NLO Electroweak Corrections to Higgs Boson Production at Hadron Colliders

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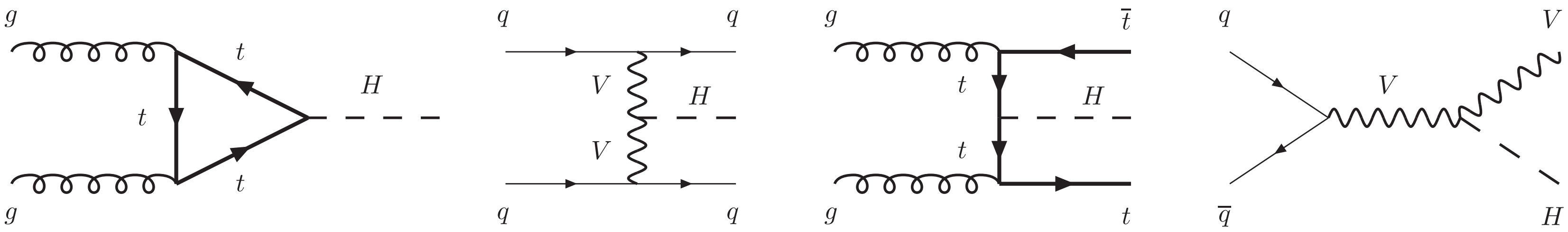
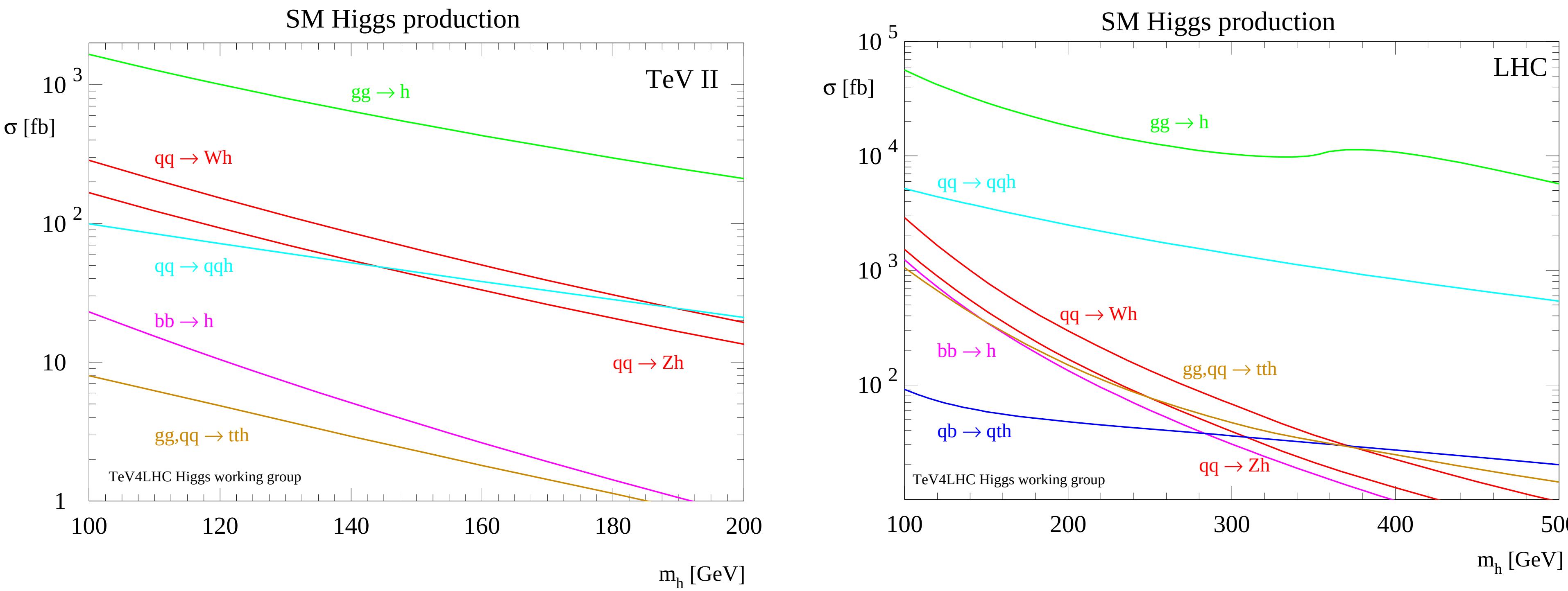
Standard Model hadronic Higgs production channels

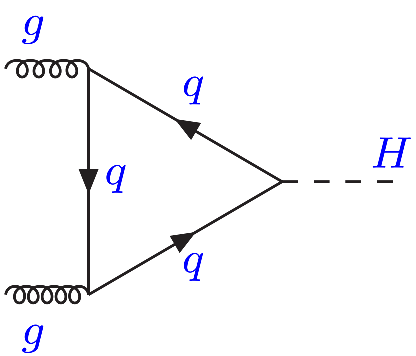




Hahn,Heinemeyer,Maltoni,Weiglein,Willenbrock [hep-ph/0607308]

Gluon-fusion ⇝ largest cross section

Lowest order (one-loop) for $gg \to H$ (in SM)



$$\sigma_{\rm LO} = \frac{G_F\, \alpha_S^2(\mu_R^2)}{288\sqrt{2}\pi} \left| \frac{3}{2} \sum_q \frac{1}{\tau_q} \left[1 + \left(1 - \frac{1}{\tau_q}\right) F(\tau_q) \right] \right|^2$$

$$F = \begin{cases} \arcsin^2 \sqrt{\tau_q}, & \tau_q \leq 1, \\ -\frac{1}{4} \left[\ln \frac{1+\sqrt{1-\tau_q^{-1}}}{1-\sqrt{1-\tau_q^{-1}}} - i\pi \right]^2 & \tau_q > 1 \end{cases} \qquad \tau_q = \frac{M_H^2}{4M_q^2}$$

Georgi-Glashow-Machacek-Nanopoulos '78

LO total cross section for $h_1 h_2 \to H$ $\Rightarrow$ $\sigma_{\rm LO} \otimes$ PDFs

- LO $\to$ strong scale dependence
- QCD corrections for reliability

QCD corrections

- NLO $\rightsquigarrow$ +80% LO at the LHC, scale dependence 20%
  - Large M_t limit: Dawson '91, Djouadi-Spira-Zerwas '91
  - Full result: Spira-Djouadi-Graudenz-Zerwas '95, Harlander-Kant '05, Anastasiou-Beerli-Bucherer-Daleo-Kunszt '06, Aglietti-Bonciani-Degrassi-Vicini '06
- NNLO $\rightsquigarrow$ +20% NLO at the LHC, scale dependence 10%
  - Large M_t limit: Harlander '00, Catani-de Florian-Grazzini '01, Harlander-Kilgore '01, Anastasiou-Melnikov '02, Ravindran-Smith-van Neerven '03
- Soft-gluon resum. NNLL $\rightarrow$ +6% NNLO at the LHC, scale dependence 8%: Catani-de Florian-Grazzini-Nason '03
- Soft-gluon resum. N^3LL $\rightarrow$ +8% NNLO at the LHC, scale dependence 3%: Moch-Vogt '05, Laenen-Magnea '06, Idilbi-Ji-Ma-Yuan '06, Ravindran '06, Ahrens-Becher-Neubert-Yang '09

NLO EW corrections

- Dominant contributions enhanced by M_t^2: Djouadi-Gambino '94
- Light-quark contributions: Aglietti-Bonciani-Degrassi-Vicini '04
- Top-quark contributions (Taylor expansion for $M_H < 2M_W$): Degrassi-Maltoni '04
- Full EW Corrections: Actis-Passarino-Sturm-U. '08

