NNLO QCD correction to vector boson production at hadron colliders.

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Based on:

S. Catani, L. Cieri, G. F., D. de Florian & M. Grazzini

Phys. Rev. Lett. 103: 082001, 2009 [arXiv:0903.2120]

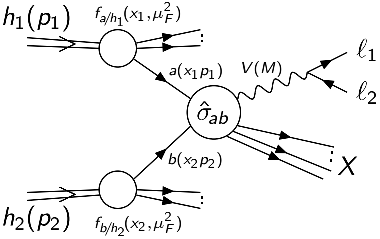
Outline

Motivations

The study of Drell-Yan lepton pair production is well motivated:

- Large production rates and clean experimental signatures:
  - Important for detector calibration.
  - Possible use as luminosity monitor.
- Transverse momentum distributions needed for:
  - Precise prediction for M_W.
  - Beyond the Standard Model analysis.
- Test of perturbative QCD predictions.
- Constrain for fits of PDFs.

The Drell-Yan process



$$h_1(p_1) + h_2(p_2) \;\rightarrow\; V(M) + X \;\rightarrow\; \ell_1 + \ell_2 + X$$

$$\textit{where} \quad V = \gamma^*, Z^0, W^\pm \quad \textit{and} \quad \ell_1\ell_2 = \ell^+\ell^-, \ell\nu_\ell$$

According to the QCD factorization theorem:

$$d\sigma(M,s) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 \; f_{a/h_1}(x_1, \mu_F^2)\, f_{b/h_2}(x_2, \mu_F^2)\, d\hat{\sigma}_{ab}(M, \hat{s}; \alpha_S, \mu_R^2, \mu_F^2).$$

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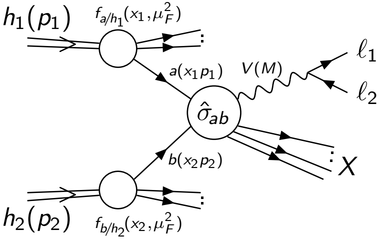

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State of the art: fixed order calculations

Historically the Drell-Yan process [Drell,Yan('70)] was the first application of parton model ideas developed for deep inelastic scattering.

- QCD corrections:
  - Total cross section known up to NNLO [Hamberg,Van Neerven,Matsuura('91)], [Harlander,Kilgore('02)]
  - Rapidity distribution known up to NNLO [Anastasiou,Dixon,Melnikov,Petriello('03)]
  - Fully exclusive NNLO calculation completed [Melnikov,Petriello('06)], [Catani,Cieri,de Florian,G.F.,Grazzini('09)]
  - Vector boson transverse-momentum distribution known up to NLO [Ellis,Martinelli,Petronzio('83)],[Arnold,Reno('89)], [Gonsalves,Pawlowski,Wai('89)]
- Electroweak correction are know at $\mathcal{O}(\alpha)$ [Dittmaier,Kramer('02)],[Baur,Wackeroth('02)], [Carloni Calame,Montagna,Nicrosini,Vicini('06)]

LO and NLO calculations

- In general LO calculations give the order of magnitude of cross sections and distributions, NLO corrections provide reliable estimate.
- At NLO the presence of infrared singularities in *real* and *virtual* corrections prevent the straightforward implementation of numerical techniques. This is a problem especially for fully exclusive quantities.
- The NLO subtraction method consists in the introduction of auxiliary QCD cross section *in a general way* exploiting the universality of the soft and collinear emission [Giele et al.('92),Frixione et al.('96), Catani,Seymour('97)]. It allows (relatively) straightforward calculations, once the QCD amplitudes are available

$$\sigma^{NLO} = \int_{m+1} d\sigma^R(\epsilon) \;+\; \int_m d\sigma^V(\epsilon)$$

$$= \int_{m+1} \left[d\sigma^R(\epsilon) - d\sigma^A(\epsilon) \right]_{\epsilon=0} \;+\; \int_m \left[d\sigma^V(\epsilon) + \int_1 d\sigma^A(\epsilon) \right]_{\epsilon=0}$$

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Fully differential NNLO calculations

