Global Fits to Thrust at NNNLL Order with Power Corrections

André H. Hoang

Max-Planck-Institute for Physics
Werner-Heisenberg-Institut
Munich

R. Abbate, M. Fickinger, V. Mateu, I.Stewart, AHH to appear

RADCOR 2009, Ascona, Oct 25 - 30, 2009

Outline

- Motivation & World average for $\alpha_s(M_z)$
- Event shapes - Thrust
- Previous work
- SCET factorization formula
- Numerical results, pictures
- New precise measurement of $\alpha_s(M_z)$

Our result: (preliminary)

$$\alpha_s(M_z) = 0.1135 \pm 0.0011 \pm 0.0006$$

Motivation

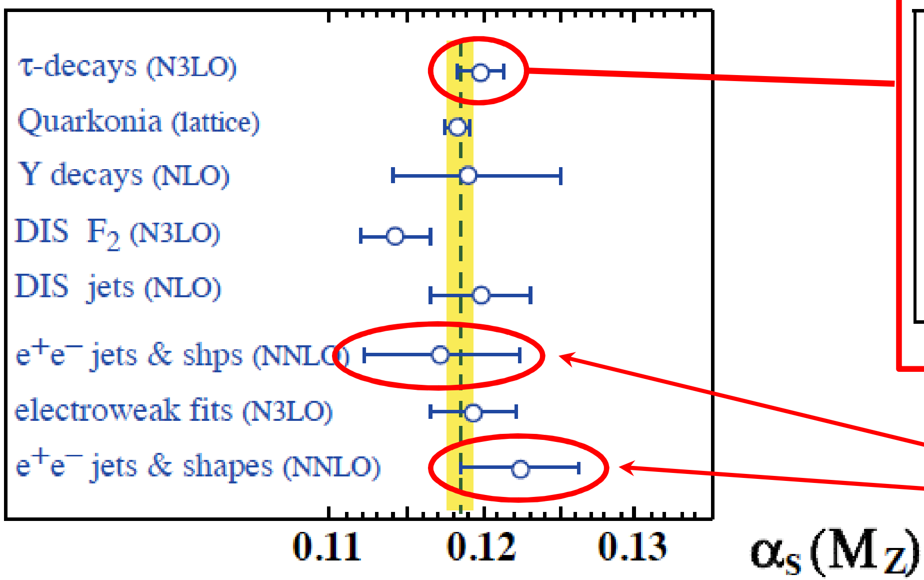
Strong Coupling: $\alpha_s(M_z)$

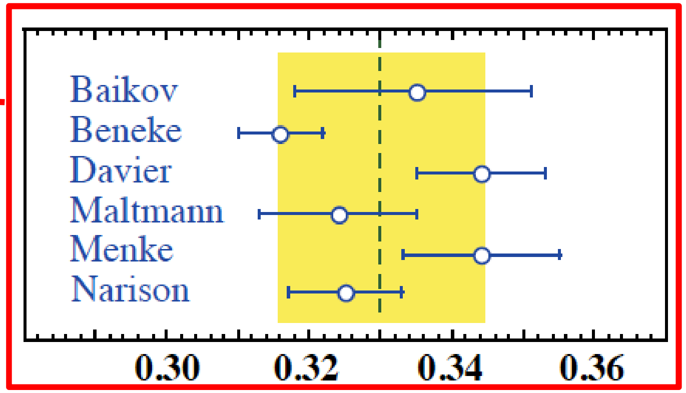
- key parameter in the SM and enters the analysis of all collider data (Tevatron, LHC, ILC)
- Important role in new physics searches (e.g. ew. precision observables, $B \to X_s \gamma$, gauge coupling unification)

Motivation

Strong Coupling: $\alpha_s(M_z)$

- recent world average Bethke, 0908.1135





event shape results at fixed order

→ Ever decreasing error from averaging, BUT – after 35 years of work – still the issue is far from being settled.

$$\alpha(M_z) = 0.1184 \pm 0.0007$$

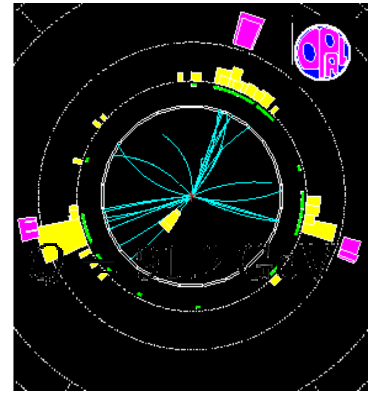
Event Shapes

→ Classic method for determining $\alpha_s(M_z)$

Single-variable jet distributions

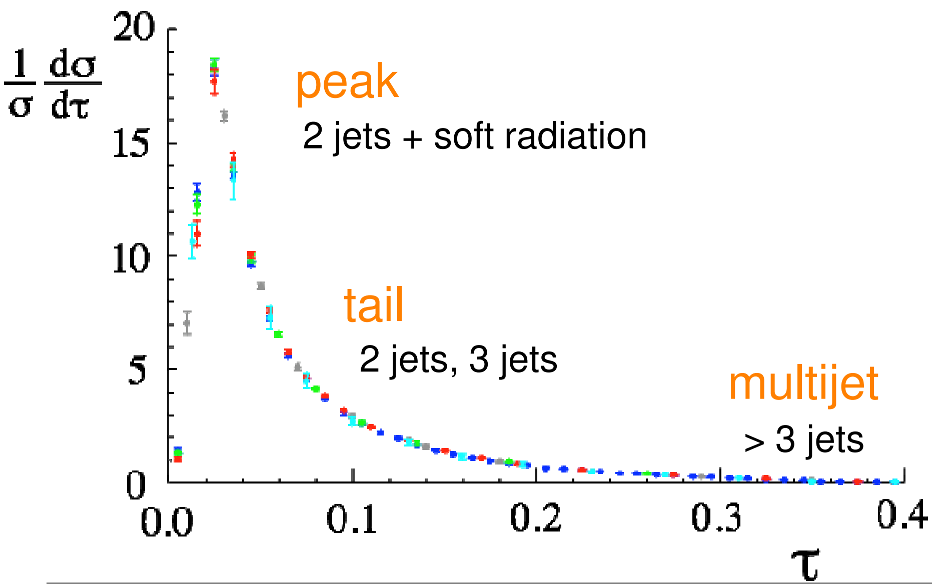
e.g. Thrust

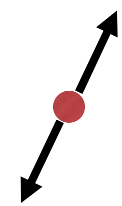
$$T = \max_{\hat{t}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|} \qquad \tau = 1 - T$$



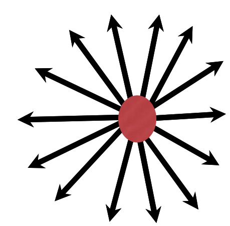
OPAL 3 jet event

ALEPH, DELPHI, L3, OPAL, SLD





$\tau = 0$



$\tau = 0.5$

Recent Developments

Theoretical Advances: (all LO in 1/Q)

1) Fixed order NNLO (3 jets final state) — Gehrmann, Gehrmann-De Ridder, Glover, Heinrich; Weinzierl
2) Fixed order NNNLO (2 jets state)
3) Proof SCET factorization theorem, massive quarks (thrust, jet masses) — Fleming, Mantry, Stewart, AHH
4) NNNLL (SCET) summation of large logs (massless thrust) — Becher, Schwartz
5) Field theory treatment of power corrections, relation to moments of non-perturbative soft function — Lee, Sterman
6) Non-perturbative soft function implementation (SCET) without $\mathcal{O}(\Lambda_{\rm QCD})$ renormalon in first power correction — Stewart, AHH; Ligeti, Stewart, Tackmann

Recent Developments

SCET Order Counting:

LL NLL NNLL NNNLL

$$\ln \frac{d\sigma}{dy} = (\alpha_s \ln)^k \ln + (\alpha_s \ln)^k + \alpha_s (\alpha_s \ln)^k + \alpha_s^2 (\alpha_s \ln)^k + \dots$$

		cusp	non-cusp	matching	alphas	Classic Counting
standard counting (emphasizes summation)	LL	1	–	tree	1	LLA
	NLL	2	1	tree	2	NLLA
	NNLL	3	2	1	3	NNLLA + LLO
	$\mathrm{N}^3\mathrm{LL}$	4^{pade}	3	2	4	NNNLLA + NLO
primed counting (emphasizes fixed order)	LL′	1	–	tree	1	LLA
	NLL′	2	1	1	2	NLLA + LLO
	NNLL′	3	2	2	3	NNLLA + NLO
	$\mathrm{N}^3\mathrm{LL}'$	4^{pade}	3	3	4	NNNLLA + NNLO

Theory error from Padé estimate of $\Gamma_3^{\rm cusp}$

Recent Developments

Recent Analyses:

Dissertori, Gehrmann, Gehrmann-De Ridder, Glover, Heinrich, Luisoni Stenzel

- NNLO[+NLLA] (for $\tau, \rho, C, B_T, B_W, y_{23}$; ALEPH tail data)
  - with m_b and QED corrections (fixed-order)
  - error band method for theory uncertainties
  - NNLO with smaller scale-dependence than NLLA+NNLO
  - hadronization corrections from MC's

$$\alpha_s(M_z) = 0.1224 \pm 0.0013 \pm 0.0011 \pm 0.0028$$

Becher, Schwartz

- NNNLL [SCET] (for τ, ALEPH+OPAL tail data)
  - massless quarks
  - error band method for theory uncertainties
  - hadronization corrections from MC's