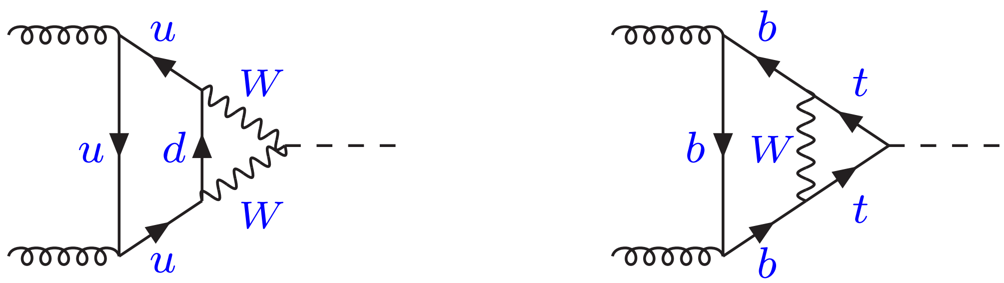
  → Smooth threshold behaviour using **complex masses** for W and Z

Mixed QCD-EW corrections $\mathcal{O}(\alpha\alpha_s)$

- Light-quark contributions (Taylor expansion for $M_H < M_W$): Anastasiou-Boughezal-Petriello '08

  → Extension to all values of M_H using the full NLO EW corrections

Computation of EW corrections



- Many mass scales $\longrightarrow$ analytical methods need expansions
- ..., but also light fermions are present $\longrightarrow$ collinear singularities

$$\Downarrow$$

- Analytical cancellation of collinear logarithms in the light fermion masses
- Feynman parametrization, algebraic manipulation
- Numerical integration

Numerical computation

Write the **finite part** in one of the following forms:

1) $\int dx \, \frac{Q(x)}{V(x)}$ $\quad V(x)$ positive definite $\rightsquigarrow$ not applicable in the physical region

2) $\frac{1}{B} \int dx \, Q(x) \ln^n V(x)$ $\quad B$ constant $\neq 0$ $\rightsquigarrow$ not always possible

3) $\int dx \, \frac{Q(x)}{V(x)} \, f\left(\frac{V(x)}{P(x)}\right)$ $\quad f(0) = 0,$ $\quad f(x) = ln^n(1+x),\ Li_n(x),\ S_{n,p}(x)$

↳ **General, always possible at 2-loop level**

Typical integrand with k Feynman variables:

$$z_1^{n_1} \cdots z_k^{n_k} V^{\mu}(z_1, \ldots, z_k) \ln^m V(z_1, \ldots, z_k), \qquad \mu = -1, -2$$

- The integration domain is finite ($\subseteq [0,1]^k$)
- V is quadratic with respect to a subset of $\{z_1, \ldots, z_k\}$, in which ...
- ... each z_i^2 is proportional to one squared external momentum.

- The quadratic is not complete
  - $\mu = -1$ and $m = 0$ ($m > 0$ can be treated similarly)

$$\frac{1}{a\,x+b} = \partial_x \frac{1}{a}\,\ln\left(1+\frac{a}{b}\,x\right)$$

  - $\mu = -2$ and $m = 0$ ($m > 0$ can be treated similarly)

$$\frac{1}{(a\,x\,y+b\,x+c\,y+d)^2} = -\partial_x\,\partial_y\,\frac{1}{a\,d-b\,c}\,\ln\left\{1+\frac{(a\,d-b\,c)\,x}{b\,(axy+bx+cy+d)}\right\}$$

- The quadratic is complete

$$V(z) = z^t\,H\,z + 2\,K^t\,z + L = (z^t - Z^t)\,H\,(z-Z) + B = Q(z) + B$$

$$Z = -K^t H^{-1}, \quad B = L - K^t H^{-1} K, \qquad \mathcal{P}^t \partial_z\, Q(z) = -Q(z), \quad \mathcal{P} = -(z-Z)/2,$$

$$V^\mu(z) = \left(\beta - \mathcal{P}^t\,\partial_z\right)\int_0^1 dy\, y^{\beta-1}\left[Q(z)\,y+B\right]^\mu$$

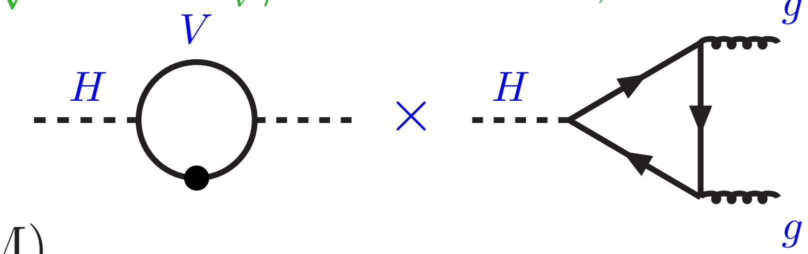
If $\mu = -1$ we choose: $\beta = 1$

$$V^{-1} = \left(1-\mathcal{P}^t\,\partial_z\right)\frac{1}{Q}\,\ln\left(1+\frac{Q}{B}\right)$$

Threshold behaviour

Square root singularities $\longrightarrow$ $1/\beta_V = 1/\sqrt{1 - 4\, M_V^2/s}$ $\quad V = W, Z$

(1-loop) $\otimes$ (H wave-function ren.)



- ”Minimal” complex-mass scheme (MCM)

$$\mathcal{A} = \frac{\mathcal{A}^W_{\text{sing}}}{\beta_W} + \frac{\mathcal{A}^Z_{\text{sing}}}{\beta_Z} + \mathcal{A}_{reg}$$

A^V_{sing} are gauge invariant

$M_V^2 \;\longrightarrow\; s_V = \mu_V^2 - i\mu_V\gamma_V$ in $\dfrac{A^V_{sing}}{\beta_V}$ $\qquad \mu_V^2 = M_V^2 - \Gamma_V^2 \qquad \gamma_V = \Gamma_V \left(1 - \frac{\Gamma_V^2}{2\, M_V^2}\right)$

Same approach as by Degrassi-Maltoni [hep-ph/0407249]

