NNLO Antenna Subtraction with One Hadronic Initial State

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Motivation:

- Tevatron and LHC: machines for QCD precision physics
  $\Rightarrow$ new discovery potential related to how good we understand what we already know
- For precise predictions we need a precise determination of
  - strong coupling constant α_s
  - parton distributions
  - quark masses
  - ...
- Need for higher order calculations: NLO, NNLO, ...

Subtraction at NLO

- For an m-jet cross section, need to integrate numerically over phase space:
  - LO:

$$d\sigma_{\rm LO} = \int_{d\Phi_m} d\sigma_{\rm tree}$$

divergent numerical integral

  - NLO:

$$d\sigma_{\rm NLO} = \int_{d\Phi_{m+1}} d\sigma^{\rm R}_{\rm NLO} + \int_{d\Phi_m} d\sigma^{V}_{\rm NLO}$$

Problem: same divergent structure as virtual part but summation occur only after phase space integration

Subtraction at NLO

- For an m-jet cross section, need to integrate numerically over phase space:
  - LO:

$$d\sigma_{\rm LO} = \int_{d\Phi_m} d\sigma_{\rm tree}$$

  - NLO:

Local counter term integral

$$d\sigma_{\rm NLO} = \int_{d\Phi_{m+1}} \left(d\sigma^{\rm R}_{\rm NLO} - d\sigma^{\rm S}_{\rm NLO}\right) + \left[\int_{d\Phi_{m+1}} d\sigma^{\rm S}_{\rm NLO} + \int_{d\Phi_m} d\sigma^{V}_{\rm NLO}\right]$$

Solution: Introduce subtraction term which reproduces $\sigma^{R}_{\rm NLO}$ in all singular limits, and can be integrated analytically

[Z. Kunszt, D. Soper]

General subtraction methods at NLO

- At NLO different subtraction methods exists
  - Dipole subtraction:

    S. Catani, M. Seymour

    NNLO: S. Weinzierl
  - ε-prescription:

    S. Frixione, Z. Kunszt, A. Signer

    NNLO: S. Frixione, M. Grazzini; V. Del Duca, G. Somogy, Z. Trocsanyi
  - Antenna Subtraction:

    D. Kosower; J. Campbell, M. Cullen, N. Glover; A. Daleo, D. Maître, T. Gehrmann

    NNLO: A. Gehrmann-De Ridder, N. Glover, T. Gehrmann

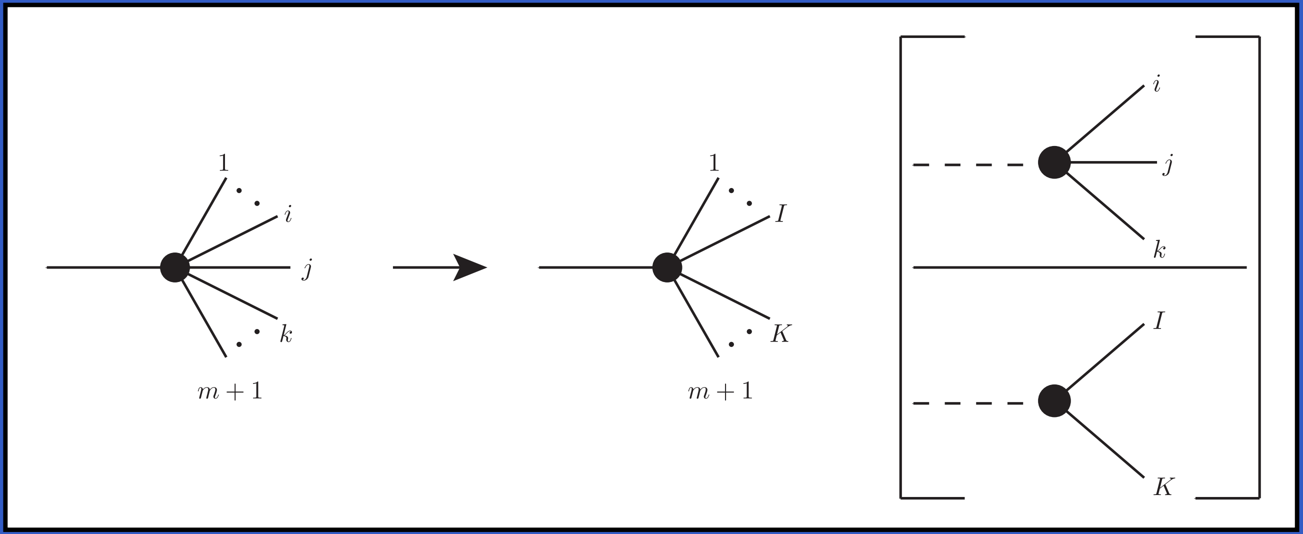
NLO antenna subtraction

- How is $\mathrm{d}\sigma^{S}_{\mathrm{NLO}}$ constructed within the antenna frame work?

  It must satisfy:

$$\mathrm{d}\sigma^{R}_{\mathrm{NLO}} \xrightarrow{\text{soft \& collinear limit}} \mathrm{d}\sigma^{S}_{\mathrm{NLO}}$$

- Exploit factorization of phase space and matrix element in soft and coll. limit:



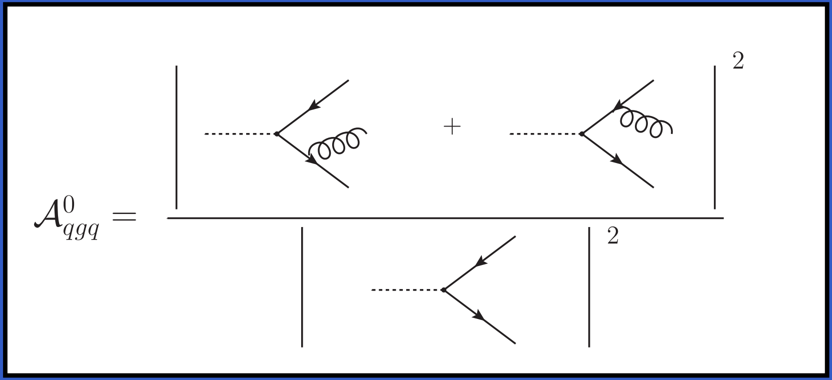
$$\sum_{m+1} \mathrm{d}\Phi_{m+1} |M_{m+1}|^2 J_m^{(m+1)} \longrightarrow \sum_{m+1} \mathrm{d}\Phi_m |M_m|^2 J_m^{(m)} \sum_j \mathrm{d}\Phi_{X^0_{ijk}} X^0_{ijk}$$