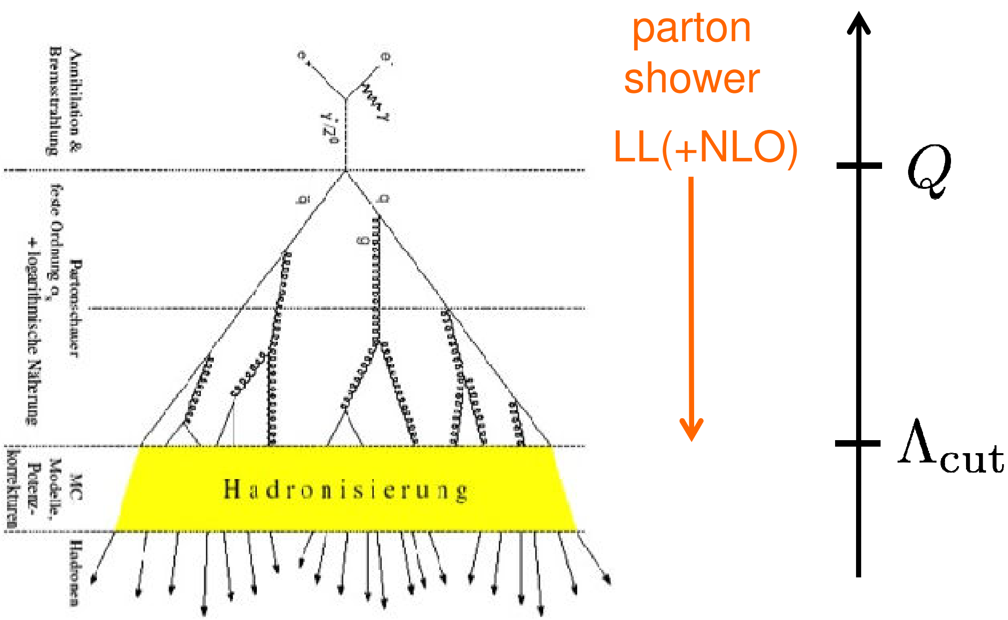
$$\alpha_s(M_z) = 0.1172 \pm 0.0013 \pm 0.0012 \pm 0.0012$$

Davison, Webber

- NNLO+NLLA (for τ, all tail data)
  - massless quarks
  - hadronization correction model with α_0
  - leading $\mathcal{O}(\Lambda_{\rm QCD})$ renormalon subtracted
  - simultaneous fit for α_0 and $\alpha_s(M_z)$

$$\alpha_s(M_z) = 0.1164 \pm 0.0022 \pm 0.0017$$

Hadronization Corrections from QCD Monte Carlos



Monte Carlo QCD:

Partonic MC results are in some (yet unspecified) scheme with an IR cutoff $\Lambda_{\rm cut} = 1\ {\rm GeV}$

→ free of IR renormalons

Analytic (multiloop) QCD:

Dim. reg. used to regularize IR momentum contributions

→ IR renormalons

→ Hadronization corrections in MC's cannot be used to estimate nonperturbative corrections for multiloop results based on dim. reg.

→ All analyses using hadronization corrections from MC's essentially fit the perturbative multiloop results to the LL(+NLO) partonic MC predictions.

Improvements over earlier work

Abbate, Fickinger, Mateu, Stewart, AHH

1) Full treatment of non-perturbative effects from field theory

   (treatment of errors from power corrections in the tail region)

2) Stable interface between perturbative an non-perturbative effects

   ($\mathcal{O}(\Lambda_{\rm QCD})$ renormalon subtraction at NNNLL)

3) Simultaneous description of peak, tail and multijet regions

   (eventually analyze ALL data)

4) Account for factorization theorm for subleading order (SCET)

5) Consistent treatment of quark mass in SCET-QCD

6) Treatment of QED effects in SCET-QCD-QED

This talk: Thrust & tail fits

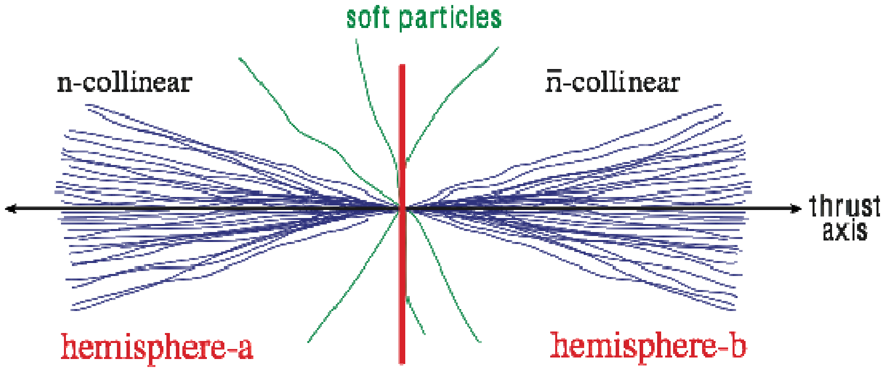
Technology and all input available for peak fits to also treat ρ and C-parameter at the same order.

Basics

e.g. $e^+e^- \to Z \to 2\,\mathrm{jets} + X_{\mathrm{soft}}$

$\to$ consider $T \approx 1$ $(\tau \ll 1)$

$m_Z^2 \gg M_{\mathrm{jet}}^2 \gg E_{\mathrm{soft}}^2$



$\mu_Q \simeq m_Z = 91.2\,\mathrm{GeV}$ "hard"

$\mu_J \simeq M_{\mathrm{jet}} \simeq 20\,\mathrm{GeV}$ "collinear"

$\mu_S \simeq E_{\mathrm{soft}} \simeq 5\,\mathrm{GeV}$ or smaller, down to Λ_{QCD} "soft"

→ 3 (or 4) distinct scales

EFT playground (SCET)

Basics

singular

$$\left(\frac{d\sigma}{d\tau}\right)^{\rm part} \sim \sum \alpha_s^n\,\delta(\tau) + \sum \alpha_s^n \frac{\ln^m(\tau)}{\tau}$$

- factorization formula
- log summation
- non-perturbative effects (soft fct. S)

non-singular

$$+ \sum \alpha_s^n\, \ln^m(\tau)$$
$$+ \sum \alpha_s^n\, \tau\, \ln^m(\tau)$$
$$+ \ldots$$

- use fixed-order
- non-perturbative effects (soft fct. S)

Lee, Stewart

$$\left(\frac{d\sigma}{d\tau}\right) = \int d\ell \left[\left(\frac{d\sigma}{d\tau}\right)^{\rm sing}_{\rm part}\left(\tau - \frac{\ell}{Q}\right) + \left(\frac{d\sigma}{d\tau}\right)^{\rm nonsing}_{\rm part}\left(\tau - \frac{\ell}{Q}\right)\right] S^{\rm mod}(\ell)$$

misses power corrections $\frac{\Lambda_{\rm QCD}}{Q}$ ➡ $\frac{\delta\alpha_s}{\alpha_s} \sim \frac{\Lambda_{\rm QCD}}{Q} = 0.5\%$

Partonic Cross Section $\left(\frac{d\sigma}{d\tau}\right)^{\rm sing}_{\rm part}$ □

Korchemsky, Sterman; Bauer etal.
Fleming, Mantry, Stewart, AHH
Schwartz

Factorization Formula

$$\left(\frac{d\sigma}{d\tau}\right)^{\rm sing}_{\rm part} \sim \sigma_0\, H(Q,\mu_Q) U_H(Q,\mu_Q,\mu_s) \int d\ell d\ell'\, U_J(Q\tau-\ell-\ell',\mu_Q,\mu_s)\, J_T(Q\ell',\mu_j)\, S_T(\ell-\Delta,\mu_s)$$

Partonic Cross Section $\left(\frac{d\sigma}{d\tau}\right)^{\rm sing}_{\rm part}$

Factorization Formula

$$\left(\frac{d\sigma}{d\tau}\right)^{\rm sing}_{\rm part} \sim \sigma_0\, H(Q,\mu_Q) U_H(Q,\mu_Q,\mu_s) \int d\ell d\ell'\, U_J(Q\tau - \ell - \ell', \mu_Q, \mu_s)\, J_T(Q\ell', \mu_j)\, S_T(\ell - \Delta, \mu_s)$$

Hard function

- Matching coefficient known at $\mathcal{O}(\alpha_s^2)$
- non-cusp anomalous dimension $\mathcal{O}(\alpha_s^3)$
- cusp anomalous dimension $\mathcal{O}(\alpha_s^3)$

Moch, Vermaseren, Vogt
Gehrmann, Huber, Maitre

Moch, Vermaseren, Vogt

Partonic Cross Section $\left(\frac{d\sigma}{d\tau}\right)^{\rm sing}_{\rm part}$

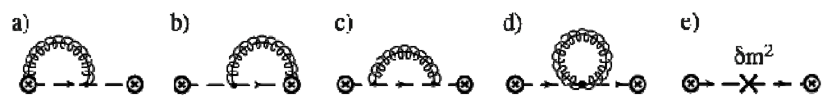
Factorization Formula

$$\left(\frac{d\sigma}{d\tau}\right)^{\rm sing}_{\rm part} \sim \sigma_0\, H(Q,\mu_Q) U_H(Q,\mu_Q,\mu_s) \int d\ell d\ell'\, U_J(Q\tau-\ell-\ell',\mu_Q,\mu_s)\, J_T(Q\ell',\mu_j)\, S_T(\ell-\Delta,\mu_s)$$

