R-evolving QCD (matrix elements)

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In collaboration with André Hoang, Ambar Jain, Iain Stewart (PRL101:151602,2008) arXiv:0908.3189 and work in progress

- OPE a la Wilson and OPE a la MS
- The renormalon problem: What is the size of their effect? How to deal with it? A new subtraction scale R
- R-RGE and MSR
- Examples: Ellis-Jaffe sum rule, heavy meson mass difference.
- A new sum rule to probe renormalons
- Applications of the sum rule
- Conclusions

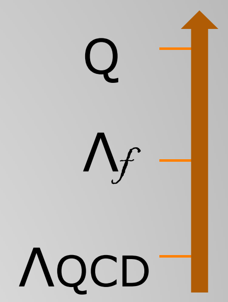
Outline

OPE a la Wilson and MS

OPE is a standard tool for perturbative calculations: it allows to separate the perturbative part of a process from a non-perturbative piece which can be calculated with different devices.

Consider a dimensionless observable σ for some hard scattering process that happens at scale $Q \gg \Lambda_{QCD}$. In Wilson OPE: $Q > \Lambda_f > \Lambda_{QCD}$

$$\sigma = C_0^W(Q,\Lambda_f)\, \theta_0^W(\Lambda_f) + C_1^W(Q,\Lambda_f)\, \theta_1^W(\Lambda_f)/Q^p + \ldots$$



- Loops are hard too evaluate because of hard cutoff
- No clear separation of scales $C_0^W(Q,\Lambda_f) \sim \Lambda_f$, $\ln \Lambda_f/Q$
- Lorentz and gauge invariance not manifest

MS is used in combination with dimensional regularization d=4-2ε.

$$\sigma = \overline{C}_0\,(Q,\mu)\overline{\theta}_0(\mu) + \overline{C}_1\,(Q,\mu)\frac{\overline{\theta}_1(\mu)}{Q^2} + \ldots$$

Now the power counting is manifest as $\overline{C}_0$ contains only $\ln^k \frac{\mu}{Q}$ but no $(\mu/Q)^k$

Power counting is manifest

Lorentz and gauge invariance are manifest

A lot of technology to make loop calculations, but

Loop integrations are done on unphysical regions and Wilson coefficients and matrix elements have unphysical contributions inside

OPE a la Wilson and MS

This phenomenon appears as “renormalons”

- How to probe them? What is the size of their effect?

- How to deal with them consistently?

Renormalons

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Renormalons

$$\alpha_s(k) = \alpha_s(\mu)\sum_{n=0}^{\infty}\left(\frac{\alpha_s(\mu)\beta_0}{4\pi}\right)^n \ln^n\left(\frac{\mu^2}{k^2}\right)$$

$$C_0(\mu,Q) \approx \overline{C}_0(\mu,Q) + \frac{1}{Q^p}\int_0^{\mu} dk\; k^{p-1}\alpha_s(k)$$

Renormalons have a power-like dependence,
but its normalization is difficult to estimate

$$\frac{\mu^p}{Q^p}\frac{2\pi}{\beta_0}\sum_{n=0}^{\infty}\left(\frac{\alpha_s(\mu)\beta_0}{2p\pi}\right)^{n+1} n! \to B(u) \sim -\frac{\mu^p}{Q^p}\frac{2\pi}{\beta_0}\frac{1}{2u-p}$$

$$\frac{\theta_1(\mu)}{Q^p} \approx \frac{\overline{\theta}_1(\mu)}{Q^p} + \frac{1}{Q^p}\int_{\mu}^{\infty} dk\; k^{p-1}\alpha_s(k)$$

$$-\frac{\mu^p}{Q^p}\frac{2\pi}{\beta_0}\sum_{n=0}^{\infty}\left(\frac{\alpha_s(\mu)\beta_0}{2p\pi}\right)^{n+1} n! \to B(u) \sim \frac{\mu^p}{Q^p}\frac{2\pi}{\beta_0}\frac{1}{2u-p}$$

There is a cancellation
between Wilson coefficient
and (higher twist)
matrix element

t'Hooft, Mueller,
Beneke, Luke, Manohar, …

We want OPE in a scheme that preserves the good feature of MS, but removes the renormalon behavior

$C_0(Q,R,\mu)=\overline{C_0}(Q,\mu)-\delta C_0(Q,R,\mu)$

$\theta_1(\mu,R)=\overline{\theta_1}(\mu)-\delta\theta_1(\mu,R)$

$$\delta C_0(Q,R,\mu)=\left(\frac{R}{Q}\right)^p\sum_{n=1}^{\infty}d_n\left(\frac{\mu}{R}\right)\left[\frac{\alpha_s(\mu)}{4\pi}\right]^n$$

If for large n the coefficient of C_0 and $\pm C_0$ behave in the same way

$$C_0(Q,R,\mu)\sim\left(\frac{\mu^p}{Q^p}-\frac{R^p}{Q^p}\frac{\mu^p}{R^p}\right)\sum_n n!\left(\frac{2\beta_0}{p}\right)^n Z$$

