Radcor 2009

Resonant Particle Production at Hadron Colliders

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outline

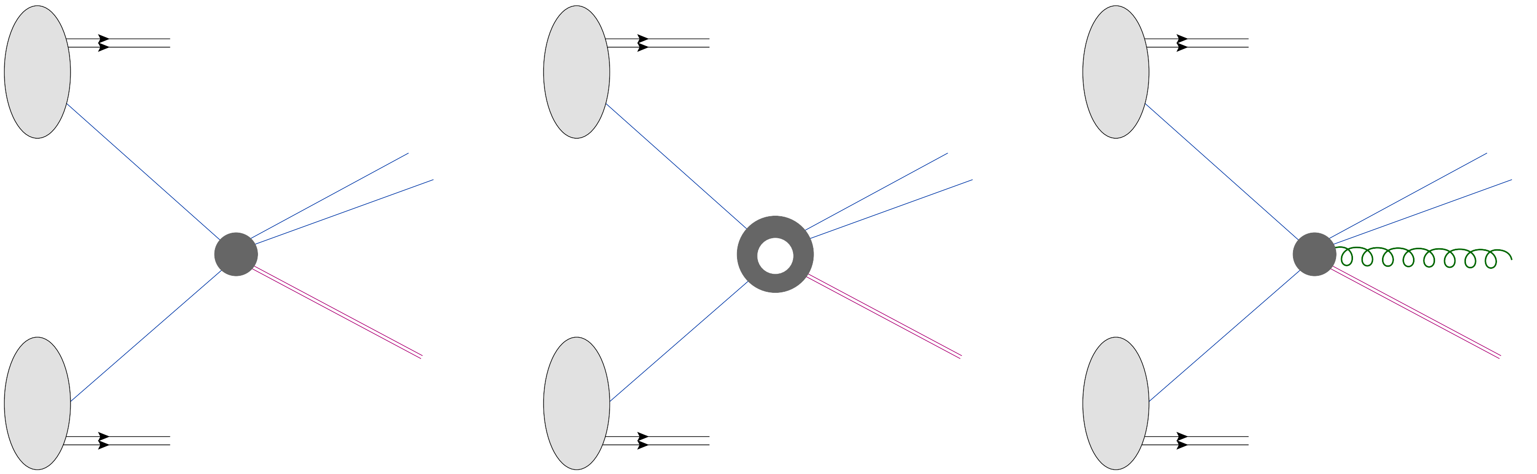
introduction
- background
- main question
- ET and “normal” approach

general approach
- virtual corrections (ET)
- real corrections (FO)

single top $ub \longrightarrow dbW$
- overview
- tree level
- virtual corrections
- real corrections

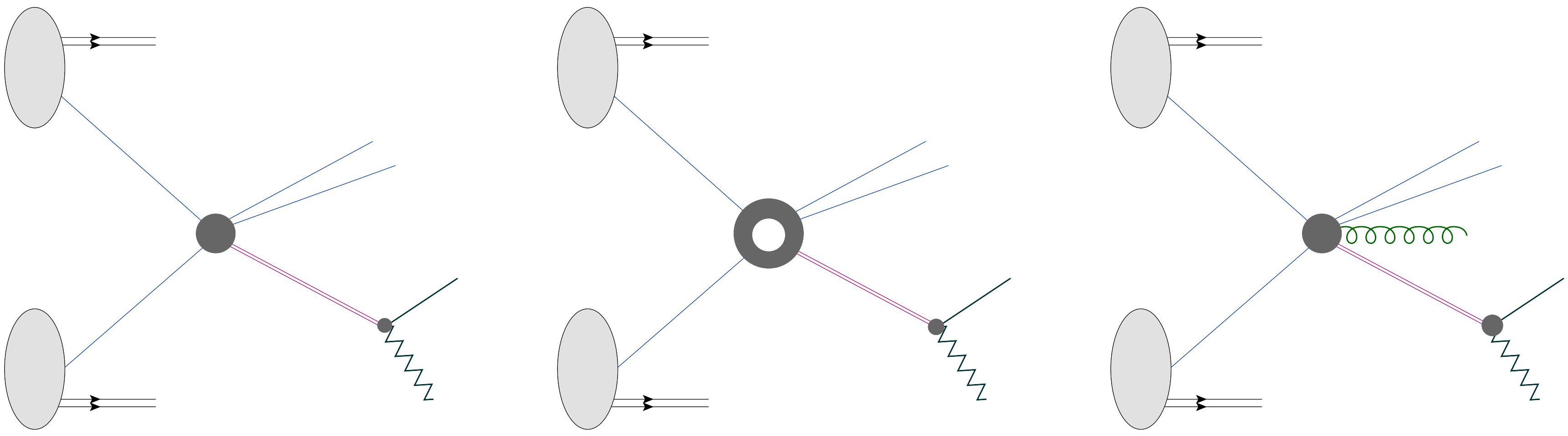
conclusions
- work in progress
- outlook

Production of an on-shell heavy (unstable) particle X: $p_X^2 = m_X^2$



- often this is a reasonable approximation but
- cuts on decay products not possible
- off-shell effects of X not taken into account

Production of an on-shell heavy (unstable) particle X, including decay: $p_X^2 = m_X^2$



- (improved) narrow width approximation, $M_{\rm decay}^2 = m_X^2$
- cuts on decay products possible
- off-shell effects of X not taken into account

