R(s), Bjorken sum rule and the Crewther Relation in order α_s^4

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(based on /very/ recent results obtained in collaboration to

P. Baikov and J. Kühn

RAD COR 2009

- intro: current status of R(s) (changes since RADCOR-2007) and NNNLO determinations of α_s

- the problem of reliability of the $\boldsymbol{\mathcal{O}(\alpha_s{}^4)}$ result for $R(s)$

- DIS sum rules and the generalized Crewther relation

  /**R. Crewther (1972); D. Broadhurst and A. Kataev (1993)**/

- (**new!**) the Bjorken sum rule in $\boldsymbol{\mathcal{O}(\alpha_s{}^4)}$ for QCD

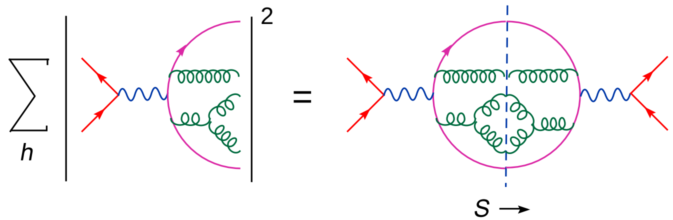
- (**new!**) results for the Bjorken sum rule and the (non-singlet) Adler function for a generic gauge group in $\boldsymbol{\mathcal{O}(\alpha_s{}^4)}$

- (**new!**) successfull test of the both results (and the quenched QED β function) with the (generalized) Crewther relation

- Conclusions and Perspectives

$$\boldsymbol{R(s) = \sigma_{tot}(e^+e^- \to hadrons)/\sigma(e^+e^- \to \mu^+\mu^-)}$$

related (via unitarity) to the correlator of the EM quark currents:



$$R(s) \approx \Im\,\Pi(s - i\delta)$$

$$3Q^2\Pi(q^2 = -Q^2) = \int e^{iqx}\langle 0|T[\;\, j^v_\mu(x) j^v_\mu(0)\;]|0\rangle dx$$

To conveniently sum the RG-logs one uses the Adler function:

$$D(Q^2) = Q^2 \frac{d}{d\,Q^2}\Pi(q^2) = Q^2 \int \frac{R(s)}{(s+Q^2)^2}\,ds$$

$$D(Q^2) = 1 + \sum_{n>1} d_n\, a_s^n \quad (a_s \equiv \alpha_s/\pi,\; \mu = Q)$$

Status of R(s) ($\overline{\text{MS}}$-scheme, $\mu^2 = s$)
since 1991 till 2007

$$\mathbf{R(s) = 1 + \frac{\alpha_s}{\pi} + (1.9857 - 0.1152\, n_f)\ \frac{\alpha_s^2}{\pi^2}}$$

$$\mathbf{+(-6.6369 - 1.2001\, n_f - 0.00518\, n_f^2)\, \frac{\alpha_s^3}{\pi^3}}$$

/Gorishnii, Kataev, Larin, (1991); in Feynman gauge
/Surguladze, Samuel, (1991); in Feynman gauge

K.Ch, (1997); in general covariant gauge /

$$d_4(N_F = 3) =$$

$$\frac{78631453}{20736} - \frac{1704247}{432}\zeta_3 + \frac{4185}{8}\zeta_3^2 + \frac{34165}{96}\zeta_5 - \frac{1995}{16}\zeta_7$$

$$\approx \mathbf{49.0757}$$

and, finally, for the very R(s):

$$1 + a_s + 1.6398\, a_s^2 + 6.3710\, a_s^3 \mathbf{-106.8798}\, a_s^4$$

or with kinematical (trivial!) π^2 terms separated

$$r_4 = 49.0757 - \underline{155.956}$$

$$d_4 = n_f^3 \left[-\tfrac{6131}{5832} + \tfrac{203}{324}\,\zeta_3 + \tfrac{5}{18}\,\zeta_5\right]$$ (“renormalon” chain /M. Beneke 1993/)

$$+\, n_f^2 \left[\tfrac{1045381}{15552} - \tfrac{40655}{864}\,\zeta_3 + \tfrac{5}{6}\,\zeta_3^2 - \tfrac{260}{27}\,\zeta_5\right]$$ /Baikov, Kühn, K.Ch. (2002)/

$$+\, \boldsymbol{n_f} \left[\boldsymbol{-\frac{13044007}{10368} + \frac{12205}{12}\,\zeta_3 - 55\,\zeta_3^2 + \frac{29675}{432}\,\zeta_5 + \frac{665}{72}\,\zeta_7}\right]$$

$$+ \left[\boldsymbol{\frac{144939499}{20736} - \frac{5693495}{864}\,\zeta_3 + \frac{5445}{8}\,\zeta_3^2 + \frac{65945}{288}\,\zeta_5 - \frac{7315}{48}\,\zeta_7}\right]$$

