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The Electroweak Model based on the Nonlinearly Realized Gauge Group

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Based on

arXiv:0903.0281 [hep-th]
arXiv:0809.1994 [hep-th]
arXiv:0807.3882 [hep-ph]
arXiv:0712.1410 [hep-th]
arXiv:0709.0644 [hep-th]
arXiv:0705.2339 [hep-th]

The nonlinearly realized massive SU(2) YM theory

The Stückelberg Mechanism (mass term via the coupling with the flat connection) is based on the non-linear representation of the gauge group

Apply the strategy based on the LFE, the hierarchy, the Weak Power Counting and the associated perturbative *Ansatz* to massive SU(2) YM theory as a testing ground towards
the formulation of the nonlinearly realized EW model

Some important features
Formulation of the LFE
STI vs LFE
Uniqueness of the tree-level vertex functional
(no anomalous couplings)

The LFE in nonlinearly realized gauge theories

Introduce a background connection V_μ and adopt a (background) Landau gauge-fixing

$$
\begin{aligned}
\mathcal{W}(\Gamma) \equiv \int d^D x \, \alpha_a^L(x) \Bigg(& -\partial_\mu \frac{\delta \Gamma}{\delta V_{a\mu}} + \epsilon_{abc} V_{c\mu} \frac{\delta \Gamma}{\delta V_{b\mu}} - \partial_\mu \frac{\delta \Gamma}{\delta A_{a\mu}} \\
& + \epsilon_{abc} A_{c\mu} \frac{\delta \Gamma}{\delta A_{b\mu}} + \epsilon_{abc} B_c \frac{\delta \Gamma}{\delta B_b} + \frac{1}{2} K_0 \phi_a + \underbrace{\frac{1}{2} \frac{\delta \Gamma}{\delta K_0} \frac{\delta \Gamma}{\delta \phi_a}}_{\text{BILINEAR!}} \\
& + \frac{1}{2} \epsilon_{abc} \phi_c \frac{\delta \Gamma}{\delta \phi_b} + \epsilon_{abc} \bar{c}_c \frac{\delta \Gamma}{\delta \bar{c}_b} + \epsilon_{abc} c_c \frac{\delta \Gamma}{\delta c_b} \\
& + \epsilon_{abc} \Theta_{c\mu} \frac{\delta \Gamma}{\delta \Theta_{b\mu}} + \epsilon_{abc} A^*_{c\mu} \frac{\delta \Gamma}{\delta A^*_{b\mu}} + \epsilon_{abc} c^*_c \frac{\delta \Gamma}{\delta c^*_b} + \frac{1}{2} \phi^*_0 \frac{\delta \Gamma}{\delta \phi^*_a} \\
& + \frac{1}{2} \epsilon_{abc} \phi^*_c \frac{\delta \Gamma}{\delta \phi^*_b} - \frac{1}{2} \phi^*_a \frac{\delta \Gamma}{\delta \phi^*_0} \Bigg) = 0 \, .
\end{aligned}
$$

Then one can focus on the ancestor variables only

Selecting the tree-level vertex functional

Introduce variables invariant under the linearized local functional equation (**bleached variables**)

$$\begin{aligned} a_\mu &= a_{a\mu}\frac{\tau_a}{2} = \Omega^\dagger(A_\mu - F_\mu)\Omega \\ &= \Omega^\dagger A_\mu \Omega - i\partial_\mu \Omega^\dagger \Omega\,. \end{aligned}$$

Work out the WPC for the ancestor variables

$$\begin{aligned} d(\mathcal{G}) \leq (D-2)n + 2 - N_A - N_c - N_V - N_{\phi_a^*} \\ - 2(N_\Theta + N_{A^*} + N_{\phi_0^*} + N_{c^*} + N_{K_0})\,. \end{aligned}$$

Uniqueness of the tree-level vertex functional

All the bleached invariants involving at least three a's generate vertices with two phi's, two A's and two derivatives. They give rise to one-loop diagrams with degree of divergence equal to 4 and any number of external A's legs.

So they maximally violate the WPC unless they combine into the YM field strength squared.

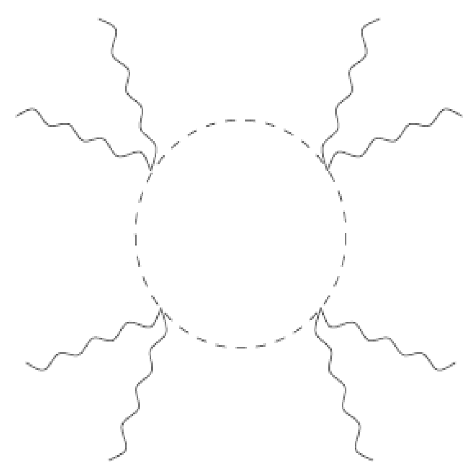


Figure 1: A weak power-counting violating graph.