NLO antenna subtraction

- NLO antenna function X^0_{ijk} contains all soft and collinear configuration of parton j emitted between two hard color-connected partons i and k

$$X^0_{ijk} = S_{ijk,IK} \frac{\left|M^0_{ijk}\right|^2}{\left|M^0_{IK}\right|^2} \quad , \quad d\Phi_{X^0_{ijk}} = \frac{d\Phi_3}{P_2}$$

- Antennae computed from matrix elements of physical processes

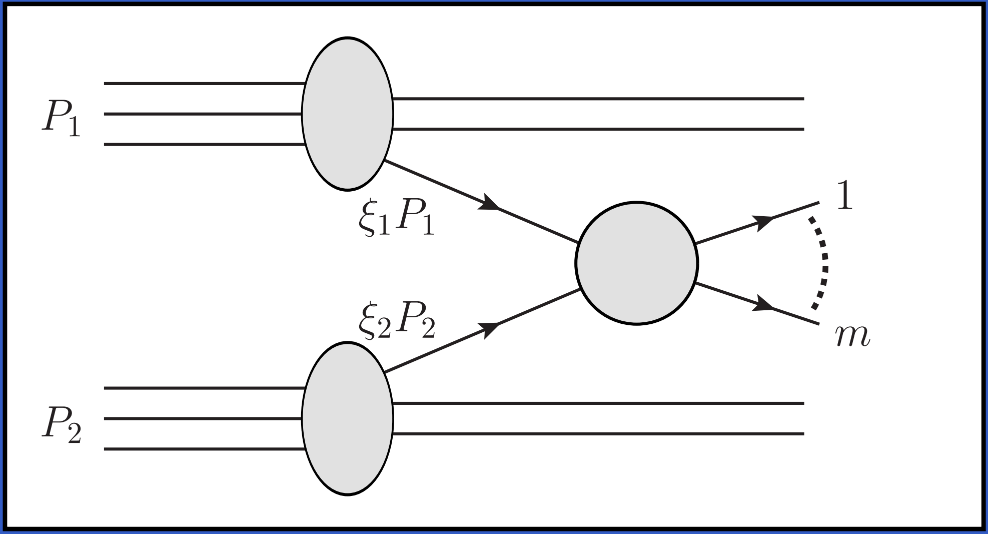


- Integrated subtraction term can be computed analytically

$$|M_m|^2\, J_m^{(m)} d\Phi_m \int d\Phi_{X^0_{ijk}}\, X^0_{ijk} \propto |M_m|^2\, J_m^{(m)} d\Phi_m \int d\Phi_3 \left|M^0_{ijk}\right|^2$$

Hadronic initial state

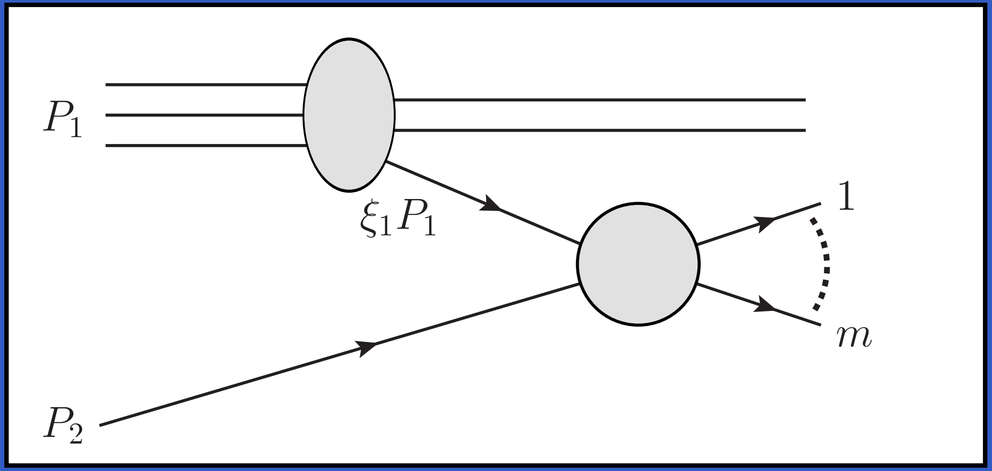
- Cross section for hadronic initial state: (pp,$p\bar{p}$)



$$d\sigma = \sum_{h_1,h_2,a,b} \int_0^1 \frac{d\xi_1}{\xi_1} \frac{d\xi_1}{\xi_2} f_a^{h_1}\left(\xi_1, \mu_F^2\right) f_b^{h_2}\left(\xi_2, \mu_F^2\right) d\hat{\sigma}_{ab}\left(\xi_1 P_1, \xi_2 P_2, \mu_F^2\right)$$

Hadronic initial state

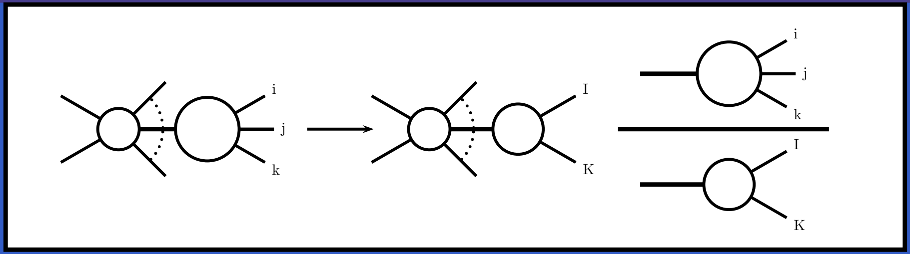
- Cross section for hadronic initial state: (ep)



$$\mathrm{d}\sigma = \sum_{h_1,a,b} \int_0^1 \frac{\mathrm{d}\xi_1}{\xi_1} \frac{\mathrm{d}\xi_1}{\xi_2} f_a^{h_1}\left(\xi_1, \mu_F^2\right) \delta\left(1-\xi_2\right) \mathrm{d}\hat{\sigma}_{ab}\left(\xi_1 P_1, \xi_2 P_2, \mu_F^2\right)$$