The cancellation of the renormalon involves the introduction of a new scale R

Renormalons cancellation and MSR

One can define a new scheme, MSR, for the Wilson coefficient C_0 such that the coefficient of the subtraction is the same MS coefficient.

$$C_0(Q,R,\mu) = \overline{C}_0(Q,\mu)\left[\overline{C}_0(R,\mu)\right]^{-(R/Q)^p}$$

The new coefficient is such that

- The value of p is derived from the difference in the dimension of θ_1 and θ_0
- Gauge and Lorentz symmetries are preserved

However the new definition is no good without an RGE

The MSR scheme

- No choice of μ,R, Q can minimize logs in coefficients and matrix elements at the same time →
- We need an R-RGE to resum gluons contributions associated to renormalons (beyond the usual μ-RGE)

NEW!!!

$$R\frac{d}{dR}\ln C_0(Q,R,R) = \bar{\gamma}[\alpha_s(R)] - \frac{R^p}{Q^p}\gamma[\alpha_s(R)]$$

Bigi, Shifman, Uraltsev; Voloshin
Hoang,Jain, IS, Stewart

A new R-RGE piece

The same expression as usual μ-AD

$$t = \frac{-2\pi}{\beta_0\alpha_s(R)}$$

$$\Lambda_{QCD}^{(0)} = R\, e^{t}$$

$$\Lambda_{QCD}^{(1)} = R\, e^{t}(-t)^{\hat{b}_1}$$

$$\hat{b}_1 = \frac{\beta_1}{2\beta_0^2}$$

$$C_0(Q,R_0,R_0) = C_0(Q,R_1,R_1)U_R(Q,R_1,R_0)U_\mu(R_1,R_0)$$

$$\ln U_R(Q,R_1,R_0) = \left(\frac{\Lambda_{QCD}^{(k)}}{Q}\right)^p \sum_{j=0}^{k} S_j(-p)^j e^{ip\pi\hat{b}_1} p^{p\hat{b}_1}\left[\Gamma\left(-p\hat{b}_1 - j, pt_0\right) - \Gamma\left(-p\hat{b}_1 - j, pt_0\right)\right]$$

$$S_0 = \frac{\gamma_0}{2\beta_0}$$

$$S_1 = \ldots$$

The MSR scheme and R-RGE

$$\ln U_R(Q,R_1,R_0) = -\int_{R_1}^{R_0} \frac{dR}{R}\frac{R^p}{Q^p}\gamma[\alpha_s(R)]$$

LO

$$\sim \frac{\gamma_0}{4\beta_0}\sum_{k=0}^{\infty}\left[\frac{\alpha_s(R_1)\beta_0}{2p\pi}\right]^{k+1} k!\left[\frac{R_1^p}{Q^p} - \frac{R_0^p}{Q^p}\sum_{l=0}^{k}\frac{1}{l!}\ln\left(\frac{R_1^p}{R_0^p}\right)\right]$$

$$\sim -\frac{\gamma_0}{4\beta_0}\int_0^{\infty} du\, e^{-\frac{4\pi}{\beta_0\alpha_s(R_1)}}\left[\frac{R_1^p}{Q^p}\frac{1}{u-\frac{p}{2}} - \frac{R_0^p}{Q^p}\frac{\left(\frac{R_1}{R_0}\right)^{2pu}}{u-\frac{p}{2}}\right]$$

Resummation of R-logs

The Ellis-Jaffe sum rule

Moments from polarized DIS in proton structure function

Broadhurst, Kataev

$$M_1(Q) = \overline{C}_B(Q,\mu)\theta_B + \overline{C}_0(Q,\mu)\frac{\hat{a}_0}{9} + ..$$