On the contrary no condition is put on the invariants containing at most two a's (also off-diagonal mass terms are allowed).Impose global $SU_R(2)$ invariance. Then the Stueckelberg mass term is selected.

Feynman rules in the Landau gauge

The classical gauge-invariant action ...

$$S = \frac{\Lambda^{(D-4)}}{g^2} \int d^D x \Big(-\frac{1}{4} G_{a\mu\nu}[a] G_a^{\mu\nu}[a] + \frac{M^2}{2} a_{a\mu}^2 \Big)$$
$$= \frac{\Lambda^{(D-4)}}{g^2} \int d^D x \Big(-\frac{1}{4} G_{a\mu\nu}[A] G_a^{\mu\nu}[A] + \frac{M^2}{2} (A_{a\mu} - F_{a\mu})^2 \Big)$$

... and the associated tree-level vertex functional (it sets the Feynman rules for the perturbative expansion)

$$\Gamma^{(0)} = S + \frac{\Lambda^{D-4}}{g^2} \int d^D x \Big(B_a (D^\mu[V](A_\mu - V_\mu))_a - \bar{c}_a (D^\mu[V] D_\mu[A] c)_a \Big)$$
$$+ \frac{\Lambda^{D-4}}{g^2} \int d^D x \, \Theta_a^\mu \, (D_\mu[A]\bar{c})_a$$
$$+ \int d^D x \Big(A_{a\mu}^* s A_a^\mu + \phi_0^* s \phi_0 + \phi_a^* s \phi_a + c_a^* s c_a + K_0 \phi_0 \Big) .$$

The STI does not yield a hierarchy

A counter-example

$$\mathcal{I} = \mathcal{S}_0\Big(\int d^Dx\,(A^*_{a\mu} + \partial_\mu \bar c_a) A^\mu_a\Big)$$
$$= \int d^Dx\,\Big(A_{a\mu}\frac{\delta S}{\delta A_{a\mu}} - (A^*_{a\mu} + \partial_\mu \bar c_a)\partial^\mu c_a\Big)$$

$$\mathcal{I}' = \int \bigg(\frac{1}{g^2}\Big(- (D[F]_\mu I_\nu)_a (D[F]^\mu I^\nu)_a + (D[F]I)^2_a - 3\epsilon_{abc}(D_\mu[F]I_\nu)_a I^\mu_b I^\nu_c - (I^2)^2 + I_{a\mu} I^\mu_b I_{a\nu} I^\nu_b\Big) + M^2 I^2 + \mathcal{S}_0((A^*_{a\mu} + \partial_\mu \bar c_a)\partial^\mu(\Omega^{-1}_{ap}\phi_p))\bigg)\,.$$

They coincide at $\vec\phi = 0$, but they have different projections on the monomial $\epsilon_{abc}\partial A^*_a c_b \phi_c$.

This is due to the antisymmetric character of the ghost fields.

$$S = -\frac{1}{g^2}\int d^Dx\,\frac{1}{4}G_{a\mu\nu}G^{\mu\nu}_a + \frac{M^2}{2}\int d^Dx\,A^2_{a\mu}\,,$$
$$I_{a\mu} = A_{a\mu} - F_{a\mu}$$

Perturbative Solution in *D* dimensions

Ansatz: Only the pole parts are subtracted by adopting

the counterterm structure

$$\widehat{\Gamma} = \Gamma^{(0)} + \Lambda_D \sum_{j \geq 1} \int d^D x \mathcal{M}^{(j)},$$

The amplitudes must be normalized as

$$\Lambda_D^{-1} \Gamma^{(n)}$$

This procedure yields symmetric amplitudes (i.e. they fullfill all the relevant symmetries of the theory).

