All-order Corrections to Multi-jet Rates using t-channel Factorised Scattering Matrix Elements

Jeppe R. Andersen (CERN)
in collaboration with Jenni Smillie (UCL)

RADCOR
October 27, 2009

What, Why, How?

What?

Develop a framework for reliably calculating many-parton rates inclusively (ensemble of $2, 3, 4, \ldots$ parton rates) and in a flexible way (jets, W+jets, Z+jets, Higgs+jets,...)

Why?

$(n+1)$-jet rate not necessarily small compared to n-jet rate

Inclusive (hard) perturbative corrections important for e.g. hard end of W $p_\perp$-spectrum.

How?

Establish universal behaviour of radiative corrections (in the so-called High Energy Limit)

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Goal

- Sufficiently *simple* model for radiative corrections that the all-order sum can be evaluated explicitly (completely exclusive)
- Sufficiently *accurate* that the description is relevant

Do we need a new approach?

Already know how to calculate. . .

- Shower MC: at most 2→2 "hard" processes with additional parton shower
- Flexible Tree level calculators:
  MadGraph, AlpGen, SHERPA,. . .
  Allow most $2 \rightarrow 4$, some $2 \rightarrow 6$ processes to be calculated at tree level.
  Interfaced with Shower MC makes for a powerful mix!
- MCFM: Many relevant $2 \rightarrow 3$ processes at up to NLO (i.e. including $2 \rightarrow 4$-contribution).
- . . . ⟨your favourite method here⟩

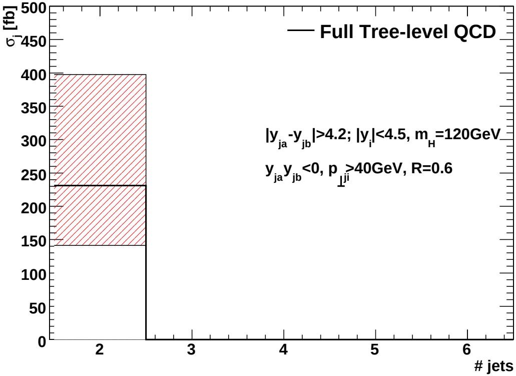
Could all be labelled “Standard Model contribution”, but give vastly different results depending on the question asked!

All Order Resummation Necessary?

Are tree-level (or generally fixed order) calculation always sufficient?

Sometimes the $(n+1)$-jet rate is as large as the n-jet rate

Higgs Boson plus n jets at the LHC at leading order



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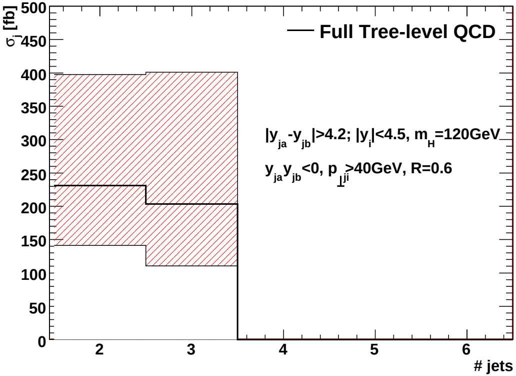

Indication that we need to go further! However, fixed order tools **exhausted** (full $2 \to 3$ with a massive leg at two loops **untenable!**).

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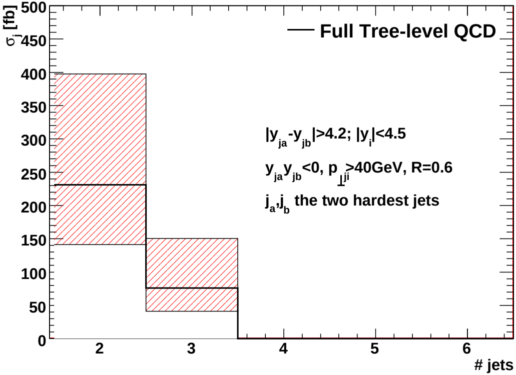

Require that the two jets passing the **rapidity cut** are **also the two hardest jets**. Reduces the 3-jet phase space and the HO corrections from real emission. Sensitivity to yet HO pert. corrections?