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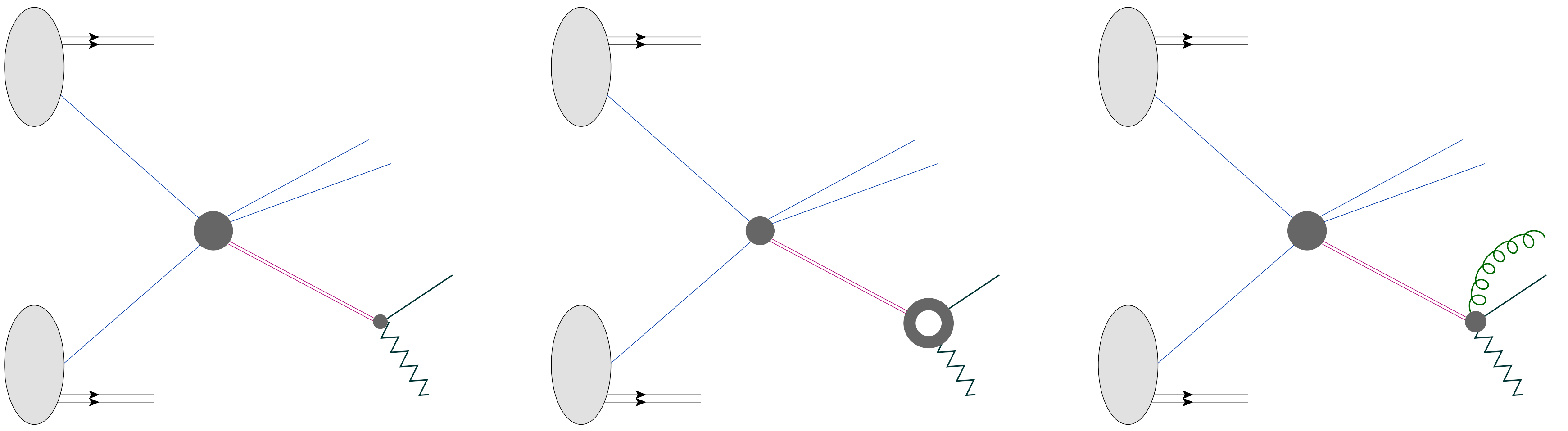

- (improved) narrow width approximation, $M_{\text{decay}}^2 = m_X^2$
- cuts on decay products possible
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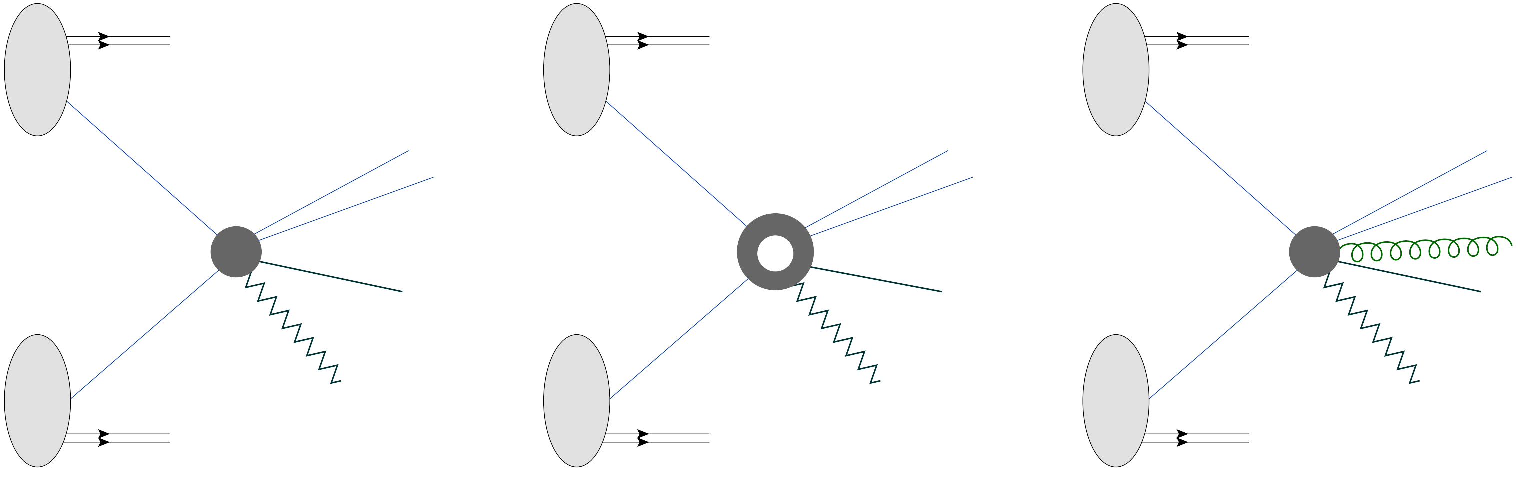
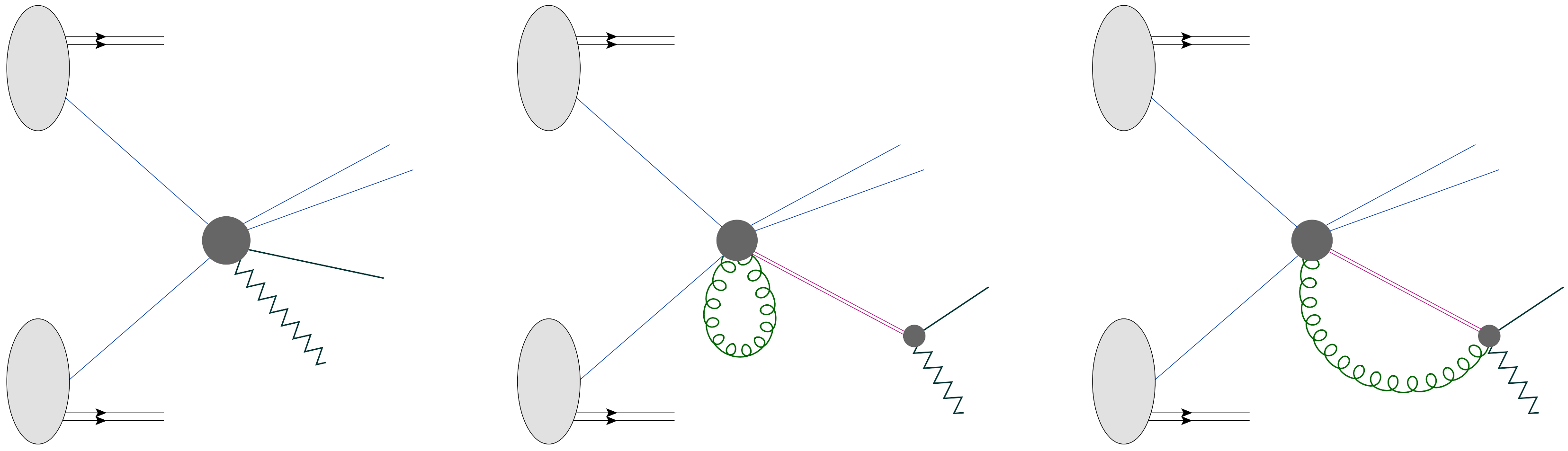
Production of an resonant heavy (unstable) particle X, including decay: $p_X^2 \sim m_X^2$



