NNLO real corrections to gluon scattering

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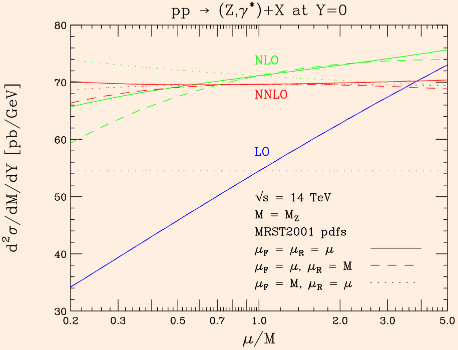
Talk structure

- Motivation - why go beyond NLO?
- Antenna subtraction at NNLO
- Antennae numerical implementation - unresolved emission from:
  - final-final emitters
  - initial-final emitters
  - initial-initial emitters
- Double real radiation counterterm
- Results
- Conclusions

- improve the theoretical prediction truncated at NLO and reduce the sensitivity of the predictions on renormalisation and factorisation scales

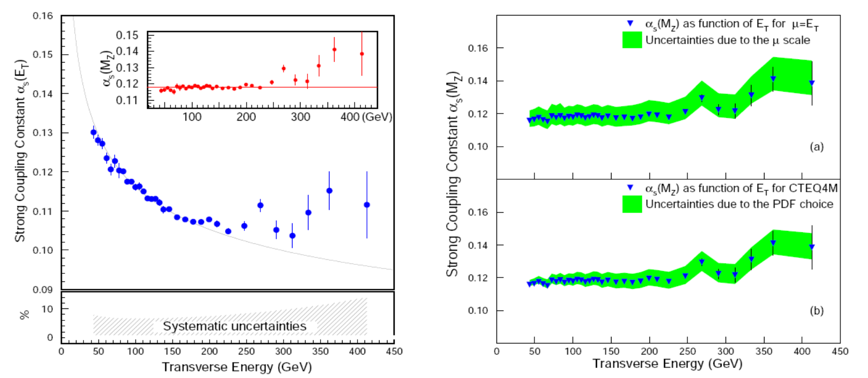


On-shell Z boson production at the LHC

[C.Anastasiou, L.Dixon, K.Melnikov, F.Petriello '03]

Why go beyond NLO?

- improvement of the information we can extract from data whenever the present theoretical uncertainties are as big or bigger than experimental errors

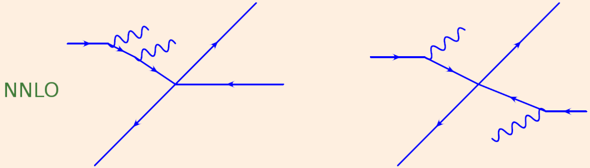


$$\alpha_s(M_Z) = 0.1178 \ ^{+6\%}_{-4\%}(scale) \ ^{+5\%}_{-5\%}(pdf)$$

- theoretical uncertainties in α_s extraction from $p\bar{p} \to$ jet are due to renormalisation scale and pdf's [CDF collaboration '01]

Why go beyond NLO?

- better description of the initial state



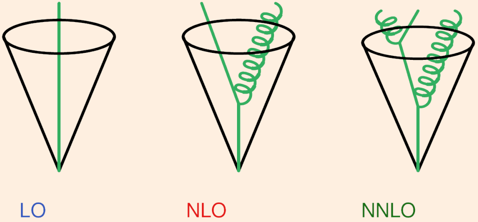
Radiative corrections coming from the initial lines

- at LO the incoming particles have no transverse momentum with respect to the beam

- initial radiation gives final state a transverse momentum kick → better and more theoretical accurate description of transverse momentum distributions

Why go beyond NLO?

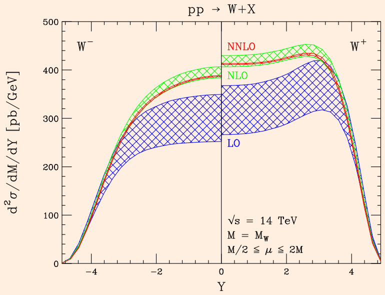
- better description of final-state jets

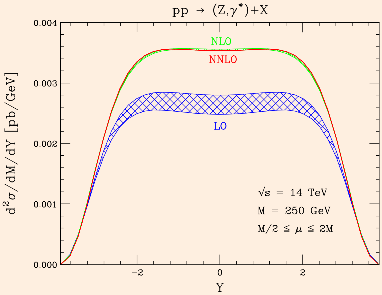


Jets modeled by extra partons

- infrared safe jet algorithm to cluster partons into jets
  - eg. anti-k_T, SIScone
- accurate predictions for jet distributions
- accurate predictions impose stronger constraints on the SM and therefore are essential to maximize the chances of discovering and understanding new physics

Precise predictions at NNLO





Gauge boson production at the LHC
[C.Anastasiou, L.Dixon, K.Melnikov, F.Petriello '03]

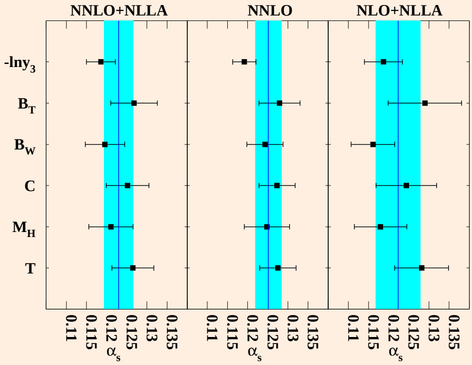
- complete stability against scale variations at NNLO
- convergence of the perturbative expansion

$e^+e^- \to 3$ jets and event shapes

Application of NNLO antenna subtraction

- parton-level event generator: EERAD3
  [A.Gehrmann-De Ridder, T.Gehrmann, E.W.N. Glover, G.Heinrich '07]
- computes jet cross sections and event shapes through to α_s^3
- independent implementation of the method by [S. Weinzierl '08]
- fixed order NNLO calculation for event shapes matched to NLLA
  [T.Gehrmann, G.Luisioni, H.Stenzel '08]
- new extractions of α_s based on NNLO or NNLO+NLLA
  [G.Dissertori, A.Gehrmann-De Ridder, T.Gehrmann, E.W.N. Glover, G.Heinrich, G.Luisioni, H.Stenzel '09]