One Loop

At one loop level
the relevant symmetries are

the linearized ST identity

the linearized local functional equation

the Landau gauge equation

Compatibility condition $\left[\mathcal{S}_0, \mathcal{W}_0\right] = 0$

Projections of the one-loop invariants on the ancestor variables

$$\mathcal{I}_1 = \frac{1}{2}\int d^Dx\, \partial_\mu A_{a\nu}\partial^\mu A^\nu_a\,,$$

$$\mathcal{I}_2 = \frac{1}{2}\int d^Dx\, (\partial A_a)^2\,,$$

$$\mathcal{I}_3 = -\frac{1}{2}\int d^Dx\, \epsilon_{abc}\partial_\mu A_{a\nu}A^\mu_b A^\nu_c\,,$$

$$\mathcal{I}_4 = \frac{1}{4}\int d^Dx\, (A^2)^2\,,$$

$$\mathcal{I}_5 = \frac{1}{4}\int d^Dx\, (A_{a\mu}A^\mu_b)(A_{a\nu}A^\nu_b)\,,$$

$$\mathcal{I}_6 = \frac{1}{2}\int d^Dx\, A^2\,,$$

$$\mathcal{I}_7 = \frac{1}{2}\int d^Dx\, V^\mu_a\Big(D^\rho G_{\rho\mu}[A] + M^2 A_\mu\Big)_a - \frac{1}{2}\int d^Dx\, \widehat{A}^*_{a\mu}\Theta^\mu_a$$
$$+\frac{1}{2}\int d^Dx\, \widehat{A}^*_{a\mu}(D^\mu[V]c)_a\,,$$

$$\mathcal{I}_8 = \int d^Dx\, (2K_0 - c_a\phi^*_a)^2\,,$$

$$\mathcal{I}_9 = \int d^Dx\, \Big(\frac{1}{2}c_a\phi^*_a A^2 - K_0 A^2\Big)\,,$$

$$\mathcal{I}_{10} = \int d^Dx\, \Big(\frac{1}{2}(D^\mu[A]\widehat{A}^*_\mu)_a c_a - \frac{1}{4}\phi^*_a c_a - \frac{1}{2}c^*_a\epsilon_{abc}c_b c_c\Big)\,,$$

$$\mathcal{I}_{11} = \int d^Dx\, \Big(c_a\phi^*_a - 2K_0\Big)\,.$$

Projections of the one-loop invariants on the ancestor amplitudes

One Loop Solution

In the bleached variables the linearized local functional equation reads

$$\frac{\delta \Gamma^{(1)}}{\delta \phi_a(x)} = 0$$

Then one needs to solve a cohomological problem in the space of bleached variables

$$\mathcal{S}_0 \Gamma^{(1)} = 0$$

One-loop counterterms

The counterterms

$$\widehat{\Gamma}^{(1)} = \frac{\Lambda^{(D-4)}}{(4\pi)^2}\frac{1}{D-4}\Big[\frac{17}{2}(\mathcal{I}_1 - \mathcal{I}_2) - \frac{67}{6}\mathcal{I}_3 + \frac{11}{4}\mathcal{I}_4 - \frac{5}{2}\mathcal{I}_5 + 3M^2\mathcal{I}_6 - 6\mathcal{I}_7 + \frac{3v^2}{128M^4}\mathcal{I}_8 - \frac{v}{8M^2}\mathcal{I}_9\Big] .$$

The nonlinearly realized EW model

Non linearly realized SU(2) x $U_Y(1)$ gauge group

Introduce the LFE for SU(2)

$U_Y(1)$ imposes constraints on the allowed bleached monomials

(once the direction of SSB is fixed)

Bleached variables with matter fields

Introduce a background connection V_μ and (for the 't Hooft gauge) an additional set of scalar sources

$$\hat{\Omega} = \frac{1}{v}\Big((v + \hat{\sigma})\mathbf{1} + i\tau_a \hat{\phi}_a\Big)$$

This yields the LFE along the same lines discussed before in the (background) 't Hooft gauge

(of an unconventional type since there are two independent mass invariants)

The bleaching also occurs with the fermionic SU(2) left doublets *L*.

$$\tilde{L} = \Omega^\dagger L$$

Notations for the fermionic matter fields

For convenience we may sometimes use a doublet notation also for the right components of the fermionic flavour eigenstates, i.e. we set

$$L \in \left\{ \begin{pmatrix} l^u_{Lj} \\ l^d_{Lj} \end{pmatrix}, \begin{pmatrix} q^u_{Lj} \\ V_{jk} q^d_{Lk} \end{pmatrix}, \quad j,k = 1,2,3 \right\}$$

$$R \in \left\{ \begin{pmatrix} l^u_{Rj} \\ l^d_{Rj} \end{pmatrix}, \begin{pmatrix} q^u_{Rj} \\ q^d_{Rj} \end{pmatrix}, \quad j = 1,2,3 \right\}$$