$$C_i(Q,R,R) \equiv \overline{C}_i(Q,R)e^{-(R^2/Q^2)\overline{C}_i(R,R)}$$

$$\theta_B = \frac{\hat{a}_8}{36} + \frac{g_A}{12}$$

$$\hat{a}_8 = 0.572$$

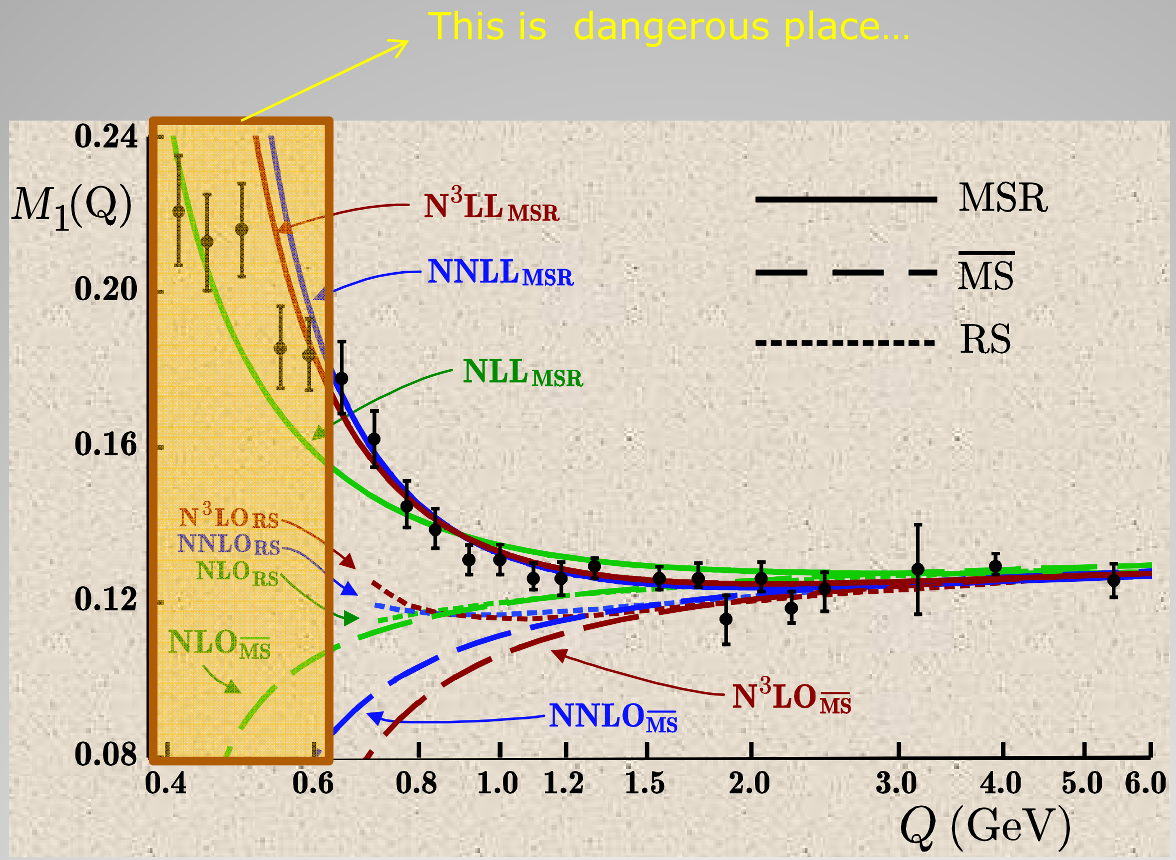
$$g_A = 1.2694$$

$$\hat{a}_0 = 0.141$$