- NNLO corrections are needed to have a good control of theoretical uncertainties. At NNLO, due to the IR divergences in the *double real*, *real-virtual* and *two-loops virtual* corrections, the situation is more complicated and still challenging.
- Several groups worked on NNLO extension of the subtraction method: [Kosower('98), Weinzierl('03), Frixione, Grazzini('05), A.& T. Gehrmann, Glover et al.('05), Somogyi, Trocsanyi, Del Duca('05)], and on different methods (sector decomposition): [Binoth,Heinrich('00), Anastasiou,Melnikov, Petriello('04)]
- Some fully completed NNLO computations exist: $e^+e^- \rightarrow 3$ *jets* [A.& T. Gehrmann, Glover, Heinrich('07),Weinzierl('08)], Higgs production in hadron collision [Anastasiou,Melnikov,Petriello('04), Catani,Grazzini('07)], vector boson production in hadron collision [Melnikov,Petriello('06), Catani,Cieri,de Florian,Ferrera,Grazzini('07)],

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- Using this method we have performed a fully exclusive NNLO calculation for vector boson production [Catani,Cieri,G.F.,de Florian, Grazzini('09)],
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V is one or more colourless particles (vector bosons, leptons, photons, Higgs bosons,. . .) [Catani,Grazzini('07)].

- Key point I: at LO the q_T of the V is exactly zero.

$$d\sigma^{V}_{(N)NLO}|_{q_T \neq 0} = d\sigma^{V+\mathrm{jets}}_{(N)LO} \quad ,$$

for $q_T \neq 0$ the NNLO IR divergences cancelled with the NLO subtraction method.

- The only remaining NNLO singularities are associated with the $q_T \rightarrow 0$ limit.
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$$d\sigma^{V}_{N^nLO} \xrightarrow{q_T \rightarrow 0} d\sigma^{V}_{LO} \otimes \Sigma(q_T/M)\, dq_T^2 = d\sigma^{V}_{LO} \otimes \sum_{n=1}^{\infty} \sum_{k=1}^{2n} \left(\frac{\alpha_S}{\pi}\right)^n \Sigma^{(n,k)} \frac{M^2}{q_T^2} \ln^{k-1} \frac{M^2}{q_T^2}\, d^2 q_T$$

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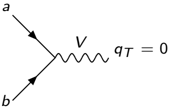

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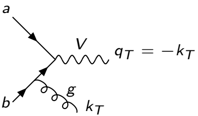

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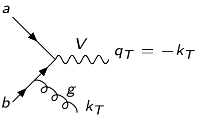

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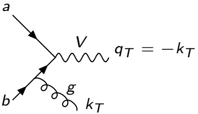

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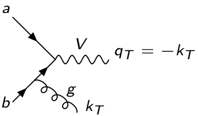

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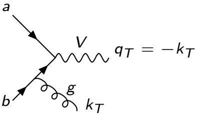

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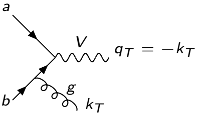

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$$\text{where} \quad \mathcal{H}^{V}_{NNLO} = \left[1 + \frac{\alpha_S}{\pi} \mathcal{H}^{V(1)} + \left(\frac{\alpha_S}{\pi} \right)^2 \mathcal{H}^{V(2)} \right]$$

- $d\sigma^{CT}$ regularizes the $q_T = 0$ singularity of $d\sigma^{V+\text{jets}}$: *double real* and *real-virtual* NNLO contributions, while *two-loops virtual* correction are contained in $\mathcal{H}^{V}_{NNLO}$.
- The choice of the counter-term has some arbitrariness but it must behave $d\sigma^{CT} \stackrel{q_T \to 0}{\longrightarrow} d\sigma^{V}_{LO} \otimes \Sigma(q_T/M) dq_T^2$. Note that $\Sigma(q_T/M)$ is universal.
- Final state partons only appear in $d\sigma^{V+\text{jets}}$ so that NNLO IR-safe cuts are included in the NLO computation: process- and observable-independent NNLO extension of the subtraction formalism.
- NLO calculation requires $d\sigma^{V+\text{jets}}_{LO}$ and $\mathcal{H}^{V(1)}$ [de Florian, Grazzini('01)].
- At NNLO we need also $d\sigma^{V+\text{jets}}_{NLO}$ [Giele et al.('93), MCFM] and $\mathcal{H}^{V(2)}$.