- tree-level background diagrams (no particle X, but same final state)
- do not want to compute one-loop background diagrams
- real background diagrams
- off-shell effects of X are taken into account, but calculation complicated

Production of an resonant heavy (unstable) particle X, including decay: $p_X^2 \sim m_X^2$



- tree-level background diagrams (no particle X, but same final state)
- use pole approximation [Stuart, Aeppli et.al.]
- within pole approximation at one loop [Fadin, Khoze, Martin]
  - factorizable corrections
  - non-factorizable corrections

  } gauge invariant separation
- real background diagrams
- off-shell effects of X are taken into account, calculation simplified

non-factorizable corrections have been extensively studied [Fadin et.al; Melnikov et.al; Beenakker et.al; Denner et.al.; Jadach et.al; . . .] but are usually neglected at hadron colliders, because:

- they seem to be more difficult to compute (not really)
- they are generally small [Beenakker et.al; Pittau]
  - resonant $\longrightarrow$ non-resonant propagator unless $E \lesssim \Gamma$ is small (soft)
  - cancellations for "inclusive" observables [Fadin, Khoze, Martin]

purpose of this work:

- do not neglect non-factorizable corrections
- try to obtain an efficient way to identify and compute minimal amount required
- why? consider e.g. top mass measurement, $\delta m_t \sim 1\ \mathrm{GeV} \lesssim \Gamma_t$

in this talk I will not consider many other (sometimes related) issues such as

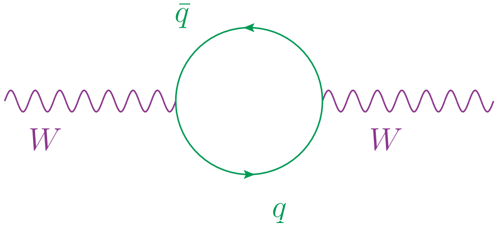
- (soft) connection of unstable particle to beam remnant
- issues related to using pole mass for unstable particle $\delta m_t \simeq \Lambda_{\mathrm{QCD}}$??

- small scale $(p_X^2 - m_X^2)/m_X^2 \sim \delta \ll 1 \to$ effective theory (ET) approach
- expand in all small parameters α and $(p_X^2 - m_X^2)/m_X^2$
- expand integrand, method of regions [Beneke, Smirnov]
- new identification [Chapovsky, Khoze, AS, Stirling]
    - factorizable corrections = hard corrections (ET, method of regions)
    - non-factorizable corrections = soft corrections (ET)
- applicable for virtual corrections and total cross section (forward scattering amplitude)
- worked out in detail for toy model and realistic applications [Beneke, Chapovsky, Falgari, Schwinn, AS, Zanderighi]
- arbitrary real corrections problematic (new scales from definition of observable)

power counting: $\alpha \sim \frac{p_X^2 - m_X^2}{m_X^2} \equiv \frac{\Delta}{m_X^2} \sim \frac{\Gamma_X}{m_X} \sim \delta \ll 1$

use method of regions [Beneke, Smirnov] and expand integrand (in principle to any order):

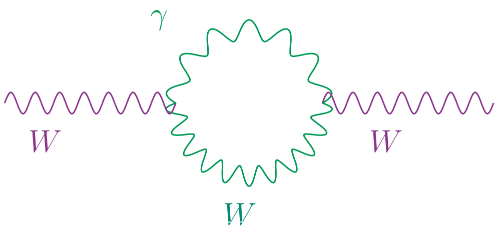
- hard corrections $p \sim m_X$ (= factorizable corrections)
- soft corrections $p \sim m_X\,\delta$ (= non-factorizable corrections)



$$\int \frac{d^d\ell}{(p+\ell)^2\ell^2}$$

hard: full

soft: $\int \frac{d^d\ell(2p\cdot\ell)}{p^2\ell^2} = 0$



$$\int \frac{d^d\ell}{(\ell^2+2p\cdot\ell+\Delta)\ell^2}$$

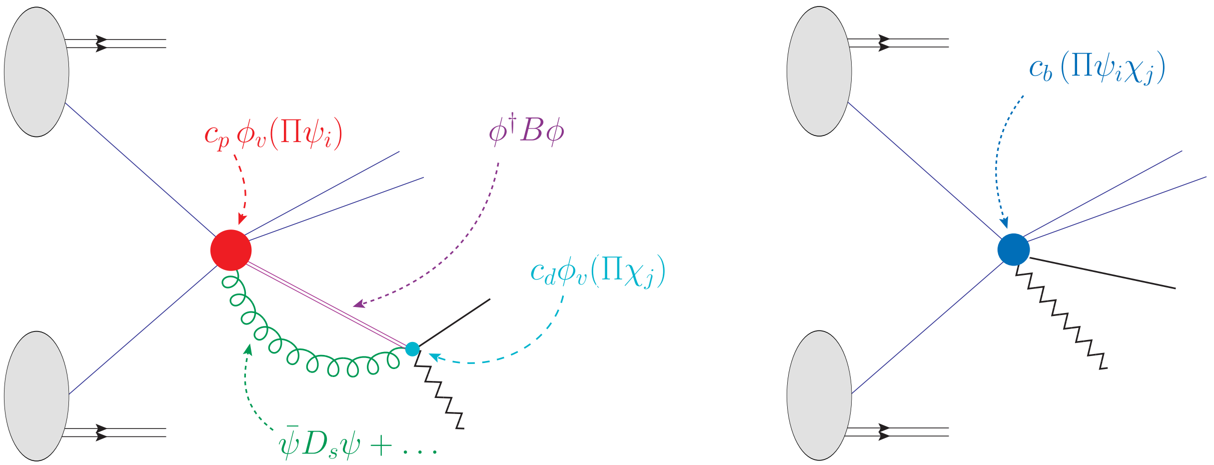
hard: $\int \frac{d^d\ell}{\ell^2(\ell^2+2p\cdot\ell)} \neq 0$

soft: $\int \frac{d^d\ell}{\ell^2(2p\cdot\ell+\Delta)} \neq 0$

- leads to resummation of hard part (= leading part in Δ) of self-energy insertions
- no issues with gauge invariance (compare fermion-loop scheme)

integrate out hard modes $\longrightarrow$ effective Lagrangian

$$\mathcal{L} = \phi^\dagger B\phi + c_p\,\phi(\Pi\psi_i) + c_d\phi(\Pi\chi_j) + c_b\,(\Pi\psi_i\chi_j) + \bar\psi D_s\psi + \ldots$$