$e^+e^- \to 3$ jets and event shapes



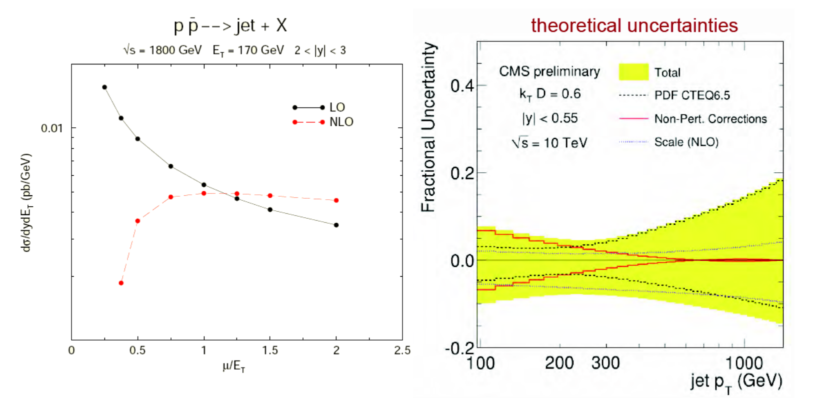
α_s measurments and total uncertainty

[G.Dissertori, A.Gehrmann-De Ridder, T.Gehrmann, E.W.N. Glover, G.Heinrich, G.Luisioni, H.Stenzel '08]

$$\alpha_s(M_Z) = 0.1224 \pm 0.0009(\text{stat}) \pm 0.0009(\text{exp}) \pm 0.0012(\text{stat}) \pm 0.0035(\text{theo})$$

One important observable - high-E_T jets

- high-E_T jet data helps to constrain α_s and the gluon pdf at large values of x



- each figure shows the theoretical uncertainties related to jet production [D. Stump et al. '03],[CMS Physics Analysis Summary '09]

One important observable - high-E_T jets

- if the cross section depends on the choice of scale, then as the scale is varied the pdf will have to change in order to be able to fit data

- doing a fit with a larger renormalization scale causes the high-x gluon to be larger since the high-E_T partonic cross section has decreased

- the scale dependence results in a shift of the pdf and, hence, makes a contribution to the pdf uncertainty

- better determination of the gluon pdf improves the theoretical predictions of any hadronic scattering process

$$\sigma = \sum_{ij} \int_0^1 dx_1 dx_2 f_i^{(h1)}(x_1,\mu) f_j^{(h2)}(x_2,\mu) \sigma_{ij\to pq}(x_1,x_2,\mu)$$

- dominant hard scattering process at LHC

- rich in potential signals of new physics

$pp \to j + X$ at NNLO

NNLO calculation for $pp \to j + X$ reaction contains:

$$\hat{\sigma}_{NNLO} \sim \int \left[|\langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)} \rangle|^2 \right]_{n+2} \mathrm{d}\Phi_{n+2} + \int \left[\langle \mathcal{M}^{(0)} | \mathcal{M}^{(1)} \rangle + \langle \mathcal{M}^{(1)} | \mathcal{M}^{(0)} \rangle \right]_{n+1} \mathrm{d}\Phi_{n+1} + \int \left[\langle \mathcal{M}^{(1)} | \mathcal{M}^{(1)} \rangle + \langle \mathcal{M}^{(0)} | \mathcal{M}^{(2)} \rangle + \langle \mathcal{M}^{(2)} | \mathcal{M}^{(0)} \rangle \right]_{n} \mathrm{d}\Phi_{n}$$

- tree level $2 \to 4$ matrix elements [F.A. Berends, W.T. Giele '87], [M.Mangano, S.J.Parke, Z.Xu '87]
- 1-loop $2 \to 3$ matrix elements [Z.Bern, L.Dixon, D.A. Kosower '93]
- 2-loop $2 \to 2$ matrix elements [C. Anastasiou, E.W.N. Glover, C.Oleari, M.E. Tejeda-Yeomans '01], [Z.Bern, A.De Freitas, L.Dixon '02]

$pp \to j + X$ at NNLO

- three-loop splitting functions required for the evolution of parton distribution functions at NNLO

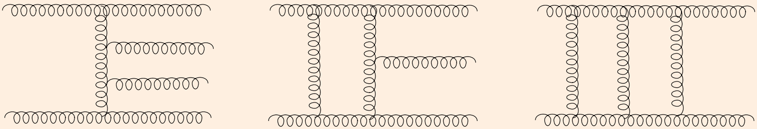
$$\frac{d}{dln\mu^2} f_i(x,\mu^2) = \sum_k \left[P_{ik}(\alpha_S(\mu^2)) \otimes f_k(\mu^2) \right](x)$$

$$P = \alpha_s P^{(0)} + \alpha_s^2 P^{(1)} + \alpha_s^3 P^{(2)} + \ldots$$

- DGLAP splitting kernels have been calculated to $\mathcal{O}(\alpha_s)^3$ and are needed for a consistent phenomenological treatment
  [S.Moch, J.A.M.Vermaseren, A.Vogt '04]

- NNLO parton distribution functions e.g.
  [A.D.Martin, R.Roberts, W.J.Stirling, R.S.Thorne, G.Watt]
  [S.Alekhin, J.Blümlein, S.Klein, S.Moch]

gluon-gluon channel



$A_6^{(0)}(gg \to gggg)$ $\quad$ $A_5^{(1)}(gg \to ggg)$ $\quad$ $A_4^{(2)}(gg \to gg)$