$U_Y(1)$ symmetry

The transformations under the (non-linearly implemented) hypercharge are ($U_Y(1)$ acts on the right on Ω)

$$e^{-i\frac{\alpha}{2}Y}\Omega e^{i\frac{\alpha}{2}Y} = \Omega V^\dagger \qquad e^{-i\frac{\alpha}{2}Y}F_\mu e^{i\frac{\alpha}{2}Y} = F_\mu + i\Omega V^\dagger\partial_\mu V\Omega^\dagger$$
$$e^{-i\frac{\alpha}{2}Y}A_\mu e^{i\frac{\alpha}{2}Y} = A_\mu \qquad e^{-i\frac{\alpha}{2}Y}B_\mu e^{i\frac{\alpha}{2}Y} = B_\mu + \frac{1}{g'}\partial_\mu\alpha$$
$$e^{-i\frac{\alpha}{2}Y}Le^{i\frac{\alpha}{2}Y} = \exp(i\frac{\alpha}{2}Y_L)L \qquad e^{-i\frac{\alpha}{2}Y}Re^{i\frac{\alpha}{2}Y} = \exp(i\frac{\alpha}{2}(Y_L+\tau_3))R,$$

where Y_L denotes the hypercharge of L and

$$V(\alpha) = \exp(i\frac{\alpha}{2}\tau_3)$$

(after the SSB direction has been chosen along the trace component of Ω)

Selection of the tree-level vertex functional

We apply the following strategy:

1. use bleached variables, so that the SU(2) gauge symmetry is automatically taken into account;

2. impose neutrality under hypercharge of the tree-level vertex functional. Since the latter is expressed in terms of bleached variables, this requirement is equivalent to electric charge neutrality;

3. select the monomials compatible with the WPC.

Two mass invariants for the vector mesons

Consider the bleached combination

$$w_\mu = \Omega^\dagger g A_\mu \Omega - g' \frac{\tau_3}{2} B_\mu + i\Omega^\dagger \partial_\mu \Omega$$

with $U_Y(1)$ transformation

$$w'_\mu = V w_\mu V^\dagger.$$

Because of hermiticity and tracelessness of w_μ the only independent mass invariants are

$$(w_\mu)^2_{11} \qquad (w^\mu)_{21}(w_\mu)_{12}$$

Vector meson mass eigenstates

Introduce the mass eigenstates

$$W^{\pm}_{\mu} = \frac{1}{\sqrt{2}}(A_{1\mu} \mp iA_{2\mu})$$

$$Z_{\mu} = \frac{1}{\sqrt{g^2 + g'^2}}(gA_{3\mu} - g' B_{\mu})$$
$$A_{\mu} = \frac{1}{\sqrt{g^2 + g'^2}}(g' A_{3\mu} + gB_{\mu}),$$

c is the cosine of the Weinberg angle, with

$$\tan\theta_W = \frac{g'}{g}$$

Take the linear combination

$$2g^2M^2\left[(w_{\mu})_{21}(w^{\mu})_{12} + \frac{1}{g^2}(1+\kappa)(w_{\mu})^2_{11}\right]$$

Then the masses of the Z and W are

$$M_W^2 = g^2M^2\,, \qquad M_Z^2 = (1+\kappa)\frac{g^2M^2}{c^2}\,,$$

The parameter *k* controls the violation of the Weinberg relation.

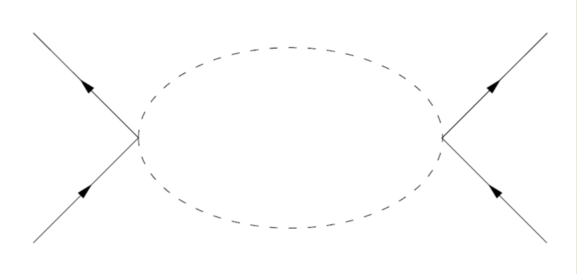
Weak Power-Counting Formula

There is a unique Lagrangian compatible with the symmetries of the theory and the Weak Power-Counting theorem

$$d(\mathcal{G}) \leq (D-2)n + 2 - N_A - N_\psi$$

D space-time dimension, *n* loop number

Massive fermions have degree one in the nonlinearly realized model since there are quadrilinear interaction vertices with two fermions and two Goldstone and thus one-loop logarithmically divergent four-fermion amplitudes



Massive Majorana Neutrinos

The peculiar UV behaviour of the fermions in the nonlinearly realized EW model implies that a Majorana mass term for neutrinos is allowed in the most general classical action compatible with the WPC and the symmetries.