- matching coefficients c_i contain effects of hard modes
- matching is done on shell, i.e. $p_X^2 = \bar{s} = m_X^2 + \mathcal{O}(\Delta)$, with $\bar{s}$ the complex position of pole. $\longrightarrow$ compare complex mass scheme [Denner, Dittmaier]
- at NLO, can take $p_X^2 = m_X^2$ for virtual corrections
- soft (and collinear . . .) d.o.f. still dynamical
- can be combined with further resummations (e.g. non-relativistic $\longrightarrow$ ET has more complicated structure)

For total cross section can proceed as for virtual (forward scattering amplitude)

- $e^+e^- \to t\bar{t}$ near threshold [Hoang et.al; Beneke et.al; Melnikov et.al; Yakovlev et.al . . .]
- $e^+e^- \to W^+W^-$ near threshold [Beneke et.al.]

For an arbitrary observable

- not clear what expansion parameter is
- observable can introduce new scales $\longrightarrow$ change in structure of ET
- case by case study for some observables possible, but not viable as general approach for hadron colliders
- base real corrections on fixed-order approach

take full real matrix element (including bg diagrams) and apply (say) subtraction method

$$\int d\Phi_{n+1}|M_{n+1}|^2 = \int d\Phi_{n+1}\left(|M_{n+1}|^2 - |M^{\text{sing}}_{n(+1)}|^2\right) + \int d\Phi_{n+1}|M^{\text{sing}}_{n(+1)}|^2$$
$$\simeq \int d\Phi_{n+1}\left(|M_{n+1}|^2 - |M^{\text{sing}}_{n(+1)}|^2\right) + \int d\Phi_{n+1}|M^{\text{sing exp}}_{n(+1)}|^2$$

$\int d\Phi_{n+1}|M^{\text{sing}}_{n(+1)}|^2$ matches singularity structure of full virtual correction $\int d\Phi_n|M^{\text{v}}_n|^2$

$\int d\Phi_{n+1}|M^{\text{sing exp}}_{n(+1)}|^2$ matches singularity structure of virtual term $\int d\Phi_n|M^{\text{v exp}}_n|^2$

we subtract something and add back something different, but difference is higher order in δ

expansion only required for n parton kinematics

for those who like gauge invariance arguments: each part is separately gauge independent politically incorrect comment about gauge invariance:

we can replace full matrix element $|M_{n+1}|^2$ by a gauge-dependent subset $|M^{\text{res}}_{n+1}|^2$ containing all leading in δ terms (i.e. all diagrams with resonant propagators)

$$\int d\Phi_{n+1}|M_{n+1}|^2 \simeq \underbrace{\int d\Phi_{n+1}\left(|M^{\text{res}}_{n+1}|^2 - |M^{\text{res sing}}_{n(+1)}|^2\right)}_{\text{gauge dependent, but only at NLO in } \delta} + \int d\Phi_{n+1}|M^{\text{sing exp}}_{n(+1)}|^2$$

if we compute at order δ^n, we end up with residual gauge dependence at order δ^{n+1}.

this is completely analogous to renormalization/factorization scale/scheme dependence.

what value for ξ ?	what value for μ ?	formally: any
$\xi \sim 1$ (parameter in $\mathcal{L}$)	$\mu \sim s_{ij}$	avoid large coefficients
setting $\xi = 10^{10}$	setting $\mu = M_{\text{Planck}}$	simply stupid !!
variation of ξ	variation of μ	estimate of h.o. corrections ??