- explicit infrared poles from loop integrations
  - pole structure agrees with prediction of [S. Catani '98]
- implicit poles in phase space regions for single and double unresolved gluon emission
- procedure to extract the infrared singularities and assemble all the parts
  - sector decomposition [C.Anastasiou, K.Melnikov, F.Petriello '03],[T. Binoth, G.Heinrich '02]
  - NNLO subtraction [V. Del Duca, G.Somogyi, Z.Trocsanyi '05],[S.Catani, M.Grazzini '07]
  - NNLO antenna subtraction [A.Gehrmann-De Ridder, T.Gehrmann, E.W.N. Glover, G. Heinrich '05]

NNLO subtraction

Structure of NNLO antenna subtraction [A.Gehrmann-De Ridder, T.Gehrmann, E.W.N. Glover, G. Heinrich '05]:

$$
\begin{aligned}
\mathrm{d}\sigma_{NNLO} &= \int_{\mathrm{d}\Phi_{m+2}} \left(\mathrm{d}\sigma^{R}_{NNLO} - \mathrm{d}\sigma^{S}_{NNLO}\right) \\
&+ \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^{V,1}_{NNLO} - \mathrm{d}\sigma^{VS,1}_{NNLO}\right) \\
&+ \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\sigma^{V,2}_{NNLO} + \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\sigma^{S}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{VS,1}_{NNLO}
\end{aligned}
$$

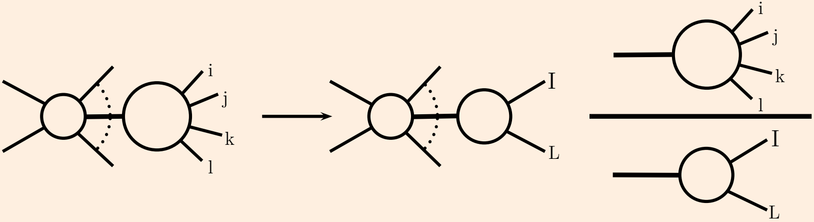
- $\mathrm{d}\sigma^{S}_{NNLO}$: real radiation subtraction term for $\mathrm{d}\sigma^{R}_{NNLO}$
- $\mathrm{d}\sigma^{VS,1}_{NNLO}$: one-loop virtual subtraction term for $\mathrm{d}\sigma^{V,1}_{NNLO}$
- $\mathrm{d}\sigma^{V,2}_{NNLO}$: two-loop virtual corrections
- subtraction terms constructed using the antenna subtraction method at NNLO
- each line above is finite numerically and free of infrared ϵ-poles

Antenna functions and types

- colour-ordered pair of hard partons (radiators) with radiation in between
  - hard quark-antiquark pair
  - hard quark-gluon pair
  - hard gluon-gluon pair
- three-parton antenna $\rightarrow$ one unresolved parton
- four-parton antenna $\rightarrow$ two unresolved partons
- can be at tree level or at one loop
- all have three antenna types
  - final-final antenna
  - initial-final antenna
  - initial-initial antenna
- all three-parton and four-parton antenna functions can be derived fom physical matrix elements, normalised to two-parton matrix elements

NNLO final-final antennae

- factorisation of both the squared matrix elements and the $(m+2)$- particle phase space → colour connected unresolved particles



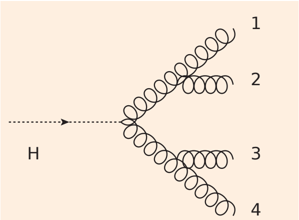
- momentum mapping: [D.A. Kosower '02]

$$
\begin{aligned}
p_I^\mu &= x p_i^\mu + r_1 p_j^\mu + r_2 p_k^\mu + z p_l^\mu \\
p_L^\mu &= (1-x) p_i^\mu + (1-r_1) p_j^\mu + (1-r_2) p_k^\mu + (1-z) p_l^\mu
\end{aligned}
$$

- phase-space factorisation:

$$
\begin{aligned}
\mathrm{d}\Phi_{m+2}(p_a, \ldots, p_i, p_j, p_k, p_l, \ldots, p_{m+2}) &= \mathrm{d}\Phi_m(p_a, \ldots, p_I, p_L, \ldots, p_{m+2}) \\
&\quad \mathrm{d}\Phi_{X_{ijkl}}(p_i, p_j, p_k, p_l)
\end{aligned}
$$

$F_4^0(1_g, 2_g, 3_g, 4_g)$ - final final emitters



- gluon-gluon antenna function:

$$X_{1234} = \frac{|M_{gggg}|^2}{|M_{gg}|^2} \equiv F_4^0(1_g, 2_g, 3_g, 4_g)$$

- each gluon pair can act as hard emitter pair or become soft

- disentangle both single and double unresolved limits to identify the radiators in the previous mapping

- exploit $\mathcal{N} = 1$ supersymmetry relation among the different triple collinear splitting functions

$$\sum_{P(a,b,c)} \left(P_{ggg\to G}(a,b,c) + 2P_{g\bar{q}q\to G}(a,b,c) + P_{q\tilde{g}\bar{q}\to G}(a,b,c)\right) =$$

$$\sum_{P(a,b,c)} \left(2P_{qgg\to Q}(a,b,c) + P_{q\tilde{g}\tilde{g}\to Q}(a,b,c) + 2P^{\text{non-iden.}}_{q\bar{q}q\to Q}(a,b,c) + P^{\text{ident.}}_{q\bar{q}q\to Q}(a,b,c)\right)$$