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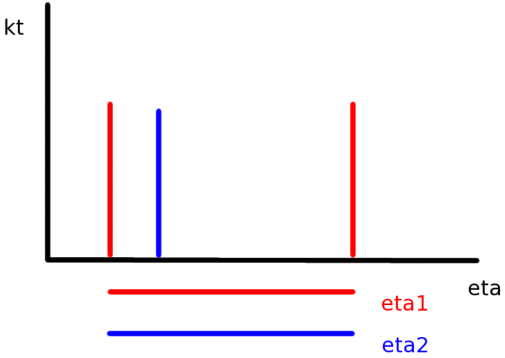

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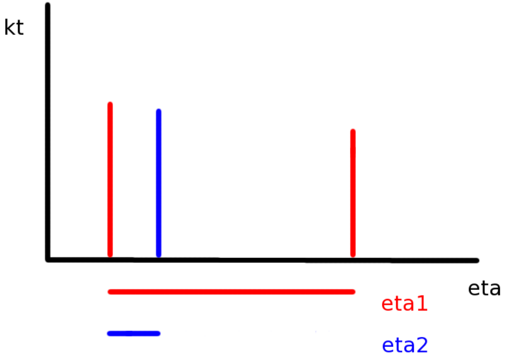

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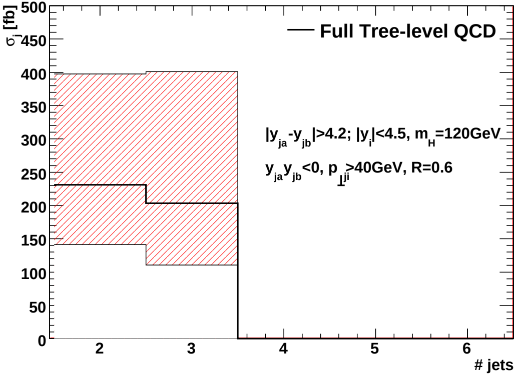

The method we develop will be applicable to both set of cuts, but crucially will allow a **stabilisation** of the perturbative series by **resummation**

Resummation and Matching

Consider the **perturbative expansion** of an observable

$$R = r_0 + r_1\alpha_s + r_2\alpha^2 + r_3\alpha^3 + r_4\alpha^4 + \cdots$$

Fixed order pert. QCD will calculate a fixed number of terms in this expansion. r_n may contain **logarithms** so that $\alpha_s \ln(\cdots)$ is large.

$$R = r_0 + \left(r_1^{LL}\ln(\cdots) + r_1^{NLL}\right)\alpha_s + \left(r_2^{LL}\ln^2(\cdots) + r_2^{NLL}\ln(\cdots) + r_2^{SL}\right)\alpha_s^2 + \cdots$$
$$= r_0 + \sum_n r_n^{LL}(\alpha_s\ln(\cdots))^n + \sum_n r_n^{NLL}\alpha_s(\alpha_s\ln(\cdots))^n + \text{sub-leading terms}$$

Need simplifying assumptions to get to all orders - useful **iff** the terms really do describe **the dominant part** of the **full pert. series.**
Matching combines **best of both worlds**:

$$R = r_0 + r_1\alpha_s + r_2\alpha^2 + \left(r_3^{LL}\ln^3(\cdots) + r_3^{NLL}\ln^2(\cdots) + r_3^{SL}\right)\alpha^3 + \cdots$$

Factorisation of QCD Matrix Elements

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It is **well known** that QCD matrix elements **factorise** in certain kinematical limits:
Soft limit $\rightarrow$ **eikonal approximation** $\rightarrow$ enters all parton shower (and much else) resummation.

Like all good limits, the eikonal approximation is applied **outside its strict region of validity**.