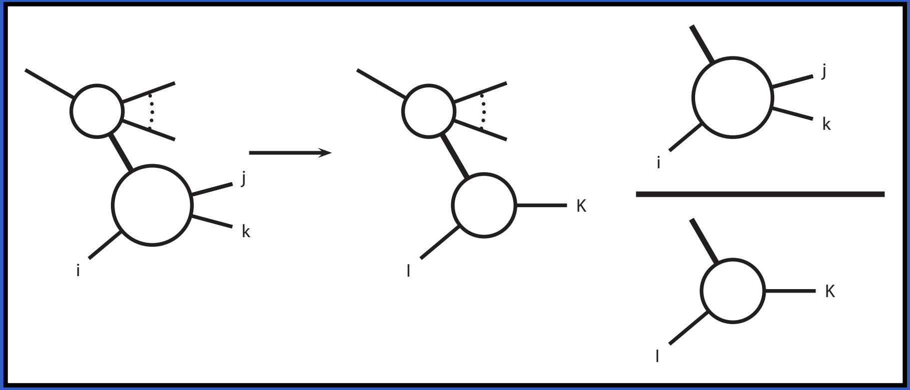
Hadronic initial state

- final-final:



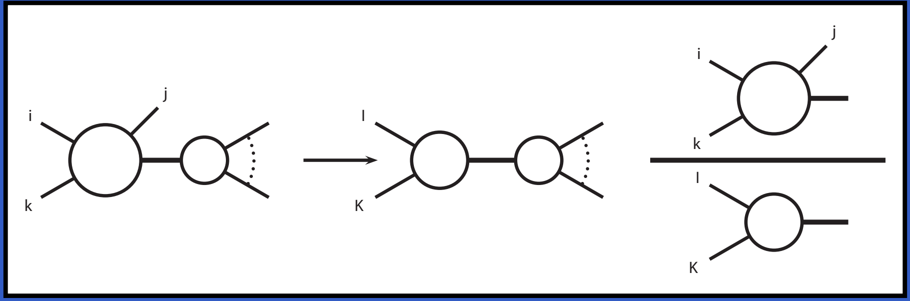
Applied to $e^+e^- \rightarrow$ 3 jets at NNLO [A. Gehrmann De-Ridder, T. Gehrmann, N. Glover, G. Heinrich; S. Weinzierl]

- initial-final:



Sufficient for e.g. DIS (2+1)-jet process [A. Daleo, T. Gehrmann, D. Maître]

- initial-initial:



Needed for vector boson plus jet production [A. Daleo, T. Gehrmann, D. Maître] [R. Boughezal, A. Gehrmann De-Ridder, M. Ritzmann]

m-jet cross section

n-parton contribution to the m-jet cross section ($p = \xi_1 P_1$, $r = \xi_2 P_2$):

$$\mathrm{d}\hat{\sigma}_{ab}^{i}\left(p,r\right) = \mathcal{N} \sum_{n} \mathrm{d}\Phi_n\left(k_1,\ldots,k_n;p,r\right) \frac{1}{S_n} \left|\mathcal{M}_n\left(k_1,\ldots,k_n;p,r\right)\right|^2 J_m^{(n)}\left(k_1,\ldots,k_n\right)$$

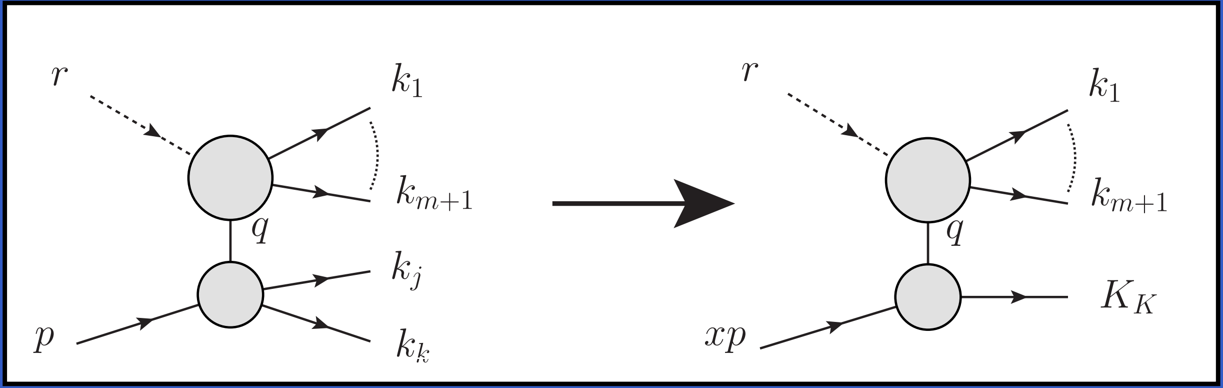
- LO: $n = m$
- NLO: $n = m+1$

- Subtraction term for initial-final singularity:

$$\mathrm{d}\hat{\sigma}^{S(if)} = \mathcal{N} \sum_{m+1} \mathrm{d}\Phi_{m+1}\left(k_1,\ldots,k_{m+1};p,r\right) \frac{1}{S_{m+1}} \times \sum_{j} X_{i,jk}^{0} \left|\mathcal{M}_m\left(k_1,\ldots,k_{m+1};xp,r\right)\right|^2 J_m^{(m)}\left(k_1,\ldots,k_{m+1}\right)$$

