## NLO QCD CORRECTIONS TO PROCESSES WITH MULTIPLE ELECTROWEAK BOSONS

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- Introdution: VBFNLO
- NLO for WWW, WWZ and WZZ production
- $W W \gamma$ and $Z Z \gamma$ at NLO
- Wrj production at NLO
- Conclusions



## Beginnings of VBFNLO: QCD corrections to VBF processes

NLO predictions for vector boson fusion processes at the LHC:
$q q \rightarrow q q H \quad$ Han, Valencia, Willenbrock (1992); Figy, Oleari, DZ (2003); Campbell, Ellis, Berger (2004)

- Higgs coupling measurements
$q q \rightarrow q q \mathrm{Z}$ and $q q \rightarrow q q \mathrm{~W}$
Oleari, DZ: hep-ph/0310156
- $\mathrm{Z} \rightarrow \tau \tau$ as background for $H \rightarrow \tau \tau$
- measure central jet veto acceptance at LHC
$q q \rightarrow q q W W, q q \rightarrow q q Z Z, q q \rightarrow q q W Z$
Jäger, Oleari, Bozzi, DZ: hep-ph/0603177,
hep-ph/0604200, hep-ph/0701105, arXiv:0907.0580
- $q q W W$ is background to $H \rightarrow W W$ in VBF
- underlying process is weak boson scattering: $W W \rightarrow W W, W W \rightarrow Z Z, W Z \rightarrow W Z$ etc.
$\Longrightarrow$ Talk by Barbara Jäger


## $q q \rightarrow q q V V: 3$ weak bosons on a quark line

- NLO corrections to $q q \rightarrow q q V V$ contain all loops with a virtual gluon attached to a quark line with one, two or three weak bosons

(a)

(c)

(e)

(b)

(d)

(f)


## Extending VBFNLO: $V V V$ and $V V j$ Production at NLO QCD

New processes implemented in 2008 release of VBFNLO:

- Triple weak boson production: $V V V=W^{ \pm} W^{\mp} W^{ \pm}, W^{+} W^{-} Z$ and $W^{ \pm} Z Z$ with leptonic decay of the weak bosons and full $H \rightarrow W W$ and $H \rightarrow$ ZZ contributions Work in collaboration with V. Hankele, S. Prestel, C. Oleari and F. Campanario

New processes already available for future releases:

- $W^{+} W^{-} \gamma$ and $Z Z \gamma$ production with leptonic decay of weak bosons Work in collaboration with G. Bozzi and F. Campanario
- $W^{ \pm} \gamma j$ production (with $W$ leptonic decay and final state photon radiation) Work in collaboration with C.Englert, F. Campanario and M. Spannowsky

Code is available at http:/ /www-itp.particle.uni-karlsruhe.de/ ~vbfnloweb

## $V V V$ Production: Motivation

- Standard Model background for SUSY processes with multi-lepton $+p_{T}$ signature
- Possibility to obtain information about quartic electroweak couplings.

- QCD corrections to $p p \rightarrow \mathrm{VVV}+\mathrm{X}$ on experimentalist's wishlist:
[The QCD, EW, and Higgs
Working Group: hep-ph/0604120]

| process <br> $(V \in\{Z, W, \gamma\})$ | relevant for |
| :--- | :--- |
|  |  |
| 1. $p p \rightarrow V V$ jet | $t \bar{t} H$, new physics |
| 2. $p p \rightarrow t \bar{t} b \bar{b}$ | $t \bar{t} H$ |
| 3. $p p \rightarrow t \bar{t}+2$ jets | $t \bar{t} H$ |
| $4 . p p \rightarrow V V b \bar{b}$ | $\mathrm{VBF} \rightarrow H \rightarrow V V, t \bar{t} H$, new physics |
| 5. $p p \rightarrow V V+2$ jets | $\mathrm{VBF} \rightarrow H \rightarrow V V$ |
| $6 . p p \rightarrow V+3$ jets | various new physics signatures |
| $7 . p p \rightarrow V V V$ | SUSY trilepton |

## Example: Contributions to $W W Z$ production

a)

b)

c)




- All resonant and non-resonant matrix elements as well as spin correlations of final state leptons and Higgs contribution included.
- Interference terms due to identical particles in the final state have been neglected.
- All fermion mass effects neglected. $(H \tau \tau$-coupling $=0)$


## 1-loop matrix elements and real emission matrix elements

Three different topologies:


I Vertex correction proportional to Born matrix element.