Will discuss the **less well-studied factorisation** of scattering amplitudes in a different kinematic limit, better suited for describing perturbative corrections from **hard parton emission**

Factorisation only **becomes exact** in a region **outside** the reach of any collider…

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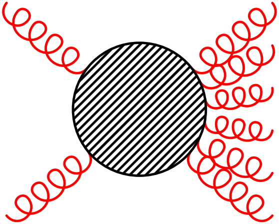
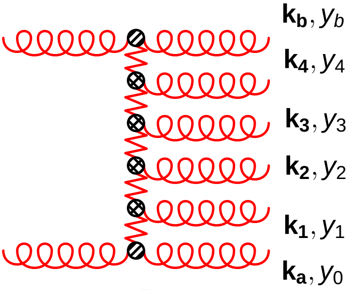
The Possibility for Predictions of *n*-jet Rates

The Power of Reggeisation



High Energy Limit

$$|\hat{t}| \quad \text{fixed}, \quad \hat{s} \to \infty$$



$$\mathcal{A}^R_{2\to 2+n} = \frac{\Gamma_{A'A}}{q_0^2} \left(\prod_{i=1}^{n} e^{\omega(q_i)(y_{i-1}-y_i)} \frac{V^{J_i}(q_i, q_{i+1})}{q_i^2 q_{i+1}^2} \right) e^{\omega(q_{n+1})(y_n - y_{n+1})} \frac{\Gamma_{B'B}}{q_{n+1}^2}$$

$q_i = \mathbf{k}_a + \sum_{l=1}^{i-1} \mathbf{k}_l$

LL: Fadin, Kuraev, Lipatov; NLL: Fadin, Fiore, Kozlov, Reznichenko

Maintain (at LL) terms of the form

$$\left(\alpha_s \ln \frac{\hat{s}_{ij}}{|\hat{t}_i|} \right)$$

to all orders in α_s.

At LL only gluon production; at NLL also quark–anti-quark pairs produced.
Approximation of any-jet rate possible.

Comparison of 3-jet scattering amplitudes

Universal behaviour of scattering amplitudes in the HE limit:

$$\forall i \in \{2,\ldots,n-1\} : y_{i-1} \gg y_i \gg y_{i+1}$$
$$\forall i,j : |p_{i\perp}| \approx |p_{j\perp}|$$

$$\left|\overline{\mathcal{M}}^{MRK}_{gg\to g\cdots g}\right|^2 = \frac{4\,s^2}{N_C^2-1}\,\frac{g^2\,C_A}{|p_{1\perp}|^2}\left(\prod_{i=2}^{n-1}\frac{4\,g^2 C_A}{|p_{i\perp}|^2}\right)\frac{g^2\,C_A}{|p_{n\perp}|^2}.$$

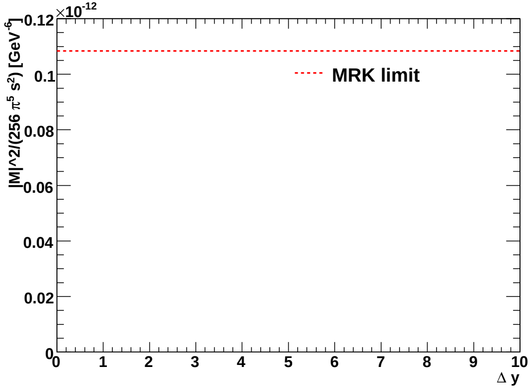
$$\left|\overline{\mathcal{M}}^{MRK}_{qg\to qg\cdots g}\right|^2 = \frac{4\,s^2}{N_C^2-1}\,\frac{g^2\,C_F}{|p_{1\perp}|^2}\left(\prod_{i=2}^{n-1}\frac{4\,g^2 C_A}{|p_{i\perp}|^2}\right)\frac{g^2\,C_A}{|p_{n\perp}|^2},$$

$$\left|\overline{\mathcal{M}}^{MRK}_{qQ\to qg\cdots Q}\right|^2 = \frac{4\,s^2}{N_C^2-1}\,\frac{g^2\,C_F}{|p_{1\perp}|^2}\left(\prod_{i=2}^{n-1}\frac{4\,g^2 C_A}{|p_{i\perp}|^2}\right)\frac{g^2\,C_F}{|p_{n\perp}|^2},$$

Allow for analytic resummation (BFKL equation).
However, how well does this actually approximate the amplitude?