Here a p=2 renormalon

$$M_1(Q) = C_B(Q,R_1,R_1)U_R^B(Q,R_1,R_0)\theta_B + C_0(Q,R_1,R_1)U_R^0(Q,R_1,R_0)\frac{\overline{a}_0}{9}$$

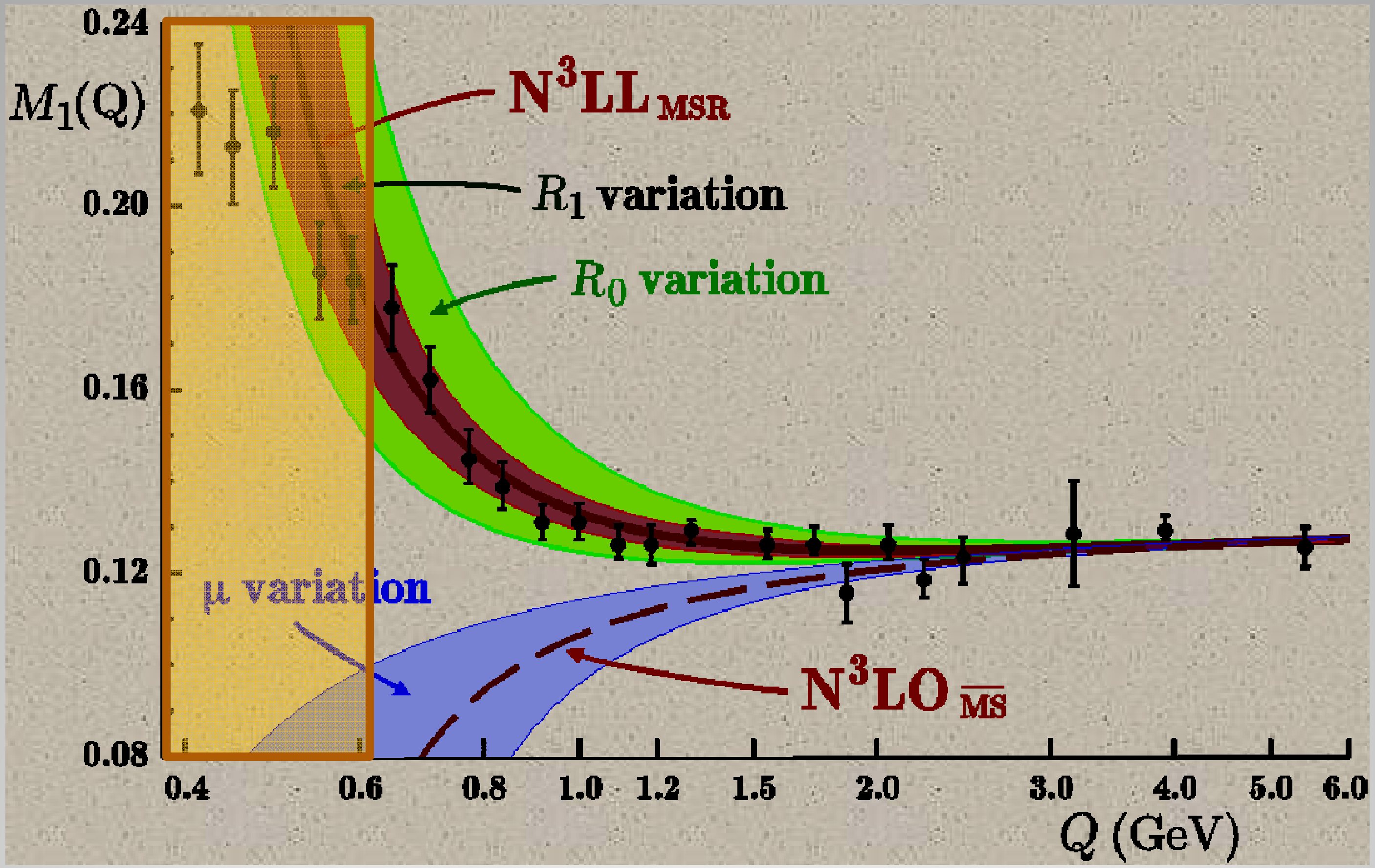
C_B and C_0 are known at 3 loops (Kodaira; Larin, Ritbergen, Vermaseren)