I-F NLO phase space factorization

- Kinematics is now: $q + p \longrightarrow k_j + k_k \quad \Rightarrow \quad q + xp \longrightarrow K_K$

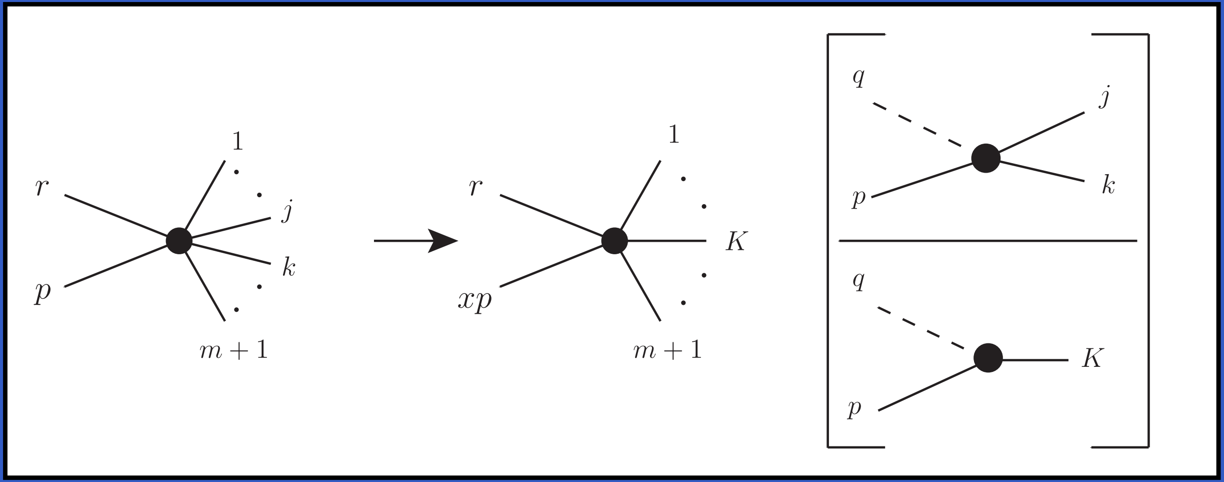


- Limits:
  - $xp \longrightarrow p \qquad K_K \longrightarrow k_k \qquad$ when j soft
  - $xp \longrightarrow p \qquad K_K \longrightarrow k_j + k_k \qquad$ when $j \,||\, k$
  - $xp \longrightarrow p - k_j \qquad K_K \longrightarrow k_k \qquad$ when $j \,||\, i$
- Phase space factorization for $m+1$ particles:

$$\mathrm{d}\Phi_{m+1}(k_1,\ldots,k_{m+1};p,r) = \mathrm{d}\Phi_m(k_1,\ldots,K_K,\ldots,k_{m+1};xp,r) \times \frac{Q^2}{2\pi}\mathrm{d}\Phi_2(k_j,k_k;p,q)\frac{\mathrm{d}x}{x}$$

I-F NLO matrix element factorization

- Obtain antennae functions by crossing final-final NLO antennae



$$\sum_{m+1} d\Phi_{m+1} |M_{m+1}|^2 J_m^{(m+1)} \longrightarrow \sum_{m+1} d\Phi_m |M_m|^2 J_m^{(m)} \sum_j \frac{Q^2}{2\pi} d\Phi_2 \frac{dx}{x} X_{i,jk}^0$$

- Again integrated subtraction term can be computed analytically:

$$\mathcal{X}_{i,jk}^0(x) = \frac{1}{C(\epsilon)} \int d\Phi_2 \frac{Q^2}{2\pi} X_{i,jk}^0 \quad , \quad C(\epsilon) = (4\pi)^\epsilon \frac{e^{-\epsilon\gamma_E}}{8\pi^2}$$

[A. Daleo, T. Gehrmann, D. Maître]

NLO integrated subtraction term

- Integrated subtraction term has to be convoluted with PDFs
- Make change of variable and obtain

$$\mathrm{d}\sigma^{S(if)}(p,r) = \sum_{m+1}\sum_{j} \frac{S_m}{S_{m+1}} \int \frac{\mathrm{d}\xi_1}{\xi_1} \int \frac{\mathrm{d}\xi_2}{\xi_2} \int_{\xi_1}^{1} \frac{\mathrm{d}x}{x} f_a^{h_1}\left(\frac{\xi_1}{x}\right) f_b^{h_2}(\xi_2) \times C(\epsilon)\, \mathcal{X}^0_{i,jk}(x)\, \mathrm{d}\hat{\sigma}^B(\xi_1 P_1, \xi_2 P_2)$$

- Mass factorization can be carried out
- Phase space integration in $\mathrm{d}\hat{\sigma}^B$ and convolutions can be done numerically

Subtraction at NNLO

- Structure of NNLO m-jet cross section

$$
\begin{aligned}
\mathrm{d}\sigma_{\mathrm{NNLO}} = & \int_{\mathrm{d}\Phi_{m+2}} \left(\mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{NNLO}} - \mathrm{d}\sigma^{\mathrm{S}}_{\mathrm{NNLO}} \right) + \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\sigma^{\mathrm{S}}_{\mathrm{NNLO}} \\
& + \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^{\mathrm{V,1}}_{\mathrm{NNLO}} - \mathrm{d}\sigma^{\mathrm{VS,1}}_{\mathrm{NNLO}} \right) + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{\mathrm{VS,1}}_{\mathrm{NNLO}} \\
& + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\sigma^{\mathrm{V,2}}_{\mathrm{NNLO}} .
\end{aligned}
$$

  - $\mathrm{d}\sigma^{S}_{\mathrm{NNLO}}$: real radiation subtraction term for $d\sigma^{R}_{\mathrm{NNLO}}$
  - $\mathrm{d}\sigma^{VS,1}_{\mathrm{NNLO}}$: one loop real subtraction term for $d\sigma^{V,1}_{NNLO}$
  - $\mathrm{d}\sigma^{V,2}_{\mathrm{NNLO}}$: two loop virtual corrections