Comparison of 3-jet scattering amplitudes

Study just a slice in phase space:



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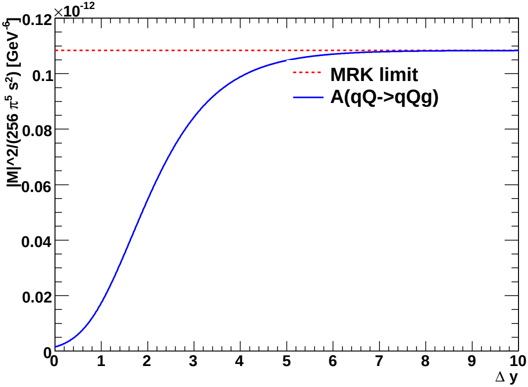

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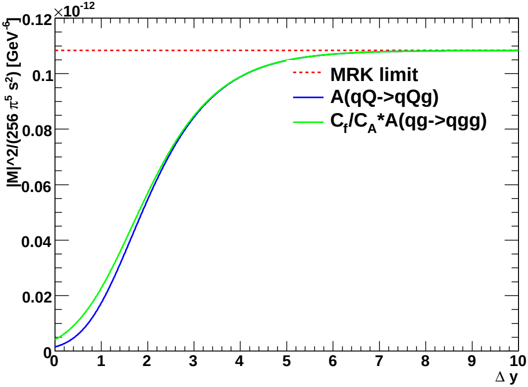

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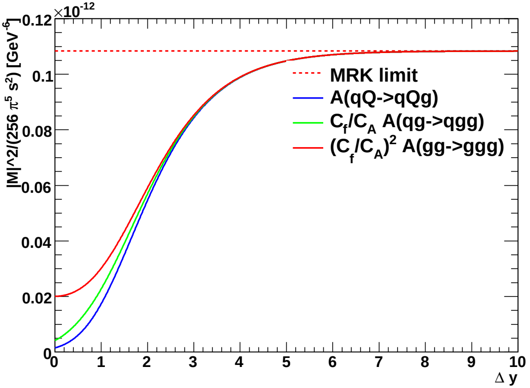

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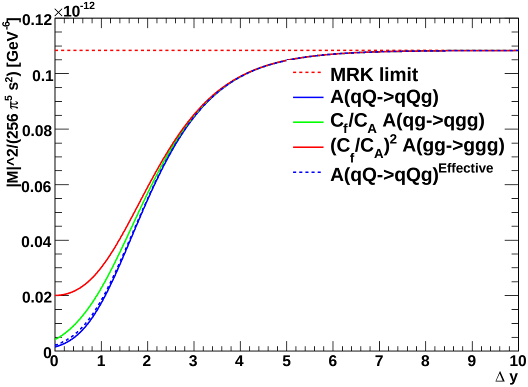
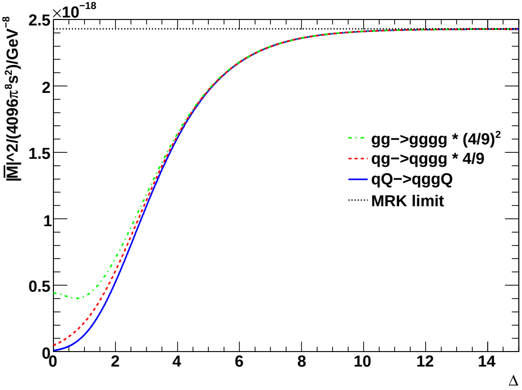


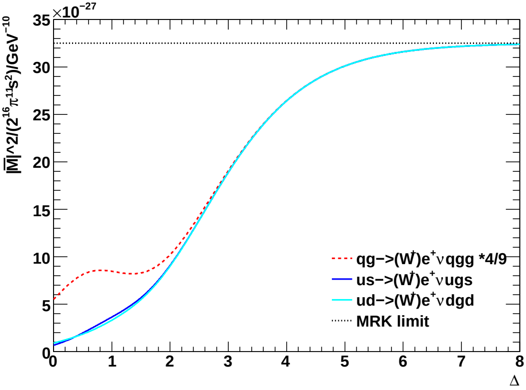
Comparison of 4-jet scattering amplitudes