$R_1 = Q$
$R_0 = 1$ GeV

Data: Osipenko et al.
RS-scheme: Campanario, Pineda

The Ellis-Jaffe sum rule



Ellis-Jaffe sum rule

Setting H=B,D $\Delta m_H^2 = m_{H^*}^2 - m_H^2$

$$\Delta m_H^2 = \overline{C}_G(m_Q,\mu)\mu_G^2(\mu) + \frac{\overline{\Sigma}_\rho(\mu)}{m_Q} + \ldots$$

Here a p=1 renormalon:
Grozin, Neubert

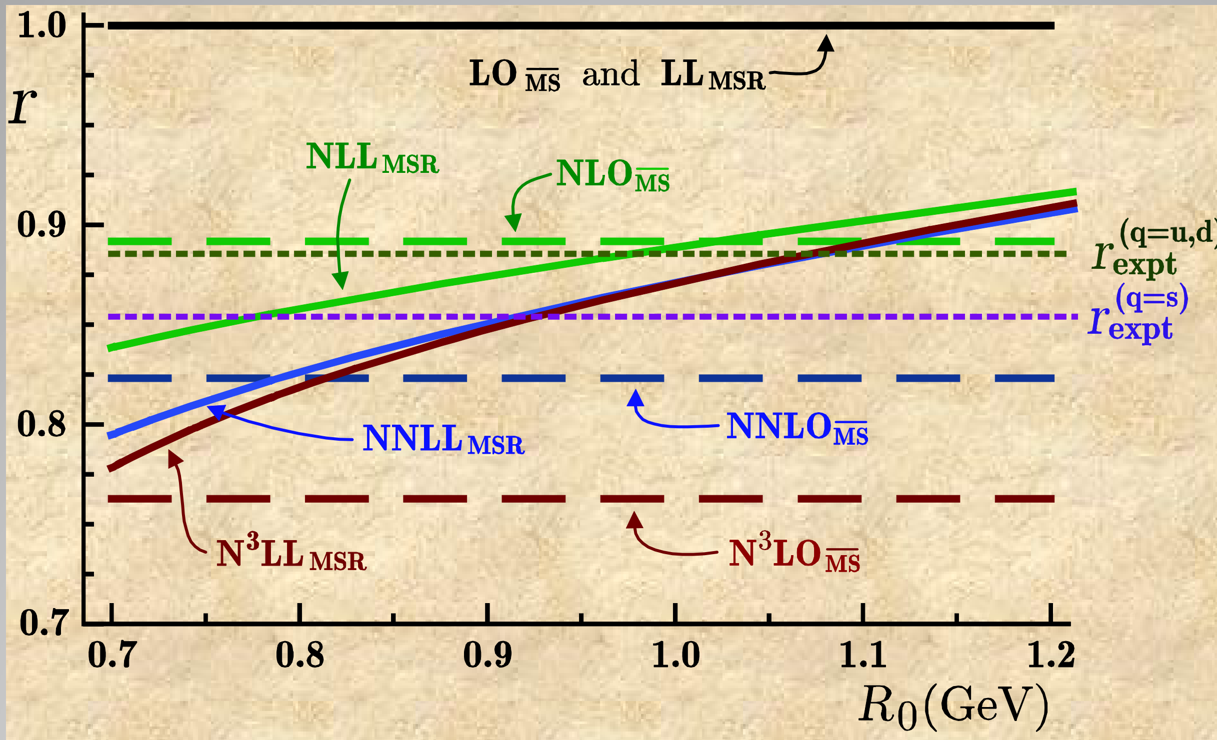
3-loops from Grozin, Marquard, Piclum, Steinhauser

The interesting observable is ratio $r = \Delta m_B^2 / \Delta m_D^2$

And using MSR

$$r = \frac{C_G(m_b,R_1,R_1)U_R(m_b,R_1,R_0)}{C_G(m_c,R_1,R_1)U_R(m_c,R_1,R_0)} + \frac{\Sigma_\rho(R_0,R_0)}{\mu_G^2(R_0)}\left(\frac{1}{m_b} - \frac{1}{m_c}\right)$$