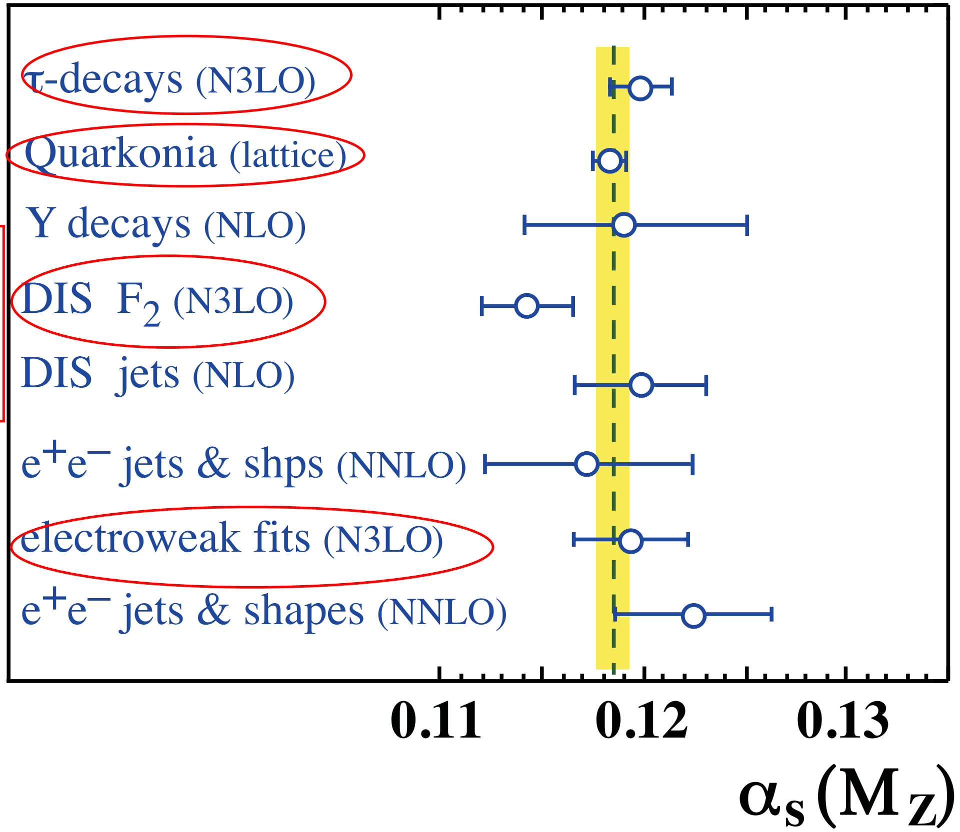
Results for the very R(s) in $\mathcal{O}(\alpha_s^4)$

$$R = 1 + a_s + (1.9857 - 0.1152 n_f)\, a_s^2 + (-6.63694 - 1.20013 n_f - 0.00518 n_f^2)\, a_s^3 +$$

$$\boxed{\boldsymbol{+(-156.61 + 18.77\, n_f - 0.7974\, n_f^2 + 0.02152\, n_f^3)\, a_s^4}}$$

World Summary of α_s 2009:

NNNLO to D(Q) and R(S)