II Maximally 4-point integrals appear.
III Up to five external legs (Pentagons):

- Two independent calculations.
- Numerically stable results with Denner Dittmaier method.


## Boxline corrections

Virtual corrections for quark line with 2 EW gauge bosons

(a)

(b)

(c)

The external vector bosons correspond to $V \rightarrow l_{1} \bar{T}_{2}$ decay currents or quark currents

Divergent terms in 4 Feynman graphs combine to multiple of corresponding Born graph

$$
\begin{aligned}
& \mathcal{M}_{\text {boxline }}^{(i)}= \\
& \quad \mathcal{M}_{B}^{(i)} F(Q) \\
& \\
& \quad\left[-\frac{2}{\epsilon^{2}}-\frac{3}{\epsilon}+\frac{4 \pi^{2}}{3}-8\right] \\
& +\frac{\alpha_{s}\left(\mu_{R}\right)}{4 \pi} C_{F} \widetilde{\mathcal{M}}_{\tau}\left(q_{1}, q_{2}\right)\left(-e^{2}\right) g_{\tau}^{V_{1} f_{1}} g_{\tau}^{V_{2} f_{2}} \\
& +\mathcal{O}(\epsilon)
\end{aligned}
$$

with $F(Q)=\frac{\alpha_{s}\left(\mu_{R}\right)}{4 \pi} C_{F}\left(\frac{4 \pi \mu_{R}^{2}}{Q^{2}}\right)^{\epsilon} \Gamma(1+\epsilon)$
$\widetilde{\mathcal{M}}_{\tau}\left(q_{1}, q_{2}\right)=\widetilde{\mathcal{M}}_{\mu \nu} \epsilon_{1}^{\mu} \epsilon_{2}^{v}$ is universal virtual $q q V V$ amplitude: use like HELAS calls in MadGraph

## Handling of IR and collinear divergences

Use tensor decomposition a la Passarino-Veltman
Split $B_{0} \cdots D_{i j}$ functions into divergent and finite parts
With $s=\left(q_{1}+q_{2}\right)^{2}, t=\left(k_{2}+q_{2}\right)^{2}=\left(k_{1}-q_{1}\right)^{2}$ we get, for example,

$$
\begin{aligned}
B_{0}\left(q^{2}\right) & =\frac{\Gamma(1+\epsilon)}{(-s)^{\epsilon}}\left[\frac{1}{\epsilon}+2-\ln \frac{q^{2}+i 0^{+}}{s}+\mathcal{O}(\epsilon)\right] \\
& =\frac{\Gamma(1+\epsilon)}{(-s)^{\epsilon}}\left[\frac{1}{\epsilon}+\widetilde{B}_{0}\left(q^{2}\right)+\mathcal{O}(\epsilon)\right] \\
D_{0}\left(k_{2}, q_{2}, q_{1}\right) & =\frac{\Gamma(1+\epsilon)}{(-s)^{\epsilon}}\left[\frac{1}{s t}\left(\frac{1}{\epsilon^{2}}+\frac{1}{\epsilon} \ln \frac{q_{1}^{2} q_{2}^{2}}{t^{2}}\right)+\widetilde{D}_{0}\left(k_{2}, q_{2}, q_{1}\right)+\mathcal{O}(\epsilon)\right] \\
D^{\mu \nu}\left(k_{2}, q_{2}, q_{1}\right) & =\frac{\Gamma(1+\epsilon)}{(-s)^{\epsilon}}\left(\frac{1}{\epsilon}\left(k_{1}^{\mu} k_{1}^{v} d_{2}\left(q_{1}^{2}, t\right)+k_{2}^{\mu} k_{2}^{v} d_{2}\left(q_{2}^{2}, t\right)\right)+\widetilde{D}^{\mu \nu}\left(k_{2}, q_{2}, q_{1}\right)+\mathcal{O}(\epsilon)\right)
\end{aligned}
$$

with $d_{2}\left(q^{2}, t\right)=1 /\left(s\left(q^{2}-t\right)^{2}\right)\left[t \ln \left(q^{2} / t\right)-\left(q^{2}-t\right)\right]$
Finite $\widetilde{D}_{i j}$ have standard PV recursion relations $\Longrightarrow$ determine them numerically

## Extension to $q q V V V$ amplitude: pentline corrections

Virtual corrections involve up to pentagons


The external vector bosons correspond to $V \rightarrow l_{1} \bar{l}_{2}$ decay currents or quark currents