Study just a slice in phase space:



Comparison of W+3-jet scattering amplitudes

Study just a slice in phase space:



Comparison of W+4-jet scattering amplitudes

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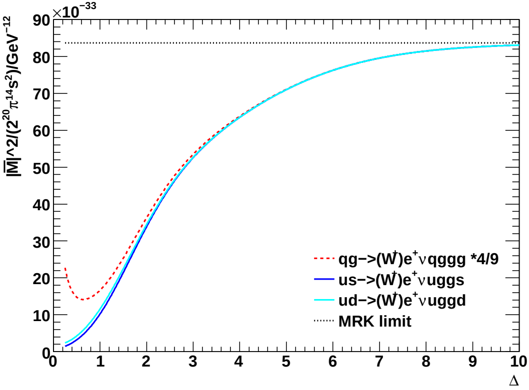
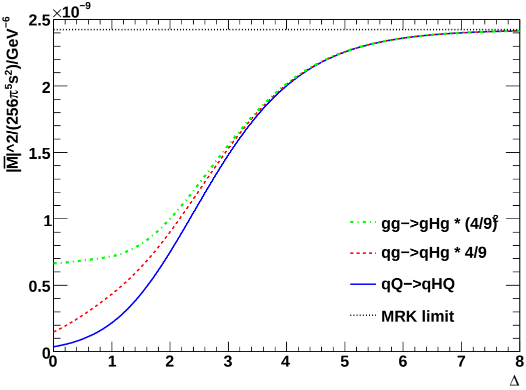


Comparison of H+2-jet scattering amplitudes

Study just a slice in phase space:



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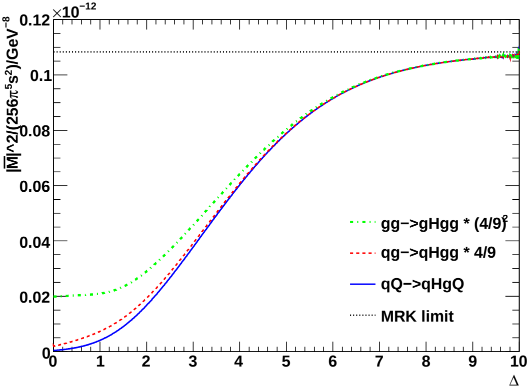


Conclusion from Study of Partonic Cross Sections

- Correct limit is obtained - but outside LHC phase space. Limit alone irrelevant.
- Universality obtained before limit is reached.

Will build frame-work which has the right MRK limit but also retains correct behaviour at smaller rapidities

Scattering of qQ-Helicity States

Start by describing quark scattering. Simple matrix element for $q(a)Q(b) \to q(1)Q(2)$:

$$M_{q^-Q^-\to q^-Q^-} = \langle 1|\mu|a\rangle \frac{g^{\mu\nu}}{t} \langle 2|\nu|b\rangle$$

***t*-channel factorised**: Contraction of (local) currents across *t*-channel pole

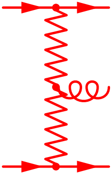
$$\left|\overline{\mathcal{M}}^t_{qQ\to qQ}\right|^2 = \frac{1}{4\,(N_C^2-1)} \left\|S_{qQ\to qQ}\right\|^2 \cdot \left(g^2\, C_F\, \frac{1}{t_1}\right) \cdot \left(g^2\, C_F\, \frac{1}{t_2}\right).$$

Extend to $2 \to n$ ...