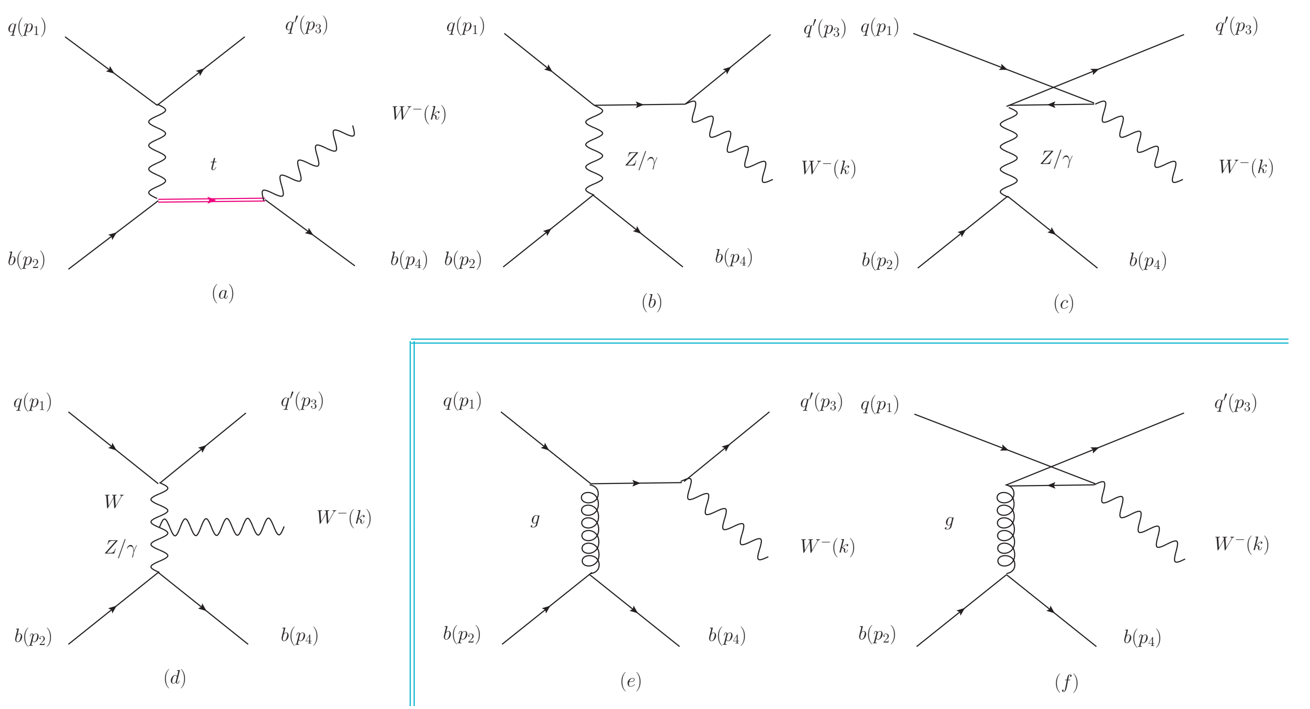
Consider single top in t-channel

- total rate and distributions at NLO [Bordes et.al; Stelzer et.al; Harris et.al; Campbell et.al; Cao et.al; . . .]
- implemented in MC@NLO [Frixione et.al.]
- comparison 5-flavour scheme vs. 4-flavour scheme [Campbell et.al.]
- EW corrections [Beccaria et.al.] and numerous studies with BSM effects

here, simply consider $u(p_1)b(p_2) \longrightarrow d(p_3)b(p_4)W^+(k) \longrightarrow d(p_3)b(p_4)e^+(p_5)\nu(p_6)$

- use (improved) narrow width for W decay
- signal is $W J_b$ pair with invariant mass $(p_W + p_{J_b})^2 \equiv s_{Wb} \sim m_t^2$
- small parameter: $\delta \equiv \dfrac{s_{Wb} - m_t^2}{m_t^2} \equiv \dfrac{\Delta}{m_t^2}$; counting: $\alpha_s^2 \sim \alpha_{ew} \sim \dfrac{\Gamma_t}{m_t} \sim \delta \ll 1$
- use 5 flavour scheme, $m_b = 0$, and "fixed" order, i.e. no parton shower etc.
- focus on $ub \longrightarrow dbW^+$ partonic process, even at NLO
- $\longrightarrow$ result by no means complete

amplitude: $\mathcal{A}^{\rm tree} = \delta_{31}\delta_{42}\left(\underbrace{g_{ew}^3\, A^{(3,0)}_{(-1)}}_{\delta^{1/2}} + \underbrace{g_{ew}^3\, A^{(3,0)}_{(0)}}_{\delta^{3/2}} + \dots\right) + \underbrace{T^a_{31}T^a_{42}\, g_{ew} g_s^2\, A^{(1,2)}}_{\delta \quad \text{signal!}}$