The sum of all QCD corrections to a single quark line is simple

$$
\begin{aligned}
\mathcal{M}_{V}^{(i)}= & \mathcal{M}_{B}^{(i)} \frac{\alpha_{s}\left(\mu_{R}\right)}{4 \pi} C_{F}\left(\frac{4 \pi \mu_{R}^{2}}{Q^{2}}\right)^{\epsilon} \Gamma(1+\epsilon) \\
& {\left[-\frac{2}{\epsilon^{2}}-\frac{3}{\epsilon}+c_{\mathrm{virt}}\right] } \\
+ & \widetilde{\mathcal{M}}_{V_{1} V_{2} V_{3}, \tau}^{(i)}\left(q_{1}, q_{2}, q_{3}\right)+\mathcal{O}(\epsilon)
\end{aligned}
$$

- Divergent terms sum to Born sub-amplitude
- Use amplitude techniques to calculate finite remainder of virtual amplitudes

Denner-Dittmaier reduction of pentagon tensors is stable: indication of numerical problems for less than $0.2 \%$ of phase space points

## Virtual corrections

Born sub-amplitude is multiplied by same factor as found for pure vertex corrections $\Rightarrow$ when summing all Feynman graphs the divergent terms multiply the complete $\mathcal{M}_{B}$

## Complete virtual corrections

$$
\mathcal{M}_{V}=\mathcal{M}_{B} F(Q)\left[-\frac{2}{\epsilon^{2}}-\frac{3}{\epsilon}+\frac{4 \pi^{2}}{3}-8\right]+\widetilde{\mathcal{M}}_{V}
$$

where $\widetilde{\mathcal{M}}_{V}$ is finite, and is calculated with amplitude techniques.
The interference contribution in the cross-section calculation is then given by

$$
2 \operatorname{Re}\left[\mathcal{M}_{V} \mathcal{M}_{B}^{*}\right]=\left|\mathcal{M}_{B}\right|^{2} F(Q)\left[-\frac{2}{\epsilon^{2}}-\frac{3}{\epsilon}+\frac{4 \pi^{2}}{3}-8\right]+2 \operatorname{Re}\left[\widetilde{\mathcal{M}}_{V} \mathcal{M}_{B}^{*}\right]
$$

The divergent term, proportional to $\left|\mathcal{M}_{B}\right|^{2}$, cancels against the subtraction terms which have the same structure as for single $W$ or $Z$ production.

## Gauge invariance tests

Numerical problems flagged by gauge invariance test: use Ward identities for pentline and boxline contributions

$$
q_{2}^{\mu_{2}} \widetilde{\mathcal{E}}_{\mu_{1} \mu_{2} \mu_{3}}\left(k_{1}, q_{1}, q_{2}, q_{3}\right)=\widetilde{\mathcal{D}}_{\mu_{1} \mu_{3}}\left(k_{1}, q_{1}, q_{2}+q_{3}\right)-\widetilde{\mathcal{D}}_{\mu_{1} \mu_{3}}\left(k_{1}, q_{1}+q_{2}, q_{3}\right)
$$

With Denner-Dittmaier recursion relations for $E_{i j}$ functions the ratios of the two expressions agree with unity (to $10 \%$ or better) at more than $99.8 \%$ of all phase space points.
Ward identities reduce importance of computationally slow pentagon contributions when contracting with $W^{ \pm}$polarization vectors

$$
J_{ \pm}^{\mu}=x_{ \pm} q_{ \pm}^{\mu}+r_{ \pm}^{\mu}
$$

choose $x_{ \pm}$such as to minimize pentagon contribution from remainders $r_{ \pm}$in all terms like

$$
J_{+}^{\mu_{1}} J_{-}^{\mu_{2}} \widetilde{\mathcal{E}}_{\mu_{1} \mu_{3}}\left(k_{1}, q_{+}, q_{-}, q_{0}\right)=r_{+}^{\mu_{1}} r_{-}^{\mu_{2}} \widetilde{\mathcal{E}}_{\mu_{1} \mu_{2} \mu_{3}}\left(k_{1}, q_{+}, q_{-}, q_{0}\right)+\text { box contributions }
$$

Resulting true pentagon piece contributes to the cross section at permille level $\Longrightarrow$ totally negligible for phenomenology