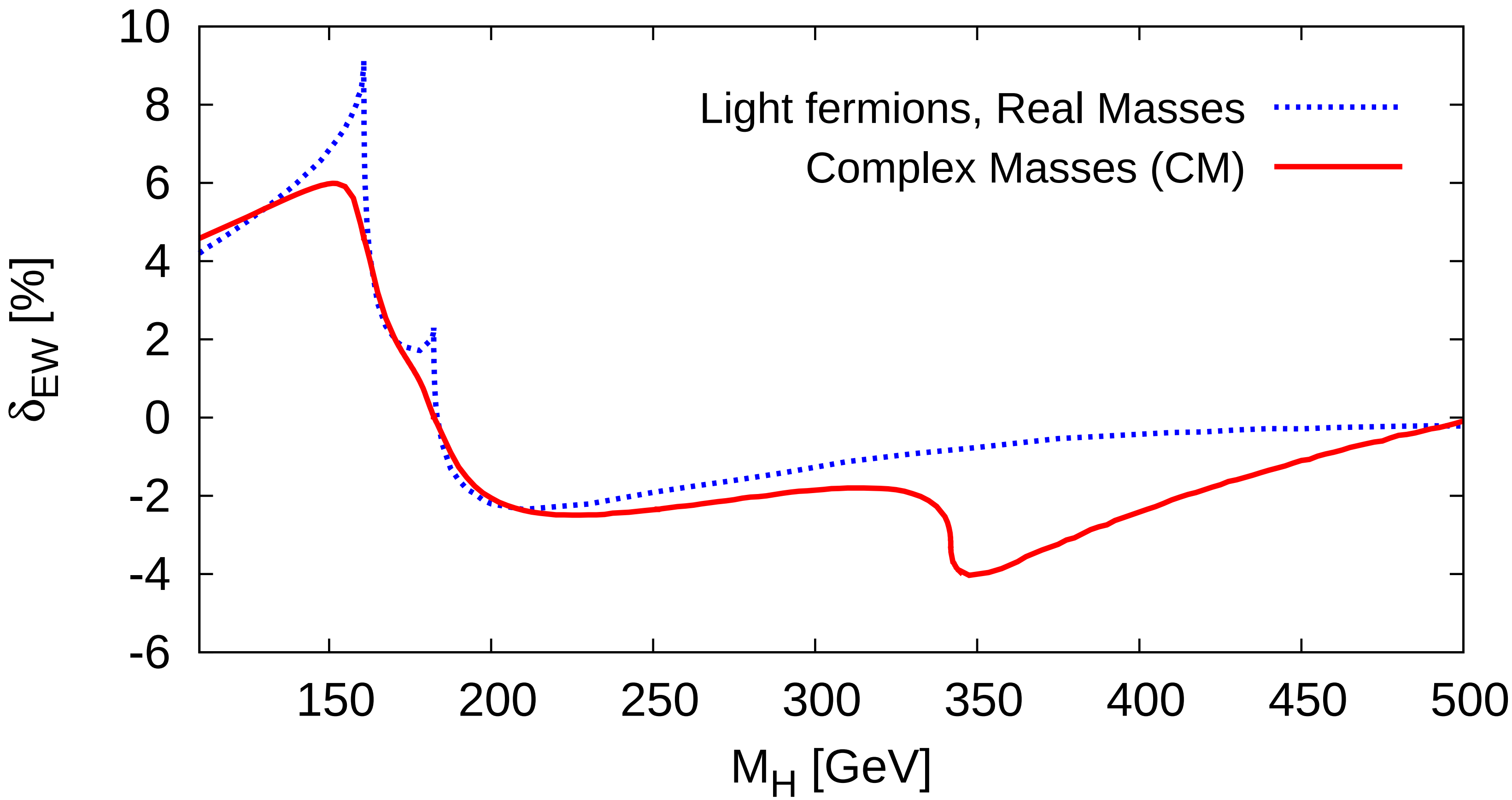
- ”Complete” complex-mass scheme (CM)

$$M_V^2 \;\longrightarrow\; s_V = \mu_V^2 - i\mu_V\gamma_V \quad \text{everywhere}$$

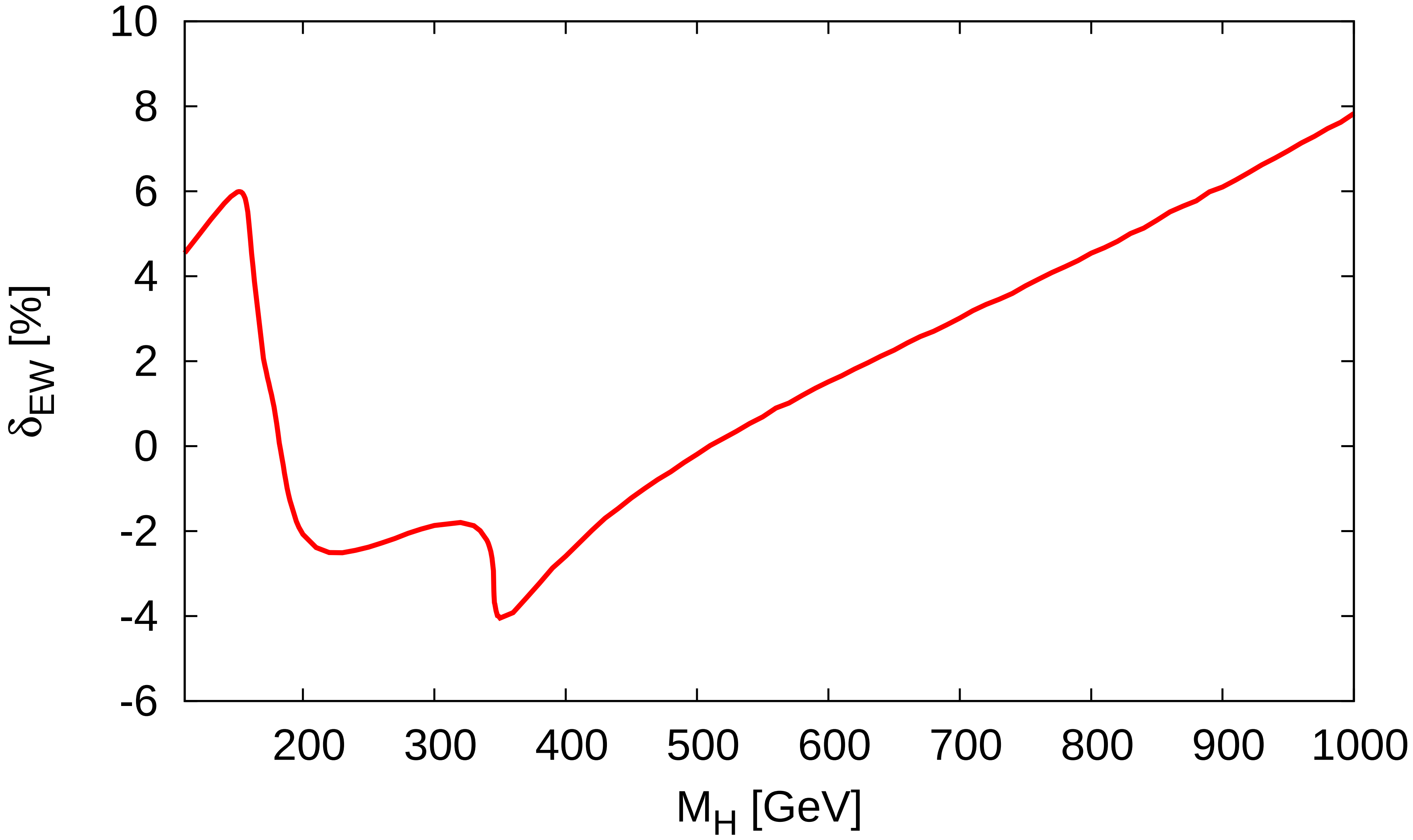
$M_V^2 \longrightarrow s_V$ also in the couplings in order to preserve gauge invariance

(Denner,Dittmaier,Roth,Wackeroth,Wieders’99-’05)

Results for EW corrections to $\sigma(gg \to H) = \sigma_{\rm LO}\,(1 + \delta_{\rm EW})$



- Light fermions: in agreement with Aglietti-Bonciani-Degrassi-Vicini
- Light fermions dominate up to 300 GeV, top-quark loop relevant at $t\bar{t}$ threshold
- Complex masses $\rightsquigarrow$
  - Cusps at WW and ZZ thresholds disappear
  - Differences with Degrassi-Maltoni already at 140 GeV


10
8
6
4
2
0
-2
-4
-6
δEW [%]
200
300
400
500
600
700
800
900
1000
MH [GeV]

Total cross section in hadron collisions

- Fold PDFs with partonic cross section

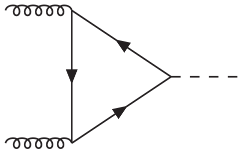
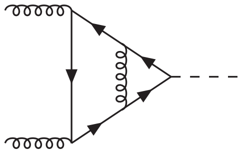
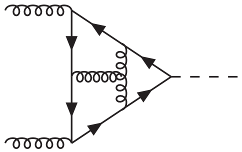
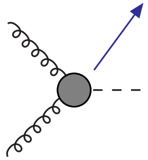
$$\sigma(h_1 h_2 \to H) = \sum_{i,j} \int_0^1 dx_1 dx_2\, f_{i,h_1}(x_1,\mu_F^2) f_{j,h_2}(x_2,\mu_F^2) \times \int_0^1 dz\, \delta\left(z - \frac{M_H^2}{s x_1 x_2}\right) z\, \sigma_{\rm LO}\, \underbrace{G_{ij}^{\rm QCD}(z,\mu_R^2,\mu_F^2)}_{\rm pQCD}$$

$$G_{ij}^{\rm QCD} = \underbrace{G_{ij}^{\rm QCD,(0)}}_{\delta_{ig}\delta_{jg}\delta(1-z)} + a_s\, G_{ij}^{\rm QCD,(1)} + a_s^2\, G_{ij}^{\rm QCD,(2)} + \ldots \qquad a_s = \frac{\alpha_s}{\pi}$$