Heavy Quark Mass Splitting



$m_b = 4.7$ GeV

$m_c = 1.6$ GeV

$\overline{\text{MS}}$: $\Delta r_{p.th.} = 0.07$

$\frac{1}{2}\sqrt{m_b m_c} < \mu < 2\sqrt{m_b m_c}$

$$\frac{\sqrt{m_b m_c}}{2} < \mu < 2\sqrt{m_b m_c}$$

MSR: $\Delta r_{p.th.} = 0.008$

$\frac{1}{2}\sqrt{m_b m_c} < R_1 < 2\sqrt{m_b m_c}$

$$\frac{\sqrt{m_b m_c}}{2} < R_1 = \mu < 2\sqrt{m_b m_c}$$

$$r = 0.860 \pm 0.065_{\Sigma_\rho} \pm 0.008_{pert}$$

Heavy Quark Mass Splitting

This phenomenon appears as “renormalons”

- How to probe them?

- How to remove them?

Renormalons

The R-RGE can be used also to probe the normalization of a renormalon. An example is provided by the O(Λ_{QCD}) renormalon in the pole mass.

$$m_{pole} = m(\mu, R) + \delta m(\mu, R)$$

MS, 1S, PS, Kinetic,Jet mass…

Beneke, Bigi,Uraltsev,Hoang,Manohar, us…

The pole mass does not depend on R.. So for μ-independent schemes

$$\frac{d}{d\ln R} m(R) = -\frac{d}{d\ln R}\delta m(R) = R\gamma_R\left[\alpha_s(R)\right]$$

We can compute m(R1)-m(R0), in a renormalon free way and of large log(R1/R0)

A new sum rule for detecting renormalons

$$m(R_1) - m_{pole} = \Lambda_{QCD} \int_{t_1}^{\infty} dt\ \gamma_R(t) \frac{d}{dt} e^{-G(t)}$$

The pole mass is obtained in the limit $R_0 \to 0$

It is interesting to see what happens with the Borel transform of this

$$B(u) = 2R \left[\sum_{\ell=0} g_\ell Q_\ell(u) - P_{1/2} \sum_{\ell=0} g_\ell \frac{\Gamma(1+\hat{b}_1 - \ell)}{(1-2u)^{1+\hat{b}_1-\ell}} \right]$$

$$P_{1/2} = \sum_{k=0} \frac{S_k}{\Gamma(1+\hat{b}_1 + k)}$$

We have a sum rule to fix renormalons! No need of bubble resummation!

$P_{1/2}$ is the normalization of the first renormalon and we can show that its series is absolutely convergent.

The the sum rule for pole mass

$$B(u) = 2R\left[\sum_{\ell=0} g_\ell Q_\ell(u) - P_{1/2}\sum_{\ell=0} g_\ell \frac{\Gamma(1+\hat{b}_1-\ell)}{(1-2u)^{1+\hat{b}_1-\ell}}\right]$$

$$P_{1/2} = \sum_{k=0} \frac{S_k}{\Gamma(1+\hat{b}_1+k)}$$

The renormalon coefficient $P_{1/2}$ depends only on R-anomalous dimension Through S_k.

However the anomalous dimension changes by a rescaling $R \longrightarrow \lambda R$

If all terms of the series where known $P_{1/2}$ would be really a constant as a function of λ. In practice we expect it to be constant for $½<\lambda<2$.

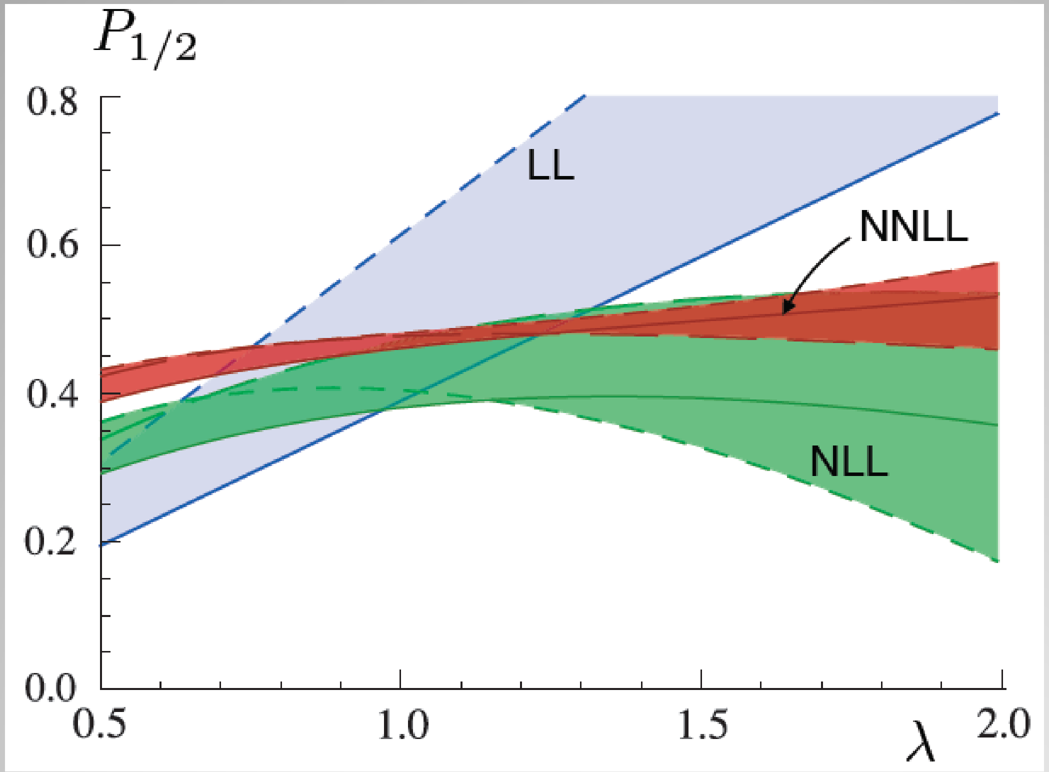
$$\gamma_0^{R'} = \lambda \gamma_0^R ,$$