## Available calculations of $V V V$ cross sections

- QCD corrections to ZZZ production without Higgs-contribution and leptonic decays. [Lazopoulos, Melnikov, Petriello; hep-ph/0703273]
- Calculation of QCD corrections to $\mathrm{ZZZ}, \mathrm{W}^{+} \mathrm{W}^{-} \mathrm{Z}, \mathrm{W}^{+} \mathrm{W}^{-} \mathrm{W}^{+}$and $\mathrm{ZZW}{ }^{+}$production without Higgs-contribution and leptonic decays. [Binoth, Ossola, Papadopoulos, Pittau; arXiv:0804:0350]
- QCD corrections to $W^{+} W^{-} Z$ production with leptonic decays and Higgs exchange.
[Hankele, DZ; arXiv:0712.3544]
Implemented into the
Fortran program VBFNLO.
- QCD corrections to $\mathrm{ZZW}^{ \pm}$and $\mathrm{W}^{ \pm} \mathrm{W}^{\mp} \mathrm{W}^{ \pm}$production with leptonic decays and Higgs graphs. [Campanario, Hankele, Oleari, Prestel, DZ; arXiv:0809.0790]
Fortran program
VBFNLO.
2008 release includes
triple weak boson
production:
arxiv:0811.4559


## Comparison of various tri-boson codes

Numerous checks on the final results. For example comparison of $\mathrm{ZZW}^{+}$in narrow width approximation and without Higgs contribution with [Binoth, Ossola, Papadopoulos, Pittau; arXiv:0804:0350]

$\Rightarrow$ Agreement at the level of the accuracy of the Monte Carlo runs.

## Input variables for LHC phenomenology

- PDFs: CTEQ6L1 at LO and CTEQ6M, $\alpha_{S}\left(m_{Z}\right)=0.118$ at NLO.
- Cuts and Masses:

$$
p_{T_{\ell}}>10 \mathrm{GeV}, \quad\left|\eta_{\ell}\right|<2.5, \quad m_{\ell^{+} \ell^{-}}>15 \mathrm{GeV}, \quad m_{H}=120 \mathrm{GeV}
$$

- Renormalization- and Factorization Scale: $\mu_{F}=\mu_{R}=3 m_{W}$.

Following results are for electrons and/or muons in the final state:
$\Longrightarrow$ Combinatorial factor of $8 / 4$ for the $\mathrm{W}^{+} \mathrm{W}^{-} \mathrm{Z} / \mathrm{ZZW}^{ \pm}$production compared to three different lepton families in the final state.

## Scale Dependence



- At LO only small $\mu_{F}$-dependence, no $\alpha_{s}\left(\mu_{R}\right)$.
- At NLO scale dependence is dominated by $\alpha_{s}\left(\mu_{R}\right)$.
- Real emission contribution drives overall scale dependence at NLO.


## Higgs mass dependence





- Cross section reflects behavior of $B R(H \rightarrow Z Z)$
- K-factor is reduced by Higgs contribution.

K-factor for $p p \rightarrow$ ZH production is about $\mathrm{K}=1.3$
[Han and Willenbrock, Phys. Lett. B 273 (1991) 167.]

## Differential cross section and $K$-factor for the highest- $p_{T}$-lepton



- K-factor increases with transverse momentum $\left(p_{T}\right)$ by almost a factor of 2 .
- Strong phase space dependence due to events with high $p_{T}$ jets recoiling against the leptons.
- Veto on jets with $p_{T}>50 \mathrm{GeV}$ leads to fairly flat K-factor.


## Extension to $W^{+} W^{-} \gamma$ and $Z Z \gamma$ Production



I


New elements of calculation:

- Different infrared divergence structure of individal loop integrals but same final virtual expressions in terms of finite parts of $C_{i j}, D_{i j}$, and $E_{i j}$ functions
- Photon isolation from jets for real emission contributions: use Frixione isolation

$$
\Sigma_{i} E_{T_{i}} \theta\left(\delta-R_{i \gamma}\right) \leq p_{T_{\gamma}} \frac{1-\cos \delta}{1-\cos \delta_{0}} \quad\left(\text { for all } \quad \delta \leq \delta_{0}\right)
$$

- Final state photon radiation becomes important: adapt phase space to this


## Scale dependence of integrated cross sections

Variation of $\mu_{F}, \mu_{R}$ about $\mu_{0}=m_{W W \gamma}$



- Behaviour similar to $V V V$ production: LO scale variation much smaller than NLO correction
- NLO scale dependence largely due to real emission contributions $\Longrightarrow$ jet veto will reduce it
- Box and pentagon contributions $\left(\tilde{\mathcal{M}}_{V}\right.$ terms $)$ are quite small: $3 \%$ and $<1 \%$ of total