- Two factorization options for QCD/ EW:
  - Partial factorization (PF): $G_{ij}^{\rm QCD} \to G_{ij}^{\rm QCD} + \delta_{\rm EW}\, G_{ij}^{\rm QCD,(0)}$
  - Complete factorization (CF): $G_{ij}^{\rm QCD} \to (1 + \delta_{\rm EW}) G_{ij}^{\rm QCD}$

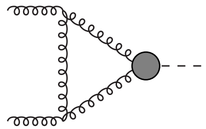
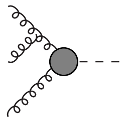
- Correct result: $G_{ij}^{\rm QCD} \to G_{ij}^{\rm QCD} + \delta_{\rm EW} G_{ij}^{\rm QCD,(0)} + a_s G_{ij}^{\rm QCD+EW,(1)} + a_s^2 G_{ij}^{\rm QCD+EW,(2)}$

QCD effective theory:

 $+$  $+$  $+ \ldots \sim$  $\left(1 + a_s \underbrace{C_{1q}} + a_s^2 \underbrace{C_{2q}} + \ldots\right)$

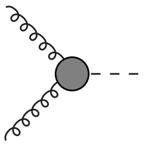
$\sim a_s \sqrt{G_F}$

Wilson coefficients

$\Rightarrow \qquad G_{ij}^{\mathrm{QCD},(1)} \sim G_{ij}^{\mathrm{QCD},(0)} C_{1q} + F_{ij}^{\mathrm{QCD},(1)} \longrightarrow$  $+$  $+ \ldots$

Anastasiou-Boughezal-Petriello '08 [0811.3458]:

Evaluation of $G_{ij}^{\mathrm{QCD+EW},(1)}$ in the effective theory:

 $\left[1+\lambda_{\mathrm{EW}}\left(1+a_s C_{1w}+a_s^2 C_{2w}+\ldots\right)+a_s C_{1q}+a_s^2 C_{2q}+\ldots\right] \qquad \delta_{\mathrm{EW}}\Big|_{M_H=0} \longrightarrow \lambda_{\mathrm{EW}}$

CF hypothesis: $C_{1w} = C_{1q}$ Result: $C_{1w} = \dfrac{7}{6} \neq C_{1q} = \dfrac{11}{4} \quad \Rightarrow$ **No CF**

However, at hadronic level

$\sigma_{\mathrm{EFF}} = \sigma_{\mathrm{CF}} + \mathrm{PDFs} \otimes \sigma_{\mathrm{LO}} \left[\delta_{\mathrm{EW}} G_{ij}^{\mathrm{QCD},(0)} (C_{1w} - C_{1q}) + \ldots\right] \sim \sigma_{\mathrm{CF}} \Rightarrow$ **Ok CF** at hadronic level

Problems with gauge invariance: $H(P) \to \gamma(p_1) + \gamma(p_2)$

Amplitude $\rightarrow$ $\mathcal{A}^{\mu\nu} = \dfrac{g^3\, s_\theta^2}{16\,\pi^2}\,(F_{\scriptscriptstyle D}\,\delta^{\mu\nu} + F_{\scriptscriptstyle P}\,p_2^\mu\,p_1^\nu).$

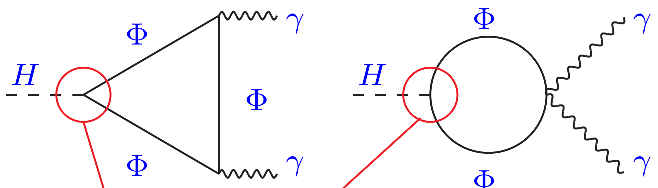
Ward Identity: $F_{\scriptscriptstyle D} + p_1 \cdot p_2\, F_{\scriptscriptstyle P} = 0$

Renormalization (Ren) $\rightarrow$ $M^2_{H,0} = M_H^2\left[1 + \dfrac{G_{\scriptscriptstyle F} M_W^2}{2\sqrt{2}\,\pi^2}\,\mathrm{Re}\,\Sigma^{(1)}_{HH}(M_H^2)\right]$

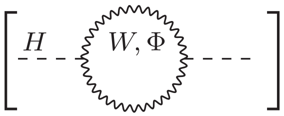
$$F_D = F_D^{(1)} \otimes (1 + \mathrm{Ren}) + F_D^{(2)} \qquad\qquad F_P = F_P^{(1)} \otimes (1 + \mathrm{Ren}) + F_P^{(2)}$$

- 2-loop level

$$\underbrace{F_D^{(2)} + p_1 \cdot p_2\, F_P^{(2)}}\ +\ \underbrace{\left(F_D^{(1)} + p_1 \cdot p_2\, F_P^{(1)}\right) \otimes \mathrm{Ren}}\ \neq\ \mathbf{0}$$



No "Re" label

$\sim M_{H,0} \ \rightarrow\ \mathrm{Re}\,\Big[$ 

$\Big]$

A gauge invariant definition of a decay width

- Unstable particles can not be asymptotic states $\rightsquigarrow$ complete process

$$pp \to gg(\to H \to \gamma\gamma) + X,$$

How to extract a pseudo-observable to be termed *Higgs partial decay width into two photons* which does not violate first principles?

From an idea of `Grassi-Kniehl-Sirlin [hep-ph:0005149]`

Complex pole: $s_H - M_H^2 + \Sigma_H(s_H) = 0$

- gauge invariant definition
- M_H real by construction

Dyson-resummed Higgs propagator

$$\Delta_H(s) = (s - s_H)^{-1} \left[1 + \Pi_H(s)\right]^{-1}, \qquad \Pi_H(s) = \frac{\Sigma_H(s) - \Sigma_H(s_H)}{s - s_H}$$

S-matrix for $i \to f$:

$$S_{fi} = V_i(s)\,\Delta_H(s) V_f(s) + B_{\rm nr}$$
$$= \left[Z_H^{-1/2}(s) V_i(s)\right] \frac{1}{s - s_H} \left[Z_H^{-1/2}(s) V_f(s)\right] + B_{\rm nr},$$

$$Z_H = 1 + \Pi_H \qquad\qquad B_{\rm nr} = \text{non-resonant background}$$

Expand the square brackets around $s = s_H$

$\Rightarrow$ Definition:

gauge invariant

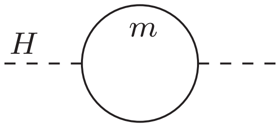
$$S(H_c \to f) = Z_H^{-1/2}(s_H) V_f(s_H) \qquad\qquad \Gamma(H_c \to f) = C \int d\Phi_f \sum_{\rm spins} \left|S(H_c \to f)\right|^2$$

$$S_{fi} = \frac{S(i \to H_c)\, S(H_c \to f)}{s - s_H} + \text{non resonant terms.}$$

Outline of the computation

Basic rule: $\lim_{\gamma,\Gamma_H \to 0} \text{Ampl}\,(s_H, m) = \text{Ampl}\,(M_H^2, \mu)$