$$\gamma_1^{R'} = \lambda\left[\gamma_1^R - 2\beta_0\gamma_0^R \ln\lambda\right] ,$$

$$\gamma_2^{R'} = \lambda\left[\gamma_2^R - (4\beta_0\gamma_1^R + 2\beta_1\gamma_0^R)\ln\lambda + 4\beta_0^2\gamma_0^R \ln^2\lambda\right] .$$

$$P_{1/2}(\lambda) = \sum_{k=0} \frac{S_k(\lambda)}{\Gamma(1+\hat{b}_1+k)}$$

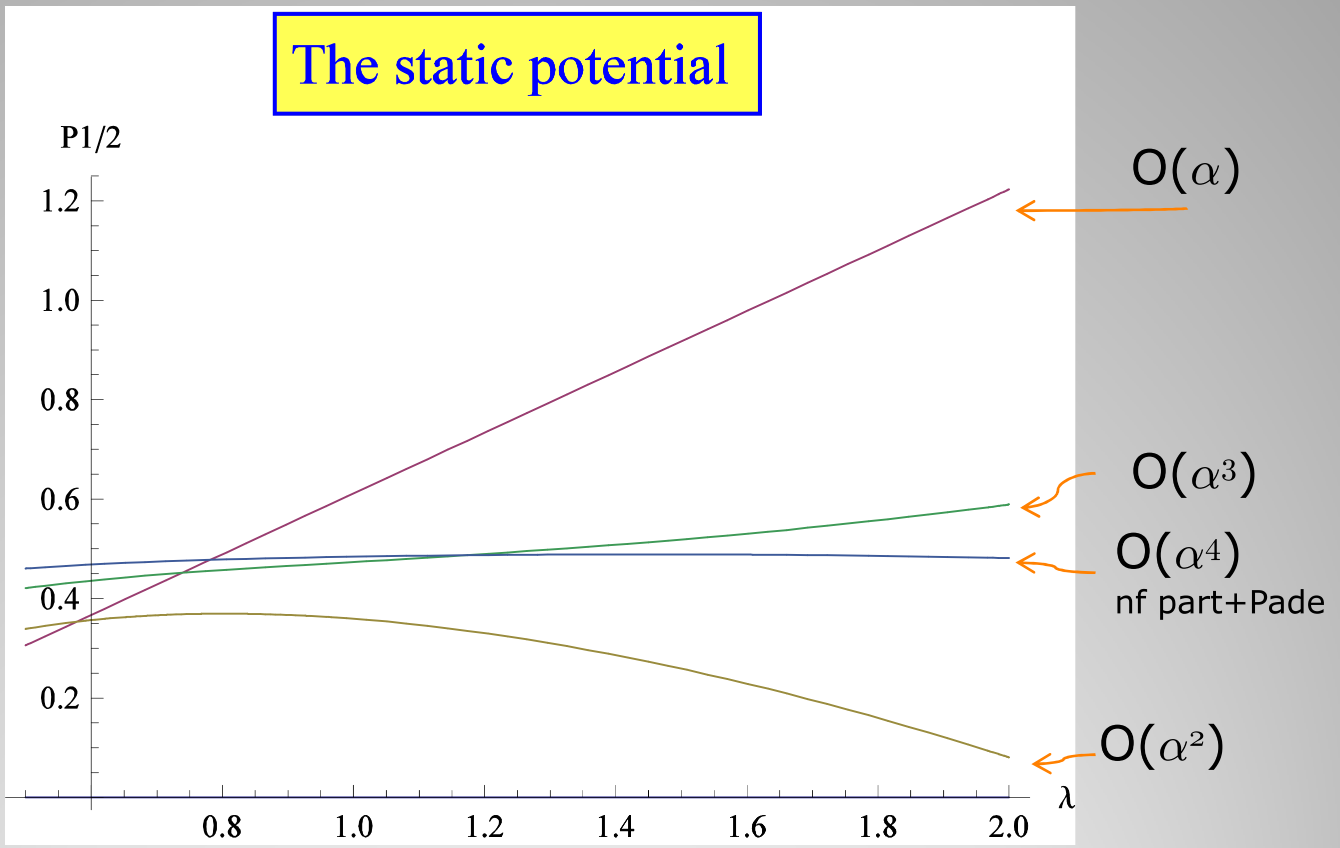
$P_{1/2}$ and the scaling of R



The renormalon of the pole mass in MSR, PS, Static

The pole mass renormalon in several schemes

Peter,
Schroeder,
AV Smirnov,
VA Smirnov,
Steinhauser



The heavy quark static potential

- It is possible to express the usual MS $\boldsymbol{\beta}$-function in terms of the renormalon free 't-Hooft $\boldsymbol{\beta}$-function

$$\beta_{MS}\left(\alpha_{MS}\right)=-2\alpha_{MS}\sum_{i=0}\beta_i\left(\frac{\alpha_{MS}}{4\pi}\right)^{i+1}$$

$$\beta_{tH}\left(\alpha_{tH}\right)=-2\left(\frac{\beta_0\alpha_{tH}^2}{4\pi}+\frac{\beta_1\alpha_{tH}^3}{(4\pi)^2}\right)$$

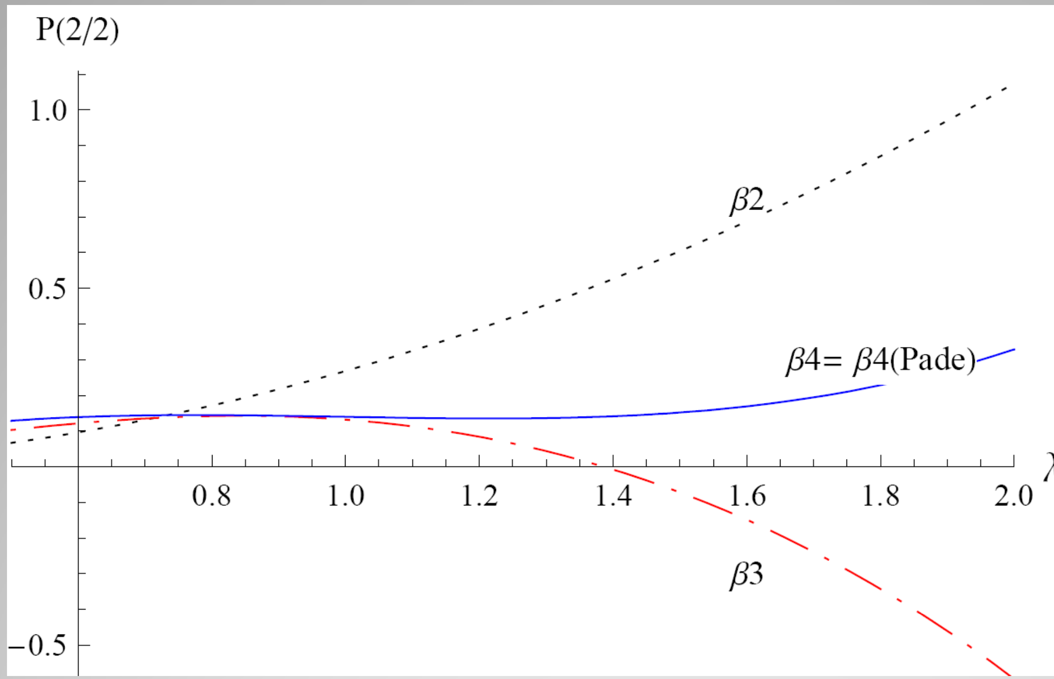
The 2 forms of $\boldsymbol{\beta}$-functions define two different couplings which can be related one-another. In order to check the renormalon behavior in the MS $\boldsymbol{\beta}$-function we consider

