中國科學院高能物理研究所
Institute of High Energy Physics Chinese Academy of Sciences

Future Colliders

Lecture-2

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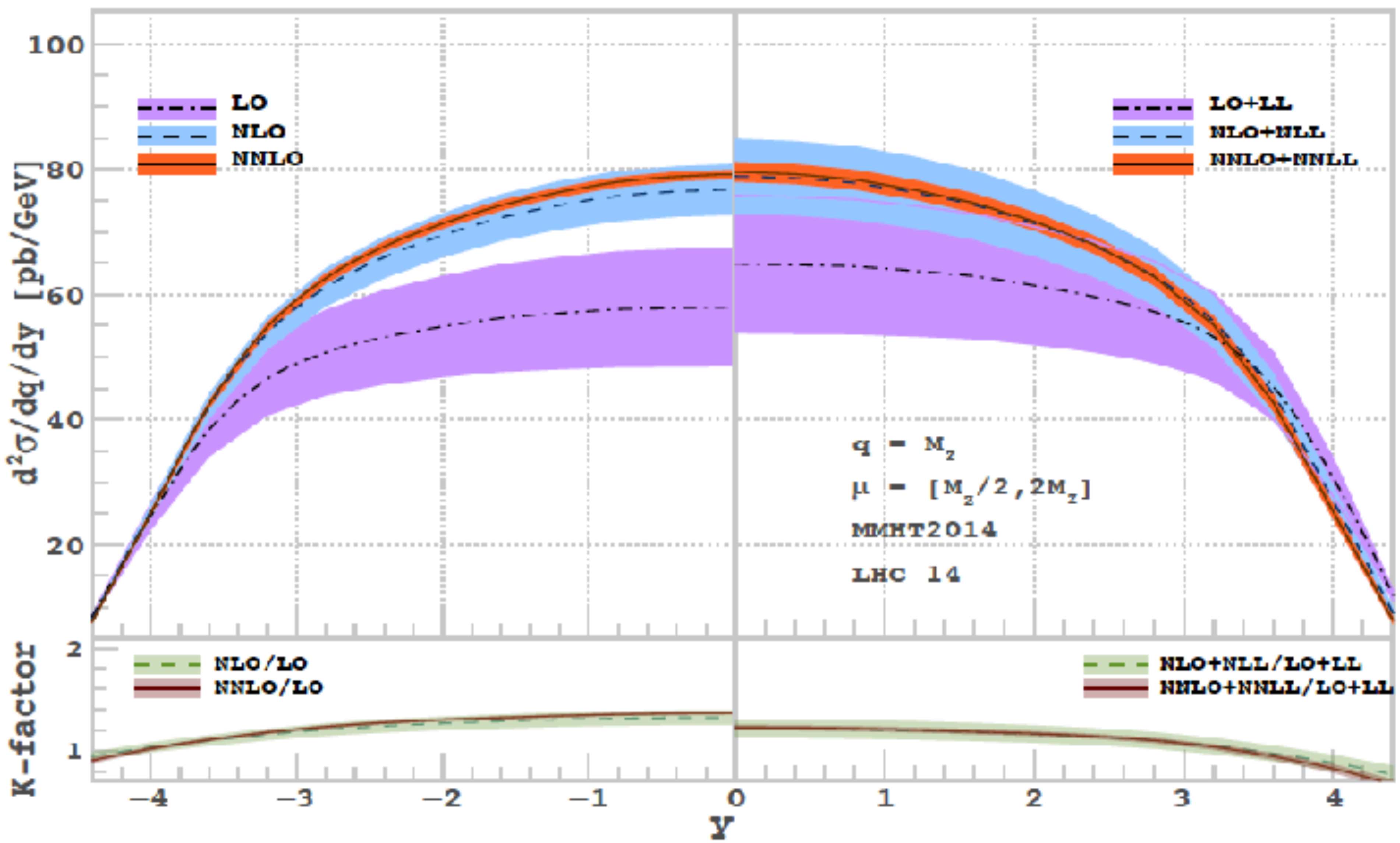
Homeworks

THINKING: Try to understand the main source of the NLO correction without calculation. Why there are more small-y Z-bosons?



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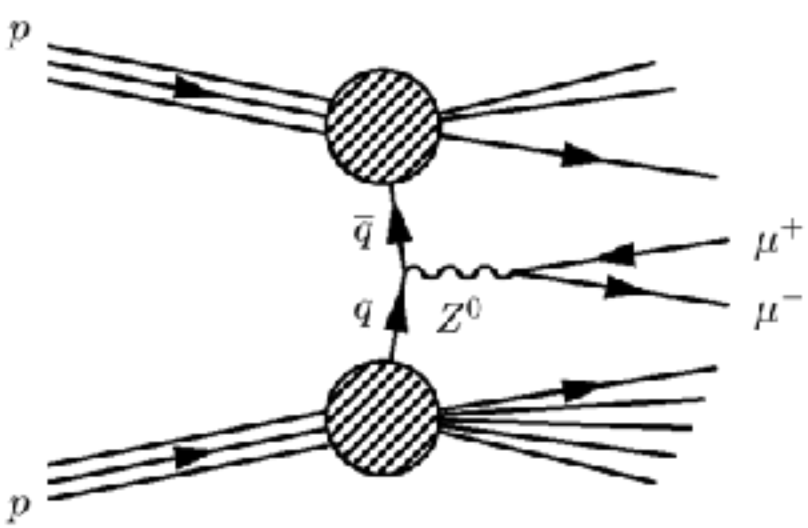
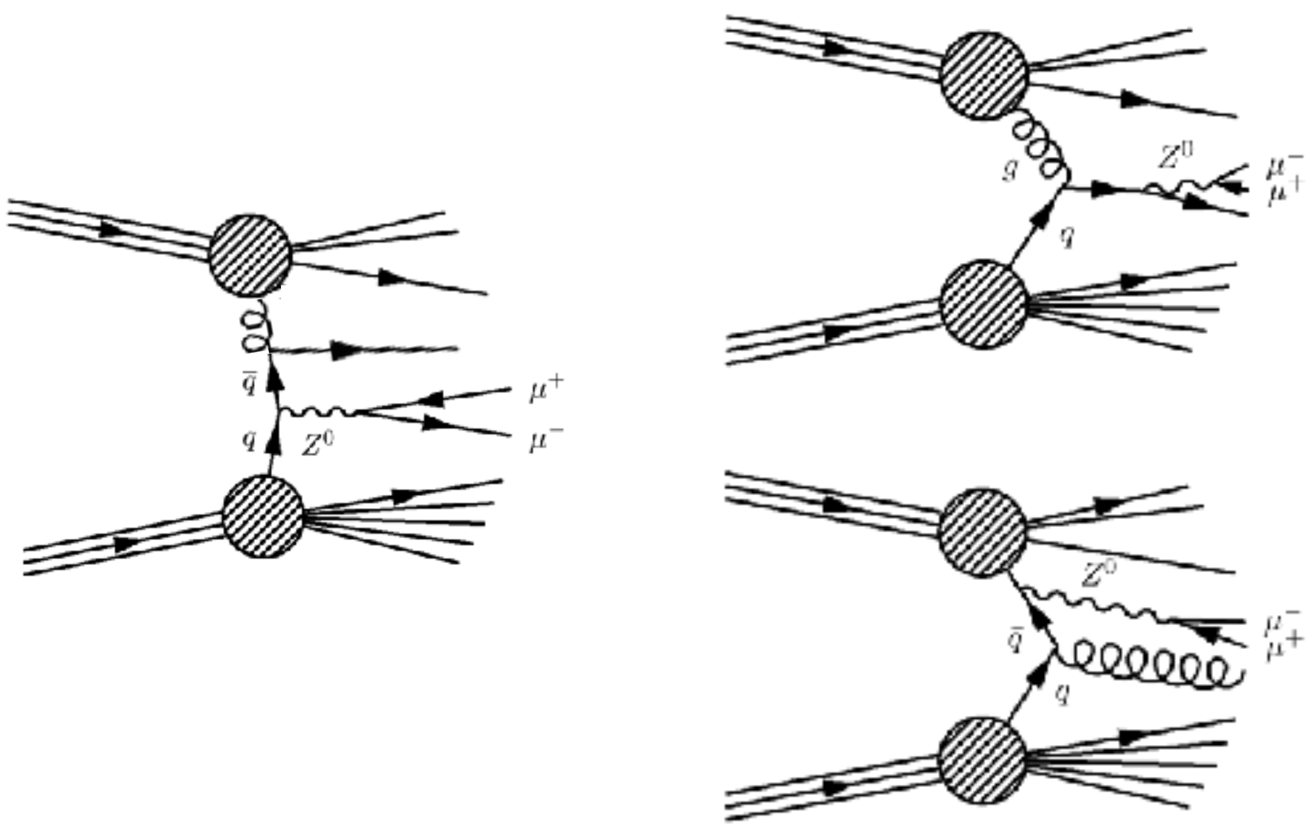
LO