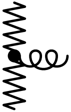
J.M.Smillie and JRA: arXiv:0908.2786

Building Blocks for an Amplitude

Identification of the **dominant contributions** to the **perturbative series** in the limit of well-separated particles

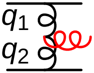


$$\frac{1}{q^2}\exp\left(\hat{\alpha}(q)\Delta y\right)$$

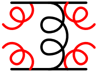


$$V^\mu(q_{i-1}, q_i)$$

$$j^\nu = \overline{\psi}\gamma^\nu\psi$$



$$V^\rho(q_1,q_2) = -(q_1+q_2)^\rho + \frac{p_A^\rho}{2}\left(\frac{q_1^2}{p_2\cdot p_A} + \frac{p_2\cdot p_B}{p_A\cdot p_B} + \frac{p_2\cdot p_n}{p_A\cdot p_n}\right) + p_A \leftrightarrow p_1 - \frac{p_B^\rho}{2}\left(\frac{q_2^2}{p_2\cdot p_B} + \frac{p_2\cdot p_A}{p_B\cdot p_A} + \frac{p_2\cdot p_1}{p_A\cdot p_1}\right) - p_B \leftrightarrow p_3.$$

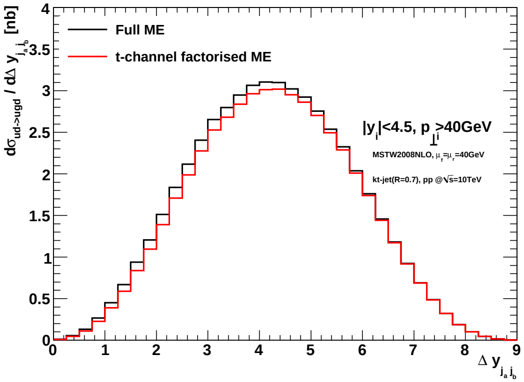


Building Blocks for an Amplitude

$p_g \cdot V = 0$ can easily be checked (gauge invariance)
The approximation for $qQ \to qgQ$ is given by

$$\left|\overline{\mathcal{M}}^t_{qQ\to qgQ}\right|^2 = \frac{1}{4\,(N_C^2-1)}\ \left\|S_{qQ\to qQ}\right\|^2 \cdot \left(g^2\,C_F\,\frac{1}{t_1}\right)\cdot\left(g^2\,C_F\,\frac{1}{t_2}\right) \cdot\left(\frac{-g^2 C_A}{t_1 t_2}\,V^\mu(q_1,q_2)V_\mu(q_1,q_2)\right).$$

3 Jets @ LHC



J.M.Smillie and JRA: arXiv:0908.2786

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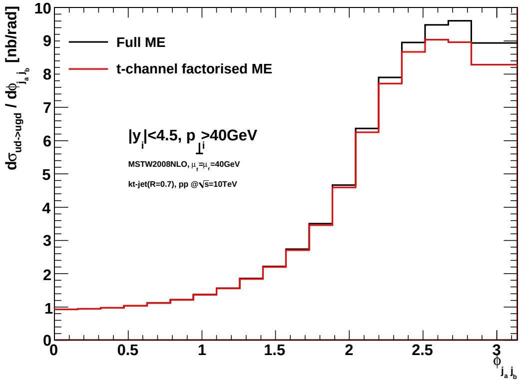

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Building Blocks for an Amplitude

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$$
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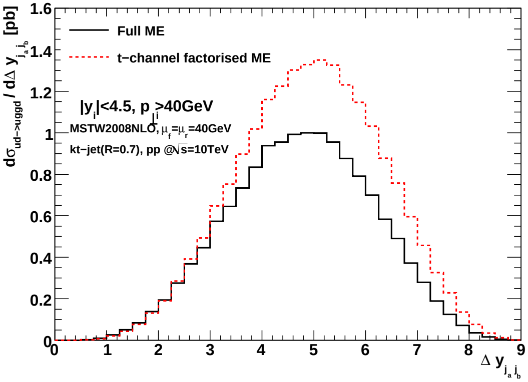
$$

4 Jets @ LHC



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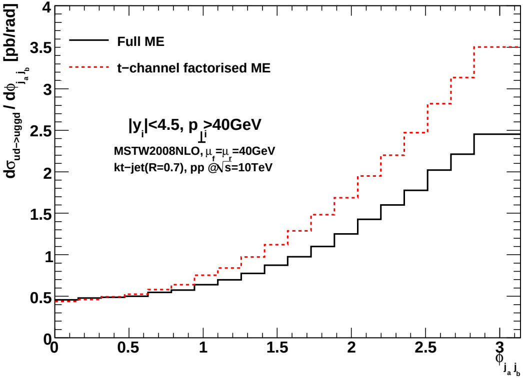