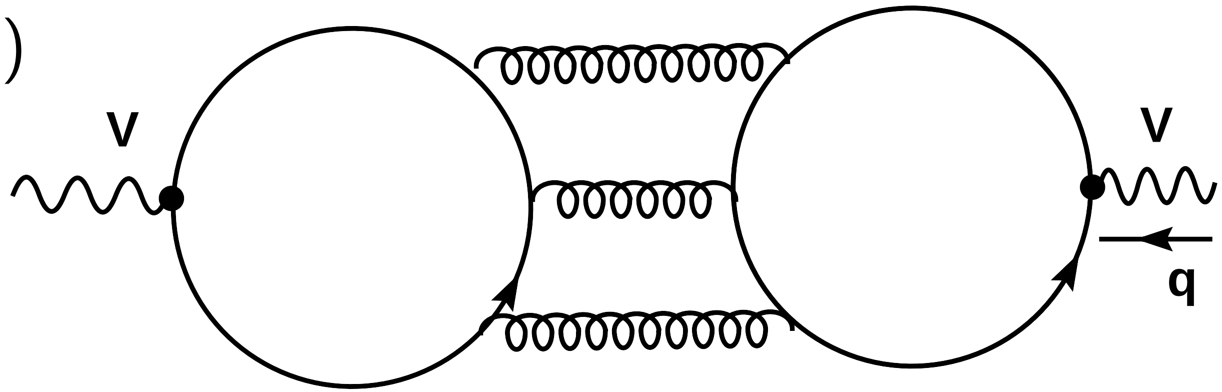


–> $\alpha_s(M_Z) = 0.1184 \pm 0.0007$

(Bethke, arXiv:0908.1135)

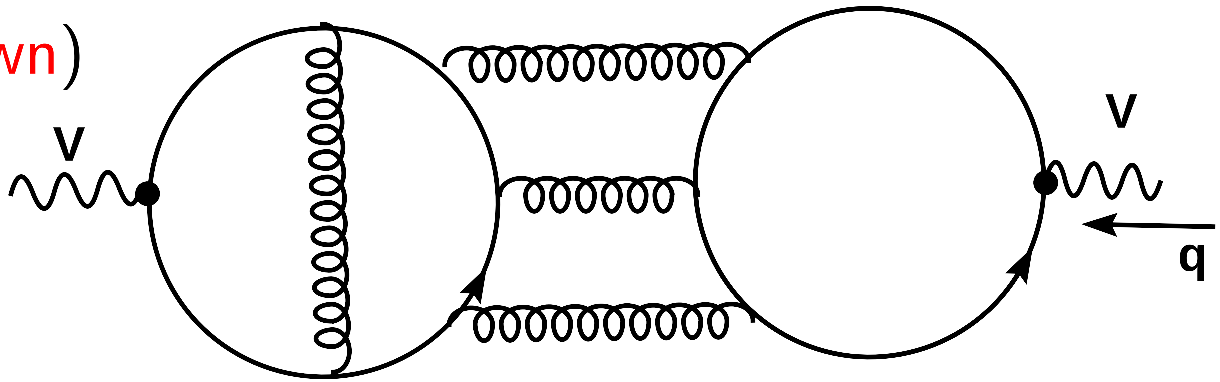
Missing Singlet terms in R(s) and $Z \rightarrow$ hadrons decays

α_s^3 (known)



$+ \ldots$

α_s^4 (unknown)



$+ \ldots$

How reliable are our results?

History of $R(s)$ teaches us to be cautious:

$\sim$ 20 years ago A. Kataev (with S. Gorishny and S. Larin) first produced a severly wrong result for the $\mathcal{O}(\alpha_s^3)$ term (corrected only by three years later!) in $R(s)$ and now he himself **(rightfully!)** rises an important question* of correctness of the first $\mathcal{O}(\alpha_s^4)$ result and calls for an urgent check of it

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* A. L. Kataev,

Is it possible to check urgently the 5-loop analytical results for the e^+e^--annihilation Adler function?

arXiv:0808.312v1

we do understand the problem since long:

"A golden rule well-known among multi-loop people says that a result of a multi-loop calculation can be trusted and considered as the result only if it is confirmed by an independent calculation preferably made by a different group and with the use of the general covariant gauge."

(cited from K. Ch., *Corrections of order* a_s^3 *to* R_{had} *in pQCD with light gluinos,* Phys. Lett. **B391**, p. 403 (1997))