Real corrections



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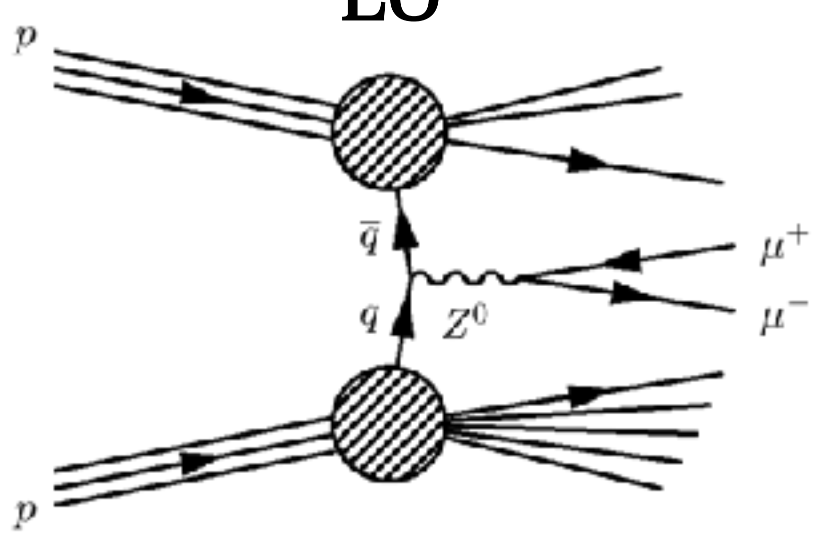
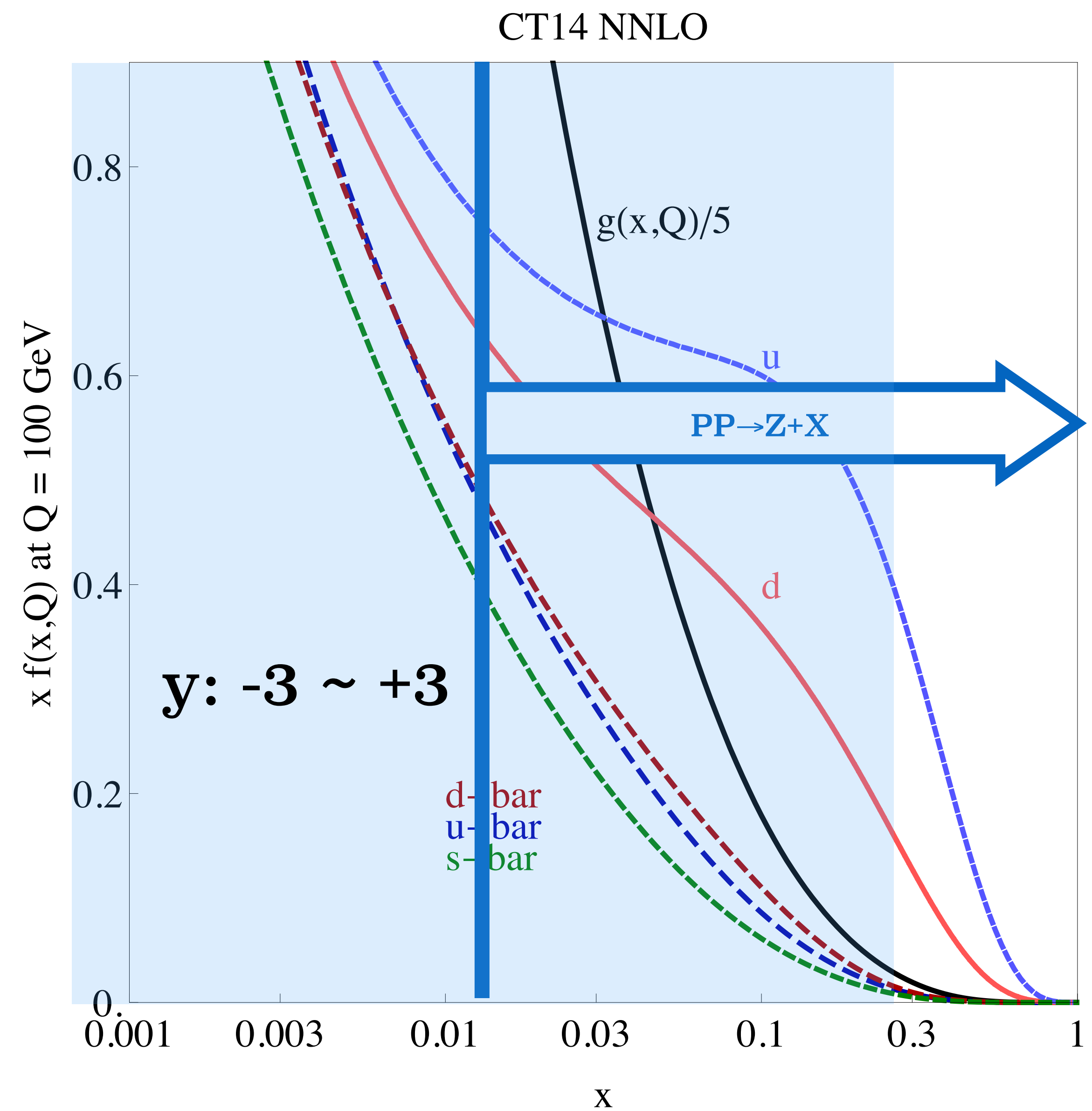