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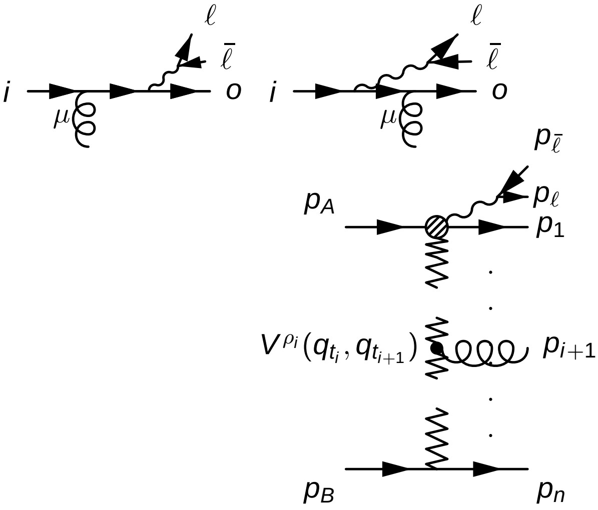
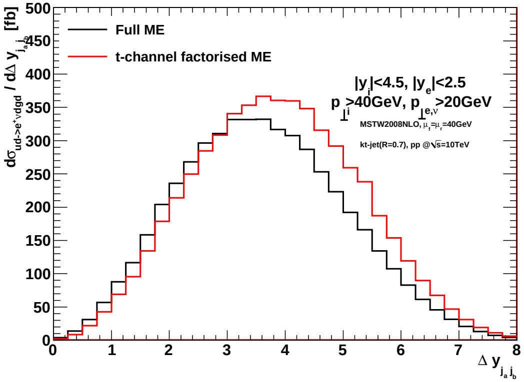
W+Jets

Two currents to calculate for $W + jets$:



W+ 3 Jets @ LHC



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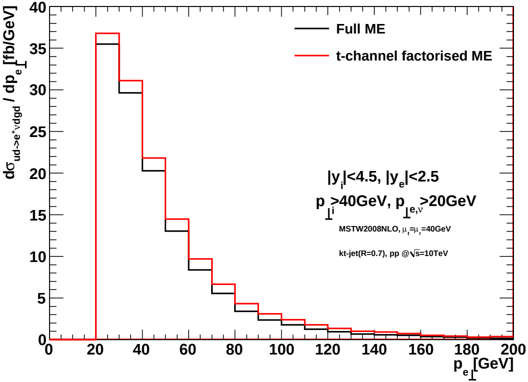

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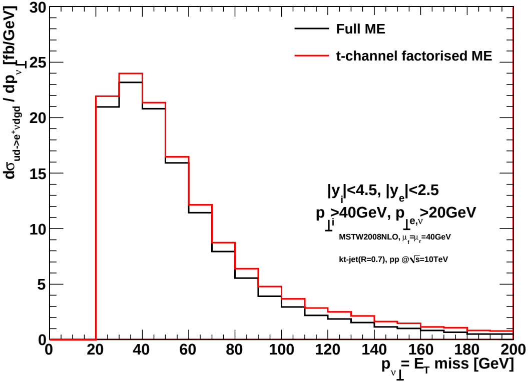

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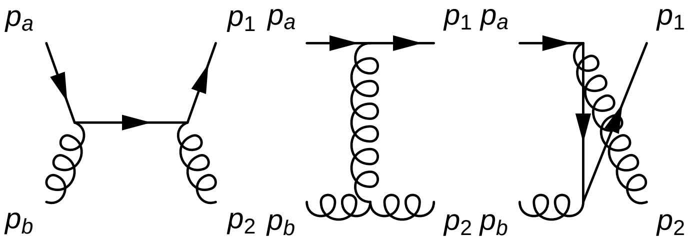
Quark-Gluon Scattering

"What happens in $2 \to 2$-processes with gluons? Surely the *t*-channel factorisation is spoiled!"