$F_4^0(1_g, 2_g, 3_g, 4_g)$ - final final emitters

$$
\begin{aligned}
&F_4^0(1,2,3,4) - \Big[D_4^0(1,2,3,4) + D_4^0(2,3,4,1) + D_4^0(3,4,1,2) + D_4^0(4,1,2,3) \\
&- A_4^0(1,2,3,4) - A_4^0(2,3,4,1) - A_4^0(3,4,1,2) - A_4^0(4,1,2,3) \\
&- \tilde{A}_4^0(1,2,4,3) - \tilde{A}_4^0(2,3,1,4) + H_4^0(2,1,4,3) + H_4^0(4,1,2,3) \\
&+ A_3^0(4,1,2)\, J_3^0((\widetilde{12}), 3, (\widetilde{14})) + A_3^0(1,2,3)\, J_3^0((\widetilde{12}), (\widetilde{23}), 4) \\
&+ A_3^0(2,3,4)\, J_3^0(1, (\widetilde{23}), (\widetilde{34})) + A_3^0(3,4,1)\, J_3^0((\widetilde{14}), 2, (\widetilde{34})) \\
&+ \frac{1}{2} G_3^0(4,1,2)\, K_3^0((\widetilde{12}), (\widetilde{14}), 3) + \frac{1}{2} G_3^0(1,2,3)\, K_3^0((\widetilde{23}), (\widetilde{12}), 4) \\
&+ \frac{1}{2} G_3^0(2,3,4)\, K_3^0((\widetilde{34}), (\widetilde{23}), 1) + \frac{1}{2} G_3^0(3,4,1)\, K_3^0((\widetilde{14}), (\widetilde{34}), 2) \\
&+ \frac{1}{2} G_3^0(2,1,4)\, K_3^0((\widetilde{14}), (\widetilde{12}), 3) + \frac{1}{2} G_3^0(3,1,2)\, K_3^0((\widetilde{12}), (\widetilde{23}), 4) \\
&+ \frac{1}{2} G_3^0(4,3,2)\, K_3^0((\widetilde{23}), (\widetilde{34}), 1) + \frac{1}{2} G_3^0(1,4,3)\, K_3^0((\widetilde{34}), (\widetilde{14}), 2) \Big]
\end{aligned}
$$

- combination finite in all single unresolved and double unresolved limits
- expression in brackets has uniquely identified hard radiators

$F_4^0(1_g, 2_g, 3_g, 4_g)$ - final final emitters

- we consider eight different mappings

$$
\begin{aligned}
&(a):\ (1,2,3,4) \to (\widetilde{123}, \widetilde{432}), && (b):\ (1,2,4,3) \to (\widetilde{124}, \widetilde{342}), \\
&(c):\ (1,4,3,2) \to (\widetilde{143}, \widetilde{234}), && (d):\ (1,4,2,3) \to (\widetilde{142}, \widetilde{324}), \\
&(e):\ (2,3,1,4) \to (\widetilde{231}, \widetilde{413}), && (f):\ (2,1,4,3) \to (\widetilde{214}, \widetilde{341}), \\
&(g):\ (4,3,1,2) \to (\widetilde{431}, \widetilde{213}), && (h):\ (4,1,2,3) \to (\widetilde{412}, \widetilde{321})
\end{aligned}
$$

- after symmetrising two new subantennae are derived: $F_{4,a}^0$,$F_{4,b}^0$
- the sum of the $F_{4,i}^0$ adds to F_4^0

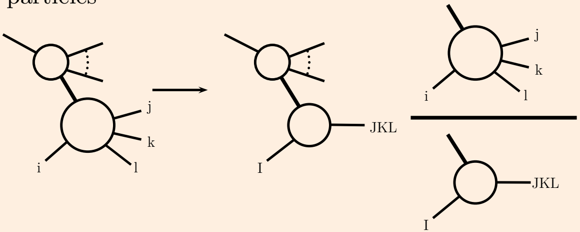
$$
\begin{aligned}
F_4^0(1,2,3,4) = {} & F_{4,a}^0(1^h,2,3,4^h) + F_{4,a}^0(1^h,4,3,2^h) + F_{4,a}^0(2^h,1,4,3^h) + F_{4,a}^0(4^h,1,2,3^h) \\
& + F_{4,b}^0(1^h,2,3^h,4) + F_{4,b}^0(1^h,4,3^h,2) + F_{4,b}^0(2^h,3,4^h,1) + F_{4,b}^0(4^h,3,2^h,1)
\end{aligned}
$$

- only full F_4^0 must be integrated analytically over the antenna phase space [A.Gehrmann-De Ridder, T.Gehrmann, E.W.N. Glover, G. Heinrich '05]

$$
\begin{aligned}
\mathcal{F}_4^0(_{1234}) = {} & 2(s_{1234})^{-2\epsilon}\left[\frac{5}{2\epsilon^4} + \frac{121}{12\epsilon^3} + \frac{1}{\epsilon^2}\left(\frac{436}{9} - \frac{11\pi^2}{3}\right) + \frac{1}{\epsilon}\left(\frac{23455}{108} - \frac{1067\pi^2}{72} - \frac{379}{6}\zeta_3\right)\right. \\
& \left. + \left(\frac{304951}{324} - \frac{7781\pi^2}{108} - \frac{2288}{9}\zeta_3 + \frac{479\pi^4}{720}\right) + \mathcal{O}(\epsilon)\right]
\end{aligned}
$$

NNLO initial-final antennae

- antenna factorisation for the initial-final situation → colour connected unresolved particles



- momentum-mapping: [A. Daleo, T. Gehrmann, D. Maître '06]

$$p_I^\mu = x p_i^\mu \qquad x = \frac{s_{ij}+s_{ik}+s_{il}+s_{jk}+s_{jl}+s_{kl}}{s_{ij}+s_{ik}+s_{il}}$$

$$p_{JKL}^\mu = p_j^\mu + p_k^\mu + p_l^\mu - (1-x)p_i^\mu$$

- phase-space factorisation:

$$\mathrm{d}\Phi_{m+2}(p_a,\ldots,p_j,p_k,p_l,\ldots,p_{m+2};p_i,r) = \mathrm{d}\Phi_m(p_a,\ldots,p_{JKL},\ldots,p_{m+2};p_I,r) \frac{Q^2}{2\pi}\mathrm{d}\Phi_3(p_j,p_k,p_l;p_i,q)\frac{dx}{x}$$

$F_4^0(\hat{1}_g, 2_g, 3_g, 4_g)$ initial-final emitters

- obtain antennae functions by crossing $1 \to 4$ NNLO antennae
- get $F_4^0(\hat{1}_g, 2_g, 3_g, 4_g)$ with:

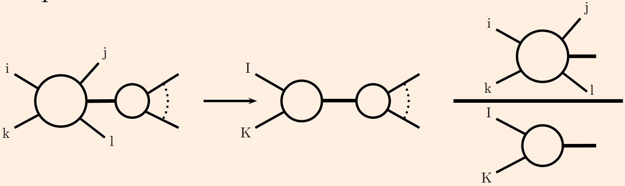
$$s_{1i} = (p_1 - p_i)^2 \qquad s_{ij} = (p_i + p_j)^2 \qquad i, j = 2, 3, 4$$
$$Q^2 = -q^2 \qquad q = p_1 - p_2 - p_3 - p_4$$

No further splitting of the antenna required because:

- in all single (double) unresolved limits four parton antenna collapses into a three (two) parton antenna with a gluon in the initial state
- crossed gluon is hard by kinematical constraints and is the initial state radiator
- final state radiator is the hardest final state gluon
- integrated antenna is the inclusive phase space integral with q^2 and $z = -\frac{q^2}{2q\cdot p}$ fixed [G. Luisioni talk]

NNLO initial-initial antennae

- antenna factorisation for the initial-initial situation → colour connected unresolved particles



- momentum-mapping: [A. Daleo, T. Gehrmann, D. Maître '06]

$$p_I^\mu = \hat{x}_i p_i^\mu \qquad \hat{x}_i = \left(\frac{s_{ik}+s_{kj}+s_{kl}}{s_{ik}}\,\frac{s_{ik}+s_{ij}+s_{il}+s_{kj}+s_{kl}+s_{jl}}{s_{ik}+s_{ij}+s_{il}}\right)^{1/2}$$

$$p_K^\mu = \hat{x}_k p_k^\mu \qquad \hat{x}_k = \left(\frac{s_{ik}+s_{ij}+s_{il}}{s_{ik}}\,\frac{s_{ik}+s_{ij}+s_{il}+s_{kj}+s_{kl}+s_{jl}}{s_{ik}+s_{kj}+s_{kl}}\right)^{1/2}$$

- phase-space factorisation:

$$\begin{aligned} d\Phi_{m+2}(p_a,\ldots,p_j,p_l,\ldots,p_{m+2}) &= d\Phi_m(\tilde{p_a},\ldots,\tilde{p}_{m+2};x_ip_i,x_kp_k) \\ &\quad \delta(x_i-\hat{x_i})\,\delta(x_k-\hat{x_k})\,[dk_j]\,[dk_l]\,dx_i\,dx_k \end{aligned}$$

$F_4^0(\hat{1}_g, \hat{2}_g, 3_g, 4_g)$ initial-initial emitters

- obtain antenna functions by crossing $1 \to 4$ NNLO antennae
- get $F_4^0(\hat{1}_g, \hat{2}_g, 3_g, 4_g)$ with:

$$s_{12} = (p_1 + p_2)^2 \qquad s_{1i} = (p_1 - p_i)^2 \qquad s_{2i} = (p_2 - p_i)^2 \qquad i = 3, 4$$
$$Q^2 = -q^2 \qquad q = p_1 + p_2 - p_3 - p_4$$

No further splitting of the antenna required because:

- in all single (double) unresolved limits four parton antenna collapses into a three (two) parton antenna with two gluons in the initial state
- crossed gluons are hard by kinematical constraints and are the initial state radiators
- integrated antenna is the inclusive phase space integral with q^2 and x_i, x_k fixed [R. Boughezal talk]

NNLO double real correction

NNLO real radiation contribution:

- all six parton tree level processes contributing to two jet final states
- gluon scattering contribution at leading coulour:

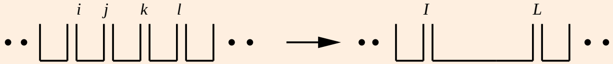
$$
\begin{aligned}
\mathrm{d}\sigma^{R}_{NNLO} \quad = \quad & N^2\, N_{born} \left(\frac{\alpha_s}{2\pi}\right)^2 \mathrm{d}\Phi_4(p_3,\ldots,p_6;p_1,p_2) \Bigg(\\
& \frac{2}{4!} \sum_{P(i,j,k,l)\in(3,4,5,6)} A_6^0(\hat{1}_g,\hat{2}_g,i_g,j_g,k_g,l_g) J_2^{(4)}(p_i,\ldots,p_l) \\
& + \frac{2}{4!} \sum_{P(i,j,k,l)\in(3,4,5,6)} A_6^0(\hat{1}_g,i_g,\hat{2}_g,j_g,k_g,l_g) J_2^{(4)}(p_i,\ldots,p_l) \\
& + \frac{2}{4!} \sum_{P_C(i,j,k,l)\in(3,4,5,6)} A_6^0(\hat{1}_g,i_g,j_g,\hat{2}_g,k_g,l_g) J_2^{(4)}(p_i,\ldots,p_l) \Bigg)
\end{aligned}
$$

- three topologies according to position of the initial state gluons

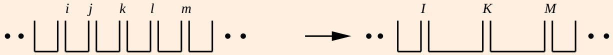
Counterterm

$$d\sigma^{S}_{NNLO} = d\sigma^{S,a}_{NNLO} + d\sigma^{S,b}_{NNLO} + d\sigma^{S,c}_{NNLO} + d\sigma^{S,d}_{NNLO} + d\sigma^{A}_{NNLO}$$

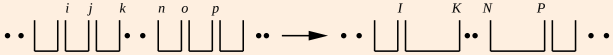
- (a) one unresolved parton → three parton antenna function X^0_{ijk}
- (b) two colour-connected unresolved partons → four parton antenna function X^0_{ijkl}



- (c) two almost colour-unconnected unresolved partons → strongly ordered product of non-independent three parton antenna functions



- (d) two colour-unconnected unresolved partons → product of independent three parton antenna functions