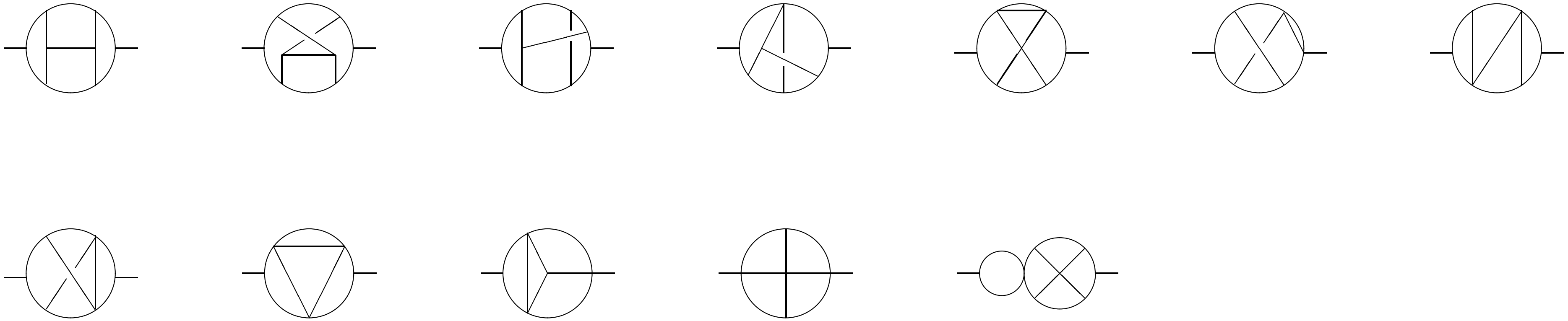
Unfortunately, at the moment we are not aware about any independent team which would be planning to check our results in full ...

In addition: we can not not compute in general gauge as it enormously (by an order of magnitude!) increases overall comlexity of calculations ...

BUT, in spite of all these sad circumstances,

very recently two powerfull checks of our results have been done !!!:

A. all non-trivial master integrals



have been successfully reproduced (with at least 3-digit accuracy) by numerical integration (with the use of quite sophisticated sector technique) by V. Smirnov, A. Smirnov and M. Tentyukov (to be published)

B. By computing $\mathcal{O}(\alpha_s{}^4)$ corrections to the polarized Bjorken sum rule and the Adler function for the general gauge group we have checked that the generalized Crewther relation

—which in order α_s^4 provide as as many as **SIX!**—

—**HIGHLY NON-TRIVIAL** constraints—

is indeed identically fulfilled by our result.

DIS Sum Rules

- the polarized Bjorken sum rule ($a_s \equiv \frac{\alpha_s}{\pi}$)

$$\boldsymbol{B}jp(Q^2) = \int_0^1 [g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)]dx = \frac{1}{6}|\frac{g_A}{g_V}|C^{Bjp}(a_s)$$

Coefficient function $C^{Bjp}(a_s)$ is fixed by OPE of two non-singlet vector currents (up to power suppressed corrections)

$$i\int TV_\alpha^a(x)V_\beta^b(0)e^{iqx}dx|_{q^2\to\infty} \approx C^{Q,abc}_{\alpha\beta\rho}A^c_\rho(0) + \ldots \qquad (1)$$

where

$$C^{Q,abc}_{\alpha\beta\rho} \sim id^{abc}\epsilon_{\alpha\beta\rho\sigma}\frac{q^\sigma}{Q^2}C^{Bjp}(a_s)$$

and $Q^2 = -q^2$

- the Gross-Llewellyn Smith sum rule

$$GLS(Q^2) = \frac{1}{2}\int_0^1 F_3^{\nu p+\overline{\nu}p}(x,Q^2)dx = 3\,C_{GLS}(a_s)$$

the function $C_{GLS}(a_s)$ comes from operator-product expansion of the axial and vector non-singlet currents

$$i\int T A_\mu^a(x) V_\nu^b(0) e^{iqx} dx|_{q^2\to\infty} \approx C_{\mu\nu\alpha}^{V,ab} V_\alpha(0) + \dots$$

where $C_{\mu\nu\alpha}^{V,ab} \sim i\delta^{ab}\epsilon_{\mu\nu\alpha\beta}\frac{q^\beta}{Q^2}C_{GLS}(a_s)$

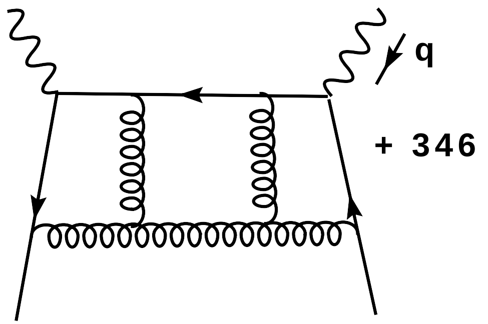
Note that both sum rules are unambiguous