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- At NNLO we need also $d\sigma^{V+\text{jets}}_{NLO}$ [Giele et al.('93), MCFM] and $\mathcal{H}^{V(2)}$.

The final result is:

$$d\sigma^{V}_{(N)NLO} = \left[d\sigma^{V+\mathrm{jets}}_{(N)LO} - d\sigma^{CT}_{(N)LO} \right] + d\sigma^{V}_{LO} \otimes \mathcal{H}^{V}_{(N)NLO} \quad ,$$

$$\textit{where} \quad \mathcal{H}^{V}_{NNLO} = \left[1 + \frac{\alpha_S}{\pi} \mathcal{H}^{V(1)} + \left(\frac{\alpha_S}{\pi} \right)^2 \mathcal{H}^{V(2)} \right]$$

- $d\sigma^{CT}$ regularizes the $q_T = 0$ singularity of $d\sigma^{V+\mathrm{jets}}$: *double real* and *real-virtual* NNLO contributions, while *two-loops virtual* correction are contained in $\mathcal{H}^{V}_{NNLO}$.
- The choice of the counter-term has some arbitrariness but it must behave $d\sigma^{CT} \xrightarrow{q_T \to 0} d\sigma^{V}_{LO} \otimes \Sigma(q_T/M) dq_T^2$. Note that $\Sigma(q_T/M)$ is universal.
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- General relation between $\mathcal{H}^{V(2)}$ and the IR finite part of the two-loops correction to a generic process is unknown: work in progress.
  We explicit computed it for the Drell-Yan process with the following method:

$$\sigma_{NNLO}^{V,tot} = \int_0^\infty dq_T^2 \frac{d\sigma_{NLO}^V}{dq_T^2}, \qquad \frac{d\sigma_{NLO}^V}{dq_T^2} = \frac{d\sigma_{NLO}^{V,(sing.)}}{dq_T^2} + \frac{d\sigma_{NLO}^{V,(fin.)}}{dq_T^2},$$

where the *"sing."* term contains all the singular contributions at $q_T = 0$ while the *"fin."* term is free of such contributions.

- We can then write

$$\sigma_{NNLO}^{V,tot} = \lim_{Q_0 \to 0} \left[\int_0^{Q_0} + \int_{Q_0}^\infty \right] dq_T^2 \frac{d\sigma_{NLO}^V}{dq_T^2}$$
$$= \sigma_{LO}^V \otimes \left(\mathcal{H}_{NNLO}^V + \lim_{Q_0 \to 0} \int_0^{Q_0} dq_T^2 \, \Sigma(q_T/M) \right) + \lim_{Q_0 \to 0} \int_{Q_0}^\infty dq_T^2 \frac{d\sigma_{NLO}^V}{dq_T^2}.$$

- This formula allows us to analytically compute $\mathcal{H}_{NNLO}^V$ from the knowledge of the NNLO total cross section and the NLO q_T distribution.

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$$= \sigma^V_{LO} \otimes \left(\mathcal{H}^V_{NNLO} + \lim_{Q_0\to 0}\int_0^{Q_0} dq_T^2\, \Sigma(q_T/M)\right) + \lim_{Q_0\to 0}\int_{Q_0}^\infty dq_T^2 \frac{d\sigma^V_{NLO}}{dq_T^2}.$$

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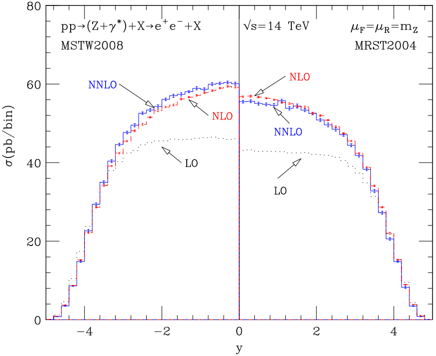
- This formula allows us to analytically compute $\mathcal{H}^V_{NNLO}$ from the knowledge of the NNLO total cross section and the NLO q_T distribution.

Numerical results at the LHC and the Tevatron



Rapidity distribution for Z production at the LHC (no cuts).