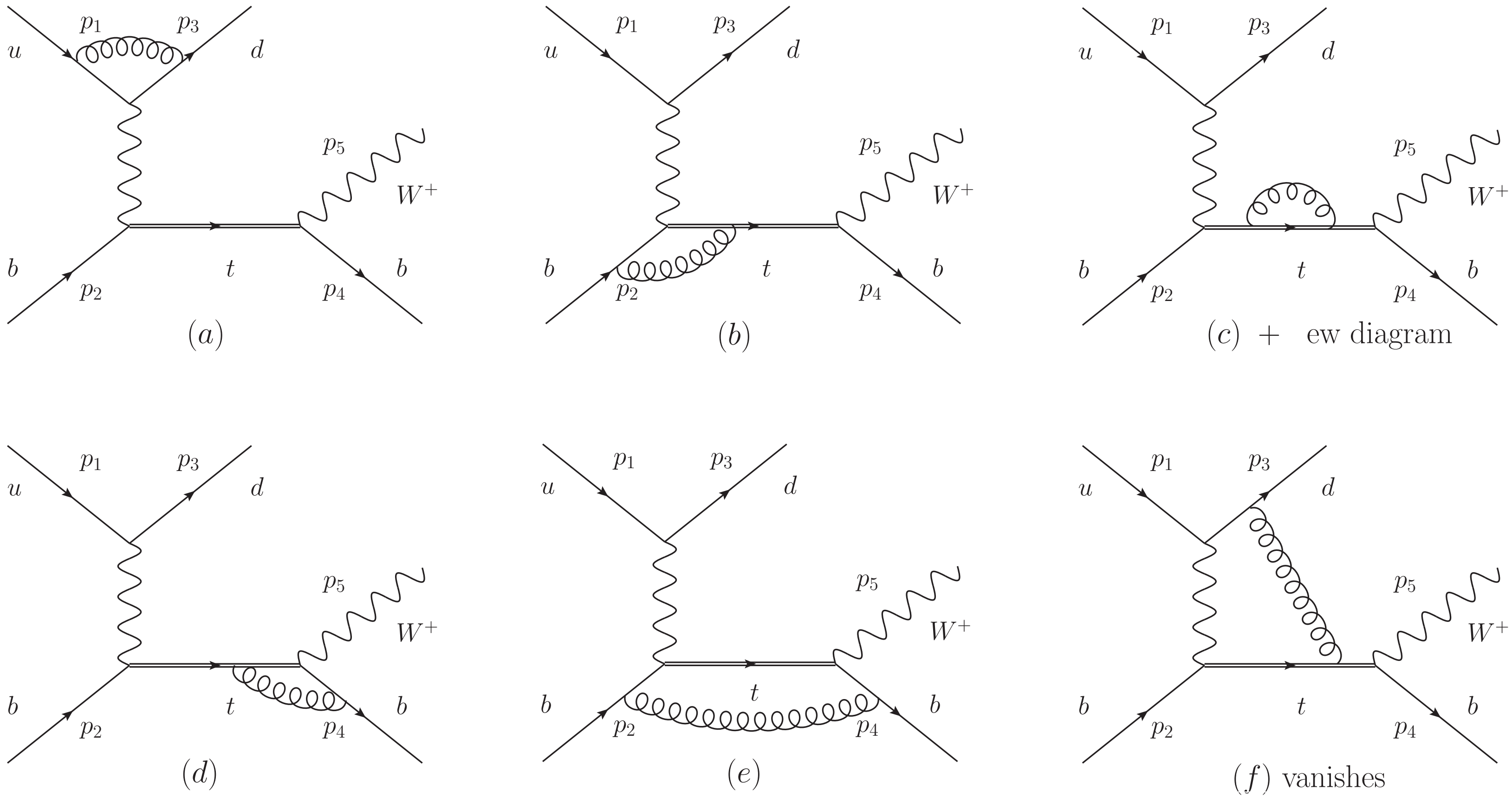


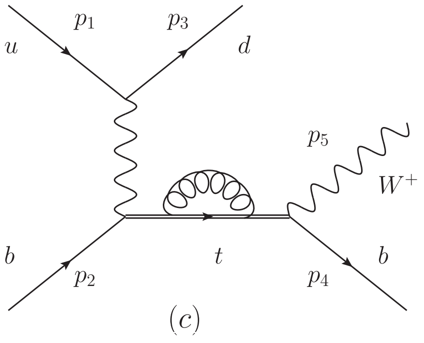
amplitude squared: (no inteference due to colour → no $\delta^{3/2}$ term)

$$|M|^2 = \underbrace{g_{ew}^6\, N_c^2\, \left|A^{(3,0)}_{(-1)}\right|^2}_{\delta} + \underbrace{g_{ew}^6\, N_c^2\, 2\,\mathrm{Re}\left(A^{(3,0)}_{(-1)}\,[A^{(3,0)}_{(0)}]^*\right)}_{\delta^2} + \underbrace{g_{ew}^2 g_s^4\, N_c\, C_F/2 \left|A^{(1,2)}\right|^2}_{\delta^2} + \dots$$

tree-level (squared) $\sim \delta$, compute all $\sim \delta^{3/2}$ contributions to $|M|^2$ ($\sim \mathcal{O}(\alpha_s)$ corrections)

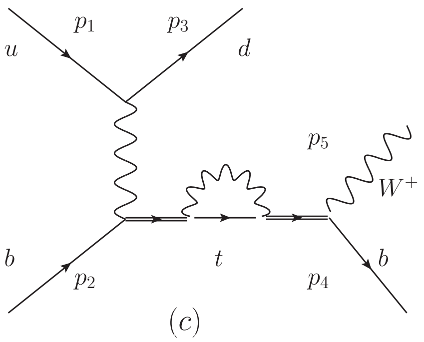
consider subset of resonant virtual diagrams (before expansion in δ this is gauge dependent)





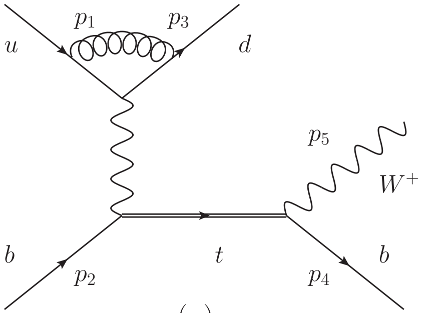
denominator	$\Delta^2\,\ell^2\,[(p_t-\ell)^2-m_t^2]$	
hard	$\Delta^2\,\ell^2\,[\ell^2-2\ell\cdot p_t]$	$\sim \dfrac{g_{ew}^3\cdot\alpha_s\cdot 1}{\delta^2\cdot 1\cdot 1}\sim 1$
soft	$\Delta^2\,\ell^2\,[-2\ell\cdot p_t+\Delta]$	$\sim \dfrac{g_{ew}^3\cdot\alpha_s\cdot\delta^4}{\delta^2\cdot\delta^2\cdot\delta}\sim\delta$

hard part of QCD self-energy is superleading, i.e. $\mathcal{O}(1)$ with LO amplitude $\sim\delta^{1/2}$
but in pole scheme this is precisely cancelled by counter term
soft part of QCD self-energy is NLO, i.e. $\mathcal{O}(\delta^{3/2})$ for $|M|^2$



denominator	$\Delta^2\,\ell^2\,[(p_t-\ell)^2-m_t^2]$	
hard	$\Delta^2\,\ell^2\,[\ell^2-2\ell\cdot p_t]$	$\sim \dfrac{g_{ew}^3\cdot\alpha_{ew}\cdot 1}{\delta^2\cdot 1\cdot 1}\sim\delta^{1/2}$
soft	$\Delta^2\,\ell^2\,[-2\ell\cdot p_t+\Delta]$	$\sim \dfrac{g_{ew}^3\cdot\alpha_{ew}\cdot\delta^4}{\delta^2\cdot\delta^2\cdot\delta}\sim\delta^{3/2}$

hard part of EW self-energy is leading, i.e. $\mathcal{O}(\delta^{1/2})\longrightarrow$ resum
soft part of EW self-energy is beyond NLO, i.e. $\mathcal{O}(\delta^2)$ for $|M|^2$

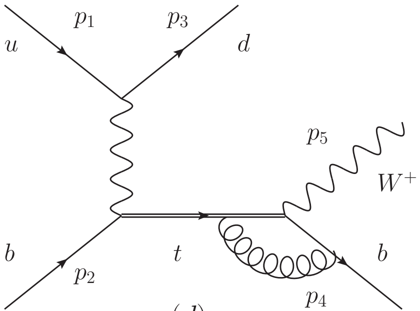


(a)

denom. $\Delta\,\ell^2\,(\ell-p_1)^2\,(\ell-p_3)^2$

hard $\Delta\,\ell^2\,(\ell-p_1)^2\,(\ell-p_3)^2 \quad \sim \dfrac{g_{ew}^3\cdot\alpha_{ew}\cdot 1}{\delta\cdot 1\cdot 1\cdot 1} \sim \delta$