Each column is numerically finite and free of IR ϵ-poles

NNLO double real subtraction

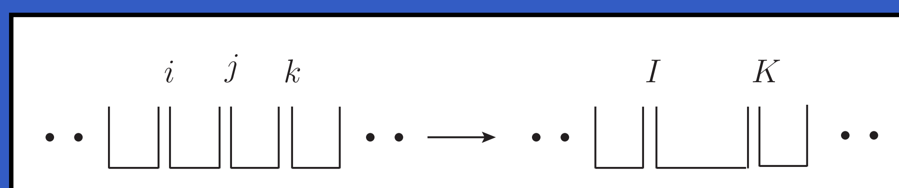
- $d\sigma^S_{\rm NNLO}$: double real subtraction $\rightarrow$ different configurations

$d\sigma^S_{\rm NNLO}$ →

- (a) one unresolved parton
- (b) two color-connected unresolved partons
- (c) two almost color-connected unresolved partons
- (d) two color-unconnected unresolved partons

[A. Gehrmann De-Ridder, T. Gehrmann, N. Glover]

(a): one unresolved parton:



- one unresolved parton but the experimental observable selects only m jets,
- three parton antenna function $X^0_{i,jk}$ can be used (like at NLO)

NNLO double real subtraction

- $d\sigma^{S}_{\mathrm{NNLO}}$: double real subtraction → different configurations

$d\sigma^{S}_{\mathrm{NNLO}}$ →

- (a) one unresolved parton
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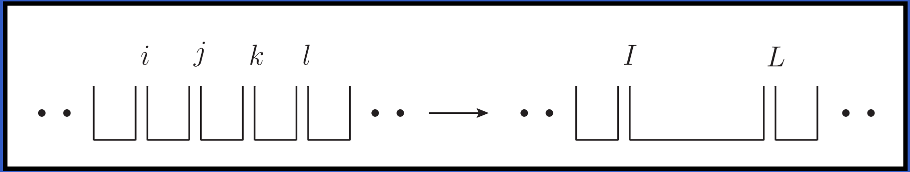
[A. Gehrmann De-Ridder, T. Gehrmann, N. Glover]

(b): two color-connected unresolved partons:



- four parton antenna function $X^{0}_{i,jkl}$
- complete set of four parton antennae for i-f configuration is now available

NNLO double real subtraction

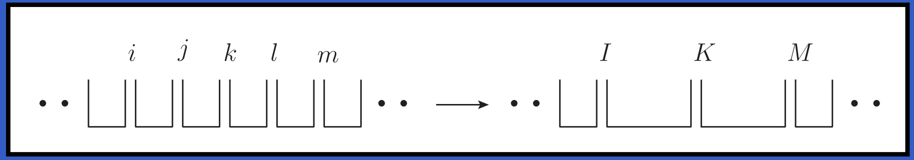
- $d\sigma^S_{\mathrm{NNLO}}$: double real subtraction $\longrightarrow$ different configurations

$d\sigma^S_{\mathrm{NNLO}}$ →

- (a) one unresolved parton
- (b) two color-connected unresolved partons
- (c) two almost color-connected unresolved partons
- (d) two color-unconnected unresolved partons

[A. Gehrmann De-Ridder, T. Gehrmann, N. Glover]

(c): two almost color-connected unresolved partons:



- share a common radiator
- accounted for by products of two tree-level three-parton antennae functions
- distinguish cases where common radiator is in the initial or final configuration

NNLO double real subtraction

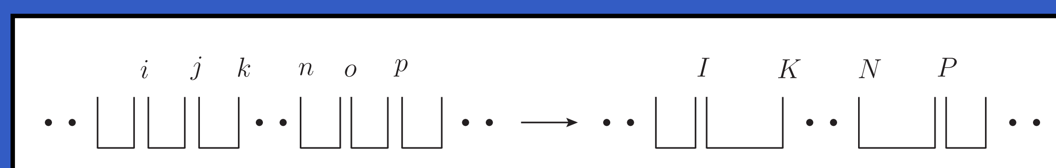
- $d\sigma^{S}_{\mathrm{NNLO}}$: double real subtraction $\rightarrow$ different configurations

$d\sigma^{S}_{\mathrm{NNLO}}$ →

- (a) one unresolved parton
- (b) two color-connected unresolved partons
- (c) two almost color-connected unresolved partons
- (d) two color-unconnected unresolved partons

[A. Gehrmann De-Ridder, T. Gehrmann, N. Glover]

(d): two color-unconnected unresolved partons:



- two well separated partons in the colour chain
- product of independent three-parton antenna functions

NNLO Antenna subtraction

- $d\sigma^{VS,1}_{\mathrm{NNLO}}$: one loop real subtraction $\longrightarrow$ several requirements

$d\sigma^{VS,1}_{\mathrm{NNLO}}$ →
- (a) remove explicit IR poles from loop
- (b) subtract single unresolved limits
- (c) remove oversubtracted terms

(a): remove poles from loop integral: [A. Gehrmann De-Ridder, T. Gehrmann, N. Glover]

- virtual correction has IR poles which have to be removed by means of the real counterpart
- subtraction term contains integrated antenna $\mathcal{X}^0_{i,jk}$

NNLO Antenna subtraction

- $d\sigma^{VS,1}_{\rm NNLO}$: one loop real subtraction $\longrightarrow$ several requirements

$d\sigma^{VS,1}_{\rm NNLO}$ →
- (a) remove explicit IR poles from loop
- (b) subtract single unresolved limits
- (c) remove oversubtracted terms

(b): subtraction of single unresolved limits:

[A. Gehrmann De-Ridder, T. Gehrmann, N. Glover]

- subtraction of singular configurations originating when the real radiation correction to the one loop amplitude becomes soft or collinear.
- subtraction term is a combination of three-parton tree-level $X^0_{i,jk}$ and three parton one-loop $X^1_{i,jk}$ antenna functions.