- Left panel: MSTW 2008 pdf. Going from NLO to NNLO the total cross section increase by about 3%: $\sigma_{NLO} = 2.030 \pm 0.001$ *nb* and $\sigma_{NNLO} = 2.089 \pm 0.003$ *nb* (errors refer to Monte Carlo numerical errors).
- Right panel: MRST 2004 pdf. Going from NLO to NNLO the total cross section decrease by about 2%: $\sigma_{NLO} = 1.992 \pm 0.001$ *nb* and $\sigma_{NNLO} = 1.954 \pm 0.003$ *nb*.
- σ_{NNLO} scale variations:
  -1.7% for $\mu_R = \mu_F = m_Z/2$,
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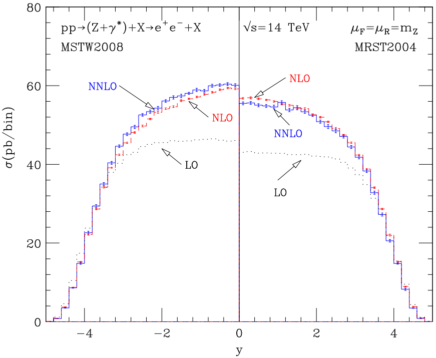

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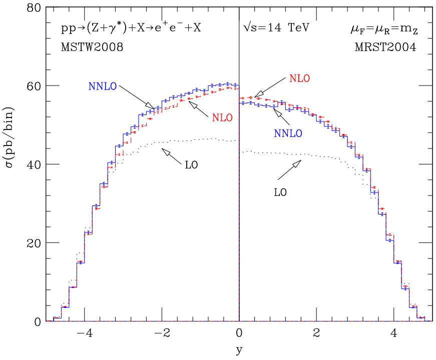

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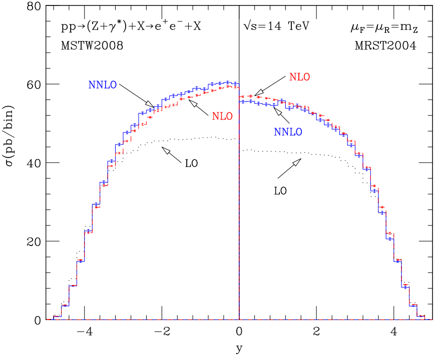

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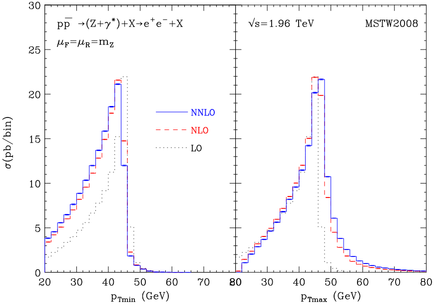

Minimum (left) and maximum (right) lepton p_T distribution for Z production at the Tevatron.

- Cuts: $p_{T\,min} \geq 20\,GeV$; $|\eta| < 2$; $70\,GeV \leq m_{e^+e^-} \leq 110\,GeV$
- At LO the distributions are kinematically bounded by $p_T < Q_{max}/2$.
- The NNLO corrections make the $p_{T_{min}}$ distribution softer, and the $p_{T_{max}}$ distribution harder.
- Accepted cross sections (errors refer to Monte Carlo numerical errors):
  $\sigma_{LO} = 103.37 \pm 0.04\,pb$,
  $\sigma_{NLO} = 140.43 \pm 0.07\,pb$,
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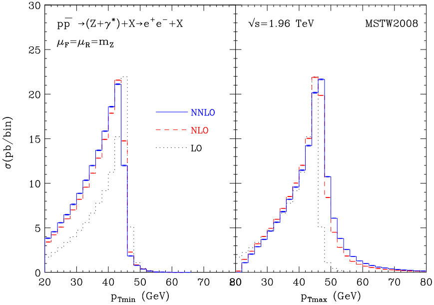

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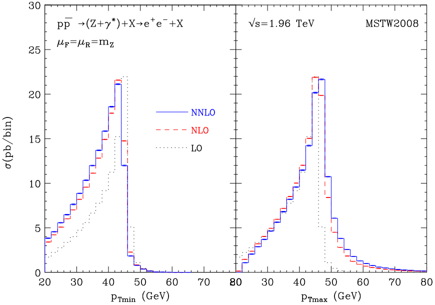

