P^{R_\alpha}_{\{k\}} c^{(i)}_{\{a\}} \hat{W}^{\{k\}}_{\{ab\}}(z,\mu) c^{(i')*}_{\{b\}}$. $c^{(i)}_{\{a\}}$ - colour basis element. $\{a\} = a_1a_2a_3a_4$.

$$\hat{W}^{\{k\}}_{\{ab\}}(z,\mu) = \langle 0|\overline{\mathrm{T}}[S_{v,b_4k_2}S_{v,b_3k_1}S^\dagger_{\bar{n},jb_2}S^\dagger_{n,ib_1}](z)\mathrm{T}[S_{n,a_1i}S_{\bar{n},a_2j}S^\dagger_{v,k_3a_3}S^\dagger_{v,k_4a_4}](0)|0\rangle,$$

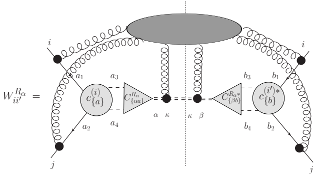
All-order diagonal colour basis for $W_{ii'}^{R_\alpha}$

- Decompose initial and final state rep product into irreducible reps: $r \otimes r' = \sum_\alpha r_\alpha$, and $R \otimes R' = \sum_\beta R_\beta$.

  Number of basis elements ($i = 1 \ldots n$) = number of pairs $P_i = (r_\alpha, R_\beta)$ of equivalent representations r_α and R_β, e.g. for $8 \otimes 8 \longrightarrow 8 \otimes 8$:
  $P_i \in \{(1,1),\ (8_S, 8_S),\ (8_A, 8_S),\ (8_A, 8_A),\ (8_S, 8_A),\ \ (10,10),\ (\overline{10},\overline{10}),\ (27,27)\}$.

- $W_{ii'}^{R_\alpha}$ is diagonal to all orders in the orthonormal basis:

$$c_{\{a\}}^{(i)} = \frac{1}{\sqrt{\dim(r_\alpha)}}\, C^{r_\alpha}_{\alpha a_1 a_2} C^{R_\beta *}_{\alpha a_3 a_4}$$



[$C^{r_\alpha}_{\alpha a_1 a_2}$ Clebsch-Gordon coefficient]

- Properties of CGKs imply that element $W_{ii'}^{R_\alpha}$ is non-zero only, if in $P_i = (r_\alpha, R_\beta)$ and $P_{i'} = (r_{\alpha'}, R_{\beta'})$ the final state reps R_β, $R_{\beta'}$ are *identical* to R_α, and r_α, r'_α are *equivalent* to R_α. This leaves only $gg[8_S]$ coupling to $HH'[8_A]$ (and interchanged), but this vanishes by Bose symmetry for the Wilson line operators.

[Explicit construction of the colour bases for all interesting cases, see MB, P. Falgari, C. Schwinn, 0907.1443 [hep-ph] appendix.]

Two-loop soft anomalous dimension at threshold

- NNLL soft-gluon resummation needs the 3-loop cusp anomalous dimension and 2-loop soft anomalous dimension in

$$\frac{d}{d\ln\mu}\hat{W}_i^{R\alpha}(L) = \left((\Gamma^r_{\rm cusp} + \Gamma^{r'}_{\rm cusp})L - 2\gamma^{R\alpha}_{W,i}\right)\hat{W}_i^{R\alpha}(L),$$

[We use a generalization of the momentum space formalism developed for Drell-Yan production by (Becher, Neubert, Xu, 2007). See P. Falgari's talk.]

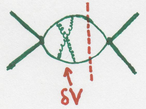
- At threshold the Wilson lines of the two heavy particle combine to a sum of single particle soft functions $C^{R_\alpha}_{\alpha a_1 a_2} S^{(R)}_{v,a_1 b_1} S^{(R')}_{v,a_2 b_2} = S^{(R_\alpha)}_{v,\alpha\beta} C^{R_\alpha}_{\beta,b_1 b_2}$. The soft function is the "square" of the soft function for the amplitude discussed recently by (Becher, Neubert, 2009; Mitov et al, 2009).
For a $2 \to 1$ process the three-particle correlations $f^{abc}T^aT^bT^c$ vanish by colour conservation. The anomalous dimension is a sum over single particle terms: $\gamma^{R_\alpha}_{W,i} = \gamma^{R_\alpha}_{H,s} + \gamma^r_s + \gamma^{r'}_s$. The 2-loop anomalous dimension satisfies Casimir scaling and can be extracted from (Becher, Neubert, 2009; Korchesmky, Radyushkin, 1992; Kidonakis, 2009).
- In Mellin space resummation formalism:

$$D^{(1)R_\alpha}_{HH'} = -C_{R_\alpha} C_A \left(\frac{460}{9} - \frac{4\pi^2}{3} + 8\zeta_3\right) + \frac{176}{9} C_{R_\alpha} T_F n_f.$$