- (A) subtracts large angle soft radiation

Counterterm - IIFFF topology

$$\mathrm{d}\sigma^{R}_{NNLO} = N^2\ N_{born} \left(\frac{\alpha_s}{2\pi}\right)^2 \mathrm{d}\Phi_4(p_3,\ldots,p_6;p_1,p_2)\,\frac{2}{4!} \sum_{P(3,4,5,6)} A_6^0(\hat{1}_g,\hat{2}_g,i_g,j_g,k_g,l_g) J_2^{(4)}(p_i,\ldots,p_l)$$

$$\mathrm{d}\sigma^{S,b}_{NNLO} = N^2\ N_{born} \left(\frac{\alpha_s}{2\pi}\right)^2 \mathrm{d}\Phi_4(p_3,\ldots,p_6;p_1,p_2)\,\frac{2}{4!} \Bigg($$

$$\Big(F_4^0(\hat{2}_g,i_g,j_g,k_g) - f_3^0(\hat{2}_g,i_g,j_g) F_3^0(\hat{\hat{2}}_g,\widetilde{(ij)}_g,k_g) - f_3^0(i_g,j_g,k_g) F_3^0(\hat{2}_g,\widetilde{(ij)}_g,\widetilde{(jk)}_g)$$

$$-f_3^0(j_g,k_g,\hat{2}_g) F_3^0(\hat{\hat{2}}_g,i_g,\widetilde{(kj)}_g)\Big) A_4^0(\hat{1}_g,\hat{\hat{2}}_g,\widetilde{(ijk)}_g,l_g) J_2^{(2)}(\widetilde{p_{ijk}},p_l))$$

$$+\Big(F_{4,a}^0(i_g,j_g,k_g,l_g) - f_3^0(i_g,j_g,k_g) f_3^0(\widetilde{(\widetilde{ij})_g},\widetilde{(\widetilde{jk})_g},l_g) - f_3^0(j_g,k_g,l_g) f_3^0(i_g,\widetilde{(jk)}_g,\widetilde{(kl)}_g)\Big)$$

$$A_4^0(\hat{1}_g,\hat{2}_g,\widetilde{(ijk)}_g,\widetilde{(lkj)}_g) J_2^{(2)}(\widetilde{p_{ijk}},\widetilde{p_{lkj}})$$

$$+\Big(F_{4,b}^0(i_g,j_g,k_g,l_g) - f_3^0(i_g,j_g,k_g) f_3^0(\widetilde{(\widetilde{ij})}_g,l,\widetilde{(jk)}_g)\Big) A_4^0(\hat{1}_g,\hat{2}_g,\widetilde{(ijl)}_g,\widetilde{(klj)}_g) J_2^{(2)}(\widetilde{p_{ijl}},\widetilde{p_{klj}})$$

$$+\Big(F_4^0(\hat{1}_g,l_g,k_g,j_g) - f_3^0(\hat{1}_g,l_g,k_g) F_3^0(\hat{\hat{1}}_g,\widetilde{(lk)}_g,j_g) - f_3^0(l_g,k_g,j_g) F_3^0(\hat{1}_g,\widetilde{(lk)}_g,\widetilde{(kj)}_g)$$

$$-f_3^0(k_g,j_g,\hat{1}_g) F_3^0(\hat{\hat{1}}_g,l_g,\widetilde{(jk)}_g)\Big) A_4^0(\hat{\hat{1}}_g,\hat{2}_g,i_g,\widetilde{(lkj)}_g) J_2^{(2)}(p_i,\widetilde{p_{lkj}}) + \text{cyclic} + \text{l.reversal} + \ldots \Bigg)$$

Counterterm - IFIFFF topology

$$\mathrm{d}\sigma^{R}_{NNLO} = N^2\, N_{born} \left(\frac{\alpha_s}{2\pi}\right)^2 \mathrm{d}\Phi_4(p_3,\ldots,p_6;p_1,p_2)\,\frac{2}{4!} \sum_{P(3,4,5,6)} A_6^0(\hat{1}_g,i_g,\hat{2}_g,j_g,k_g,l_g) J_2^{(4)}(p_i,\ldots,p_l)$$

$$\begin{aligned}
\mathrm{d}\sigma^{S,b}_{NNLO} = {} & N^2\, N_{born} \left(\frac{\alpha_s}{2\pi}\right)^2 \mathrm{d}\Phi_4(p_3,\ldots,p_6;p_1,p_2)\,\frac{2}{4!} \Bigg(\\
& \Big(F_4^0(\hat{2}_g,j_g,k_g,l_g) - f_3^0(\hat{2}_g,j_g,k_g)\, F_3^0(\hat{\hat{2}}_g,\widetilde{(jk)}_g,l_g) - f_3^0(j_g,k_g,l_g)\, F_3^0(\hat{2}_g,\widetilde{(jk)}_g,\widetilde{(kl)}_g) \\
& - f_3^0(k_g,l_g,\hat{2}_g)\, F_3^0(\hat{\hat{2}}_g,j_g,\widetilde{(kl)}_g)\Big) A_4^0(\hat{1}_g,i_g,\hat{\hat{2}}_g,\widetilde{(jkl)}_g)\, J_2^{(2)}(p_i,\widetilde{p_{jkl}}) \\
& + \Big(F_4^0(\hat{1}_g,l_g,k_g,j_g) - f_3^0(\hat{1}_g,l_g,k_g)\, F_3^0(\hat{\hat{1}}_g,\widetilde{(kl)}_g,j_g) - f_3^0(l_g,k_g,j_g)\, F_3^0(\hat{1}_g,\widetilde{(lk)}_g,\widetilde{(kj)}_g) \\
& - f_3^0(k_g,j_g,\hat{1}_g)\, F_3^0(\hat{\hat{1}}_g,l_g,\widetilde{(jk)}_g)\Big) A_4^0(\hat{\hat{1}}_g,i_g,\hat{2}_g,\widetilde{(lkj)}_g)\, J_2^{(2)}(p_i,\widetilde{p_{lkj}}) \\
& + \Big(F_4^0(\hat{1}_g,i_g,\hat{2}_g,j_g) - F_3^0(\hat{1}_g,3_g,\hat{2}_g)\, F_3^0(\hat{\hat{1}}_g,\hat{\hat{2}}_g,\tilde{j}_g) \\
& - F_3^0(\hat{2}_g,j_g,\hat{1}_g)\, F_3^0(\hat{\hat{1}}_g,\tilde{i}_g,\hat{\hat{2}}_g)\Big) A_4^0(\hat{\hat{1}}_g,\hat{\hat{2}}_g,\tilde{k}_g,\tilde{l}_g)\, J_2^{(2)}(\widetilde{p_k},\widetilde{p_l}) \\
& + \Big(F_4^0(\hat{1}_g,i_g,\hat{2}_g,l_g) - F_3^0(\hat{1}_g,3_g,\hat{2}_g)\, F_3^0(\hat{\hat{1}}_g,\hat{\hat{2}}_g,\tilde{l}_g) \\
& - F_3^0(\hat{2}_g,l_g,\hat{1}_g)\, F_3^0(\hat{\hat{1}}_g,\tilde{i}_g,\hat{\hat{2}}_g)\Big) A_4^0(\hat{\hat{1}}_g,\hat{\hat{2}}_g,\tilde{k}_g,\tilde{j}_g)\, J_2^{(2)}(\widetilde{p_k},\widetilde{p_j}) + \ldots \Bigg)
\end{aligned}$$