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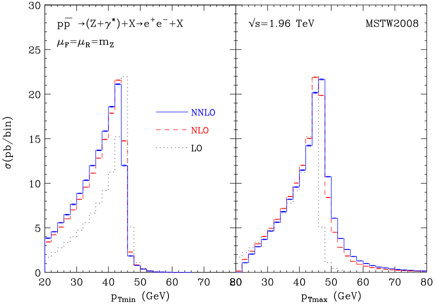

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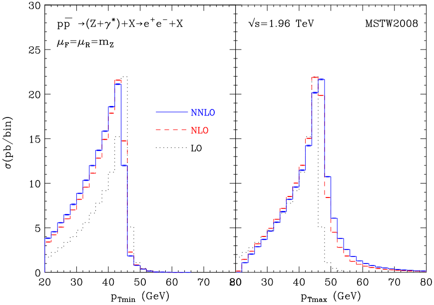

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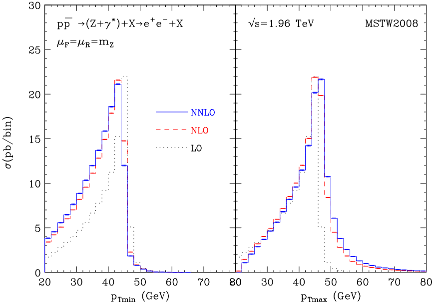

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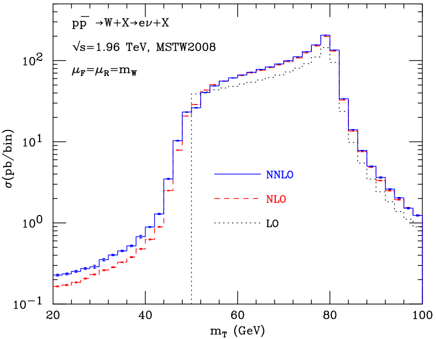

Transverse mass distribution for W production at the Tevatron:

$$m_T = \sqrt{2 p_T^l p_T^{miss} (1 - \cos \phi_{l\nu})}$$

Cuts: ${p_T}^{miss} \geq 25\ GeV$; $|\eta| < 2$; ${p_T}^l \geq 20\ GeV$

- LO distribution bounded at $m_T = 50$ GeV. At LO the W is produced with $q_T = 0$ therefore, the requirement $p_T^{\mathrm{miss}} > 25$ GeV sets $m_T \geq 50$ GeV.
- Around this region there are perturbative instabilities from LO to NLO and to NNLO.
- The origin of such instabilities are (integrable) logarithmic singularities near the boundary (Sudakov shoulder [Catani,Webber('97)]).
- Below the boundary, the $\mathcal{O}(\alpha_S^2)$ corrections are large (e.g. +40% at $m_T \sim 30$ GeV). This is not unexpected: in this region the $\mathcal{O}(\alpha_S^2)$ result is only a NLO calculation.
- Accepted cross sections (errors refer to Monte Carlo numerical errors):
  $\sigma_{LO} = 1.61 \pm 0.001\ nb$
  $\sigma_{NLO} = 1.550 \pm 0.001\ nb$
  $\sigma_{NNLO} = 1.586 \pm 0.002\ nb$
- σ_{NNLO} scales variation:
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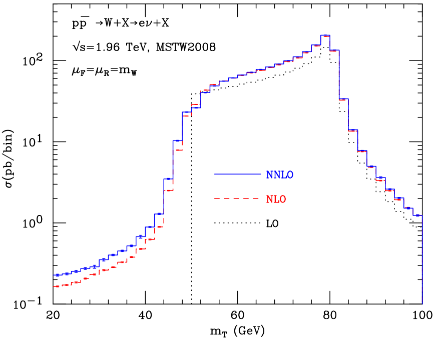

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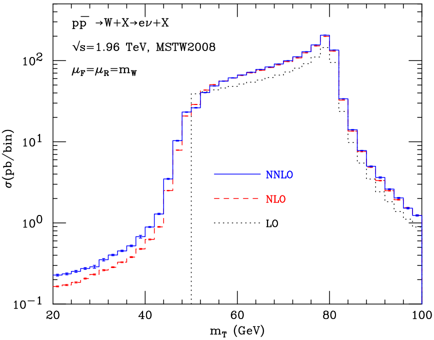