Jet function

- Matrix element known at $\mathcal{O}(\alpha_s^2)$ — Becher, Neubert
- non-cusp anomalous dimension $\mathcal{O}(\alpha_s^3)$ — Moch, Vermaseren, Vogt

$$J_n(Qr_n^+,\mu) = \frac{-1}{8\pi N_c Q}\,\mathrm{Disc}\int d^4x\, e^{ir_n\cdot x}\,\langle 0|\mathrm{T}\,\bar\chi_{n,Q}(0)\,\bar{n}\!\!\!/\,\chi_n(x)|0\rangle$$



$$\mu\frac{d}{d\mu}J(y,\mu) = \gamma_J(y,\mu)\,J(y,\mu) = \Big[2\Gamma^{\rm cusp}(\alpha_s)\ln(iy\mu^2 e^{\gamma_E}) + \gamma_J(\alpha_s)\Big]J(y,\mu)$$

Partonic Cross Section $\left(\frac{d\sigma}{d\tau}\right)^{\rm sing}_{\rm part}$ □

Factorization Formula

$$\left(\frac{d\sigma}{d\tau}\right)^{\rm sing}_{\rm part} \sim \sigma_0\, H(Q,\mu_Q) U_H(Q,\mu_Q,\mu_s) \int d\ell d\ell'\, U_J(Q\tau-\ell-\ell',\mu_Q,\mu_s)\, J_T(Q\ell',\mu_j)\, S_T(\ell-\Delta,\mu_s)$$

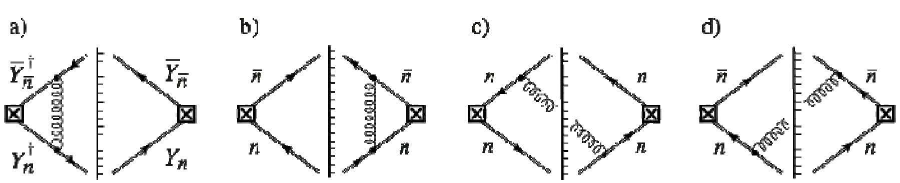
Soft function

- analytically known at $\mathcal{O}(\alpha_s)$
- numerically known at $\mathcal{O}(\alpha_s^2)$

Schwartz
Fleming, Mantry, Stewart, AHH

Becher, Schwartz
Kluth, AHH

$$S_{\rm hemi}(\ell^+,\ell^-,\mu) = \frac{1}{N_c}\sum_{X_s} \delta(\ell^+ - k_s^{+a})\delta(\ell^- - k_s^{-b})\langle 0|\overline{Y}_{\bar n}\, Y_n(0)|X_s\rangle\langle X_s|Y_n^\dagger\, \overline{Y}_{\bar n}^\dagger(0)|0\rangle$$



Partonic Cross Section $\left(\frac{d\sigma}{d\tau}\right)^{\rm sing}_{\rm part}$ □

Factorization Formula

$$\left(\frac{d\sigma}{d\tau}\right)^{\rm sing}_{\rm part} \sim \sigma_0\, H(Q,\mu_Q) U_H(Q,\mu_Q,\mu_s) \int d\ell d\ell'\, U_J(Q\tau - \ell - \ell', \mu_Q, \mu_s)\, J_T(Q\ell', \mu_j)\, S_T(\ell - \Delta, \mu_s)$$

Gap subtraction

Stewart, AHH

- $\mathcal{O}(\Lambda_{\rm QCD})$ renormalon in the threshold of the $\overline{\rm MS}$ partonic soft function
- gap parameter Δ introduced to subtract the renormalon
- allows to define renormalon-free first moment Ω_1 of the soft function

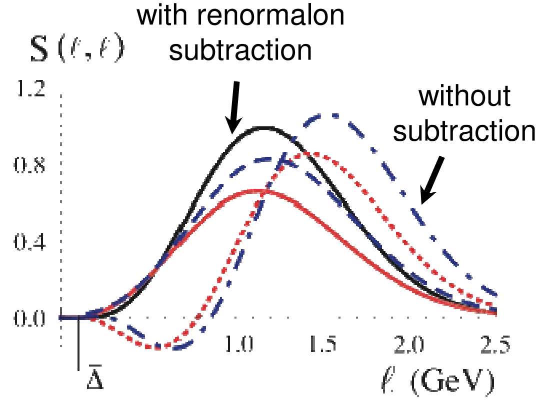
$$\Delta = \bar{\Delta}(R) + R\,[a_0\,\alpha_s(R) + a_1\,\alpha_s^2(R) + \ldots]$$

renormalon-free evolves with R-RGE

subtraction

Kluth, AHH

$\longrightarrow$ talk by I. Scimemi



Partonic Cross Section $\left(\frac{d\sigma}{d\tau}\right)^{\rm sing}_{\rm part}$ □

Factorization Formula

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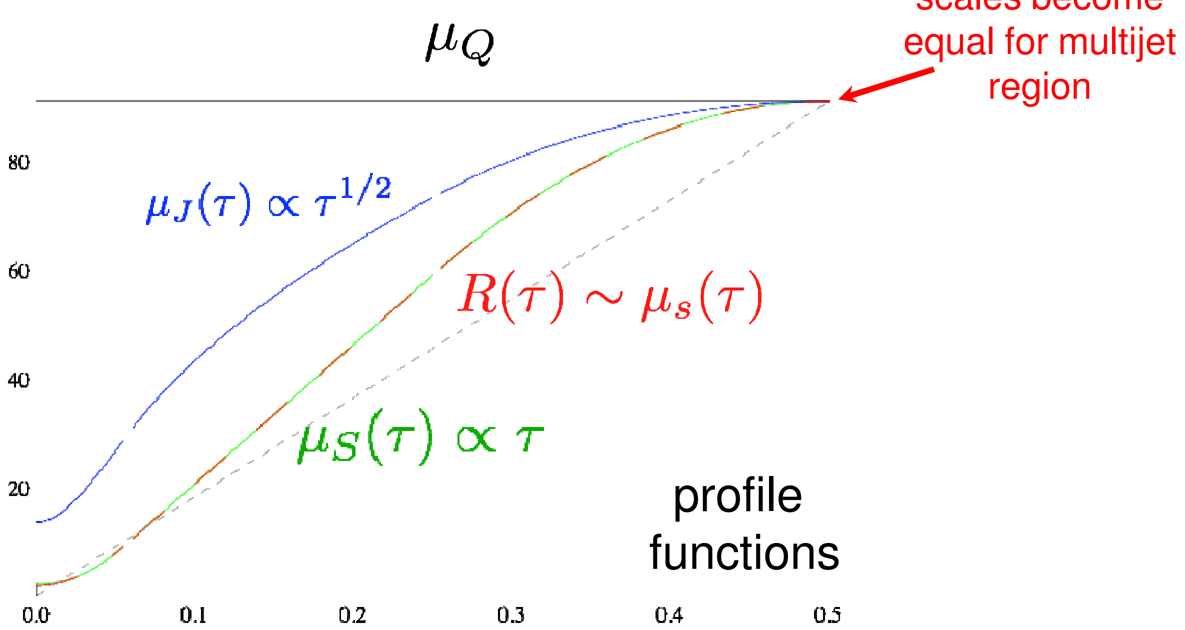
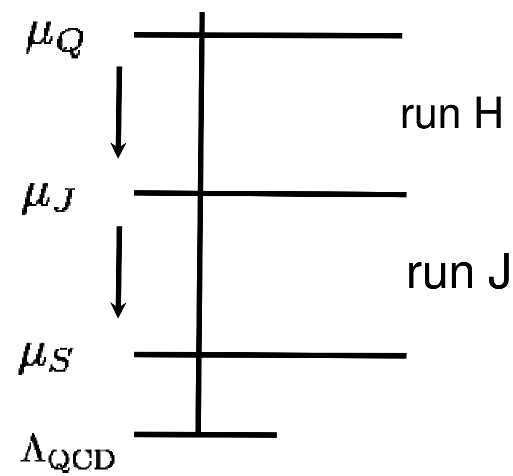
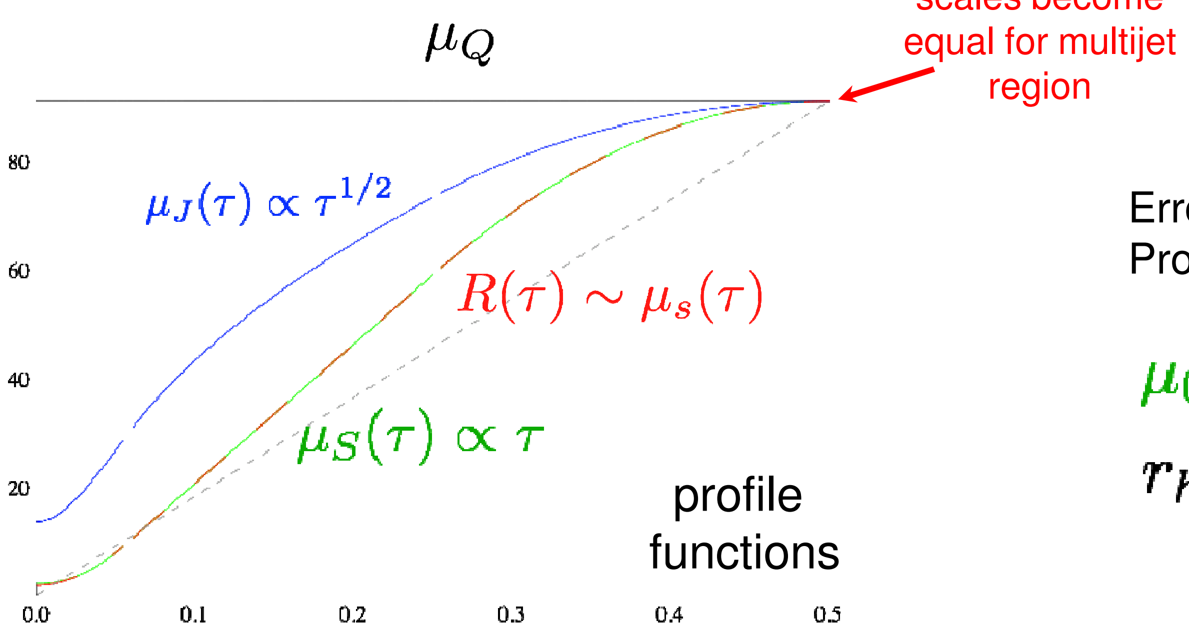