Example:



$$= \Delta - \int_0^1 dx \ln \chi, \qquad \chi = -s_H\, x\,(1-x) + m^2$$

$$s_H = M_H^2 - i\Gamma_H M_H, \qquad m^2 = \mu^2 - i\gamma\mu;$$

If $\text{Re}\chi < 0$ and $\text{Im}\chi > 0$ (second quadrant):

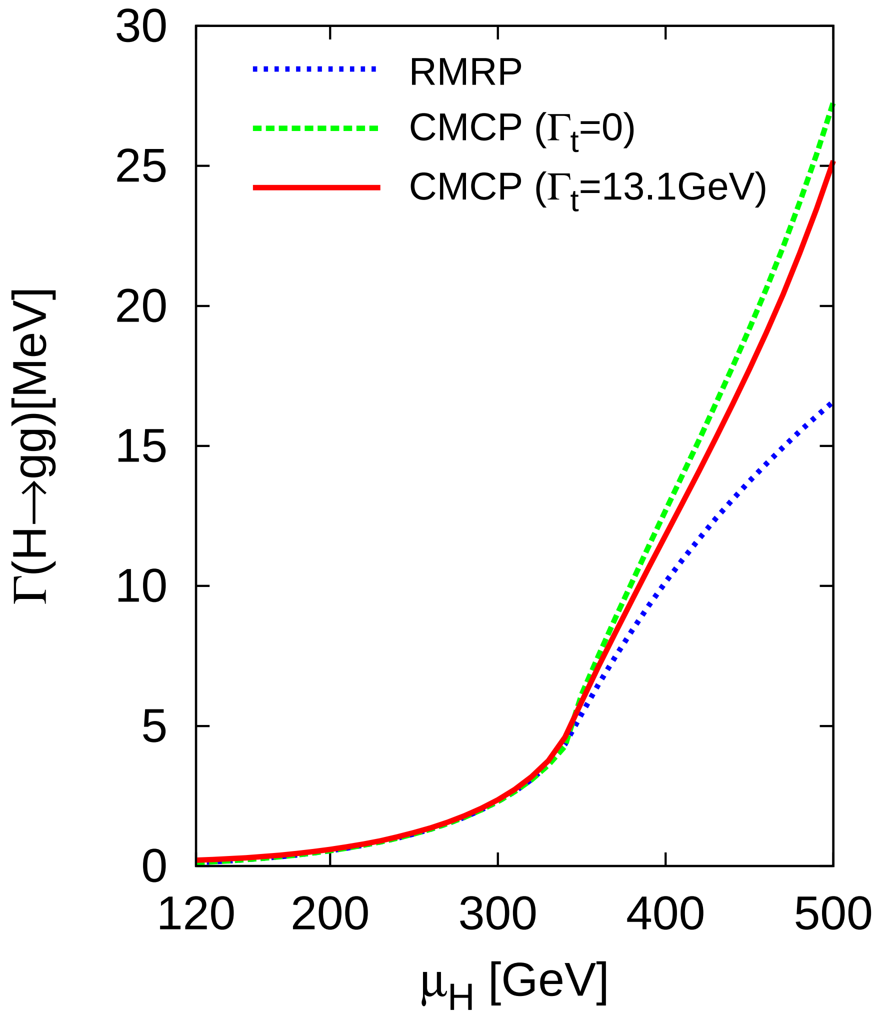
$$\lim_{\gamma,\Gamma_H \to 0} \text{Im}[\ln \chi] = \pi \quad \neq \quad \begin{array}{l}\text{Feynman prescription for}\\ \text{real masses } (\mu^2 \to \mu^2 - i0)\end{array} \quad = -\pi$$

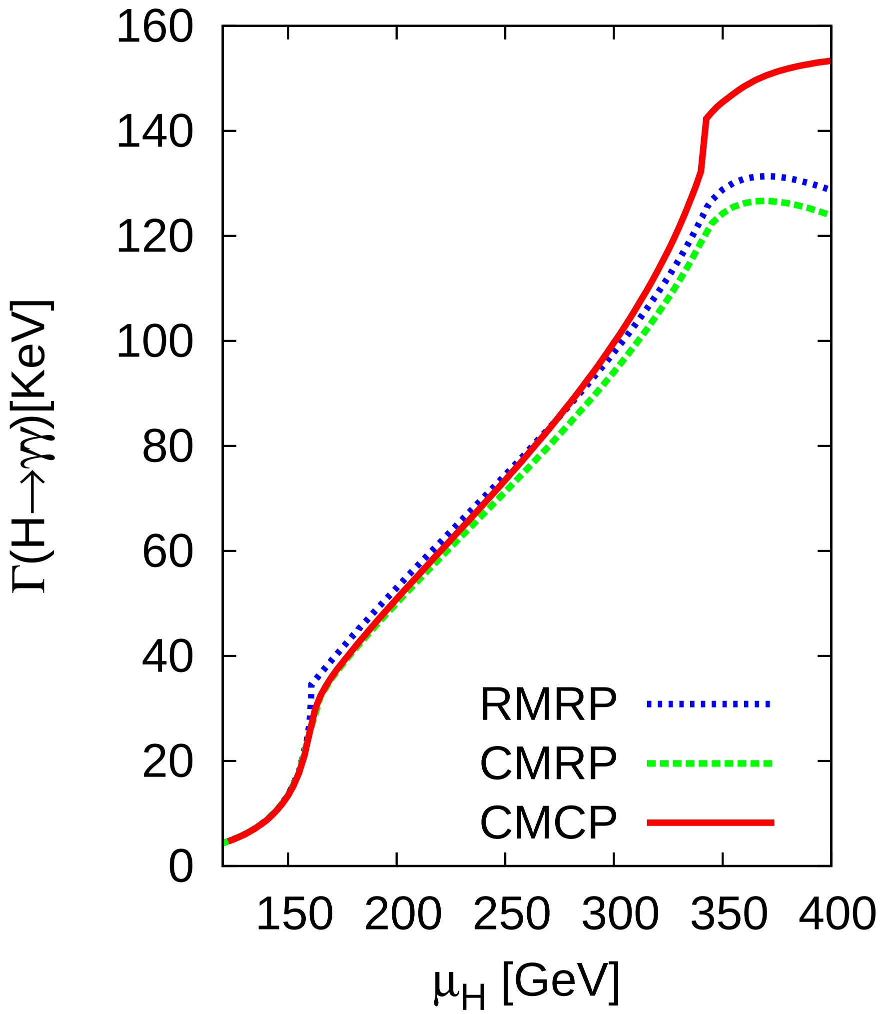
- Analytical continuation on the second Riemann sheet:

$$\ln(z) \to \ln^-(z) = \ln(z) - 2\,i\,\pi \underbrace{\theta(-\text{Re}z)\,\theta(\text{Im}z)}_{\text{second quadrant}} \quad \Leftrightarrow \quad \begin{array}{l}\text{move the cut on the}\\ \text{positive imaginary axis}\end{array}$$

- Distortion of the x integration contour if the cut crosses $[0, 1]$

Numerical effects





RMRP= real masses and momenta; CMCP= complex masses and momenta;
CMRP= complex masses, real momenta;

Summary

- Two-loop EW corrections for $gg \to H$: $-4\% < \delta_{\mathrm{EW}} < 6\%(8\%)$
- Impact on Higgs production at LHC: $+5\%$ at $M_H = 120$ GeV (for CF)
- (To be) used in HiggsNNLO, FEHiP, HIGLU
- Gauge invariant definition of production cross section: negligible numerical effects below $t\bar{t}$ threshold, but sizable for large M_H
- **Computational techniques**:
  - Analytical extraction of **collinear logarithms** at two-loop level
  - Reliable numerical computation of two-loop diagrams with **complex masses**
  - Analytical continuation and contour distortion for diagrams with complex masses and momenta