$$
\begin{aligned}
y_z^{\mathrm{LO}} &= \frac{1}{2}\log\frac{E+p}{E-p} \\
&= \frac{1}{2}\log\frac{x_1+x_2+x_1-x_2}{x_1+x_2-x_1+x_2} \\
&= \frac{1}{2}\log\frac{x_1}{x_2}
\end{aligned}
$$



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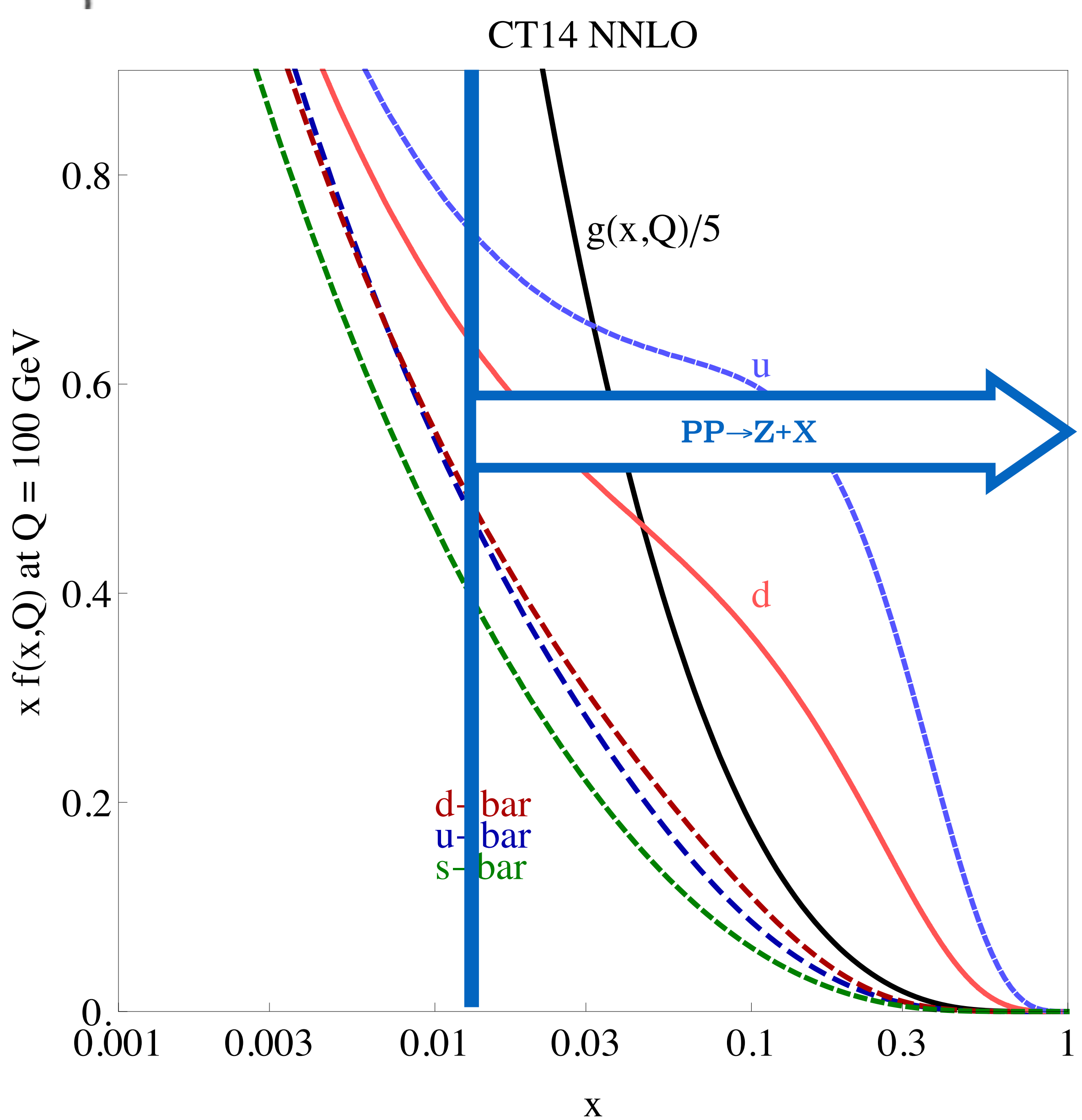