$$\frac{\alpha_{MS}-\alpha_{tH}}{\alpha_{tH}}\equiv f(\alpha_{tH})$$

and apply the sum rule P2/2 to

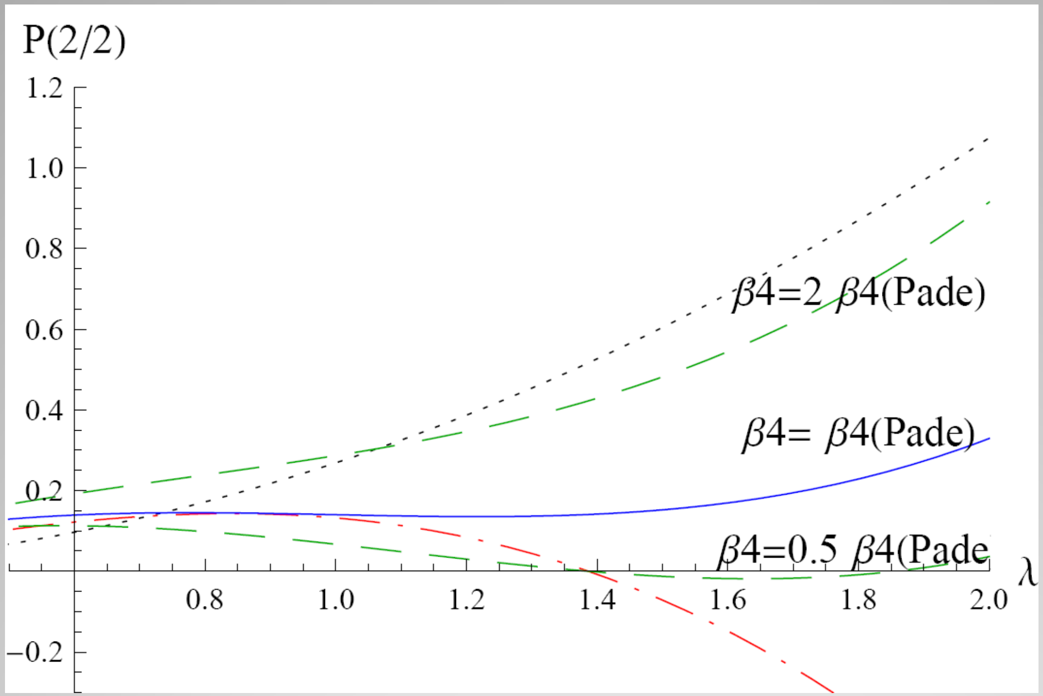
$$\tilde{f}(\alpha_{tH}(R))=R^2 f(\alpha_{tH}(R))$$

Asymptotic behavior of the QCD β-function



β4 comes from Pade apprximants : J.R. Ellis et al PRD57(1998)2665

Asymptotic behavior of the β-function



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Asymptotic behavior of the β-function

- OPE in MSR: same technical advantages as MS.
- A new IR cut is reintroduced in OPE through a new scale R
- Logs of the infrared cut R can be resummed with R-RGE
- IR renormalons are removed from Wilson coefficients
- Great improvement of the convergence of perturbative expansion
- We can estimate the higher order contributions with the variation of R_0

Conclusions I

- The R-RGE can be used to understand the renormalon structure of perturbation series
- If “enough” terms of the perturbative series are known one can evaluate the normalization of a renormalon. (The word “enough” depends on the position of the renormalon)
- Many applications are still to be worked out.

Thanks

Conclusions II

Renormalons cancellation and MSR

We want OPE in a scheme that preserves the good feature of MS, but removes the renormalon behavior

$C_0(Q,R,\mu)=\overline{C}_0(Q,\mu)-\delta C_0(Q,R,\mu)$

$$\delta C_0(Q,R,\mu)=\left(\frac{R}{Q}\right)^p \frac{C_1(Q,\mu)}{C_1(R,\mu)}\sum_{n=1}^{\infty} d_n\left(\frac{\mu}{R}\right)\left[\frac{\alpha_s(\mu)}{4\pi}\right]^n$$

This effect should be included if the information about the higher twist operator Wilson coefficient is available. We omit it in the following

If for large n the coefficient of C_0 and $\pm C_0$ behave in the same way

$$C_0(Q,R,\mu)\sim\left(\frac{\mu^p}{Q^p}-\frac{R^p}{Q^p}\frac{\mu^p}{R^p}\right)\sum_n n!\left(\frac{2\beta_0}{p}\right)^n Z$$

The cancellation of the renormalon involves the introduction of a new scale R

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Consider a dimensionless observable σ for some hard scattering process that happens at scale $Q \gg \Lambda_{QCD}$. In Wilson OPE: $Q > \Lambda_f > \Lambda_{QCD}$

$$\sigma \sim \int dk \frac{k^{p-1} f(k^2, \Lambda^2_{\rm QCD})}{(k^2+Q^2)^{p/2}}$$

$$= \int_{\Lambda_f}^{\infty} dk \frac{k^{p-1} f(k^2, 0) + \dots}{(k^2+Q^2)^{p/2}} + \int_0^{\Lambda_f} dk\, k^{p-1} f(k^2, \Lambda_{\rm QCD}) \Big[\frac{1}{Q^p} + \dots\Big]$$

$$\sigma = C_0^W(Q, \Lambda_f)\bar{\theta}_0^W(\Lambda_f) + \bar{C}_1(Q, \Lambda_f)\frac{\theta_1(\Lambda_f)}{Q^p} + \dots$$

MS is used in combination with dimensional regularization d=4-2ε.

$$\sigma \sim \mu^{2\epsilon} \int dk^{d-3} \frac{k^{p-1} f(k^2, \Lambda^2_{\rm QCD})}{(k^2+Q^2)^{p/2}}$$

$$= \mu^{2\epsilon} \int dk^{d-3} \frac{k^{p-1} f(k^2, 0) + \ldots}{(k^2+Q^2)^{p/2}} \quad + \mu^{2\epsilon} \int dk^{d-3}\, k^{p-1} f(k^2, \Lambda_{\rm QCD}) \left[\frac{1}{Q^p} + \ldots \right]$$

$$\sigma = \bar{C}_0(Q,\mu)\bar{\theta}_0(\mu) + \bar{C}_1(Q,\mu)\frac{\bar{\theta}_1(\mu)}{Q^p} + \ldots$$

OPE a la Wilson and MS