/modulo higher twists!/

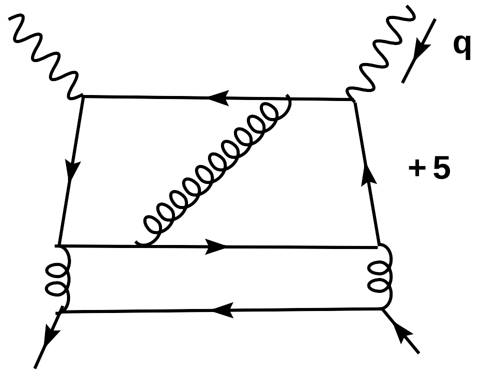
predictions of QCD which *in principle* could be confronted with experimental data

As is well-known, the evaluation of L-loop corrections to a CF of OPE could be done in terms of massless L-loop propagators (S. Gorisny, S. Larin and F. Tkachov (1982)) $\Longrightarrow$ **one could use techniques developed for** $R(s)$

$Bjp(Q)$ and $GLS(Q)$



$Bjp(Q)$



At order α_s^3 both CF's were computed in early nineties.The next order is contributed by about 54 thousand of 4-loop diagrams . . . (cmp. to $\approx$ 20 thousand of 5-loop diagrams contributing to $R(s)$ at the same order)

The Crewther relation states that in the conformal invariant limit ($\beta \equiv 0$) $C_{Bjp}(a_s)$ is related to the (nonsinglet) Adler function via the following beautiful equality

$$\boxed{\boldsymbol{C_{Bjp}(a_s(Q^2))C_D^{NS}(a_s(Q^2))|_{c-i} = 1}}$$

its generalization for real QCD reads:

$$\boxed{\boldsymbol{C^{Bjp}(a_s)C_D^{NS}(a_s) = 1 + \frac{\beta(a_s)}{a_s}\Big[K_1\,a_s + K_2\,a_s^2 + K_3\,a_s^3 + \ldots\Big]}}$$

Note that similar relation connects also the CF of the Gross-Llewellyn Smith sum rule to the full Adler function;

Main ingredients of the derivation: the AVV 3-point function and constraints on it from (approximate) conformal invariance + Adler-Bardeen anomaly theorem

Crewther Relation: (short) bibliography

discovered: R.J. Crewther, *Phys. Rev. Lett.* **28**, 1421 (1972).
S.L. Adler, C.G. Callan, D.J. Gross and R. Jackiw, *Phys. Rev.* D **6**, 2982 (1972).

generalized for "real" QCD:
D.J. Broadhurst and A.L. Kataev, *Phys. Lett.* B **315**, 179 (1993).

further developed:
G.T. Gabadadze and A.L. Kataev,*JETP Lett.* **61**, 448 (1995).)
S.J. Brodsky, G.T. Gabadadze, A.L. Kataev and H.J. Lu, *Phys. Lett.* B **372**, 133 (1996).

proven:
R.J. Crewther, *Phys. Lett.* B **397**, 137 (1997).
V. M. Braun, G. P. Korchemsky and D. Müller, Prog. Part. Nucl. Phys. **51**, 311 (2003)

discovered: R.J. Crewther, *Phys. Rev. Lett.* **28**, 1421 (1972).
S.L. Adler, C.G. Callan, D.J. Gross and R. Jackiw, *Phys. Rev.* D **6**, 2982 (1972).

"downgraded" from ideal, conformal-invariant paradise to the dirty world of real QCD by **David Broadhurst and Andrey Kataev**:
D.J. Broadhurst and A.L. Kataev, *Phys. Lett.* B **315**, 179 (1993).

further developed: ...
proven: ...

Note that

the generalization would not be possible without analytical $\mathcal{O}(\alpha_s^3)$ calculations of $D(q)$ and C^{Bjp}; the latter would not be possible without dedicated people:

(S. Gorishny, A. Kataev, S. Larin; M. Samuel, L. Surguladze; J. Vermaseren, S. Larin, F. Tkachov)

and without dedicated tools:

SCHOONSCHIP / M. Veltman/ and FORM 2 / J. Vermaseren/

Last but not the least:

21 century $\mathcal{O}(\alpha_s^4)$ calculations like we are doing would hardly be feasible without excellent possibilities for dealing with gigantic data streams offered by FORM 3 and, especially, such its versions as

ParFORM and T-FORM

M. Tentyukov et al. "ParFORM: Parallel Version of the Symbolic Manipulation Program" , cs/0407066.

M. Tentyukov, H. M. Staudenmaier, and J. A. M. Vermaseren. "ParFORM: Recent development". *Nucl. Instrum. Meth.*, A559:224–228, 2006.