Direct calculation ($q^- g^- \to q^- g^-$):

$$M = \frac{g^2}{\hat{t}} \times \frac{p_{2\perp}^*}{|p_{2\perp}|} \left(t^2_{ae} t^b_{e1} \sqrt{\frac{p_b^-}{p_2^-}} - t^b_{ae} t^2_{e1} \sqrt{\frac{p_2^-}{p_b^-}} \right) \langle b|\sigma|2\rangle \times \langle 1|\sigma|a\rangle.$$

Complete *t*-channel factorisation!

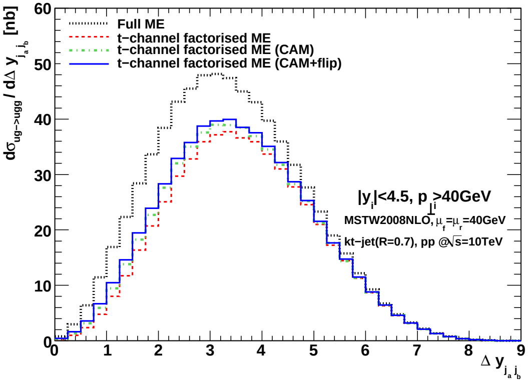
Quark-Gluon Scattering

The t-channel current generated by a helicity non-flipping gluon is that of a quark with a colour factor

$$\frac{1}{2}\left(C_A - \frac{1}{C_A}\right)\left(\frac{p_b^-}{p_2^-} + \frac{p_2^-}{p_b^-}\right) + \frac{1}{C_A}$$

instead of C_F. Tends to C_A in MRK limit.

Quark-Gluon Scattering



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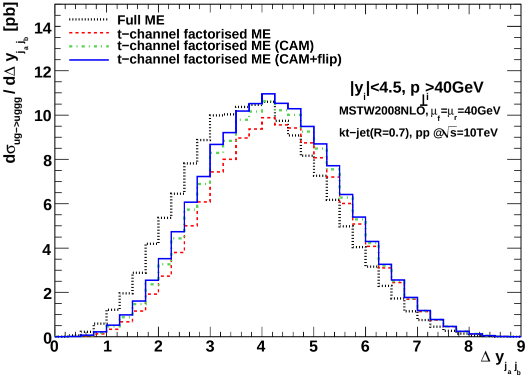


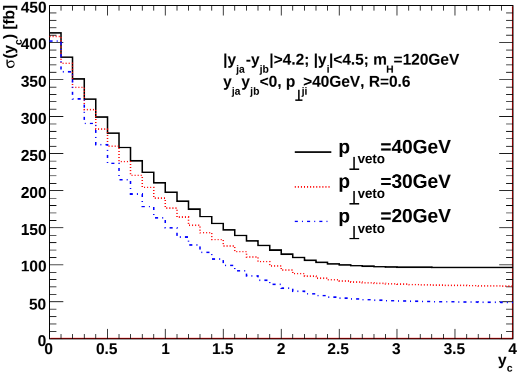
J.M.Smillie and JRA

All-Orders and Regularisation

- Have prescription for $2 \to n$ matrix element, including virtual corrections
- Organisation of cancellation of IR (soft) divergences easy
- Can calculate the sum over the n-particle phase space explicitly ($n \sim 25$) to get the all-order corrections

V. Del Duca, C.D. White, JRA arXiv:0808.3696, J.M. Smillie, JRA arXiv:0908.2786

Effect of Central rapidity jet veto in H+diJets



$$\forall j \in \{\text{jets with } p_{j\perp} > p_{\perp,\text{veto}}\} \setminus \{a,b\} \;:\; \left| y_j - \frac{y_a + y_b}{2} \right| > y_c$$

Outlook and Conclusions

Conclusions

- Emerging framework for the study of processes with multiple hard jets
- For each number of particles n, the approximation to the matrix element (real and virtual) is sufficiently simple to allow for the all-order summation to be constructed as an explicit sum over n-particle final states (exclusive studies possible)
- Resummation based on approximation which really does capture the behaviour of the scattering processes at the LHC
- Matching will correct the approximation where the full matrix element can be evaluated