soft $\Delta\,\ell^2\,(-2\ell\cdot p_1)\,(-2\ell\cdot p_3) \quad = 0$

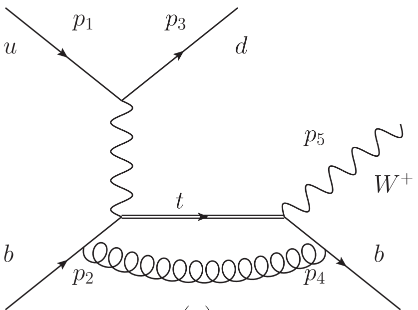


(d)

denom. $\Delta\,\ell^2\,(\ell-p_4)^2\,[(p_t-\ell)^2-m_t^2]$

hard $\Delta\,\ell^2\,(\ell-p_4)^2\,[\ell^2-2\ell\cdot p_t] \quad \sim \dfrac{g_{ew}^3\cdot\alpha_s\cdot 1}{\delta\cdot 1\cdot 1\cdot 1} \sim \delta$

soft $\Delta\,\ell^2\,(-2\ell\cdot p_1)\,[-2\ell\cdot p_t+\Delta] \quad \sim \dfrac{g_{ew}^3\cdot\alpha_{ew}\cdot\delta^4}{\delta\cdot\delta^2\cdot\delta\cdot\delta} \sim \delta$



(e)

denom. $\ell^2\,(\ell-p_2)^2\,(\ell-p_4)^2\,[(p_t-\ell)^2-m_t^2]$

hard $\ell^2\,(\ell-p_2)^2\,(\ell-p_4)^2\,[\ell^2-2\ell\cdot p_t] \quad \sim \dfrac{g_{ew}^3\cdot\alpha_s\cdot 1}{1\cdot 1\cdot 1\cdot 1} \sim \delta^2$

soft $\ell^2\,(-2\ell\cdot p_1)\,(-2\ell\cdot p_4)\,[-2\ell\cdot p_t+\Delta] \quad \sim \dfrac{g_{ew}^3\cdot\alpha_s\cdot\delta^4}{\delta^2\cdot\delta\cdot\delta\cdot\delta} \sim \delta$

explicit calculations and results are very simple!

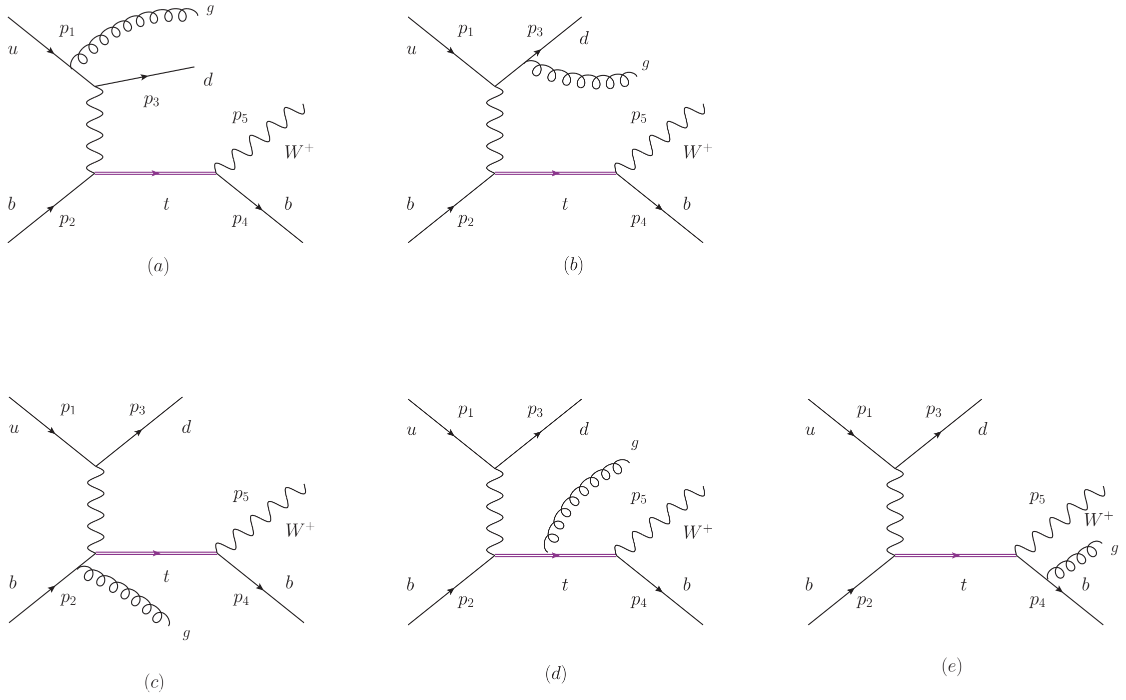
$$\mathcal{A}^{(1),\mathrm{soft}} = \mathcal{A}^{(0)} \delta V^{\mathrm{soft}}$$

$$\delta V^{\mathrm{soft}} = \frac{\alpha_s C_F}{2\pi} \left(-\frac{\Delta}{\mu m_t} \right)^{-2\epsilon} \left[\frac{1}{\epsilon} \left(1 - \ln \frac{s_{2t} s_{4t}}{m_t^2\, s_{24}} \right) + 2 + \mathrm{Li}_2 \left(1 - \frac{s_{2t} s_{4t}}{m_t^2\, s_{24}} \right) \right]$$

$$\mathcal{A}^{\mathrm{hard},(b)} = \mathcal{A}^{(0)} \delta V^{\mathrm{hard},(b)} + \frac{\alpha_s C_F}{2\pi} \frac{-i g_{ew}^4 \langle 46 \rangle \langle 3|2|1 \rangle [25]}{(s_{13} + M_W^2)\, \Delta} \frac{m_t^2}{m_t^2 - s_{2t}} \ln \frac{s_{2t}}{m_t^2}$$

$$\begin{aligned} \delta V^{\mathrm{hard},(b)} = \frac{\alpha_s C_F}{2\pi} \Bigg[& -\frac{1}{2\epsilon^2} + \frac{1}{\epsilon} \left(\ln \frac{s_{2t}}{m_t \mu} - \frac{1}{2} \right) + \mathrm{Li}_2 \left(1 - \frac{m_t^2}{s_{2t}} \right) - 2 - \frac{\pi^2}{24} \\ & - \frac{1}{2} \ln^2 \frac{s_{2t}}{m_t \mu} + \frac{1}{8} \ln^2 \frac{m_t^2}{\mu^2} + \frac{s_{2t}}{4(m_t^2 - s_{2t})} \ln \frac{m_t^2}{\mu^2} \\ & + \frac{1}{2} \ln \frac{s_{2t}}{m_t \mu} \left(2 - \frac{s_{2t}}{m_t^2 - s_{2t}} - \ln \frac{m_t^2}{\mu^2} \right) \Bigg] \end{aligned}$$

consider only subset of real diagrams (in general gauge dependent)
could use full real amplitude, difference is $\mathcal{O}(\alpha_s \delta^2) = \mathcal{O}(\delta^{5/2})$ for $|M|^2$