Consider

$$\tilde{L} = \Omega^\dagger L = \begin{pmatrix} \tilde{\nu} \\ \tilde{e}_L \end{pmatrix}$$

Each of the components is bleached. Therefore one can construct the following Majorana mass term

$$\bar{\tilde{\nu}} \tilde{\nu}$$

Feynman rules

The bosonic sector

$$\begin{aligned}\mathcal{L}_{YM} &= -\frac{1}{4}\Big(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc}A^b_\mu A^c_\nu)^2 - \frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 \\ &+ M^2\, T_r\Big[(g\Omega^\dagger A_\mu \Omega - i\partial_\mu \Omega^\dagger \Omega - g' B_\mu \frac{\tau_3}{2})(g\Omega^\dagger A^\mu \Omega - i\partial^\mu \Omega^\dagger \Omega - g' B^\mu \frac{\tau_3}{2})\Big] \\ &+ \frac{1}{2}k\ M^2\, T_r\Big[(g\Omega^\dagger A_\mu \Omega - i\partial_\mu \Omega^\dagger \Omega - g' B_\mu \frac{\tau_3}{2})\tau_3\Big] \\ &\qquad T_r\Big[(g\Omega^\dagger A^\mu \Omega - i\partial^\mu \Omega^\dagger \Omega - g' B^\mu \frac{\tau_3}{2})\tau_3\Big]\end{aligned}$$

The fermionic sector

$$\begin{aligned}\mathcal{L}_{ferm} &= \sum_i \Big(\overline{L'}^L_i i\gamma^\mu D_\mu L'^L_i + \overline{Q'}^L_i i\gamma^\mu D_\mu Q'^L_i\Big) \\ &+ \sum_i \Big(\overline{l'}^R_i i\gamma^\mu D_\mu l'^R_i + \overline{u'}^R_i i\gamma^\mu D_\mu u'^R_i + \overline{d'}^R_i i\gamma^\mu D_\mu d'^R_i\Big) \\ &- \sum_{ij}\Big(\overline{\tilde{L}'}{}^L_i G^l_{ij} l'^R_j + \overline{\tilde{Q}'}{}^L_i G^u_{ij} u'^R_j + \overline{\tilde{Q}'}{}^L_i G^d_{ij} d'^R_j + h.c.\Big)\end{aligned}$$

All anomalous symmetric interactions are forbidden by the WPC.

SU(2) custodial symmetry

The SU(2) custodial symmetry is broken

a) in the fermionic sector (as in the Standard Model);
b) in the gauge boson sector (at $g'=0$).

In the most general solution compatible with the symmetries and the WPC k is an independent parameter which affects the radiative corrections.

Requiring SU(2) custodial symmetry in the gauge boson sector implies a fine-tuning of the parameter k to zero.

The QED Ward identity

The electric charge is defined as usual

$$Q = I_3 + \frac{1}{2}Y$$

The associated Ward identity is linear

$$\begin{aligned}
&\frac{1}{g'}\Box b_0 + \Bigg(-\frac{1}{g'}\partial^\mu \frac{\delta}{\delta B^\mu} - \frac{1}{g}\partial_\mu \frac{\delta}{\delta A_{3\mu}} - \frac{1}{g}\partial_\mu \frac{\delta}{\delta V_{3\mu}} \\
&+A_{2\mu}\frac{\delta}{\delta A_{1\mu}} - A_{1\mu}\frac{\delta}{\delta A_{2\mu}} + iQL\frac{\delta}{\delta L} - i\bar{L}Q\frac{\delta}{\delta \bar{L}} + iQR\frac{\delta}{\delta R} - i\bar{R}Q\frac{\delta}{\delta \bar{R}} \\
&+\phi_2\frac{\delta}{\delta \phi_1} - \phi_1\frac{\delta}{\delta \phi_2} + b_2\frac{\delta}{\delta b_1} - b_1\frac{\delta}{\delta b_2} + c_2\frac{\delta}{\delta c_1} - c_1\frac{\delta}{\delta c_2} \\
&+\bar{c}_2\frac{\delta}{\delta \bar{c}_1} - \bar{c}_1\frac{\delta}{\delta \bar{c}_2} + V_{2\mu}\frac{\delta}{\delta V_{1\mu}} - V_{1\mu}\frac{\delta}{\delta V_{2\mu}} + \Theta_{2\mu}\frac{\delta}{\delta \Theta_{1\mu}} - \Theta_{1\mu}\frac{\delta}{\delta \Theta_{2\mu}} \\
&+A^*_{2\mu}\frac{\delta}{\delta A^*_{1\mu}} - A^*_{1\mu}\frac{\delta}{\delta A^*_{2\mu}} + \phi^*_2\frac{\delta}{\delta \phi^*_1} - \phi^*_1\frac{\delta}{\delta \phi^*_2} + c^*_2\frac{\delta}{\delta c^*_1} - c^*_1\frac{\delta}{\delta c^*_2} \\
&-iQL^*\frac{\delta}{\delta L^*} + i\bar{L}^*Q\frac{\delta}{\delta \bar{L}^*}\Bigg)\Gamma = 0\,,
\end{aligned}$$