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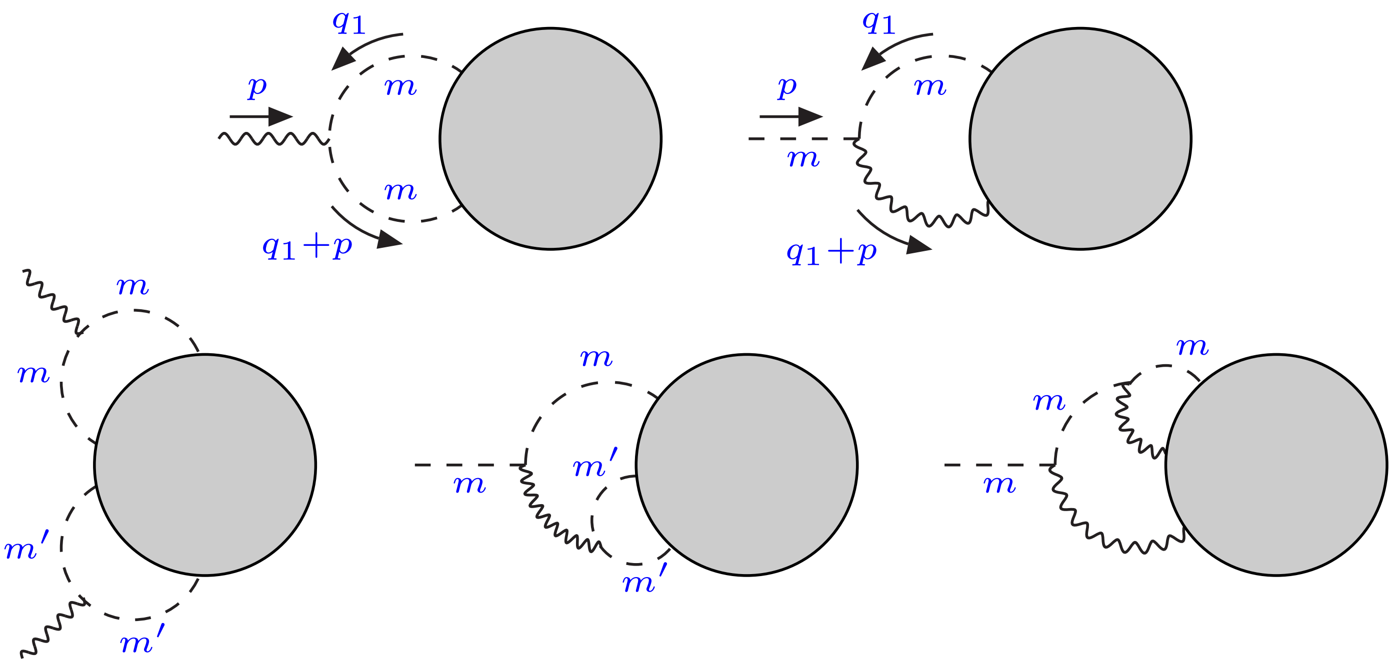
Treatment of collinear divergences

They come from the coupling of light particles (m) with massless particles (wavy)



⟹ Single divergence

⟹ Double divergence

- Single divergence: Subtraction method

$$J_1 = \frac{\mu^{4-n}}{i\,\pi^2} \int d^n q_1 \, \frac{1}{(q_1^2 + m^2)[(q_1+p)^2 + m^2][(q_1-q_2)^2 + M^2]}.$$

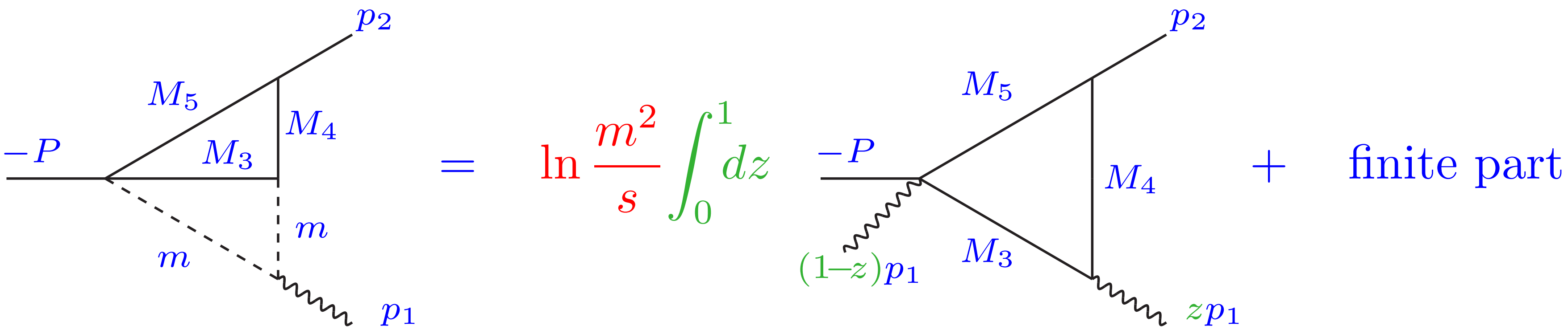
After parametrization

$$J_1 = \int_0^1 dz \int_0^z dy\, \frac{1}{V}, \qquad V = [\, A - y\,(q_2+p)^2\,]\, y + m^2\,(1-y), \qquad A = (q_2 + p\, z)^2 + M^2.$$

Add and subtract: $V_0^{-1} = (A\, y + m^2)^{-1}$

$$\begin{aligned} J_1 &= \int_0^1 dz \int_0^z dy\, \frac{1}{A\,y + m^2} + \int_0^1 dz \int_0^z dy\, \left(\frac{1}{V} - \frac{1}{V_0}\right) \\ &= -\ln\frac{m^2}{s} \int_0^1 dz \frac{1}{A} + \int_0^1 dz \frac{1}{A} \ln\frac{Az}{s} + \int_0^1 dz \int_0^z \frac{dy}{y} \left[\frac{1}{A - y\,(q_2+p)^2} - \frac{1}{A}\right] + \mathcal{O}(m^2). \end{aligned}$$

Example:



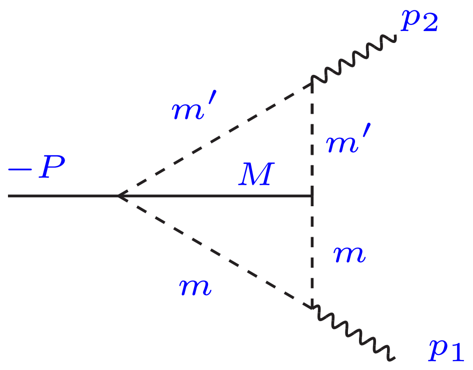
The coefficients of the log are 1-loop functions