NNLO Antenna subtraction

- $d\sigma^{VS,1}_{\mathrm{NNLO}}$: one loop real subtraction $\longrightarrow$ several requirements

$d\sigma^{VS,1}_{\mathrm{NNLO}}$

(a) remove explicit IR poles from loop

(b) subtract single unresolved limits

(c) remove oversubtracted terms

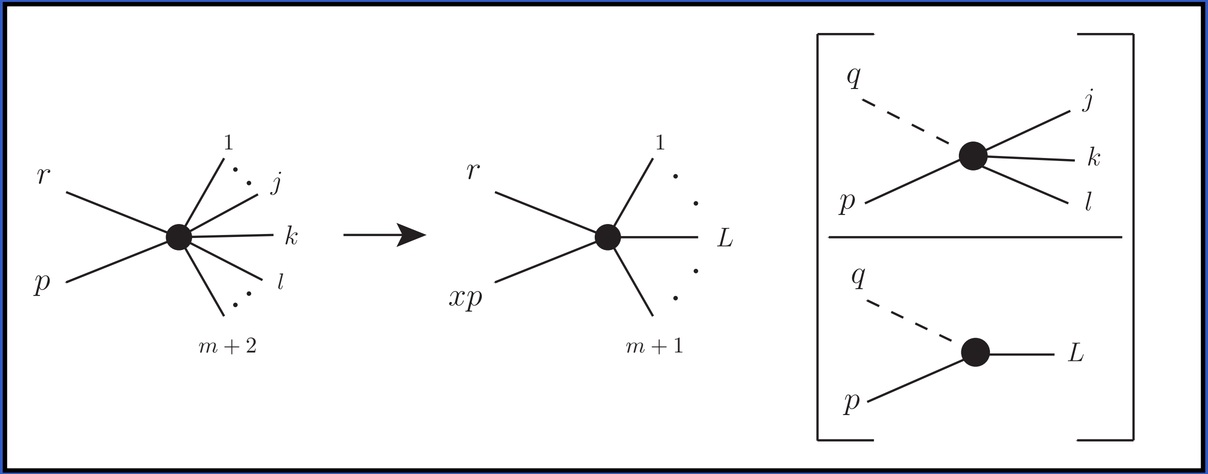
[A. Gehrmann De-Ridder, T. Gehrmann, N. Glover]

(c): remove oversubtracted terms:

- remove terms which are common to both previous contributions and are oversubtracted
- subtraction term contains initial-final and final-final antenna

I-F NNLO: double real radiation

- Obtain antennae functions by crossing final-final NNLO antennae



- Phase space factorization similar to NLO, with one particle more

$$d\Phi_{m+2}(k_1,\ldots,k_j,k_k,k_l,\ldots,k_{m+2};p,r) =$$
$$d\Phi_m(k_1,\ldots,K_L,\ldots,k_{m+2};xp,r)\,\frac{Q^2}{2\pi}\,d\Phi_{X_{i,jkl}}(k_j,k_k,k_l,p,q)\,\frac{dx}{x}$$

- Again integrated subtraction term can be computed analytically

- $2 \longrightarrow 3$ particle phase space

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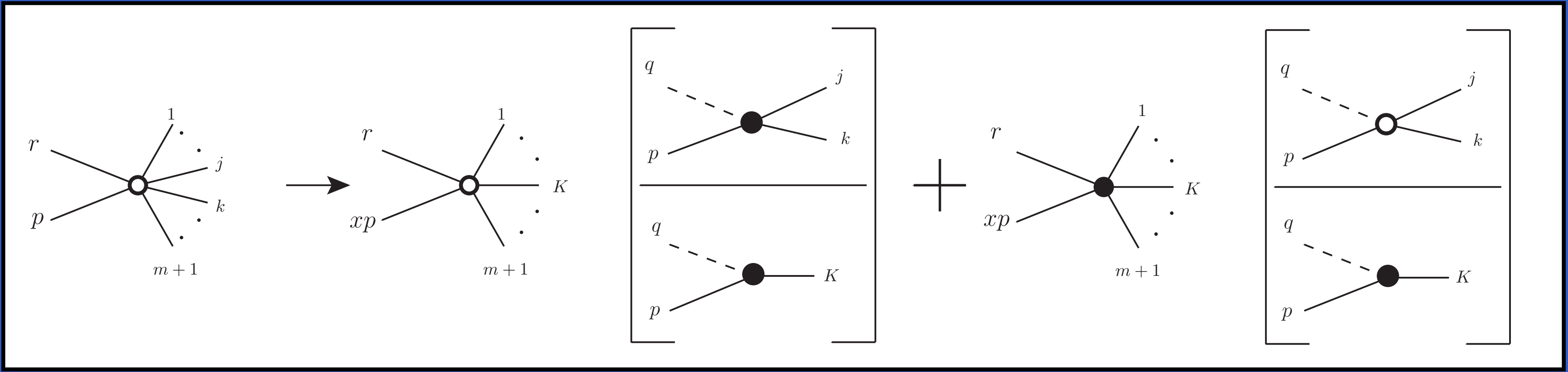
I-F NNLO: one-loop real radiation



- Single unresolved limit of 1-loop amplitude:

$$Loop_{m+1} \stackrel{j \text{ unresolved}}{\longrightarrow} Split_{tree} \times Loop_m + Split_{loop} \times Tree_m$$

[Z. Bern, L.D. Dixon, D. Dunbar, D. Kosower; S. Catani, M. Grazzini; D. Kosower, P. Uwer]
[Z. Bern, V. Del Duca, W.B. Kilgore, C.R. Schmidt]
[Z. Bern, L.D. Dixon, D. Kosower; S. Badger, E.W.N. Glover]

- Thus: $$X^1_{i,jk} = S_{i,jk;I,K} \frac{\left|\mathcal{M}^1_{i,jk}\right|^2}{\left|\mathcal{M}^0_{I,K}\right|^2} - X^0_{i,jk} \frac{\left|\mathcal{M}^1_{I,K}\right|^2}{\left|\mathcal{M}^0_{I,K}\right|^2}$$