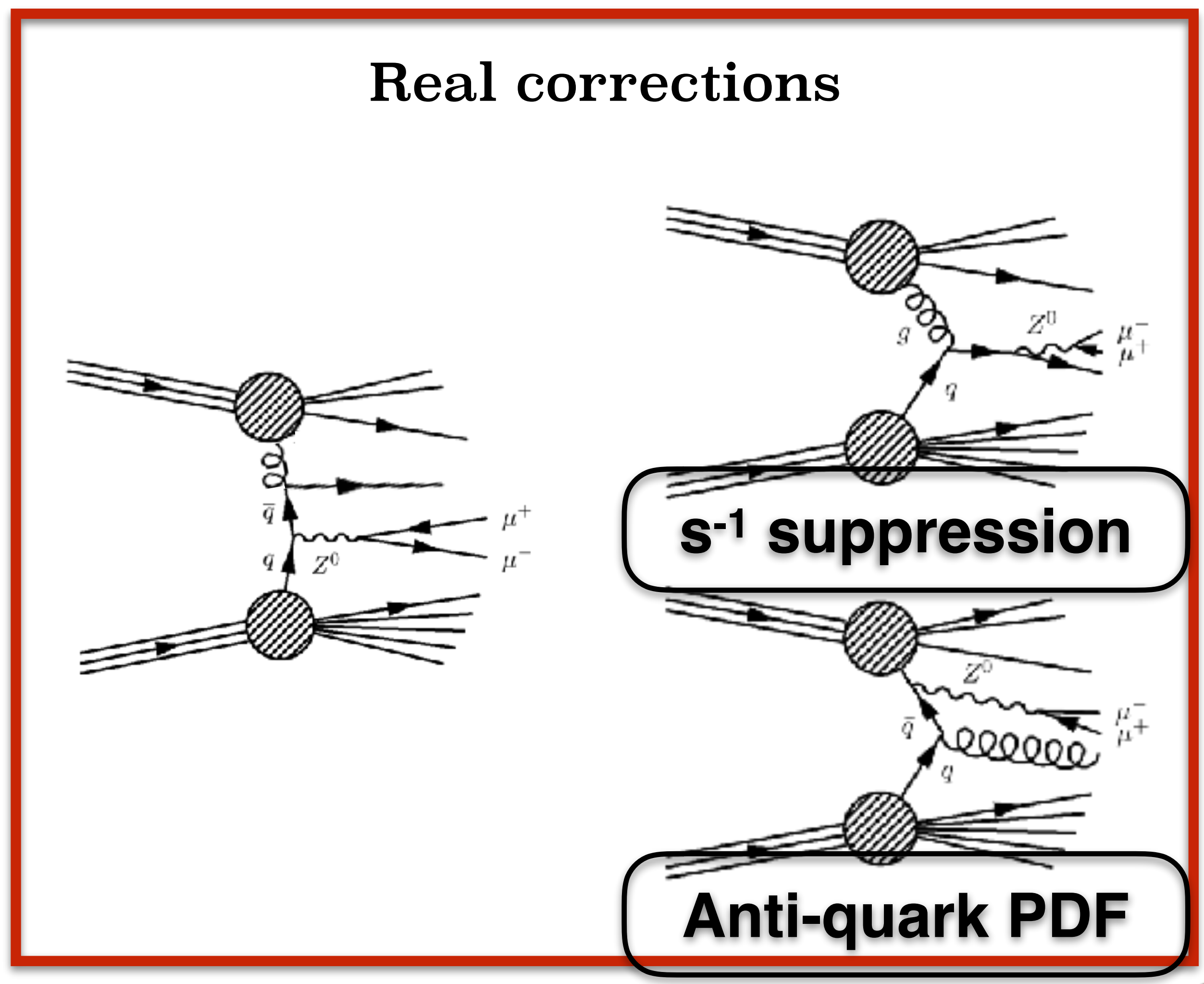

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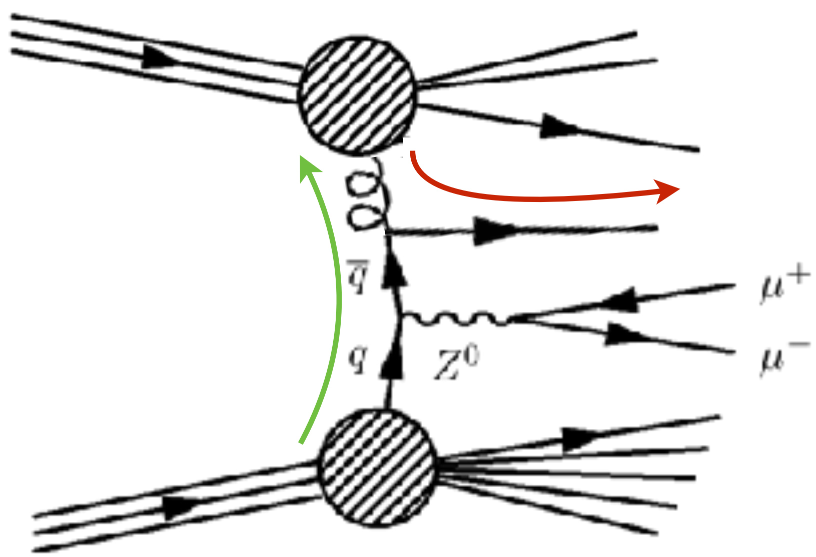
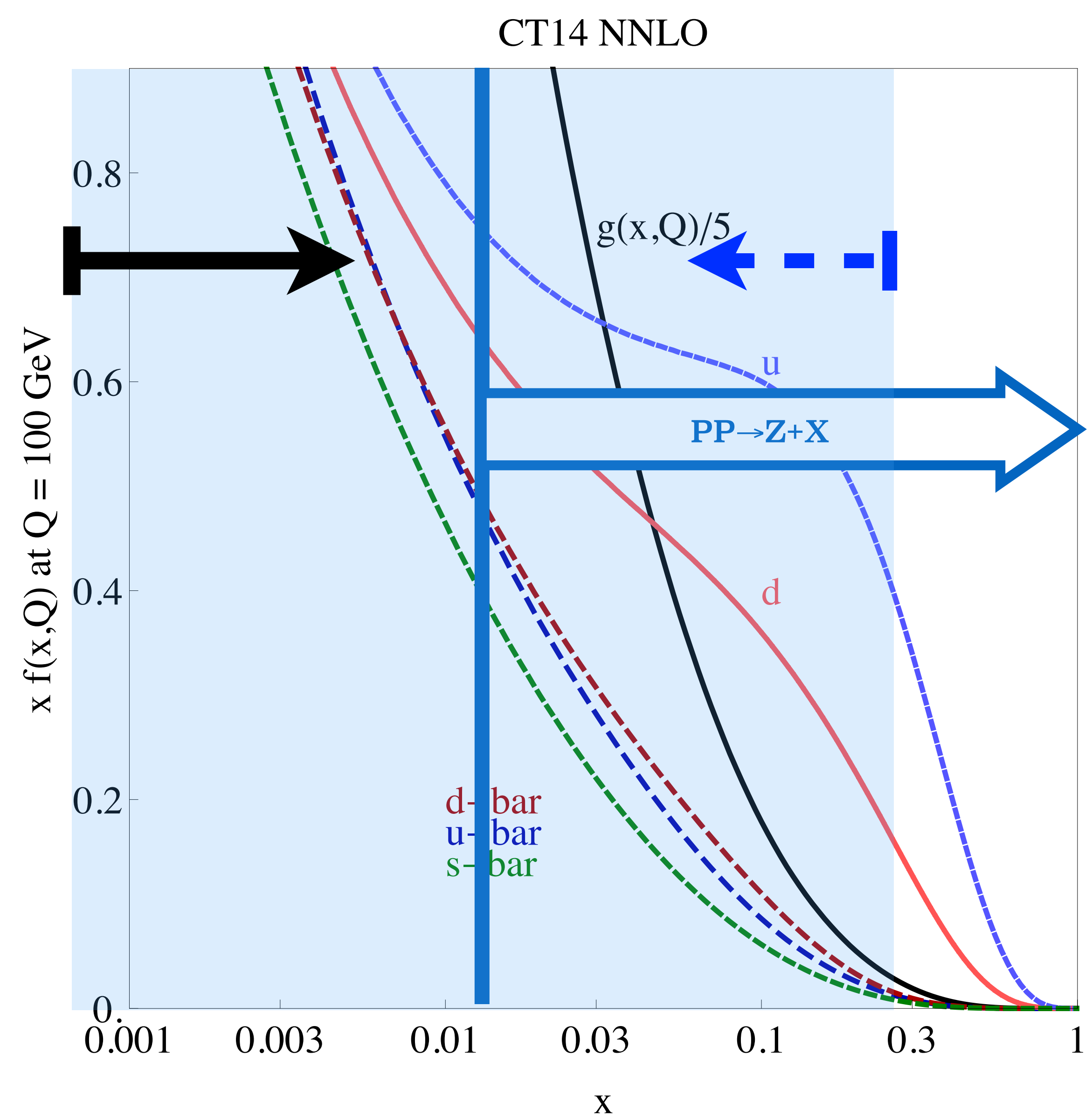
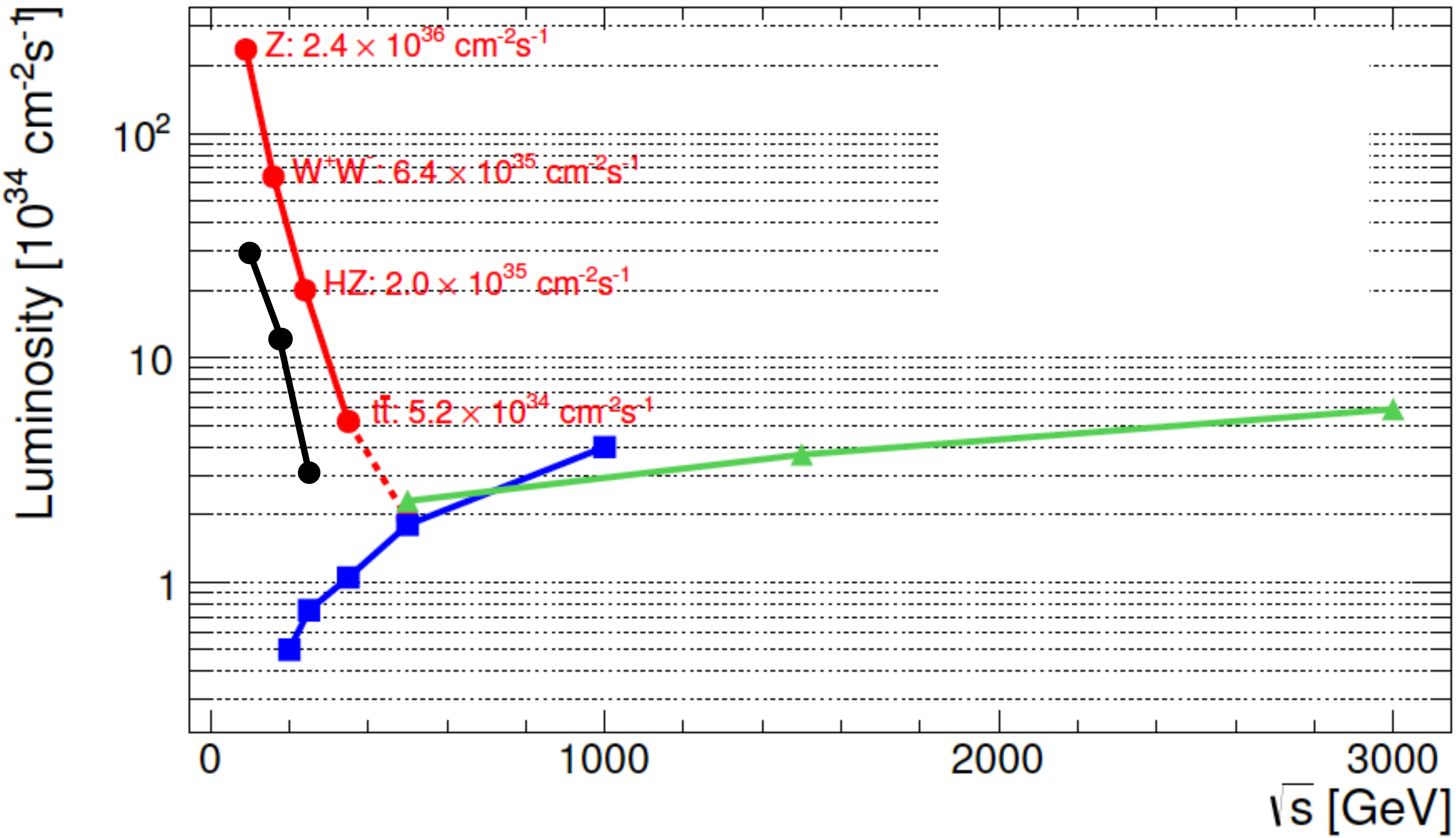