Partonic Cross Section $\left(\frac{d\sigma}{d\tau}\right)^{\text{sing}}_{\text{part}}$ □

Factorization Formula

$$\left(\frac{d\sigma}{d\tau}\right)^{\text{sing}}_{\text{part}} \sim \sigma_0\, H(Q,\mu_Q) U_H(Q,\mu_Q,\mu_s) \int d\ell d\ell'\, U_J(Q\tau-\ell-\ell',\mu_Q,\mu_s)\, J_T(Q\ell',\mu_j)\, S_T(\ell-\Delta,\mu_s)$$



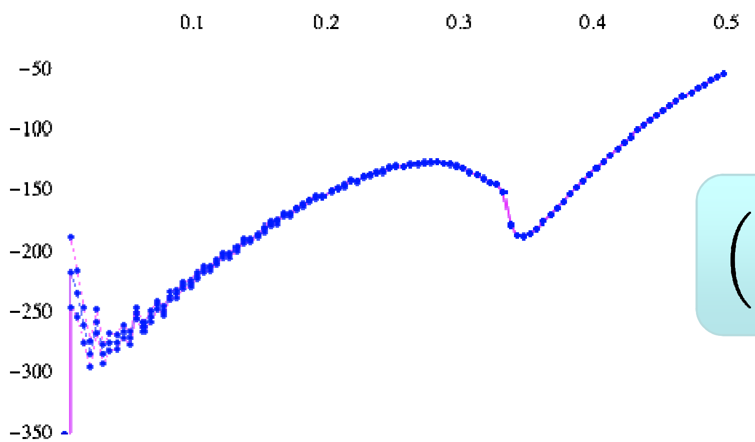
Errors from varying Profile Parameters:

$\mu_0 \quad n_1 \quad \tau_2 \quad \epsilon_J$

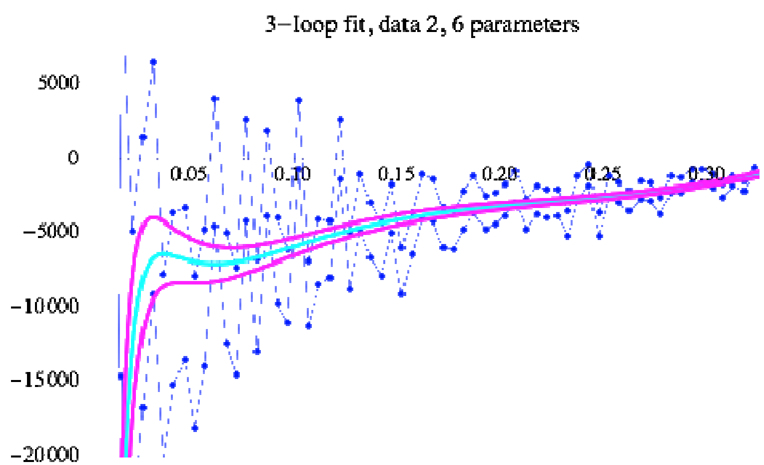
$r_h = \mu_h/Q \quad n_s$

Partonic Cross Section $\left(\frac{d\sigma}{d\tau}\right)^{\rm nonsing}_{\rm part}$

$\mathcal{O}(\alpha_s^2)$



$\mathcal{O}(\alpha_s^3)$



$$\left(\frac{d\sigma}{d\tau}\right)^{\rm nonsing}_{\rm part} = \left(\frac{d\sigma}{d\tau}\right)^{\rm full}_{\rm part} - \left(\frac{d\sigma}{d\tau}\right)^{\rm sing}_{\rm part}$$

$\mathcal{O}(\alpha_s^2)$: EVENT-2

$\mathcal{O}(\alpha_s^3)$: EERAD3

Errors from statistical uncertainties and scale setting:

$\epsilon_2,\ \epsilon_3$ $\qquad n_s$

Soft Function Model

→ Expand non-perturbative soft function in terms of a complete set of basis functions to reduce bias in the soft function parametrization.

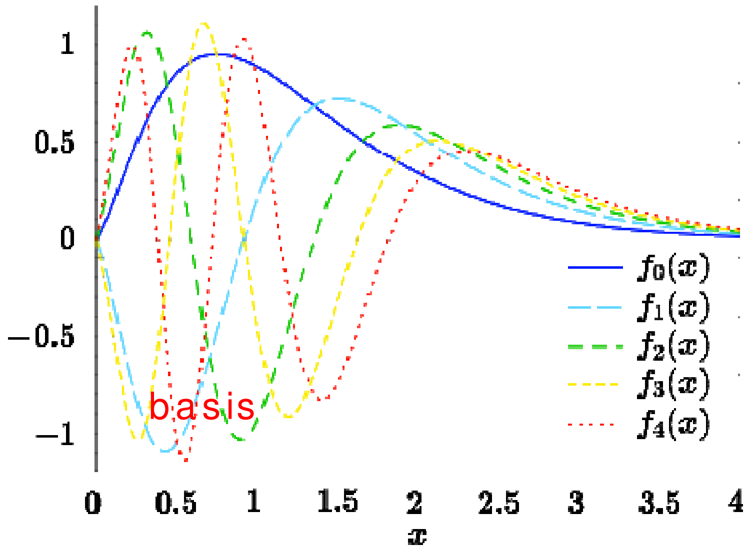
$$F(\ell) = \frac{1}{\lambda} \sum_{n=0}^{\infty} c_n \, f_n(\ell/\lambda)$$

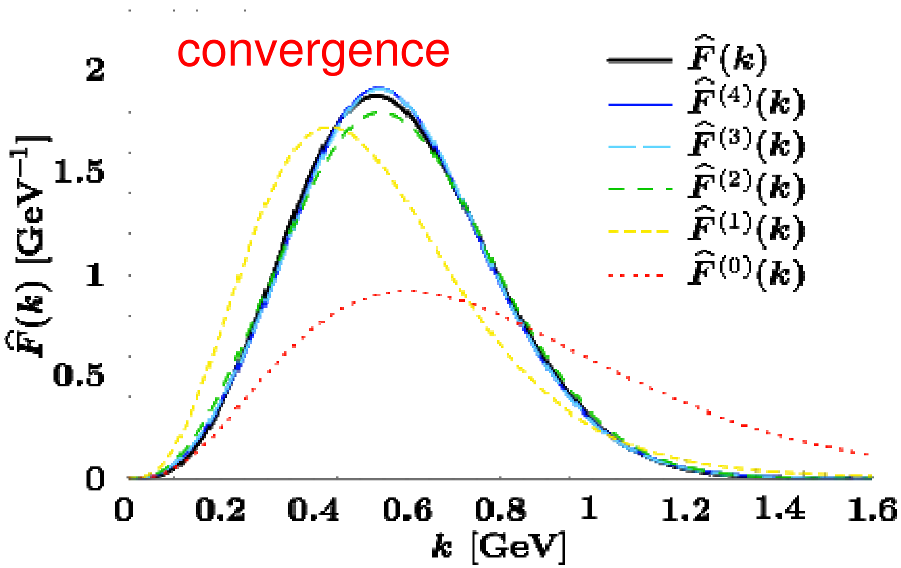
$$\Omega_1 = \int d\ell \, \ell F(\ell)$$

λ: width of the soft function

$c_0, c_1, \ldots$: expansion coeffients

$\rightarrow c_0$ sufficient for tail fits





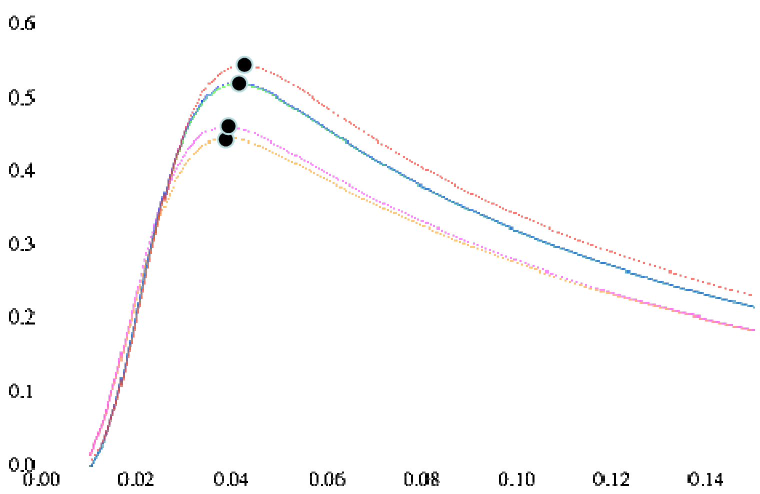
First Look at the Results

Impact of gap subtraction:

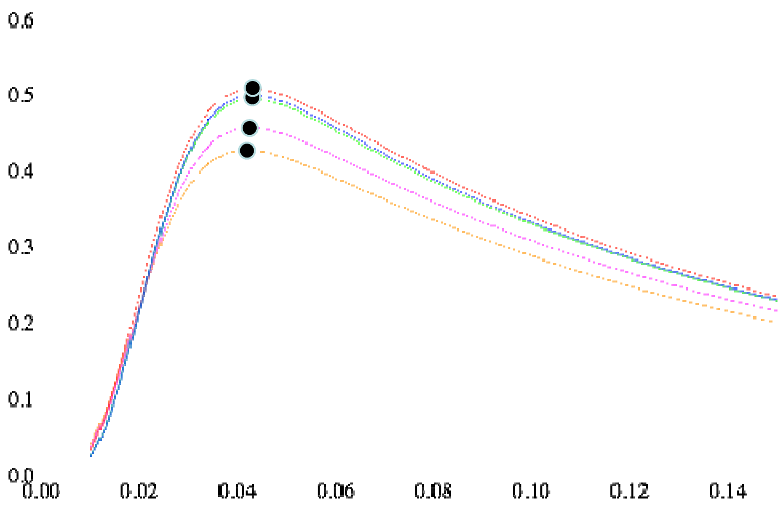
$\mathrm{NLL'}$ NNLL $\mathrm{NNLL'}$ $\mathrm{N^3LL}$ $\mathrm{N^3LL'}$

$$\frac{1}{\sigma}\tau\frac{d\sigma}{d\tau}$$

soft function without gap



with gap subtraction



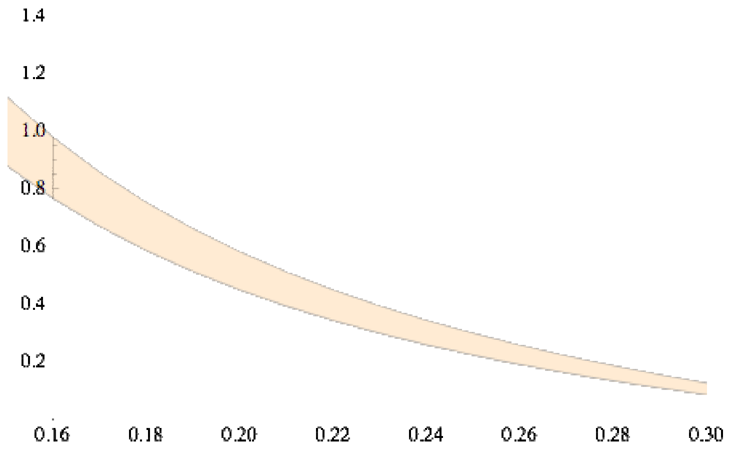
First Look at the Results

Impact of gap subtraction:

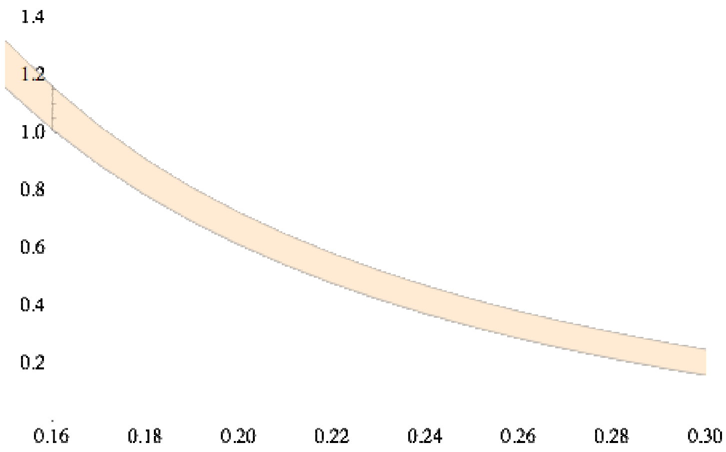
NLL′

$\frac{1}{\sigma}\tau\frac{d\sigma}{d\tau}$

summing logs only



with soft function and gap subtraction



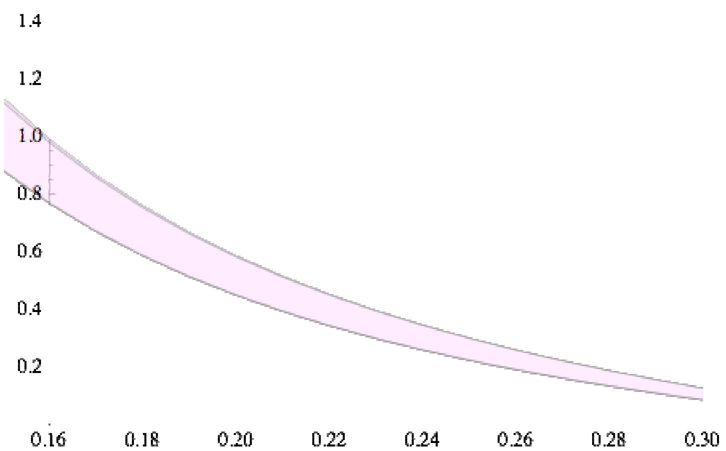
First Look at the Results

Impact of gap subtraction:

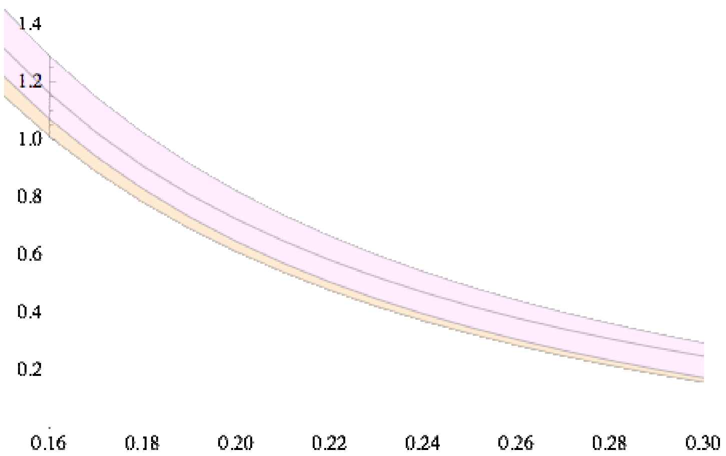
NLL′ NNLL

$$\frac{1}{\sigma}\tau\frac{d\sigma}{d\tau}$$

summing logs only



with soft function and gap subtraction



First Look at the Results

Impact of gap subtraction:

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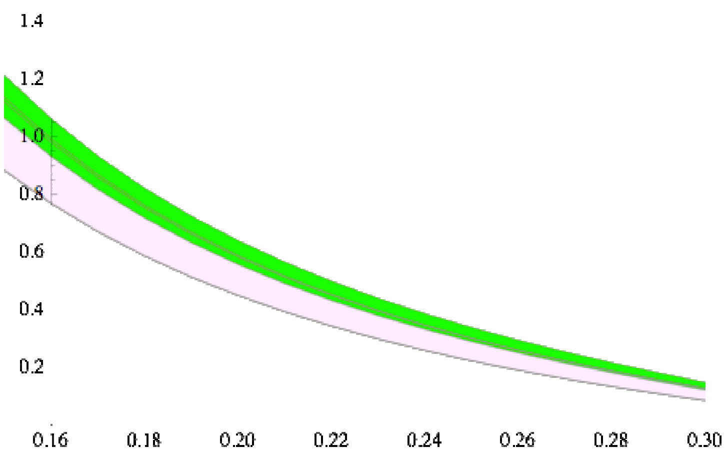
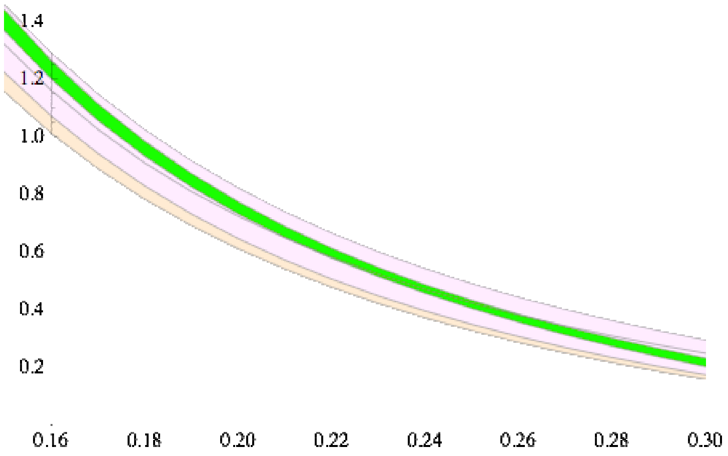


with soft function and gap subtraction



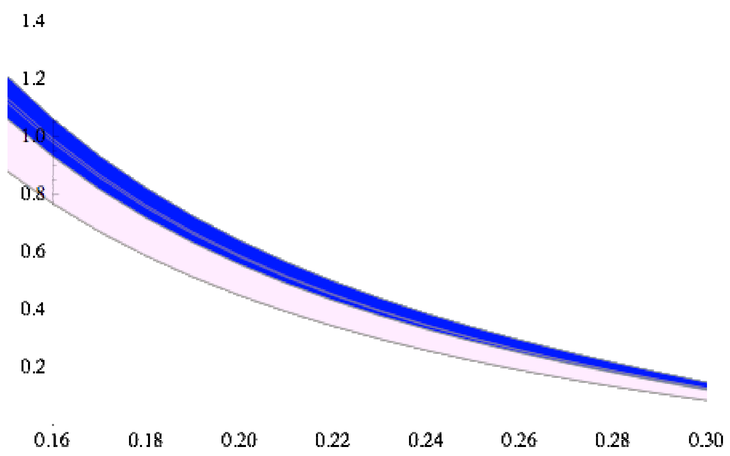
First Look at the Results

Impact of gap subtraction:

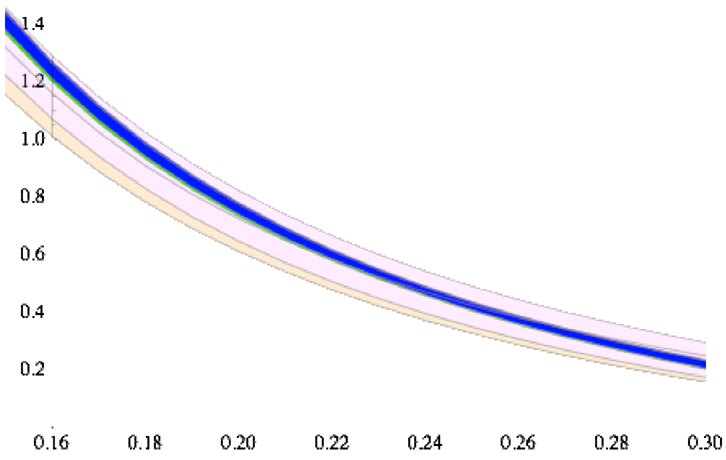
NLL' NNLL NNLL' N^3LL

$$\frac{1}{\sigma}\tau\frac{d\sigma}{d\tau}$$

summing logs only



with soft function and gap subtraction



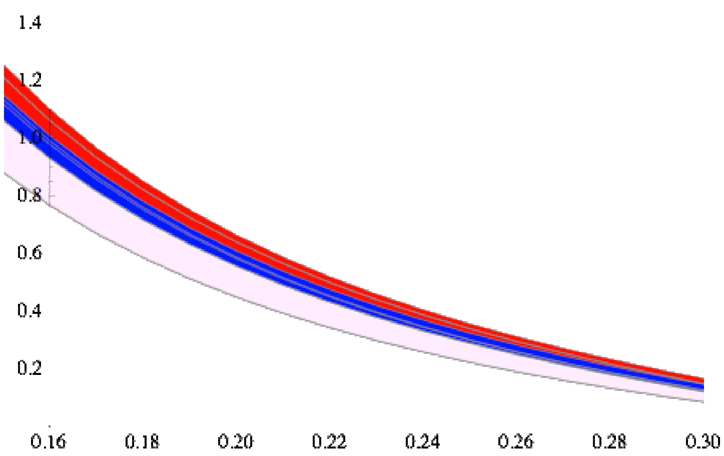
First Look at the Results

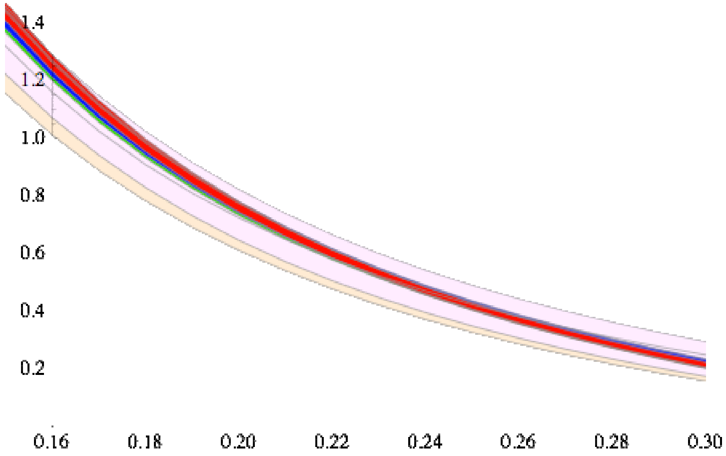
Impact of gap subtraction:

NLL′ NNLL NNLL′ N^3LL N^3LL'

$$\frac{1}{\sigma}\tau\frac{d\sigma}{d\tau}$$

summing logs only | **with soft function and gap subtraction**





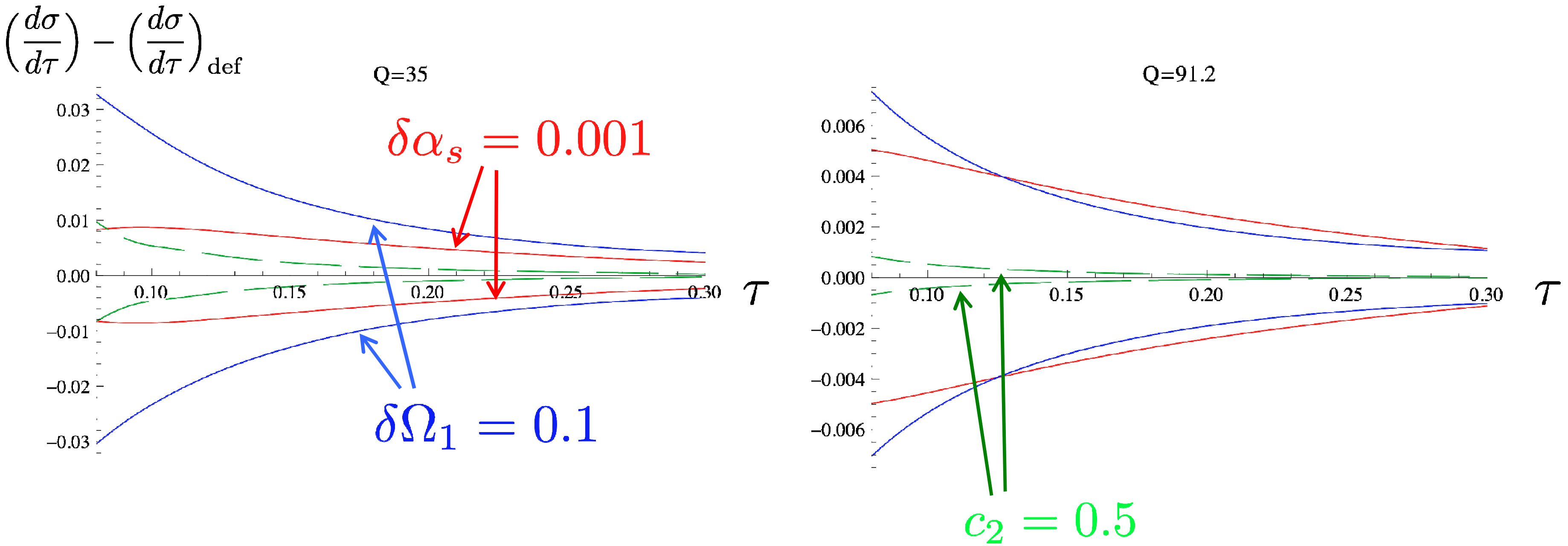
Degeneracy: α_s vs Ω_1

Why global fits for all Q values are needed:

→ Degeneracy lifted by fitting to data from multiple Q values simultaneously

default cross section

$$\alpha_s(M_z) = 0.114$$
$$\Omega_1 = 0.67$$
$$c_1 = 1$$



Our Fit Procedure: $d\sigma/d\tau$ bins

- χ^2-fit accounts for
  - statistical experimental errors
  - systematic experimental errors (uncorrelated & minimal overlap within each data set)
  - all data $Q > 35\ \mathrm{GeV}$ included in a global fit
- Theory errors
  - 500 different χ^2-fits with random scan over 13-dimensional theory parameter space (flat probability over each parameter range)
  - around 120 such fits carried out
- Downward fluctuations for $Q > M_z$ data
  - 28 data bins with “too” low values or unnaturally small experimental errors identified and removed from the fit (mostly located at $\tau > 0.33$)