This follows since $Q\phi_0 = 0$

Perturbative Solution in *D* dimensions

We apply the same *Ansatz* as before, i.e. subtract the pure pole part of properly normalized amplitudes and get the counterterms of the form

$$\widehat{\Gamma} = \Gamma^{(0)} + \Lambda_D \sum_{j \geq 1} \int d^D x \mathcal{M}^{(j)}$$

The amplitudes must be normalized as $\Lambda_D^{-1} \Gamma^{(n)}$

We adopt a pragmatic approach to γ_5

- introduce an anticommuting γ_D
- no statements are made on the analiticity properties of traces involving γ_D
- at the end we take the limit *D* = 4; since the theory is not anomalous one never meets poles in *D* - 4.

This procedure sets up the symmetric perturbative scheme for the loop expansion

Summary

- For a certain class of nonrenormalizable gauge theories based on the nonlinearly realized gauge group the infinite number of divergences which show up at each loop order can be tamed by functional methods (LFE).
- These theories enjoy in the perturbative loop expansion a stability property of a novel type, encoded in the notion of ancestor amplitudes plus the Weak Power-Counting condition.
- A working framework for the perturbative computation in the loop expansion for this class of models has been provided.
- This strategy applies to the nonlinearly realized EW model (no Higgs in the perturbative physical spectrum).
- If this scheme turns out to be phenomenologically viable (need of the evaluation of the radiative corrections!) the Higgs mass need not be constrained by present electroweak data.
- The value of the parameter Λ emerging from the fit with the electroweak precision data might give information on the energy scale where new physics is expected.

The table of bleached variables – gauge sector

$$
\begin{aligned}
a_\mu &= a_{a\mu}\frac{\tau_a}{2} = \Omega^\dagger (A_\mu - F_\mu)\Omega \\
&= \Omega^\dagger A_\mu \Omega - i\partial_\mu \Omega^\dagger \Omega\,.
\end{aligned}
$$

$$
\begin{aligned}
v_\mu &= a_{a\mu}\frac{\tau_a}{2} = \Omega^\dagger (V_\mu - F_\mu)\Omega \\
&= \Omega^\dagger V_\mu \Omega - i\partial_\mu \Omega^\dagger \Omega\,.
\end{aligned}
$$

Gauge variables

$$
\widetilde{I} = \Omega^\dagger I \Omega\,,
$$

$$
\widetilde{B}_a, \widetilde{\bar{c}}_a, \widetilde{c}_a, \widetilde{\Theta}_{a\mu}, \widetilde{A^*}_{a\mu}, \widetilde{c}^*_a\,.
$$

Variables in the adj. representation under the local left transformations

The table of bleached variables - Goldstone fields sector

$$K = K_0 - i \frac{\delta \Gamma^{(0)}}{\delta \phi_a} \tau_a = K_0 + i K_a \tau_a \, ,$$
$$\tilde{K} = \Omega^\dagger K \, ,$$
$$\Omega^* = \phi_0^* + i \phi_a^* \tau_a \, ,$$
$$\tilde{\Omega}^* = \Omega^\dagger \Omega^*$$

SU(2) doublets

$$\widetilde{\phi_0^*} = \frac{1}{v_D} (\phi_0 \phi_0^* + \phi_a \phi_a^*) \, ,$$
$$\widetilde{\phi_a^*} = \frac{1}{v_D} (\phi_0 \phi_a^* - \phi_a \phi_0^* - \epsilon_{abc} \phi_b^* \phi_c) \, .$$

$$\widetilde{K_0} = \frac{1}{v_D} \Big(\frac{v_D^2 K_0}{\phi_0} - \phi_a \frac{\delta}{\delta \phi_a} \Big(\Gamma^{(0)} \Big|_{K_0 = 0} \Big) \Big)$$