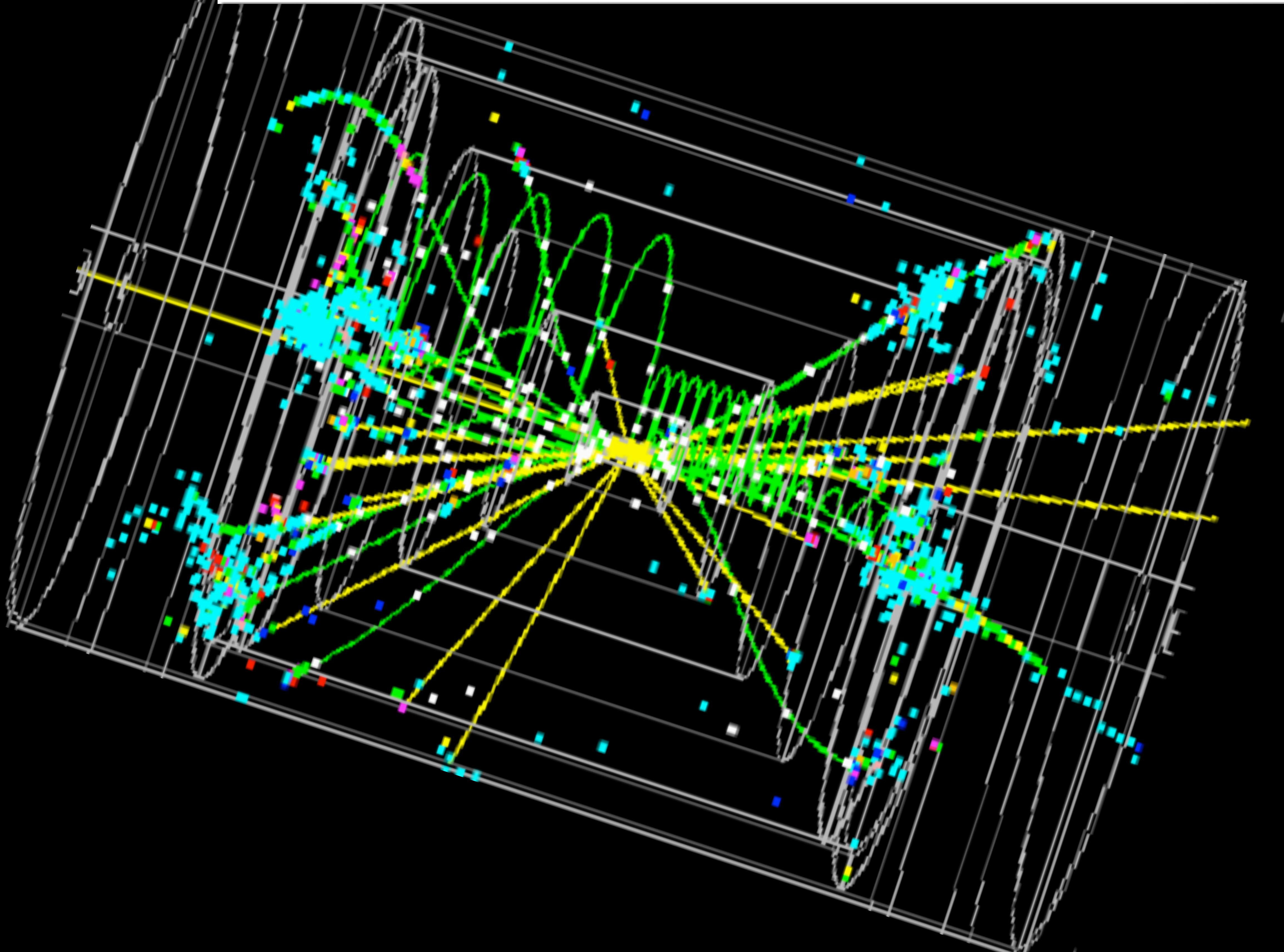
- 2-body phase space — 10^{-2} suppression;
- Gluon PDF — large enhancement at mid-x region;
- Collinear enhancement, $p_j \sim z p_g$.