Counterterm - IFFIFF topology

$$d\sigma^{R}_{NNLO} = N^2\ N_{born} \left(\frac{\alpha_s}{2\pi}\right)^2 d\Phi_4(p_3,\ldots,p_6;p_1,p_2)\,\frac{2}{4!}\sum_{P(3,4,5,6)} A_6^0(\hat{1}_g,i_g,j_g,\hat{2}_g,k_g,l_g)J_2^{(4)}(p_i,\ldots,p_l)$$

$$d\sigma^{S,b}_{NNLO} = N^2\ N_{born} \left(\frac{\alpha_s}{2\pi}\right)^2 d\Phi_4(p_3,\ldots,p_6;p_1,p_2)\,\frac{2}{4!}\Bigg($$

$$\Big(F_4^0(\hat{1}_g,i_g,j_g,\hat{2}_g) - f_3^0(\hat{1}_g,i_g,j_g)\,F_3^0(\hat{\hat{1}}_g,\widetilde{(ij)}_g,\hat{2}_g) - f_3^0(i_g,j_g,\hat{2}_g)\,F_3^0(\hat{1}_g,\widetilde{(ji)}_g,\hat{\hat{2}}_g)\Big)$$

$$A_4^0(\hat{\hat{1}}_g,\hat{\hat{2}}_g,\tilde{k}_g,\tilde{l}_g)\,J_2^{(2)}(\widetilde{p_k},\widetilde{p_l})$$

$$+\Big(F_4^0(\hat{2}_g,k_g,l_g,\hat{1}_g) - f_3^0(\hat{2}_g,k_g,l_g)\,F_3^0(\hat{\hat{2}}_g,\widetilde{(kl)}_g,\hat{1}_g) - f_3^0(k_g,l_g,\hat{2}_g)\,F_3^0(\hat{1}_g,\widetilde{(lk)}_g,\hat{\hat{2}}_g)\Big)$$

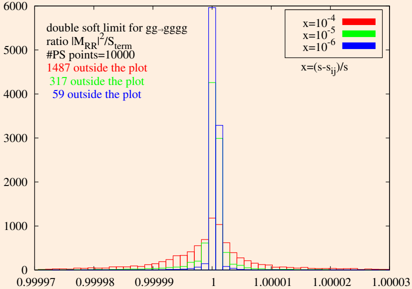
$$A_4^0(\hat{\hat{1}}_g,\hat{\hat{2}}_g,\tilde{j}_g,\tilde{i}_g)\,J_2^{(2)}(\widetilde{p_j},\widetilde{p_i})$$

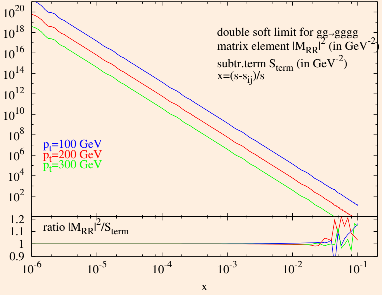
$$+\Big(F_4^0(\hat{1}_g,j_g,\hat{2}_g,k_g) - F_3^0(\hat{1}_g,j_g,\hat{2}_g)\,F_3^0(\hat{\hat{1}}_g,\hat{\hat{2}}_g,\tilde{k}_g) - F_3^0(\hat{2}_g,k_g,\hat{1}_g)\,F_3^0(\hat{\hat{1}}_g,\tilde{j}_g,\hat{\hat{2}}_g)\Big)$$

$$A_4^0(\hat{\hat{1}}_g,\tilde{i}_g,\hat{\hat{2}}_g,\tilde{l}_g)\,J_2^{(2)}(\widetilde{p_i},\widetilde{p_l})$$

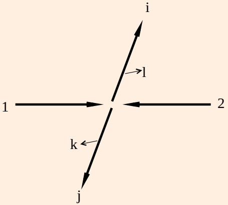
$$+\ldots\Bigg)$$

Double soft limit

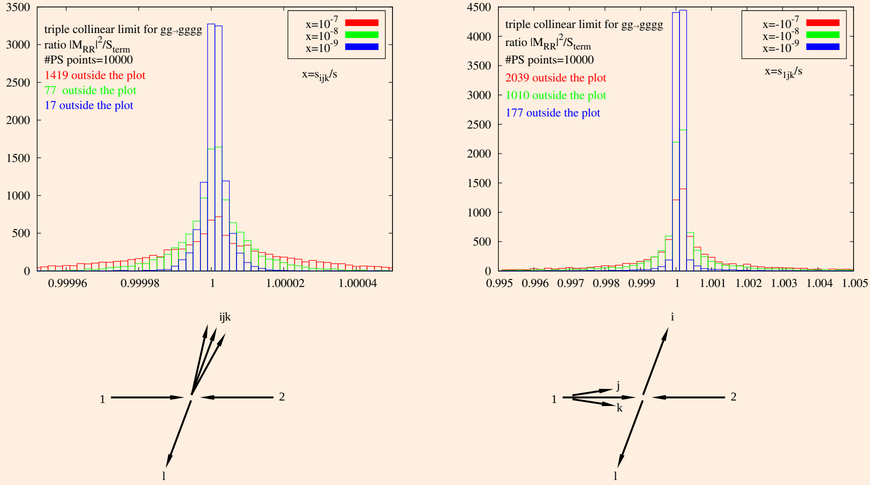




- double k, l soft limit when $s_{ij} \approx s$
- infrared behaviour of subtraction term coincides with the matrix element
- $S_{term} \xrightarrow{l_g, k_g \to 0} |M_{gg \to gggg}|^2$