Initial-final antenna functions

$q \to gq$	$A^0_{q,gq}$	$A^1_{q,gq}$ $\tilde{A}^1_{q,gq}$ $\hat{A}^1_{q,gq}$
$q \to ggq$	$A^0_{q,ggq}$ $\tilde{A}^0_{q,ggq}$	
$q \to q'\bar{q}'q$	$B^0_{q,q'\bar{q}'q}$ $\bar{B}^0_{q,q'\bar{q}'q}$	
$q \to q\bar{q}q$	$C^0_{q,q\bar{q}q}$ $C^0_{\bar{q},\bar{q}q\bar{q}}$ $C^0_{\bar{q},q\bar{q}\bar{q}}$	
$q \to gg$	$D^0_{q,gg}$	$D^1_{q,gg}$ $\hat{D}^1_{q,gg}$
$q \to ggg$	$D^0_{q,ggg}$	
$q \to q'\bar{q}'$	$E^0_{q,q'\bar{q}'}$	$E^1_{q,q'\bar{q}'}$ $\tilde{E}^1_{q,q'\bar{q}'}$ $\hat{E}^1_{q,q'\bar{q}'}$
$q \to q'\bar{q}'g$	$E^0_{q,q'\bar{q}'g}$ $\tilde{E}^0_{q,q'\bar{q}'g}$	
$q \to qq'$	$E^0_{q,qq'}$	$E^1_{q,qq'}$ $\tilde{E}^1_{q,qq'}$ $\hat{E}^1_{q,qq'}$
$q \to qq'g$	$E^0_{q,qq'g}$ $\tilde{E}^0_{q,qq'g}$	
$q \to qg$	$G^0_{q,qg}$	$G^1_{q,qg}$ $\tilde{G}^1_{q,qg}$ $\hat{G}^1_{q,qg}$
$q \to qgg$	$G^0_{q,qgg}$ $\tilde{G}^0_{q,qgg}$	
$q \to qq'\bar{q}'$	$H^0_{q,qq'\bar{q}'}$	

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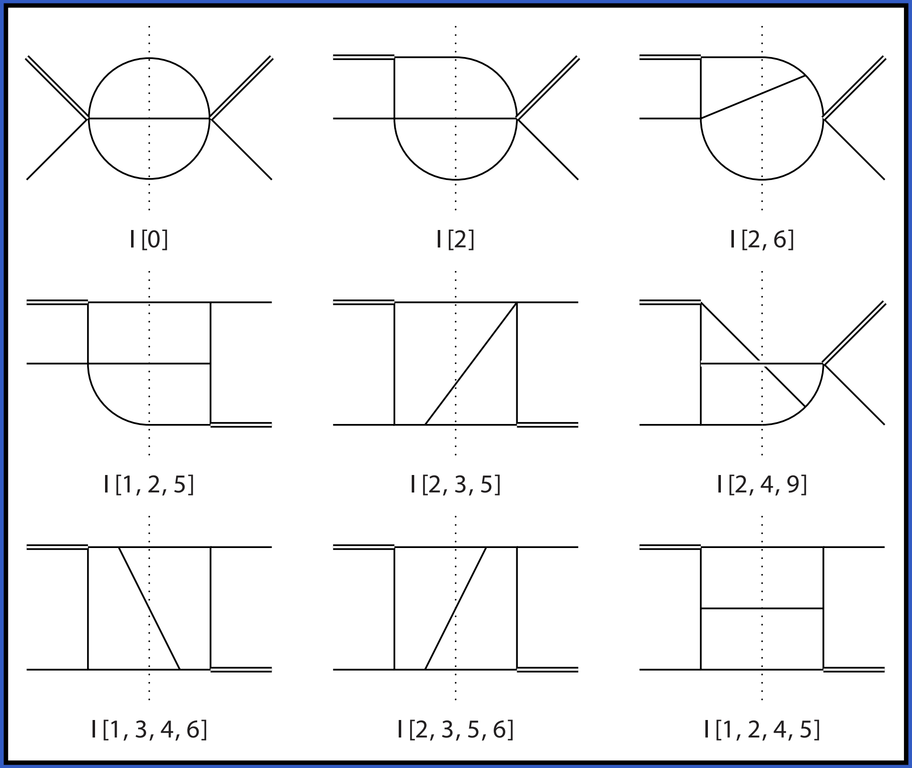
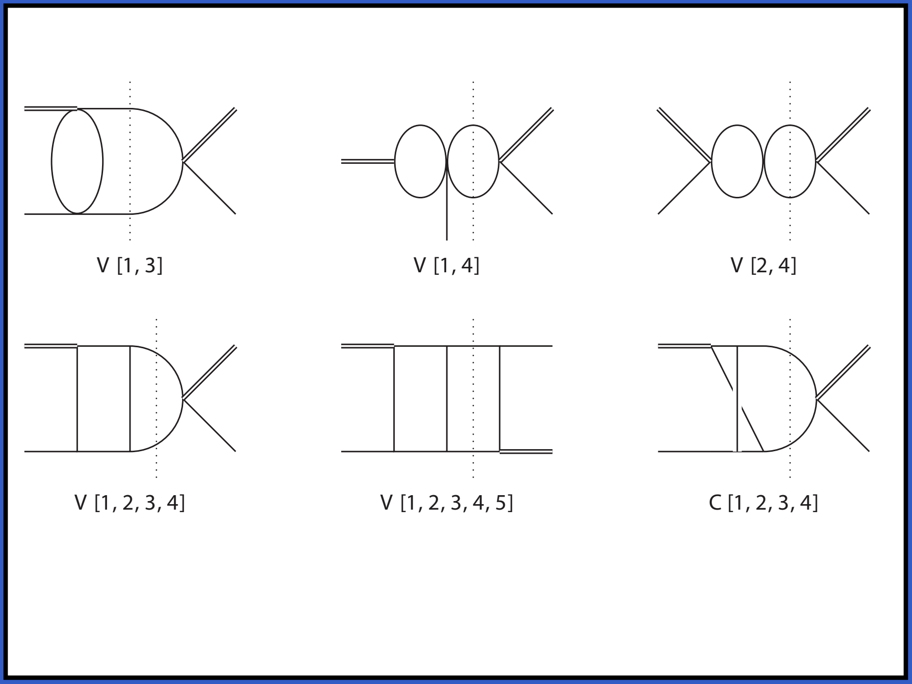
Integrated antenna computation