M. Tentyukov and J. A. M. Vermaseren. "The multithreaded version of FORM", hep-ph/0702279"

Which exactly constraints come from the Crewther relation?

$$C^{Bjp}(a_s)C_D^{NS}(a_s) = 1 + \frac{\beta(a_s)}{a_s}\Big[K_1\, a_s + K_2\, a_s^2 + K_3\, a_s^3 + \ldots\Big]$$

If it is valid at order a_s^n, then at the next order a_s^{n+1}, we have

$$\left(d_{n+1} - C_{n+1}^{Bjp} + \text{interference terms}\right) a_s^{n+1} = \beta_0\, a_s\Big[K_n\, a_s^n\Big]$$

$$\alpha_s^1 : (d_1 - C_1) : C_F \Longleftrightarrow K_0 \equiv 0 \leftarrow \text{ one constraint}$$

$$\alpha_s^2 : (d_2 - C_2) : C_F^2, T\, C_F, C_F\, C_A \Longleftrightarrow K_1 : C_F \leftarrow \text{ two constraints}$$

$$\alpha_s^3 : (d_3 - C_3) : C_F^3\,, C_F^2 C_A\,, C_F C_A^2\,, C_F^2 T\,, C_F C_A T\,, C_F T^2$$

$$\Updownarrow$$

$$K_2 : C_F^2, C_F C_A, C_F T \leftarrow \text{ three constraints}$$

At last, at $\mathcal{O}(\alpha_s^4)$ there exist exactly 12 color strtuctures:

$$C_F^4\,,\ C_F^3C_A\,,\ C_F^2C_A^2\,,\ C_FC_A^3\,,\ C_F^3T_Fn_f\,,\ C_F^2C_AT_Fn_f\,,$$
$$C_FC_A^2T_Fn_f\,,\ C_F^2T_F^2n_f^2\,,\ C_FC_AT_F^2n_f^2\,,\ C_FT_F^3n_f^3\,,\ d_F^{abcd}d_A^{abcd}\,,\ n_fd_F^{abcd}d_F^{abcd}$$

while the coefficient K_3 is contributed by only **6** color structures:

$$C_FT^2\,,C_F\,C_A^2\,,C_F^2\,T\,,C_F\,C_A\,T\,,C_F^2\,C_A\,,C_F^3$$

Thus, we have 12-6 $=$ **6** constraints on the difference

$$\boldsymbol{d_4 - (C^{Bjp})_4}$$

3 of them are very simple: the above difference cannot contain

$$C_F^4,\ d_F^{abcd}d_A^{abcd} \quad n_fd_F^{abcd}d_F^{abcd}$$

remaining three are a bit more complicated

Calculations of the Adler $D^{NS}(a_s)$-function and the CF $C^{Bjp}(a_s)$ of the Bjorken sum rule for the polarized DIS have been just finished <u>for a generic gauge group at $\boldsymbol{\mathcal{O}(\alpha_s{}^4)}$</u>

$$C^{Bjp} = 1 \quad - \quad a_s + a_s^2\,[-4.583 + 0.3333\,n_f] + a_s^3\left[-41.44 + 7.607\,n_f - 0.1775\,n_f^2\right]$$
$$+ \quad a_s^4\left[-479.4 + 123.4\,n_f - 7.697\,n_f^2 + 0.1037\,n_f^3\right]$$

$$C^{Bjp}(n_f = 3) = 1 - 1.a_s - 3.583a_s^2 - 20.22a_s^3 \; \boxed{\text{-175.7}}\, a_s^4$$