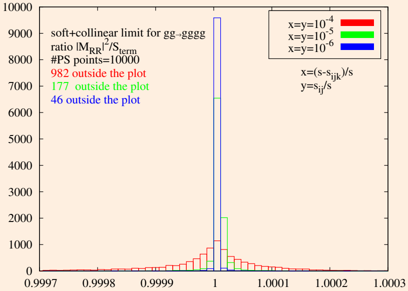


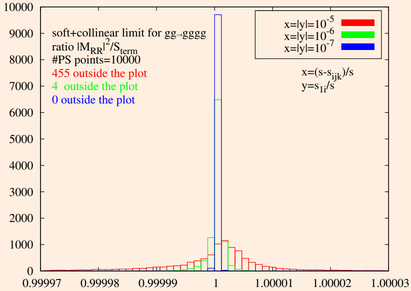
Triple collinear limit

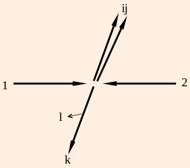


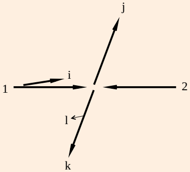
- generate phase space points with small triple invariant s_{ijk} or s_{1jk} mass

Soft and collinear limit



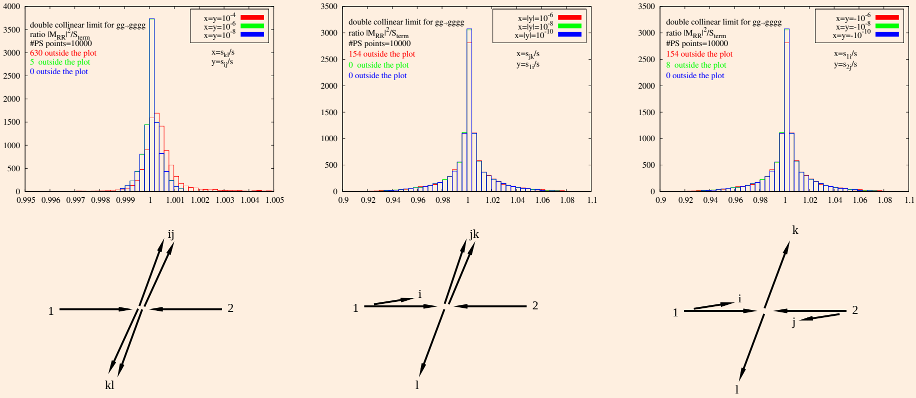






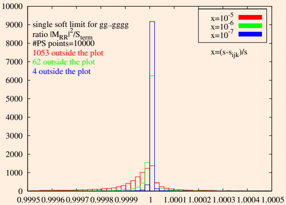
- generate phase space points with $s_{ijk} \approx s$ and $s_{ij} \to 0$ or $s_{1i} \to 0$

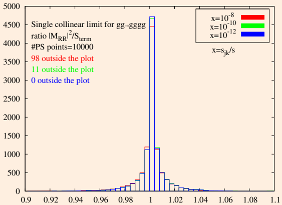
Double collinear limit

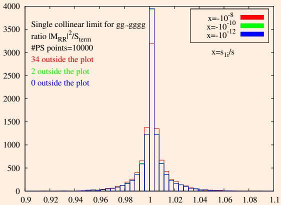


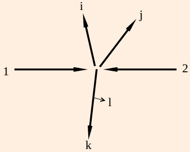
- generate phase space points with two independent $s_{ij}, s_{kl} \to 0$ simultaneously

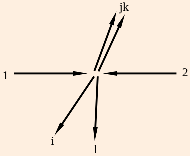
Singly unresolved limits

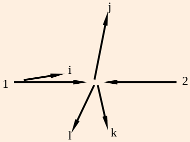












- generate phase space points with $s_{ijk} \approx s$
- $\mathrm{d}\sigma^{A}_{NNLO}$ subtracts large angle soft gluon radiation
- generate phase space points with s_{jk} or s_{1i} small

Conclusions

$$
\begin{aligned}
\mathrm{d}\sigma_{NNLO} &= \int_{\mathrm{d}\Phi_{m+2}} \left(\mathrm{d}\sigma^{R}_{NNLO} - \mathrm{d}\sigma^{S}_{NNLO}\right) + \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\sigma^{S}_{NNLO} \\
&+ \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^{V,1}_{NNLO} - \mathrm{d}\sigma^{VS,1}_{NNLO}\right) + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{VS,1}_{NNLO} \\
&+ \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\sigma^{V,2}_{NNLO}
\end{aligned}
$$

- $\mathrm{d}\sigma^{S}_{NNLO}$ for gluon channel [this talk]
- infrared structure of the double real (NNLO) emission written in terms antenna functions
- $\int_{\mathrm{d}\Phi_{m+2}} \left(\mathrm{d}\sigma^{R}_{NNLO} - \mathrm{d}\sigma^{S}_{NNLO}\right)$ finite and integrable in four dimensions
- $\int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\sigma^{S}_{NNLO}$ becomes possible with new results [G. Luisioni talk],[R. Boughezal talk]

Conclusions

Future work:

- derive mixed real-virtual counterterm
- go beyond leading colour approximation
- include remaining channels:
  - 4g2q processes
  - 2g4q processes
  - 6q processes