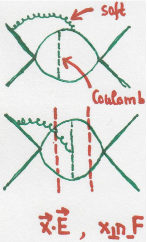
[Differs from ansatz by Moch, Uwer, 2008; confirmed by Czakon et al., 2009]

NNLL at $\mathcal{O}(\alpha_s^2)$ for $t\bar{t}$

2-loop anomalous dimension not enough for NNLL resummation.
Consider $\alpha_s^2 \ln\beta$ terms for $t\bar{t}$:



- $\hat{\sigma}^{(0)} \times \alpha_s^2 \ln\beta$ from singular heavy-quark potentials ($1/r^2$ etc.)
  Can be obtained from $e^+e^- \longrightarrow t\bar{t}X$ calculation [MB, Signer, Smirnov, 1999] (+ colour, spin adjustment)
  Diagonal in the singlet-octet basis.



- $\hat{\sigma}^{(0)} \times \alpha_s^2 \ln^{2,1}\beta$ from sub-leading (non-eikonal) β-suppressed soft interactions in SCET and NRQCD. Implies new soft functions with operator insertions between Wilson lines.
  Off-diagonal in the singlet-octet basis.
  Corresponds to three-particle correlations.
  Vanish for the total cross section to all orders in α_s (Lorentz-invariance + scaling), but not for the amplitude.
- Above + diagonal leading soft function consistent with (Ferroglia et al., 2009). No extra terms for $t\bar{t}$ total cross section, though.

[NNLL resummation for the total cross section requires non-relativistic factorization. 2-loop anomalous dimension from (Ferroglia et al., 2009) applies to relativistic production.]

Expansion of the partonic top-pair production section

$$\hat{\sigma}(\beta)^{(2)} = \hat{\sigma}(\beta)^{(0)} \left(\frac{\alpha_s}{4\pi}\right)^2 \left[\frac{c_{20}}{\beta^2} + \frac{1}{\beta}\left\{c_{12}\ln^2\beta + c_{11}\ln\beta + c_{10}\right\} + c_{04}\ln^4\beta + c_{03}\ln^3\beta + c_{02}\ln^2\beta + c_{01}\ln^1\beta\right]$$

- c_{20} – double Coulomb
  c_{12} – Coulomb× 1-loop soft
  c_{11} – Coulomb× soft, running Coulomb potential
  c_{10} – ..., 1-loop Coulomb potential, 1-loop hard matching coefficient [extracted from Czakon, Mitov, 2008]
  c_{04}, c_{03}, c_{02} – 2-loop soft c_{01} – ..., NRQCD logs from singular potentials

  Obtained with two largely independent methods. [See M. Czakon's talk]

Example: $gg \longrightarrow [t\bar{t}]_8 X$

$$\hat{\sigma}(\beta)^{(2)}_{gg\to[t\bar{t}]_8 X} = \hat{\sigma}(\beta)^{(0)} \left(\frac{\alpha_s}{4\pi}\right)^2 \left[\frac{\pi^4}{27\beta^2} + \frac{\pi^2}{\beta}\left\{-32\ln^2\beta + \left(\frac{118}{9} - 32\ln 2\right)\ln\beta + \frac{46}{9}\ln 2 - \frac{5\pi^2}{18} - \frac{127}{27}\right\}\right.$$
$$+ 4608\log^4\beta + \left(27648\ln 2 - \frac{65152}{3}\right)\log^3\beta + \left(59904\ln 2^2 - 96576\ln 2 - 3088\pi^2 + \frac{204944}{3}\right)\log^2\beta$$
$$\left. + \left(55296\ln 2^3 - 137952\ln 2^2 - 9264\pi^2\ln 2 + 202352\ln 2 + 33120\zeta(3) + \frac{65908\pi^2}{9} - \frac{1244776}{9}\right)\log\beta\right]$$

Conclusion

1) Factorization of soft and Coulomb gluon summation proved (SCET $\times$ NRQCD
   Leading soft function diagonal to all orders in a simple colour basis
   2-loop anomalous dimension at threshold determined.

2) Soft gluon resummation at NNLL for total cross section possible.
   For complete NNLL combine with non-relativstic log resummation.

3) Top pair cross section at threshold known at $\mathcal{O}(\alpha_s^2)$ at threshold up to the constant term.