Transverse mass distribution for W production at the Tevatron:

$$m_T = \sqrt{2 p_T^l p_T^{miss} (1 - \cos\phi_{l\nu})}$$

Cuts: ${p_T}^{miss} \geq 25\ GeV$; $|\eta| < 2$; ${p_T}^{l} \geq 20\ GeV$

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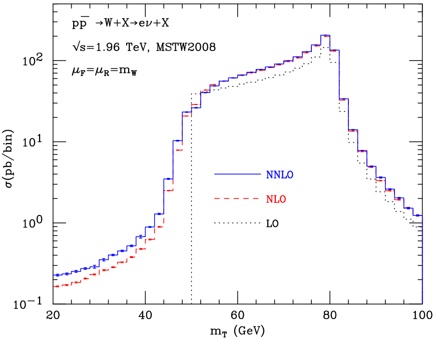

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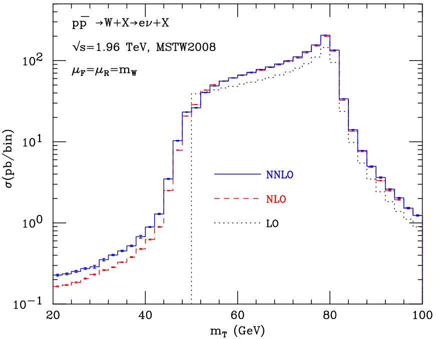

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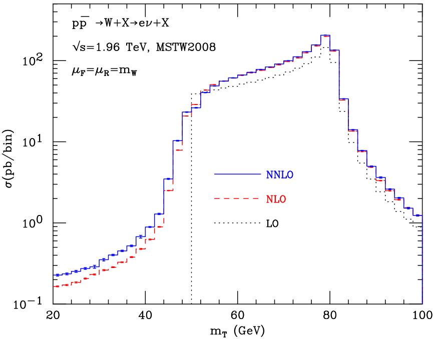

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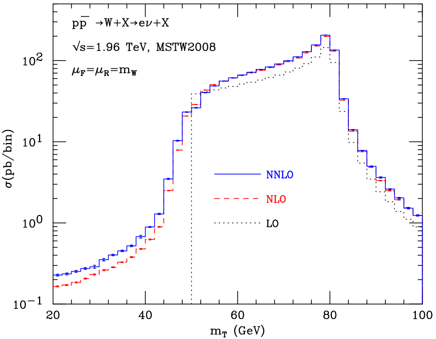

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Conclusions and Perspectives

- We have presented a fully exclusive NNLO QCD calculation for vector boson production in hadron collisions [Catani, Cieri, G.F., de Florian, Grazzini: [arXiv:0903.2120]], based on the [Catani, Grazzini('07)] NNLO extension of the subtraction formalism.
- We have implemented the calculation in a parton level Monte Carlo. The program allows the user to apply arbitrary kinematical cuts on the final state and on the associated jet activity computing the required distributions in the form of bin histograms.
- Our computation parallels, with a complete independent method, the one by [Melnikov,Petriello('06)]. In the quantitative studies we have carried out, the two computations gives results in numerical agreement.
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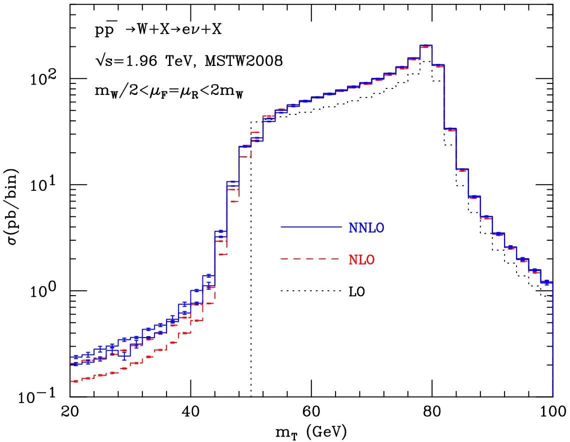
Back up slides



Transverse mass distribution for W production at the Tevatron:

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Transverse momentum resummation

$$\frac{d\hat{\sigma}_{ab}}{dq_T^2} = \frac{d\hat{\sigma}_{ab}^{(res)}}{dq_T^2} + \frac{d\hat{\sigma}_{ab}^{(fin)}}{dq_T^2};$$