Quiz



Z: 2.4 × 10^36 cm^-2s^-1
W+W-: 6.4 × 10^35 cm^-2s^-1
HZ: 2.0 × 10^35 cm^-2s^-1
tt̄: 5.2 × 10^34 cm^-2s^-1
Luminosity [10^34 cm^-2s^-1]
10^2
10
1
0
1000
2000
3000
√s [GeV]

Lecture 2: Higgs Physics at Higgs Factory (I)

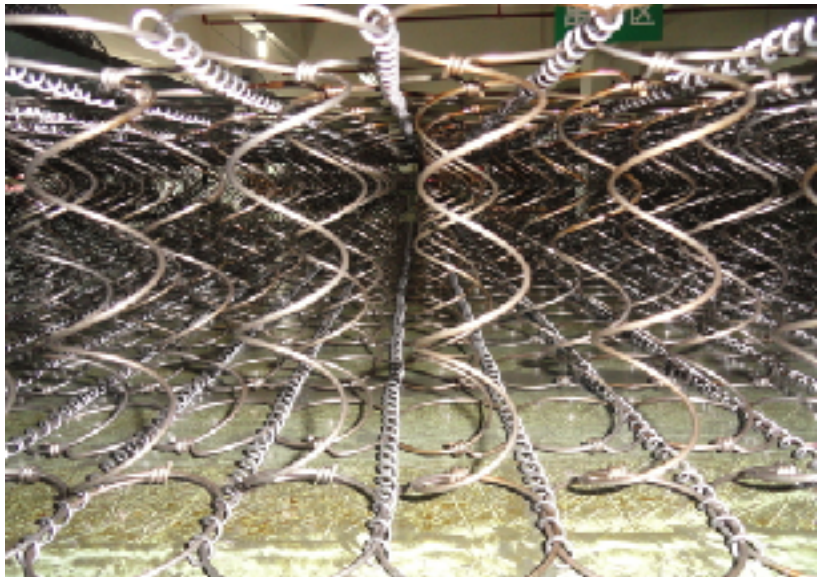


The SM Higgs Boson

- The EWSB in the SM (tree-level).

$$V(\varphi) = \mu^2 \left(\varphi^\dagger \varphi\right) + \lambda \left(\varphi^\dagger \varphi\right)^2$$

$$\varphi = \begin{pmatrix} G^+ \\ v + \frac{1}{\sqrt{2}}\left(H^0 + iG^0\right) \end{pmatrix} \quad \langle \varphi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$$



$$D_\mu \varphi =$$

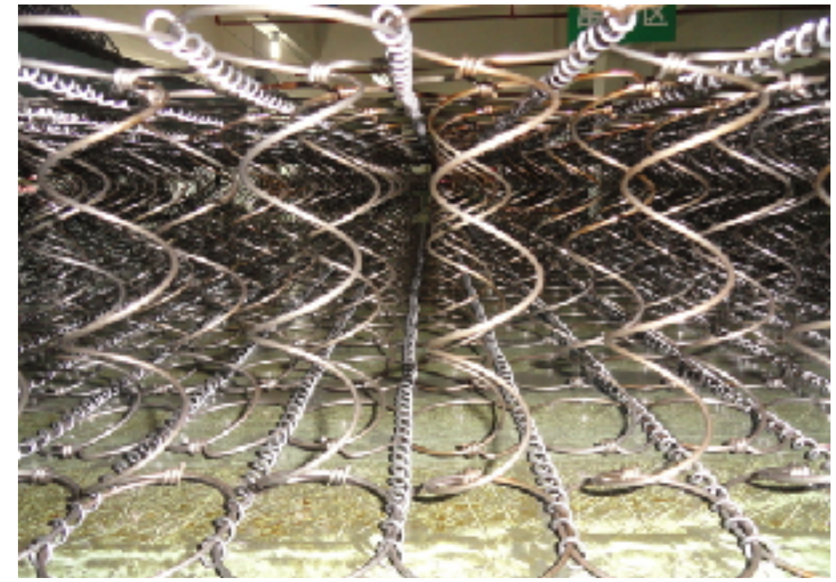
$$\begin{pmatrix} \partial_\mu G^+ + \frac{i}{2}\left(g W_\mu^3 + g' B_\mu\right) G^+ + \frac{ig}{2} W_\mu^+ \left(\sqrt{2}v + H^0 + iG^0\right) \\ \frac{1}{\sqrt{2}}\partial_\mu H^0 + \frac{i}{\sqrt{2}}\partial_\mu G^0 + \frac{ig}{\sqrt{2}} W_\mu^- G^+ + \frac{i}{2\sqrt{2}}\left(-g W_\mu^3 + g' B_\mu\right)\left(\sqrt{2}v + H^0 + iG^0\right) \end{pmatrix}$$

$$= \begin{pmatrix} \partial_\mu G^+ + ieA_\mu G^+ + \frac{ie\left(c_W^2 - s_W^2\right)}{2 s_W c_W} Z_\mu G^+ + \frac{ie}{2 s_W} W_\mu^+ \left(\sqrt{2}v + H^0 + iG^0\right) \\ \frac{1}{\sqrt{2}}\partial_\mu H^0 + \frac{i}{\sqrt{2}}\partial_\mu G^0 + \frac{ie}{\sqrt{2} s_W} W_\mu^- G^+ - \frac{ie}{2\sqrt{2} s_W c_W} Z_\mu^0 \left(\sqrt{2}v + H^0 + iG^0\right) \end{pmatrix}$$

$$\begin{pmatrix} A^0 \\ Z^0 \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B \\ W^3 \end{pmatrix} \qquad \frac{e^2 v^2}{2 s_W^2} W_\mu^- W^{+\mu} + \frac{e^2 v^2}{4 s_W^2 c_W^2} Z_\mu^0 Z^{0\mu}$$

The SM Higgs Boson

- The EWSB in the SM (tree-level).