- Double divergence: Double subtraction

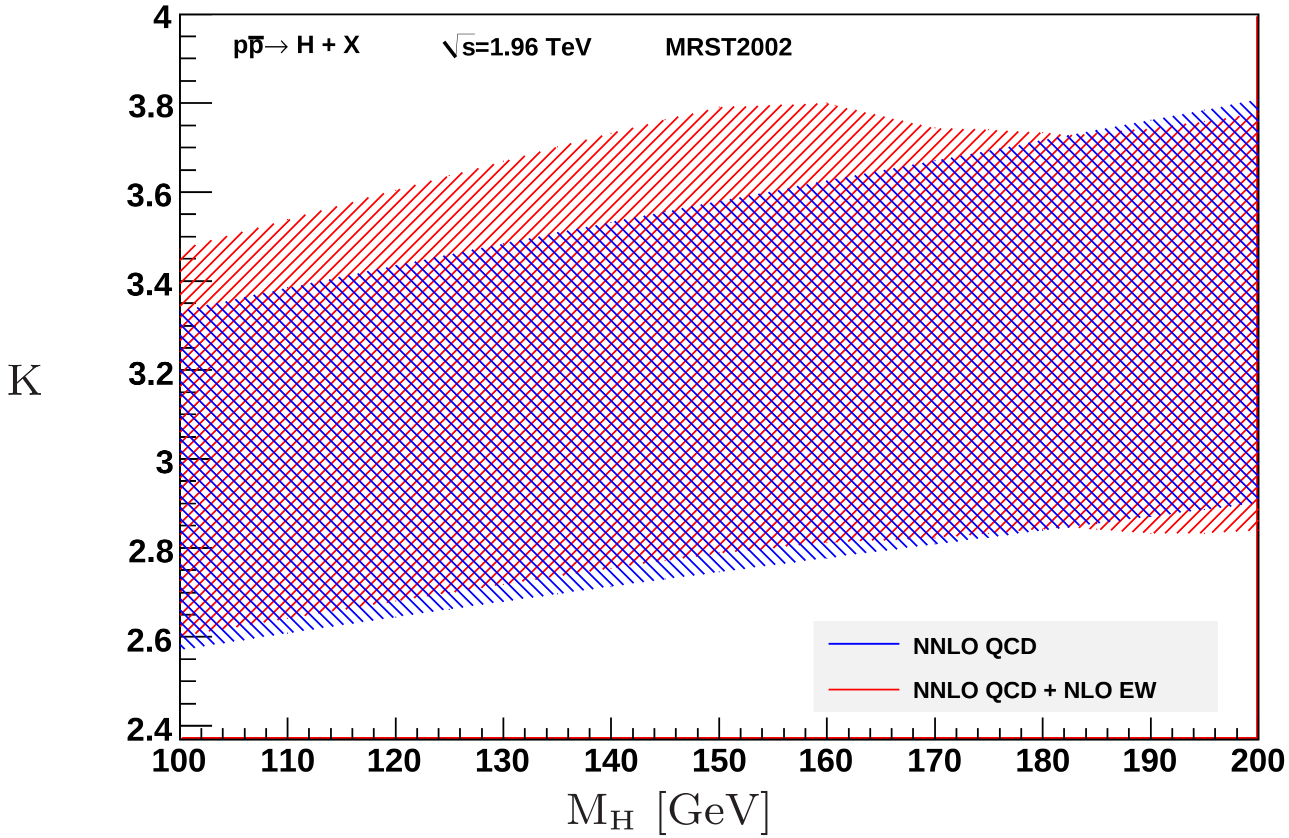
$$\int_0^1 dxdy \frac{1}{xya(x,y)+\lambda b(x,y)} = \int_0^1 dxdy \Bigg\{ \frac{1}{xya(x,y)+\lambda b(x,y)}\Bigg|_{x,y} + \frac{1}{xya(x,0)+\lambda b(x,0)}\Bigg|_{x} + \frac{1}{xya(0,y)+\lambda b(0,y)}\Bigg|_{y} + \frac{1}{xya(0,0)+\lambda b(0,0)} \Bigg\}, \qquad \lambda \to 0$$

$$f(z)|_z = f(z) - f(z)|_{z^2=\lambda z=0}$$

- First term $\longrightarrow$ set $\lambda = 0$
- Second (third) term $\longrightarrow$ integrate in y (x) $\longrightarrow$ $\ln(\lambda)$
- Last term $\longrightarrow$ integrate in x and y $\longrightarrow$ $\ln^2(\lambda)$



$$= \ln\frac{m^2}{s}\ln\frac{m'^2}{s}\mathrm{Li}_2\left(\frac{s}{M^2}\right) + \left(\ln\frac{m^2}{s} + \ln\frac{m'^2}{s}\right)\left[\mathrm{Li}_3\left(\frac{s}{M^2}\right) + 2\,S_{12}\left(\frac{s}{M^2}\right) - \ln\frac{M^2}{s}\mathrm{Li}_2\left(\frac{s}{M^2}\right)\right] + \text{ finite part}$$



Tevatron

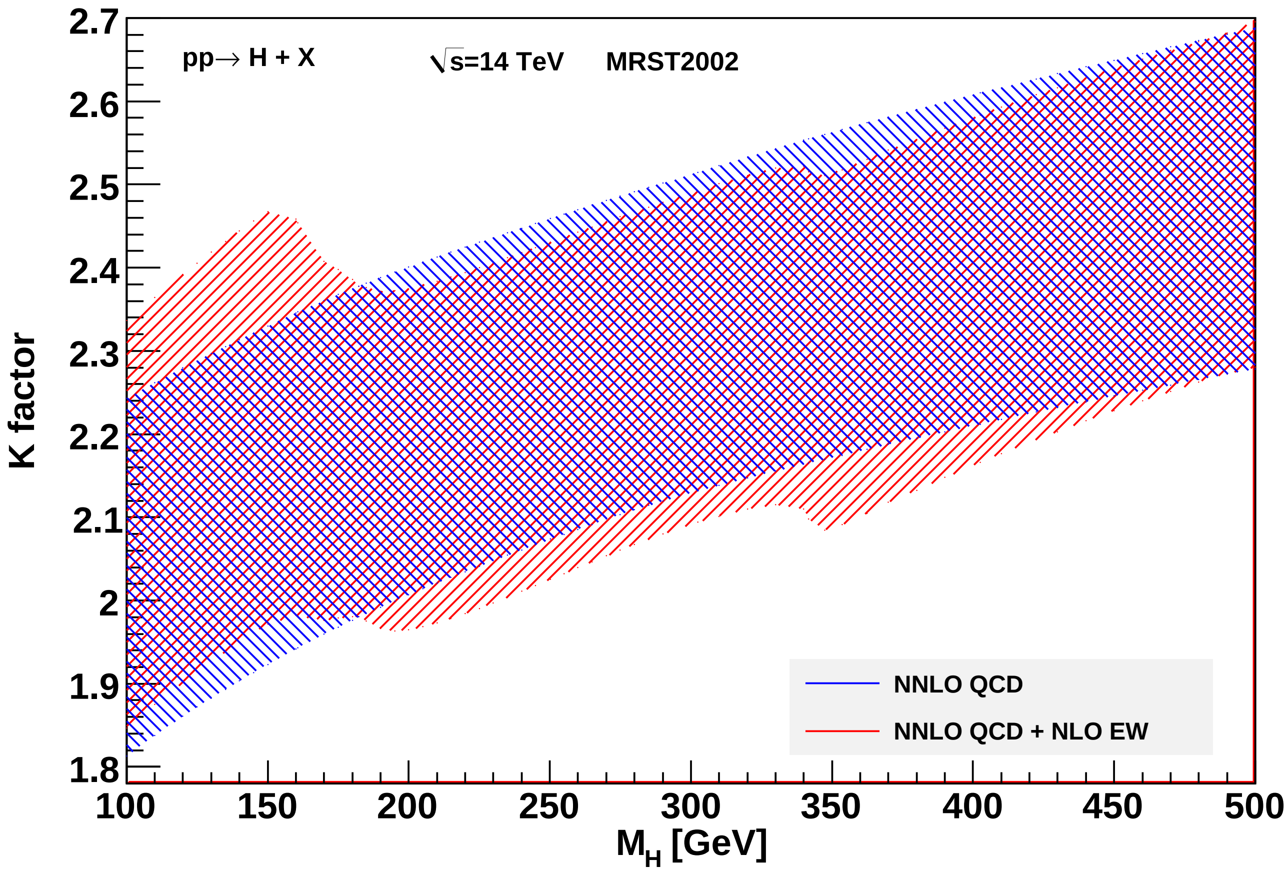
$$K = \frac{LO+NLO+NNLO}{LO}$$

M_H [GeV]	$\delta_{\rm CF}$ [%]	$\delta_{\rm PF}$ [%]
120	$+4.9$	$+1.6$
140	$+5.7$	$+1.8$
160	$+4.8$	$+1.5$
180	$+0.5$	$+0.1$
200	-2.1	-0.6

using HiggsNNLO by Grazzini

- NLO EW $\rightsquigarrow$ Stronger sensitivity on the Higgs mass
- NLO EW smaller than NNLL (Catani,de Florian,Grazzini,Nason'03)