The finite component $\left(\lim_{Q_T\to 0}\int_0^{Q_T^2} dq_T^2 \left[\frac{d\hat{\sigma}_{ab}^{(fin)}}{dq_T^2}\right]_{f.o.} = 0\right)$ ensure to reproduce the fixed order calculation at large q_T

Resummation holds in impact parameter space:

$$\frac{d\hat{\sigma}_{ab}^{(res)}}{dq_T^2} = \sigma^{(0)}\frac{M^2}{\hat{s}}\int_0^\infty db\,\frac{b}{2}J_0(bq_T)\,\mathcal{W}_{ab}(b,M),$$

$q_T \ll M \Leftrightarrow Mb \gg 1, \quad \log M^2/q_T^2 \gg 1 \Leftrightarrow \log Mb \gg 1$

In the Mellin moments space we have the exponentiated form:

$$\mathcal{W}_N(b,M) = \mathcal{H}_N(\alpha_S)\times\exp\left\{\mathcal{G}_N(\alpha_S,L)\right\} \qquad \text{where} \qquad L\equiv\log\left(\frac{M^2b^2}{b_0^2}\right)$$

$$\mathcal{G}_N(\alpha_S,L) = L\,g^{(1)}(\alpha_S L) + g_N^{(2)}(\alpha_S L) + \frac{\alpha_S}{\pi}g_N^{(3)}(\alpha_S L)+\cdots; \qquad \mathcal{H}_N(\alpha_S) = \left[1+\frac{\alpha_S}{\pi}\mathcal{H}_N^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2\mathcal{H}_N^{(2)}+\cdots\right]$$

LL ($\sim\alpha_S^n L^{n+1}$): $g^{(1)}$, $(\sigma^{(0)})$; NLL ($\sim\alpha_S^n L^n$): $g_N^{(2)}$, $\mathcal{H}_N^{(1)}$; NNLL ($\sim\alpha_S^n L^{n-1}$): $g_N^{(3)}$, $\mathcal{H}_N^{(2)}$;

Note that thanks to the exponentiation the perturbative approach is now valid for $\alpha_S L \sim 1$

We computed the function $\mathcal{H}_N^{(2)}$ recently. In the study presented here we have performed the resummation up to NLL matched with the LO calculation.

The q_T resummation formalism

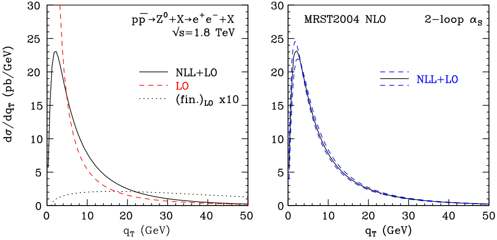
The main distinctive features of the formalism we are using are [Catani,de Florian, Grazzini('01)], [Bozzi,Catani,de Florian, Grazzini('03,'06,'08)]:

- Resummation performed at partonic cross section level: PDF evaluated at $\mu_F \sim M$: no PDF extrapolation in the non perturbative region, study of renormalization and factorization scale dependence as in fixed-order calculations.
- Possible to make prediction without introducing non perturbative effects: Landau singularity of the QCD coupling regularizated using a *minimal prescription* [Laenen,Sterman,Vogelsang('00)].
- Resummed effects exponentiated in a universal Sudakov form factor $\mathcal{G}_N(\alpha_S, L)$; process-dependence factorized in the hard scattering coefficient $\mathcal{H}_N(\alpha_S)$.
- Perturbative unitarity constrain and resummation scale Q:

$$\ln\left(\frac{M^2 b^2}{b_0^2}\right) \to \widetilde{L} \equiv \ln\left(\frac{Q^2 b^2}{b_0^2}+1\right) \;\Rightarrow\; \exp\left\{\mathcal{G}_N(\alpha_S, \widetilde{L})\right\}\Big|_{b=0} = 1 \;\Rightarrow\; \int_0^\infty dq_T^2 \left(\frac{d\hat{\sigma}}{dq_T^2}\right)_{NLL+LO} = \hat{\sigma}_{NLO}^{(tot)}$$

  - avoids unjustified higher-order contributions in the small-b region: no need for unphysical switching from resummed to fixed-order results.
  - allows to recover *exactly* the total cross-section upon integration on q_T
  - variations of the resummation scale $Q \sim M$ allows to estimate the uncertainty from uncalculated logarithmic corrections at higher orders.