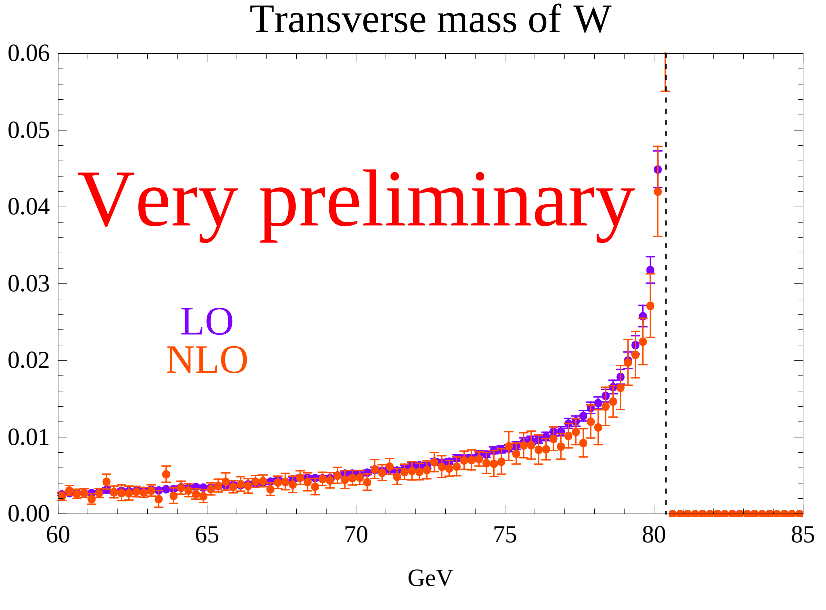
The only ET-input is the resummed propagator for the top.

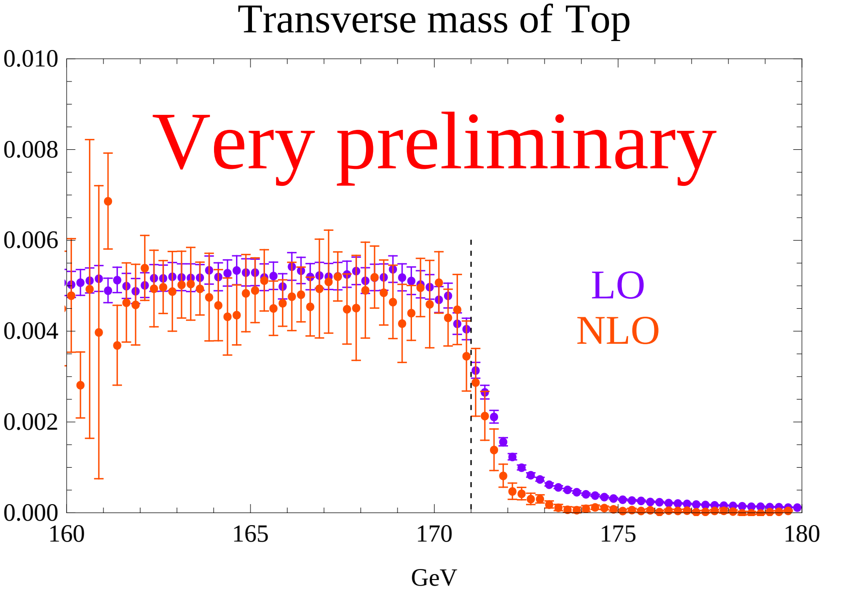
LHC $\sqrt{s} = 10\ \mathrm{TeV}$, only partonic subprocess $ub \to dWb$

define jets: $k_\perp$ cluster algorithm $\Rightarrow$ J_b with $|k_{J_b\,\perp}| > 40\ \mathrm{GeV}$
J_q with $|k_{J_q\,\perp}| > 40\ \mathrm{GeV}$

top window: $150\ \mathrm{GeV} < \sqrt{(k_{J_b} + k_e + k_\nu)^2} < 200\ \mathrm{GeV}$
$\not{E}_\perp > 40\ \mathrm{GeV}$, $|k_{e\,\perp}| > 20\ \mathrm{GeV}$, $m_t = 171.3\ \mathrm{GeV}$, MSTW 2008, NLO pdf

transverse mass of W (improved narrow width) and t (off-shell effects taken into account)





- using ET inspired approach, the computational effort to include off-shell effects for unstable particles is modest
- single top (work in progress)
  - preliminary comparison to [Campbell et.al.] confirms off-shell effects $\mathcal{O}(\alpha_s\,\delta)$ are relatively small for total cross section
  - perform full analysis for generic observables
  - consider effects of $\log\Gamma_t/m_t$
  - higher order contributions in Δ are not too difficult to compute and can be numerically important (e.g. QCD “background”)
  - full calculation beyond $\mathcal{O}(\delta^{3/2})$ for $|M|^2$ would require two-loop matching coefficient
- the really interesting process is top pair production (outlook)
  - off-shell effects at tree level have been considered [Kauer, Zeppenfeld]
  - (inclusive) non-factorizable corrections to invariant mass distributions are small [Beenakker et.al.]
  - but cancellations between real and virtual contributions are disturbed by cuts
  - to have confidence in a top mass measurement with $\delta m_t \sim 1\ \mathrm{GeV}$ these corrections have to be considered