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$$\Big(\partial_\mu G^+ + ieA_\mu G^+ + \frac{ie\left(c_W^2 - s_W^2\right)}{2s_W c_W} Z_\mu G^+ + \frac{ie}{2s_W} W_\mu^+ \left(\sqrt{2}v + H^0 + iG^0\right) \Big)$$

THINKING: Try to give the result when the scalar belongs to the (2j+1, y) representation of the SU(2)×U(1) group.

$$\begin{pmatrix} A^0 \\ Z^0 \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B \\ W^3 \end{pmatrix} \qquad \frac{e^2 v^2}{2s_W^2} W_\mu^- W^{+\mu} + \frac{e^2 v^2}{4s_W^2 c_W^2} Z_\mu^0 Z^{0\mu}$$

The SM Higgs Boson

- The EWSB in the SM (tree-level).

$$\mathcal{L}_{\text{Yukawa}} = -Y_u^{ij}\epsilon_{ab}\bar{Q}_i^a\varphi^{b*}u_{Rj} - Y_d^{ij}\bar{Q}_i^a\varphi_a d_{Rj} - Y_e^{ij}\bar{L}_i^a\varphi_a e_{Rj} + \text{h.c.}$$

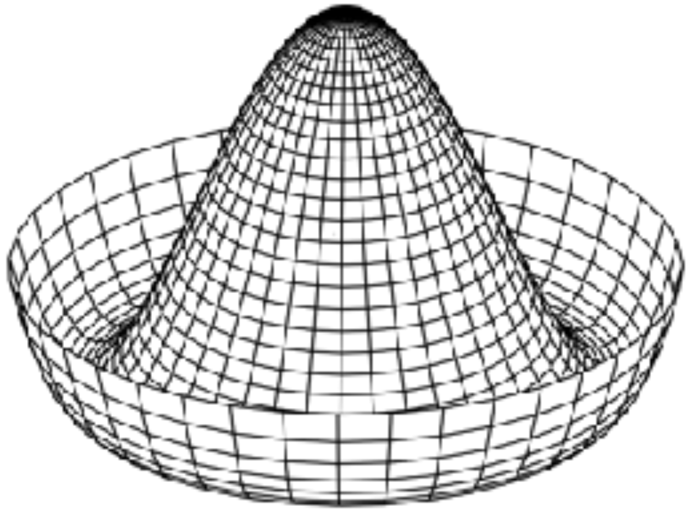
$$Y_u^{ij} v\bar{u}_{Li}u_{Rj} + Y_d^{ij} v\bar{d}_{Li}d_{Rj} + Y_e^{ij} v\bar{e}_{Li}e_{Rj} + \text{h.c.}$$

Polar decomposition theorem:

$$u_L \to U_L^u u_L,\ \ u_R \to U_R^u u_R,\ \ d_L \to U_L^d d_L,\ \ d_R \to U_R^d d_R$$

$$\mathcal{L}_{\text{Yukawa}} = -\frac{m_{ui}}{v}\epsilon_{ab}\bar{Q}_i^a\varphi^{b*}u_{Ri} - \frac{m_{di}}{v}\bar{Q}_i^a\varphi_a d_{Ri} - \frac{m_{ei}}{v}\bar{L}_i^a\varphi_a e_{Ri} + \text{h.c.}$$

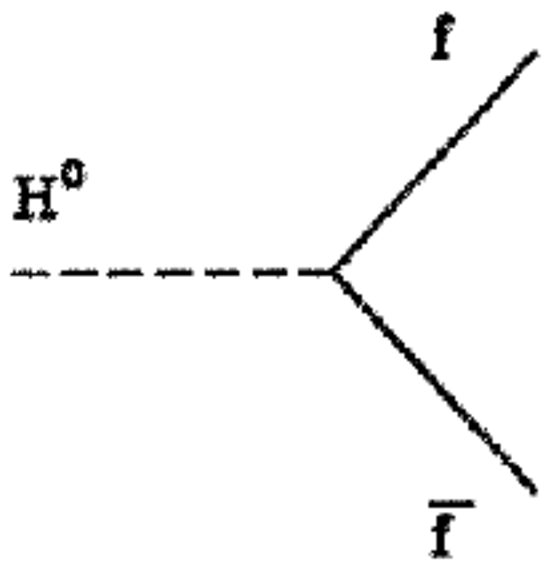
$$V_{CKM} = U_L^{u\dagger}U_L^d$$



The SM Higgs Boson

- The vertices with Higgs boson.

$$\mathcal{L}_{\text{Yukawa}} = -\frac{m_{ui}}{v}\epsilon_{ab}\bar{Q}_i^a\varphi^{b*}u_{Ri} - \frac{m_{di}}{v}\bar{Q}_i^a\varphi_a d_{Ri} - \frac{m_{ei}}{v}\bar{L}_i^a\varphi_a e_{Ri} + \text{h.c.}$$



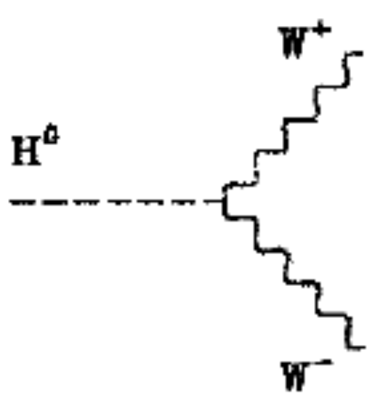
$$-i\frac{m_f}{\sqrt{2}v} \propto m_f$$

	u-quark	d-quark	s-quark	c-quark	b-quark	t-quark	e	μ	τ
mass (GeV)	0.0022	0.0047	0.096	1.28	4.18	160	0.000511	0.105	1.776
Yukawa	0.000009	0.00002	0.0004	0.005	0.017	0.65	0.000002	0.0004	0.0072

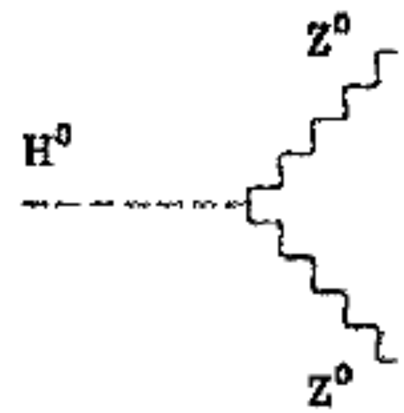
The SM Higgs Boson

- The vertices with Higgs boson.