Resummed results: q_T spectrum of Drell-Yan e^+e^- pairs at $\sqrt{s} = 1.8\ TeV$



- Left side: NLL+LO result compared with fixed LO result.
  Resummation cure the fixed order divergence at $q_T \to 0$.
- Right side: variation of factorization and renormalization scales as in customary fixed-order calculations: $\sim 5\%$ at low q_T, $\sim 9\%$ at $q_T \sim 50\ GeV$.
- Finite LO component contribution is: $\lesssim 1\%$ near the peak, $\sim 8\%$ at $q_T \sim 20\ GeV$, $\sim 60\%$ at $q_T \sim 50\ GeV$.
- Integral of the NLL+LO curve reproduce the total NLO cross section to better 1% (check of the code).

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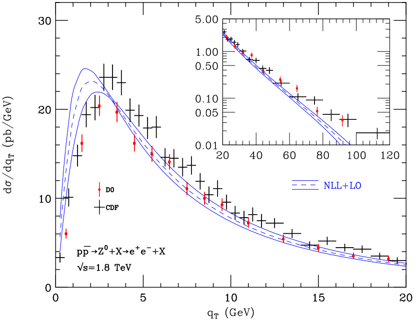

- CDF data: $66\ GeV < M^2 < 116\ GeV$, $\sigma_{tot} = 248 \pm 11\ pb$ [CDF Collaboration ('00)]
  D0 data: $75\ GeV < M^2 < 105\ GeV$, $\sigma_{tot} = 221 \pm 11\ pb$ [D0 Collaboration ('00)]
- Our calculation implements $\gamma^* Z$ interference and finite-width effects. Here we use the narrow width approximation (differences within 1% level).
- NLL+LO resummed result fits reasonably well also in the $q_T \lesssim 20\ GeV$ (without a model for non-perturbative effects).
- Suppression in the large-q_T region ($q_T \lesssim 60\ GeV$) (strong dependence from the resummation scale).

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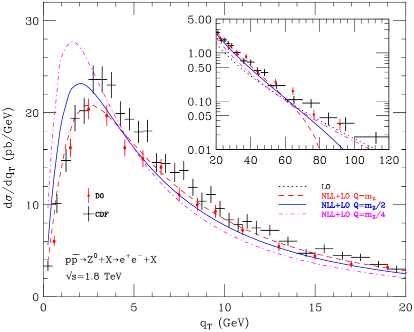
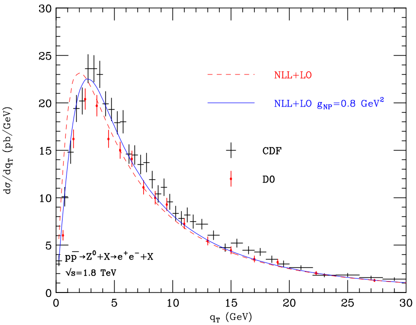


- NLL+LO results for different values of the resummation scale Q (estimate of higher-order logarithmic contributions).
- We vary $Q = m_Z/2$, $\quad m_Z/4 \le Q \le m_Z$: uncertainty $\pm 12 - 15\%$ in the region $q_T \gtrsim 20\ GeV$ (it dominate over the renormalization and factorization scale variations).
- We expect a sensible reduction once the complete NNLL+NLO calculation will be available.

Non perturbative effects: q_T spectrum of Drell-Yan e^+e^- pairs at $\sqrt{s}$=1.8 TeV



- Up to now result in a complete perturbative framework.
- Non perturbative effects parametrized by a NP form factor $S_{NP} = \exp\{-g_{NP} b^2\}$:

$$\exp\{\mathcal{G}_N(\alpha_S, \widetilde{L})\} \quad \rightarrow \quad \exp\{\mathcal{G}_N(\alpha_S, \widetilde{L})\}\, S_{NP}$$

$$g_{NP} = 0.8\, GeV^2$$ [Kulesza et al.('02)]

- With NP effects there is a better agreement with the data.
- Quantitative impact of such NP effects is within perturbative uncertainties.