  $\rightsquigarrow$ at $M_H = 120$ GeV: $+$ 4.9% (NLO EW) versus $+$ 12% (NNLL)



LHC

$$K = \frac{LO+NLO+NNLO}{LO}$$

M_H [GeV]	$\delta_{\rm CF}$ [%]	$\delta_{\rm PF}$ [%]
120	+4.9	+2.4
150	+5.9	+2.8
200	−2.1	−1.0
310	−1.7	−0.9
410	−0.8	−0.8

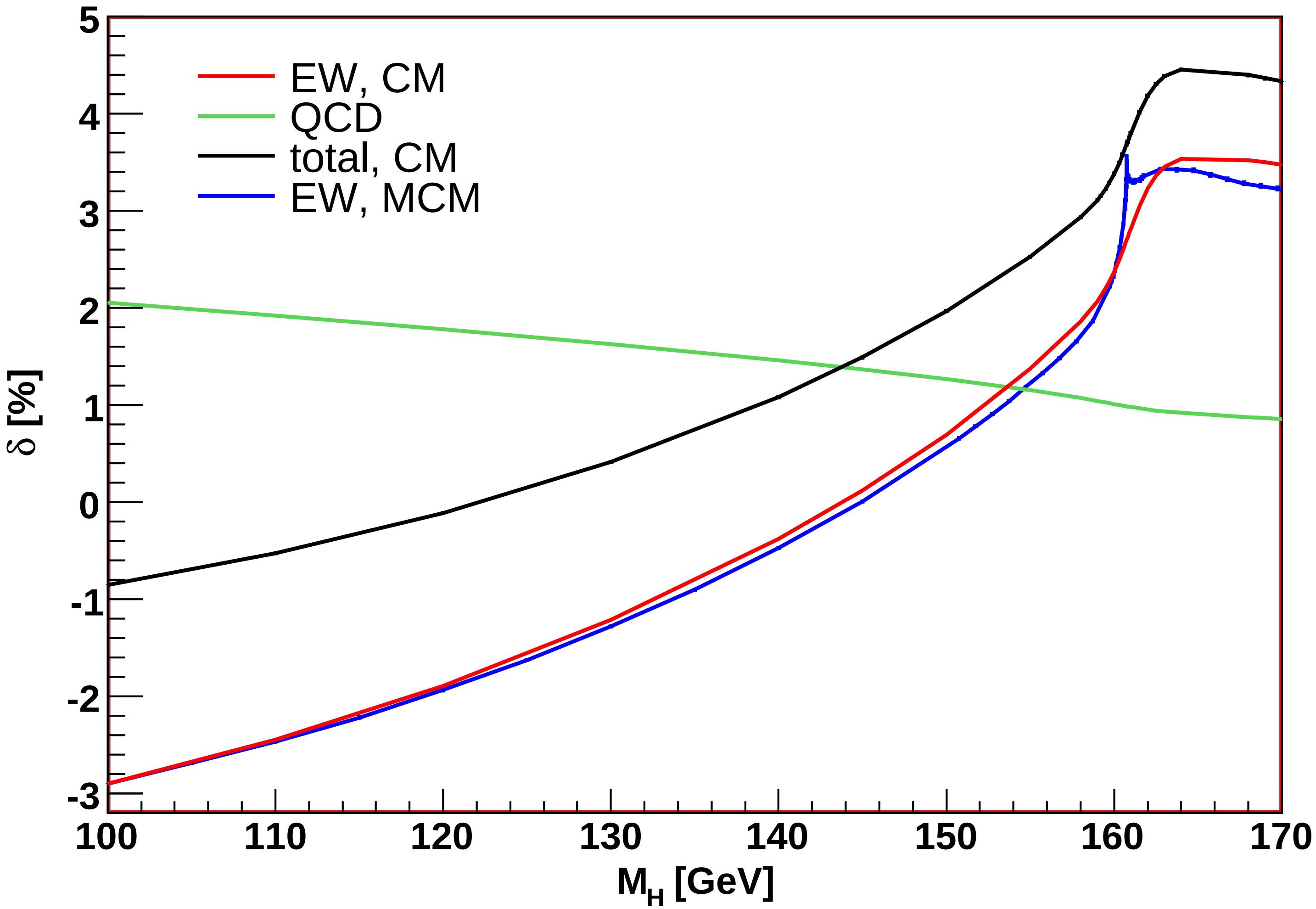
using HiggsNNLO by Grazzini

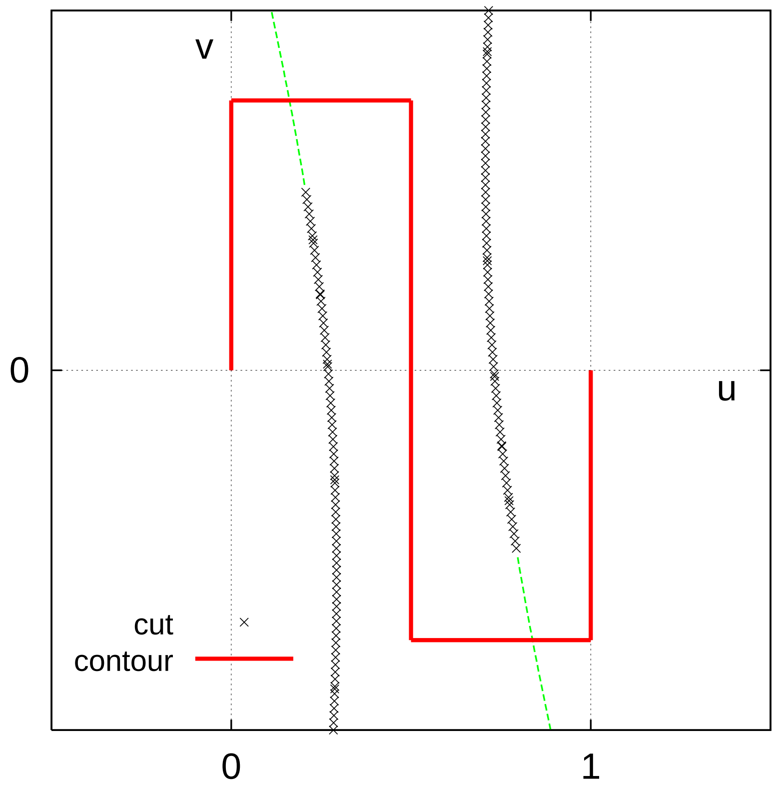
- NLO EW $\rightsquigarrow$ Stronger sensitivity on M_H (WW and $\bar{t}t$ threshold visible)
- NLO EW same order as NNLL (Catani,de Florian,Grazzini,Nason'03)
- for large M_H: NLO EW is negative $\rightsquigarrow$ screening NNLL

M_H [GeV]	$\delta_{\mathrm{EW}}[\%]$ Aglietti & al.	$\delta_{\mathrm{EW}}[\%]$ Actis & al.
150	+7.3	+5.9
156	+7.7	+5.7
160	+6.9	+4.8
166	+4.1	+2.9
170	+3.1	+2.0
176	+2.4	+1.1
180	+2.0	+0.5
186	+0.2	−0.7

Results for the decay width of $H \to \gamma\gamma$

$$\Gamma(H \to \gamma\gamma) = \Gamma_0\,(1 + \delta)$$

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