- Reduce phase space integrals to master integrals
- Integration over inclusive 2- or 3-particle phase space [C. Anastasiou, K. Melnikov]
- Functions of q^2 and $x = -\frac{q^2}{2\,p\cdot q}$

Computation of master integrals

- 9 real and 6 virtual masters:





Computation of master integrals

- Masters computed using differential equations
- Example: ($d = 4 - 2\epsilon$)

$$\begin{cases} x \frac{\partial I[2]}{\partial x} = & -\frac{d-4}{2} I[2] + \frac{3d-8}{2} \left(1 + \frac{1}{x-1}\right) \frac{I[0]}{Q^2} \\ Q^2 \frac{\partial I[2]}{\partial Q^2} = & (d-4)\, I[2] \qquad \Rightarrow I[2] \propto \left(Q^2\right)^{-2\epsilon} \end{cases}$$

  - boundary condition from explicit computation at $x = 1$
  - putting all together:

$$I[2] = \frac{2^{-7+4\epsilon}}{\pi^{3-2\epsilon}} \frac{\Gamma(1-\epsilon)^3}{\Gamma(3-3\epsilon)\,\Gamma(2-2\epsilon)} \frac{3\epsilon-2}{1-2\epsilon} (1-x)^{1-2\epsilon} x^{\epsilon} (Q^2)^{-2\epsilon} {}_2F_1(1-2\epsilon, 1-\epsilon, 2-2\epsilon, 1-x)$$

- For simple masters exact result in $\epsilon \longrightarrow$ expanded with HypExp

[T. Huber, D. Maître]

- For the others expansion up to needed power of ϵ

Check with DIS structure functions

- Completed full set of integrated $2 \to 3$ tree-level and $2 \to 2$ one-loop antennae
- Cross check with NNLO DIS structure functions
  - DIS cross section for photon exchange

$$\frac{d^2\sigma}{dx\,dy} = \frac{2\pi\alpha^2}{Q^4} s\left[\left(1+(1-y)^2\right) F_2(x,Q^2) - y^2 F_L(x,Q^2)\right]$$

  - Checks $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ type antenna functions
  - At NLO (before mass factorization) [E. Zijlstra, W. van Nerveen; S. Moch, J. Vermaseren, A. Vogt]

$$\frac{1}{C_f}\left(F_{2,q}^{(1)} - \frac{d-1}{d-2} F_{L,q}^{(1)}\right) = 4\mathcal{A}_{q,gq}^{0} + 8\delta\left(1-z\right) F_q^{(1)}$$

$$\frac{1}{d-2}\left(F_{2,g}^{(1)} - \frac{d-1}{d-2} F_{L,g}^{(1)}\right) = -4\mathcal{A}_{g,q\bar{q}}^{0}$$

Check with DIS structure functions

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  - At NLO (before mass factorization) [E. Zijlstra, W. van Nerveen; S. Moch, J. Vermaseren, A. Vogt]

$$\frac{1}{C_f}\left(F_{2,q}^{(1)} - \frac{d-1}{d-2} F_{L,q}^{(1)}\right) = \ldots (1-z)\, F_q^{(1)}$$

$$\frac{1}{d-2}\left(F_{2,g}^{(1)} - \ldots F_{L,g}^{(1)}\right) = -4\mathcal{A}^0_{g,q\bar{q}}$$

Full agreement!

Check with ϕ-DIS structure functions

- Structure functions for a scalar particle coupling only to gluons
- Permits to check integrated $\mathcal{F}$, $\mathcal{G}$ and $\mathcal{H}$-type antenna functions
  - DIS cross section for scalar exchange has only one structure function: $T_{\phi,i}$, for $i = q, g$
  - Some example

$$
\begin{aligned}
T^{(1)}_{\phi,g} &= 2N\,\mathcal{F}^0_{g,gg} + 2n_f\mathcal{G}_{g,q\bar{q}} + 4\,\delta\,(1-z)\,F^{(1)}_g \\
\frac{1}{C_f\,(1-\epsilon)}\,T^{(1)}_{\phi,q} &= -4N\,\mathcal{G}^0_{q,qg} \\
T^{(2)}_{\phi,g}\Big|_{N^2} &= \mathcal{F}^0_{g,ggg} + 4\mathcal{F}^1_{g,gg} + \delta\,(1-z)\left(8F^{(2)}_g + 4F^{(1)}_g\right)
\end{aligned}
$$

[S. Moch, J. Vermaseren, A. Vogt]

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$$T^{(1)}_{\phi,g} = 2N\,\mathcal{F}^{0}_{g,gg} + \ldots_{,q\bar{q}} + 4\,\delta\,(1-z)\,F^{(1)}_{g}$$

$$\frac{1}{C_f\,(1-\epsilon)}\,T^{(1)}_{\phi,q} = -4N\,\ldots$$

$$T^{(2)}_{\phi,g}\Big|_{N^2} = \ldots_{gg} + 4\mathcal{F}^{1}_{g,gg} + \delta\,(1-z)\left(8F^{(2)}_{g} + 4F^{(1)}_{g}\right)$$

Full agreement!

[S. Moch, J. Vermaseren, A. Vogt]

Conclusions and outlook

- Antenna subtraction scheme
  - subtraction method based on collecting all IR and collinear radiation between two pair of color connected hard partons
  - final-final case applied successfully at NNLO for $e^+e^- \to$ 3-jet
  - all ingredient for initial-final subtraction now available
  - cross check of initial-final antennae with DIS structure functions is completed
- Potential applications:
  - NNLO DIS (2+1)-jet production
  - contribution to hadron-collider jet production