FAC/PMS prediction due to Kataev ans Starshenko (almost 15 years old!) for the $\mathcal{O}(\alpha_s^4)$ term at $n_f = 3$ is $\boxed{-130\ a_s^4}$

$$C^{Bjp}(n_f = 4) = 1 - a_s - 3.25a_s^2 - 13.85a_s^3 \; -102.4\,a_s^4$$

$$C^{Bjp}(n_f = 5) = 1 - a_s - 2.917a_s^2 - 7.84a_s^3 \; -41.96\,a_s^4$$

$$C^{Bjp}(n_f = 6) = 1 - a_s - 2.583a_s^2 - 2.185a_s^3 \; +6.2\,a_s^4,$$

K&S for $n_f = 6$: $\boxed{+22\ a_s^4}$

	d_4	$(1/C^{Bjp})_4$
C_F^4	$\frac{4157}{2048} + \frac{3}{8}\,\zeta_3$	$\frac{4157}{2048} + \frac{3}{8}\,\zeta_3$
$\frac{d_F^{abcd}\,d_F^{abcd}}{(d_R/n_f)}$	$-\frac{13}{16} - \ \zeta_3 + \frac{5}{2}\,\zeta_5$	$-\frac{13}{16} - \ \zeta_3 + \frac{5}{2}\,\zeta_5$
$\frac{d_F^{abcd}\,d_A^{abcd}}{d_R}$	$\frac{3}{16} - \frac{1}{4}\,\zeta_3 - \frac{5}{4}\,\zeta_5$	$\frac{3}{16} - \frac{1}{4}\,\zeta_3 - \frac{5}{4}\,\zeta_5$
$T_n^3 C_F$	$-\frac{6131}{972} + \frac{203}{54}\,\zeta_3 + \frac{5}{3}\,\zeta_5$	$-\frac{605}{972}$
$T_n^2 C_F^2$	$\frac{5713}{1728} - \frac{581}{24}\,\zeta_3 + \frac{125}{6}\,\zeta_5 + 3\,\zeta_3^2$	$\frac{869}{576} - \frac{29}{24}\,\zeta_3$
$T_n^2 C_F C_A$	$\frac{340843}{5184} - \frac{10453}{288}\,\zeta_3 - \frac{170}{9}\,\zeta_5 - \frac{1}{2}\,\zeta_3^2$	$\frac{165283}{20736} + \frac{43}{144}\,\zeta_3 - \frac{5}{12}\,\zeta_5 + \frac{1}{6}\,\zeta_3^2$
$T_n C_F^3$	$\frac{1001}{384} + \frac{99}{32}\,\zeta_3 - \frac{125}{4}\,\zeta_5 + \frac{105}{4}\,\zeta_7$	$-\frac{473}{2304} - \frac{391}{96}\,\zeta_3 + \frac{145}{24}\,\zeta_5$
$T_n C_F^2 C_A$	$\frac{32357}{13824} + \frac{10661}{96}\,\zeta_3 - \frac{5155}{48}\,\zeta_5 - \frac{33}{4}\,\zeta_3^2 - \frac{105}{8}\,\zeta_7$	$-\frac{17309}{13824} + \frac{1127}{144}\,\zeta_3 - \frac{95}{144}\,\zeta_5 - \frac{35}{4}\,\zeta_7$
$T_n C_F C_A^2$	$-\frac{(\cdots)}{(\cdots)} + \frac{8609}{72}\,\zeta_3 + \frac{18805}{288}\,\zeta_5 - \frac{11}{2}\,\zeta_3^2 + \frac{35}{16}\,\zeta_7$	$-\frac{(\cdots)}{(\cdots)} - \frac{59}{64}\,\zeta_3 + \frac{1855}{288}\,\zeta_5 - \frac{11}{12}\,\zeta_3^2 + \frac{35}{16}\,\zeta_7$
$C_F^3\,C_A$	$-\frac{253}{32} - \frac{139}{128}\,\zeta_3 + \frac{2255}{32}\,\zeta_5 - \frac{1155}{16}\,\zeta_7$	$-\frac{8701}{4608} + \frac{1103}{96}\,\zeta_3 - \frac{1045}{48}\,\zeta_5$
$C_F^2\,C_A^2$	$-\frac{592141}{18432} - \frac{43925}{384}\,\zeta_3 + \frac{6505}{48}\,\zeta_5 + \frac{1155}{32}\,\zeta_7$	$-\frac{435425}{55296} - \frac{1591}{144}\,\zeta_3 + \frac{55}{9}\,\zeta_5 + \frac{385}{16}\,\zeta_7$
$C_F\,C_A^3$	$\frac{(\cdots)}{(\cdots)} - \frac{(\cdots)}{(\cdots)}\,\zeta_3 - \frac{77995}{1152}\,\zeta_5 + \frac{605}{32}\,\zeta_3^2 - \frac{385}{64}\,\zeta_7$	$\frac{(\cdots)}{(\cdots)} - \frac{(\cdots)}{(\cdots)}\,\zeta_3 - \frac{12545}{1152}\,\zeta_5 + \frac{121}{96}\,\zeta_3^2 - \frac{385}{64}$