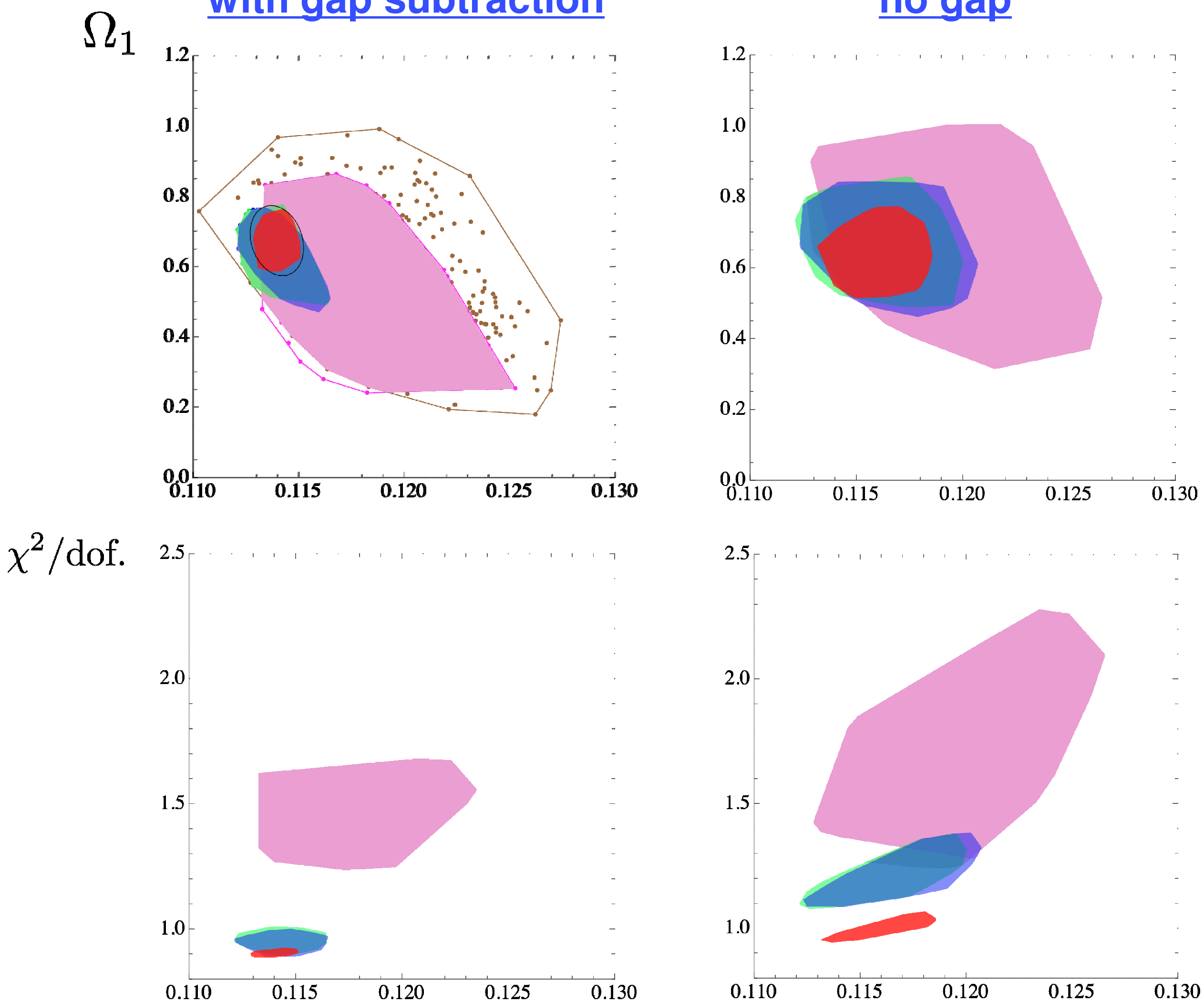
scale variation: μ_0, n_1, τ_2, ϵ_J, $r_h = \mu_h/Q$, n_s

2-loop: s_1, ϵ_2

3-loop: $\Gamma_3^{\rm cusp}$, H_3, J_3, S_3, ϵ_3

Fits to $d\sigma/d\tau$ bins: theory errors

with gap subtraction   no gap



$\alpha_s(M_z)$

"standard" data set:

$Q \geq 35\ \text{GeV}$

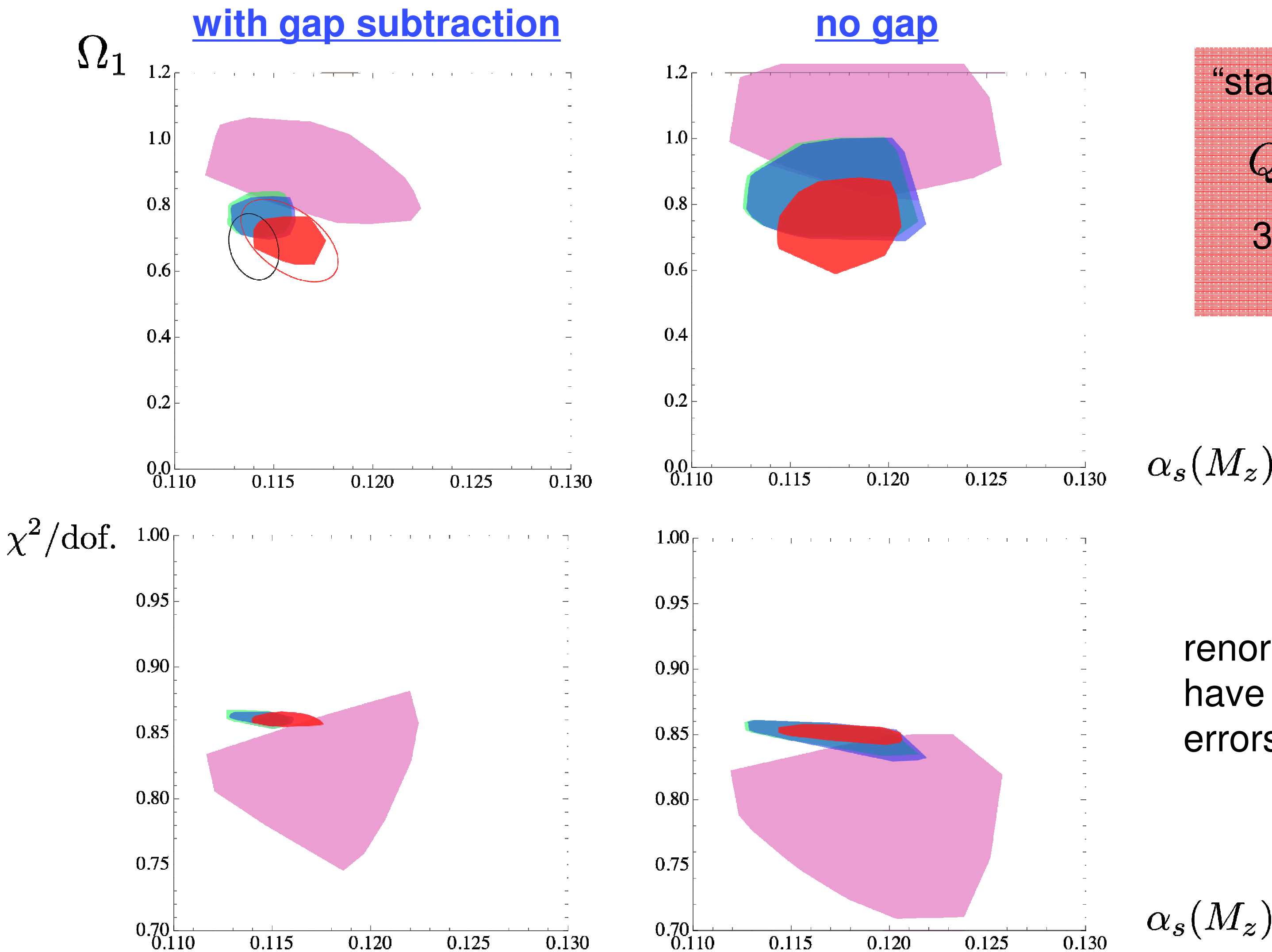
$\frac{6}{Q} \leq \tau \leq 0.33$

488 bins

$\alpha_s(M_z)$

renormalon-free results have smaller theory errors and better fits

Fits to $M_1 = \int d\tau\, \tau \frac{d\sigma}{d\tau}$: theory errors



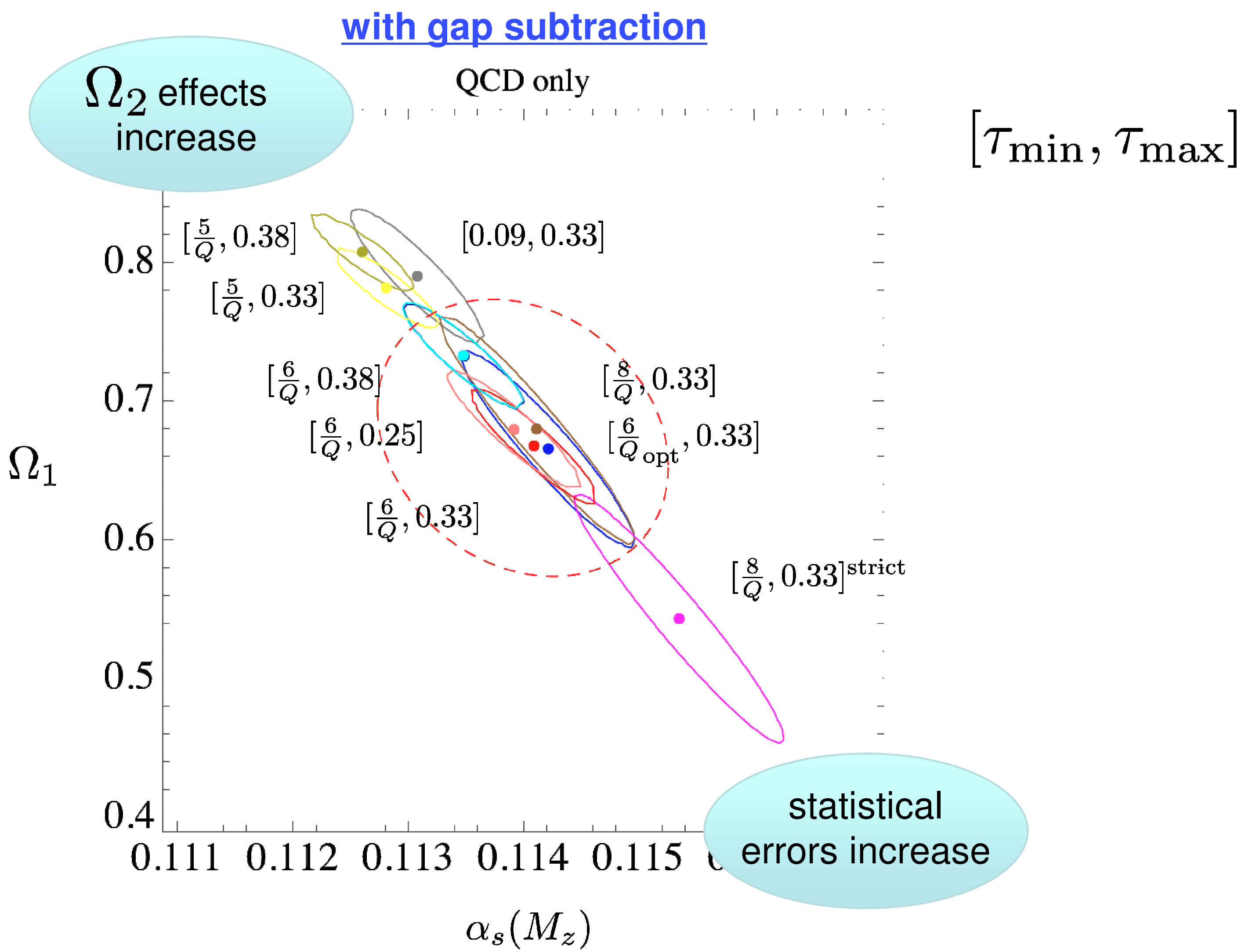
"standard" data set:

$Q \geq 35$ GeV

35 data points

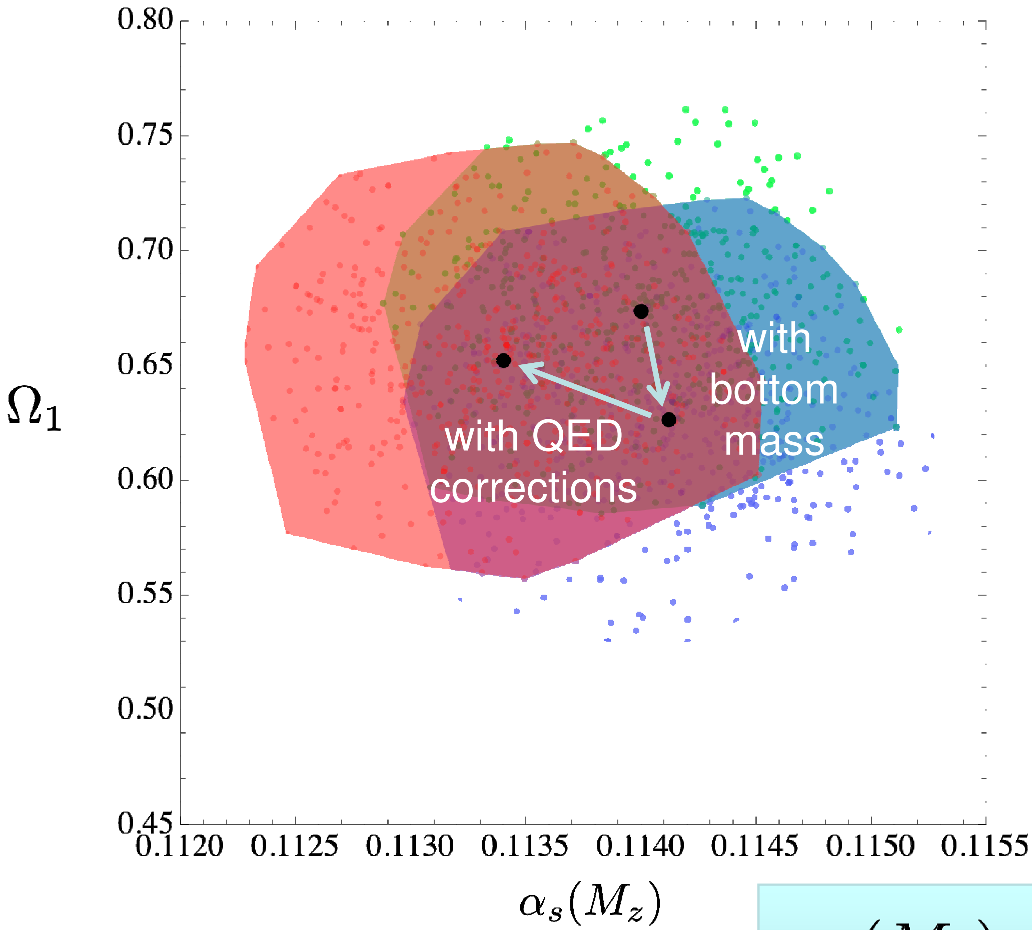
renormalon-free results have smaller theory errors and better fits

Fits to $d\sigma/d\tau$ bins: diff. datasets



Fits to $d\sigma/d\tau$ bins: m_b and QED

with gap subtraction



"standard" data set:

$Q \geq 35$ GeV

$\frac{6}{Q} \leq \tau \leq 0.33$

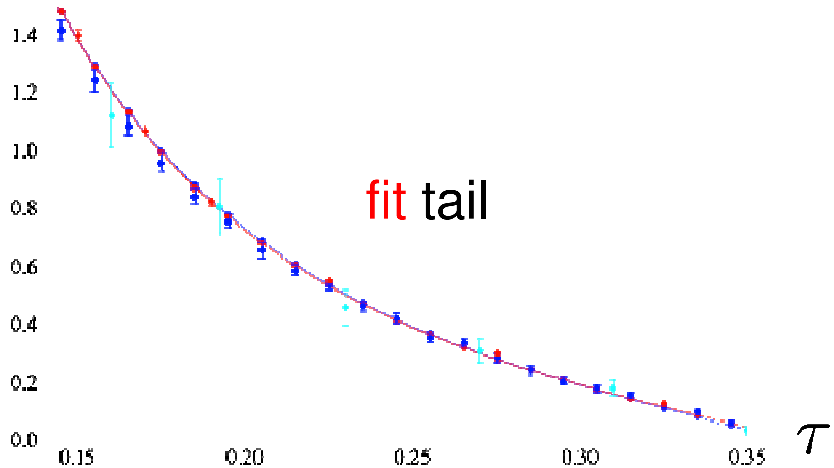
488 bins

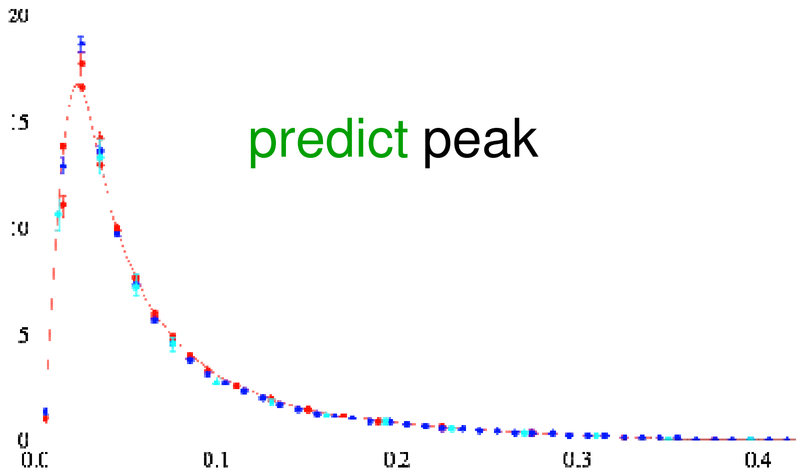
Our result: (preliminary)

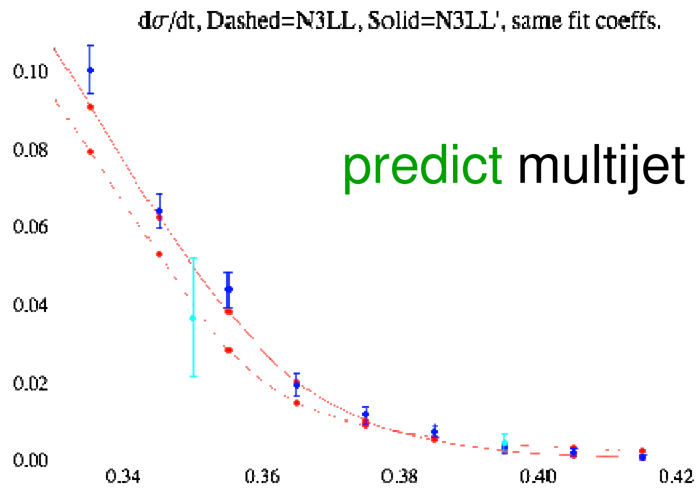
$$\alpha_s(M_z) = 0.1135 \pm 0.0011 \pm 0.0006$$

Peak and Multijet Regions

$Q = 91.2\ \mathrm{GeV}$







Conclusions & Outlook

$\alpha_s(M_z)$

- No entirely consistent overall picture concerning methods to determine $\alpha_s(M_z)$
- Current world average essentially comes from the lattice

Event shapes:

- SCET provides powerful formalism to provide predictions with high precision for jet observables
- Consistent field theory implementation of non-perturbative effects
- Presented results applicable to jet masses and C-parameter
- Soft function determination from peak fit
- Bottom quark mass from $Q < 35\ \mathrm{GeV}$ data
- Future: hadron event shapes

Tail Fits for Thrust:

$$\alpha_s(M_z) = 0.1135 \pm 0.0011 \pm 0.0006$$