## NLO Corrections to Distributions: $p_{T}$ of photon




Strong phase space dependence of K-factors (depends on LO scale choice)

## NLO QCD Corrections to $W \gamma j$ Production

- Provide NLO QCD corrections including leptonic $W$ decay, e.g.

$$
p p \rightarrow e^{+} v_{e} \gamma j, \quad p p \rightarrow e^{-} \bar{v}_{e} \gamma j
$$

- Sizable cross section at LHC (1.2 pb) and Tevatron ( 15 fb ) for $p_{T j}, p_{T \gamma}>50 \mathrm{GeV}$ and separation cuts (later)
- Measurement of anomalous $W W \gamma$ coupling: veto on jets in $W \gamma$ events requires good knowledge of cross section and distributions: want NLO
- Photon isolation à la Frixione probed at NLO level

- Initial and final state photon radiation. Final radiation from lepton is important
- Virtual corrections up to pentagons
- External gluon already at tree level $\Longrightarrow$ nonabelian boxes with three gluon vertex
- Larger number of subtraction terms


## Virtual Corrections: nonabelian Contributions

Example: non-abelian extension of boxline graphs. Keep modular structure of calculation

(1)

(2)

(3)

(4)

$$
\begin{gathered}
\left(C_{F}-\frac{1}{2} C_{A}\right)\left(A_{1}(12)+A_{3}(12)\right) \\
+C_{F} \quad\left(A_{2}(12)+A_{4}(12)\right)
\end{gathered}
$$



$$
C_{A}\left(A_{5}+A_{6}+A_{7}\right)
$$

Combine to two boxline amplitudes $M_{V}(12)$ and $M_{V}(21)$ and new nonabelian combination

$$
\begin{aligned}
M_{V}(12, \text { boxline }) & =\left(C_{F}-\frac{1}{2} C_{A}\right) \sum_{i=1,4} A_{i}(12) \\
M_{V}(n a) & =\frac{1}{2} C_{A}\left(A_{2}(12)+A_{4}(12)+A_{2}(21)+A_{4}(21)\right)+C_{A}\left(A_{5}+A_{6}+A_{7}\right)
\end{aligned}
$$

## Scale dependence: LHC and Tevatron

Identify lepton, photon and one or more jets with $k_{T}$-algorithm ( $D=0.7$ )

$$
p_{T j, \gamma} \geq 50 \mathrm{GeV}, \quad\left|y_{j}\right| \leq 4.5,\left|\eta_{\gamma}\right| \leq 2.5, \quad p_{T l} \geq 20 \mathrm{GeV}, \quad\left|\eta_{l}\right| \leq 2.5 \quad R_{l, \gamma}, R_{l, j}>0.2
$$

Frixione isolation of photons with $\delta_{0}=1$
Cross sections are for $W \rightarrow e \nu_{e}$ only




Scale variation at LHC for $\mu_{F}=\mu_{R}=2^{ \pm 1} \cdot 100 \mathrm{GeV}$ :
$\pm 11 \%$ at LO reduced to $\pm 7 \%$ at NLO Almost flat behaviour for veto of additional jets of $p_{T}>50 \mathrm{GeV}$ should be taken as accidental and not as a measure of NLO uncertainties

## NLO corrections to distributions



- Clear shape changes of distributions when going from LO to NLO
- Average K-factor of 1.4 at LHC is significantly larger than LO scale variation


## Conclusions

- NLO QCD corrections to $p p \rightarrow \mathrm{VVV}+\mathrm{X}$ are Standard Model background processes for new-physics searches and are sensitive to quartic electroweak couplings.
- All off-shell diagrams as well as the Higgs-contributions have been considered.
- New results for $W W \gamma$ and $Z Z \gamma$ production will appear soon.
- First results on $W \gamma j$ production at NLO have been published in EPL.
- Latest release of VBFNLO includes NLO QCD corrections for $W^{+} W^{-} Z, Z Z W^{ \pm}$and $W^{ \pm} W^{\mp} W^{ \pm}$ production at hadron colliders: arxiv:0811.4559

Code is available at
http:/ /www-itp.particle.uni-karlsruhe.de/~vbfnloweb

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V. Hankele, B. Jäger, M. Worek, C. Oleari, K. Arnold, G. Bozzi, F. Campanario, C. Englert, T. Figy, G. Klämke, M. Kubocz, S. Plätzer, S. Prestel, M. Rauch, H. Rzehak, M. Spannowsky