$$C^{Bjp}(\alpha_s)C_D^{NS}(\alpha_s) = 1 + \frac{\beta(\alpha_s)}{\alpha_s}C_F\Big[K_1\,\alpha_s + K_2\,\alpha_s^2 + K_3\,\alpha_s^3 + \ldots\Big]$$

$$K_1 = -\tfrac{21}{8} + 3\zeta_3$$

$$\begin{aligned} K_2 \;=\;& n_f T && (\tfrac{163}{24} - \tfrac{19}{3}\zeta_3) \\ & +C_A && (-\tfrac{629}{32} + \tfrac{221}{12}\zeta_3) \\ & +C_F && (\tfrac{397}{96} + \tfrac{17}{2}\zeta_3 - 15\zeta_5) \end{aligned}$$

$$\begin{aligned} K_3 \;=\;& n_f^2 T^2 && (-\frac{307}{18} + \frac{203}{18}\zeta_3 + 5\zeta_5) \\ & +C_A^2 && (-\frac{406043}{2304} + \frac{18007}{144}\zeta_3 + \frac{2975}{48}\zeta_5 - \frac{77}{4}\zeta_3^2) \\ & +C_F n_f T && (-\frac{7729}{1152} - \frac{917}{16}\zeta_3 + \frac{125)}{2}\zeta_5 + 9\zeta_3^2) \\ & +C_A n_f T && (\frac{1055}{9} - \frac{(2521)}{36}\zeta_3 - \frac{125)}{3}\zeta_5 - 2\zeta_3^2) \\ & +C_A C_F && (\frac{99757}{2304} + \frac{8285)}{96}\zeta_3 - \frac{(1555}{12}\zeta_5 - \frac{105}{8}\zeta_7) \\ & +C_F^2 && (\frac{2471}{768} + \frac{61}{8}\zeta_3 - \frac{715}{8}\zeta_5 + \frac{315}{4}\zeta_7) \end{aligned}$$

Comments:

The Crewther test is highly non-trivial:

- four-loop box-type diagrams (in propagator kinematics) versus five loop propagators

- **No IR-trickery is neccessary** in calculation of $C_{Bjp}(a_s)$

- As a result we he have been able to check that $C_{Bjp}(a_s)$ is indeed gauge-independent (the Adler finction was computed in the simplest, Feynman gauge only!)

- in the course of our calculations we have had to extend the Larin treatment of Hooft-Veltman γ_5 at 4-loop level (a natural object for the dim. reg., which really appears in the course of calculations, is $\gamma^{[\mu\nu\alpha]}$ instead of $\gamma_5\gamma^\mu$ with anticommuting γ_5; the mismatch should be corrected by the Larin factor)

CONCLUSION

- The Adler $D^{NS}(a_s)$-function and the CF $C^{Bjp}(a_s)$ of the Bjorken sum rule for the polarized DIS have been both analytically evaluated for a generic gauge group at $\boldsymbol{\mathcal{O}(\alpha_s{}^4)}$
- The generalized Crewther relation puts as many as 6 highly non-tivial constraints on the difference

$$\boldsymbol{d_4 - (C^{Bjp})_4}$$

which **are all** fulfilled!

- all our master integrals have been **independently** checked by numerical integration

- Thus, our results for $D^{NS}(a_s)$ (and, thus, for the very ratio $R(s)$) should be considered as **correct** beyond any (reasonable) doubt

Perspectives

- Calculation of the Gross-Llewellyn Smith sum rule have been just finished and its results are now under analysis

- Once calculations for D^{SI} are completed (presumably within a year) we will have another strong test of all the machinery

- Calculations of the Bjorken sum rule for the unpolarized DIS as well as the Ellis-Jaffe sum rule (most complicated due to the appearance of the flavour-singlet axial-vector current/non-abelian anomaly!/) are in reach of our methods