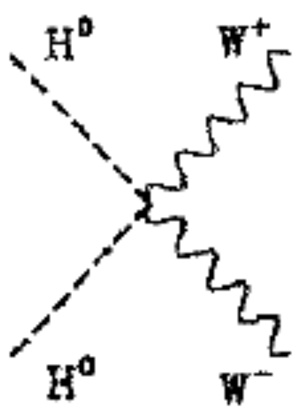
$$\left| \begin{pmatrix} \partial_\mu G^+ + ieA_\mu G^+ + \frac{ie\left(c_W^2 - s_W^2\right)}{2s_W c_W} Z_\mu G^+ + \frac{ie}{2s_W} W_\mu^+ \left(\sqrt{2}v + H^0 + iG^0\right) \\ \frac{1}{\sqrt{2}}\partial_\mu H^0 + \frac{i}{\sqrt{2}}\partial_\mu G^0 + \frac{ie}{\sqrt{2}s_W} W_\mu^- G^+ - \frac{ie}{2\sqrt{2}s_W c_W} Z_\mu^0 \left(\sqrt{2}v + H^0 + iG^0\right) \end{pmatrix} \right|^2$$



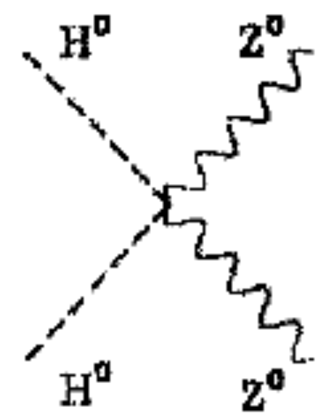
$$\frac{iem_W}{s_W} \sim 0.3vi$$



$$\frac{iem_Z}{s_W c_W} \sim 0.4vi$$



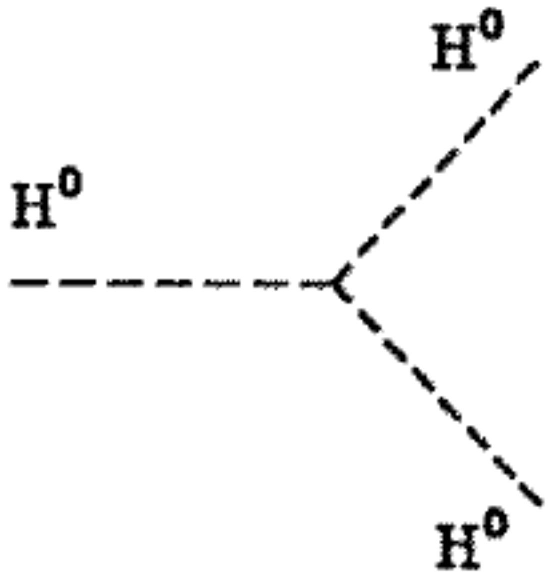
$$\frac{ie^2}{2s_W^2} \sim 0.2i$$



$$\frac{ie^2}{2s_W^2 c_W^2} \sim 0.28i$$

The SM Higgs Boson

- The vertices with Higgs boson.



$$-6\sqrt{2}i\lambda v = -\frac{3iem_h^2}{2m_W s_W}$$
$$\sim -1.1vi$$

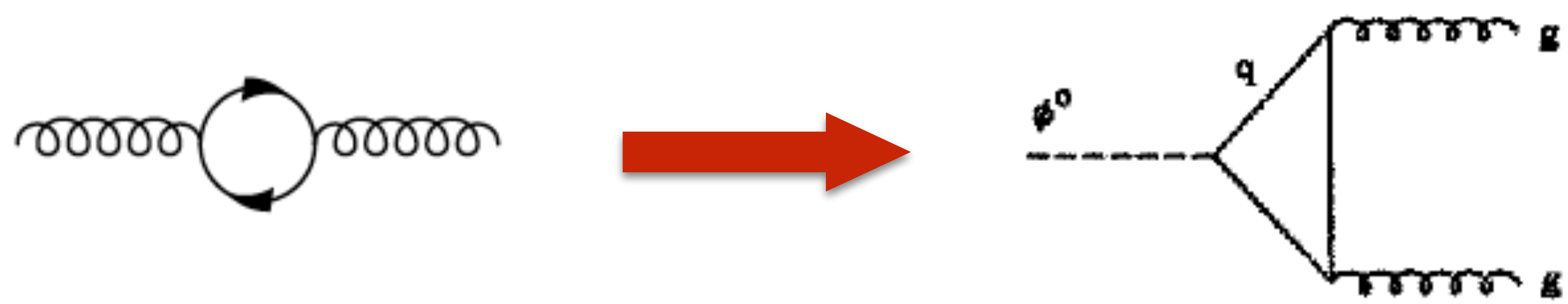
$$-6i\lambda = -\frac{3ie^2m_h^2}{4m_W^2 s_W^2}$$
$$\sim -0.77i$$

The SM Higgs Boson

- Low energy theorem:

$$\lim_{p_h \to 0} \mathcal{M}(A \to B + h) = \frac{1}{\sqrt{2}v} \left(\sum_f m_f \frac{\partial}{\partial m_f} + \sum_{V=W,Z} m_V \frac{\partial}{\partial m_V} \right) \mathcal{M}(A \to B)$$

- The vertices with Higgs boson (loop induced).



$$\begin{aligned}
\lim_{p_h \to 0} \mathcal{M}(g \to g + h) &\approx \frac{m_t}{\sqrt{2}v} \frac{\partial}{\partial m_t} \mathcal{M}(g \to g) \\
&= \frac{2\alpha_S}{\pi} \delta^{ab} T_F (p^2 g^{\mu\nu} - p^\mu p^\nu) \frac{m_t}{\sqrt{2}v} \frac{1}{3m_t} \\
&= \frac{\alpha_S}{3\sqrt{2}\pi v} \delta^{ab} (p^2 g^{\mu\nu} - p^\mu p^\nu) \\
\Rightarrow \mathcal{L}_{\text{eff}} &= \frac{\alpha_S}{12\sqrt{2}\pi v} h G^a_{\mu\nu} G^{a,\mu\nu}
\end{aligned}$$

The SM Higgs Boson

- Low energy theorem:

$$\lim_{p_h \to 0} \mathcal{M}(A \to B + h) = \frac{1}{\sqrt{2}v} \left(\sum_f m_f \frac{\partial}{\partial m_f} + \sum_{V=W,Z} m_V \frac{\partial}{\partial m_V} \right) \mathcal{M}(A \to B)$$

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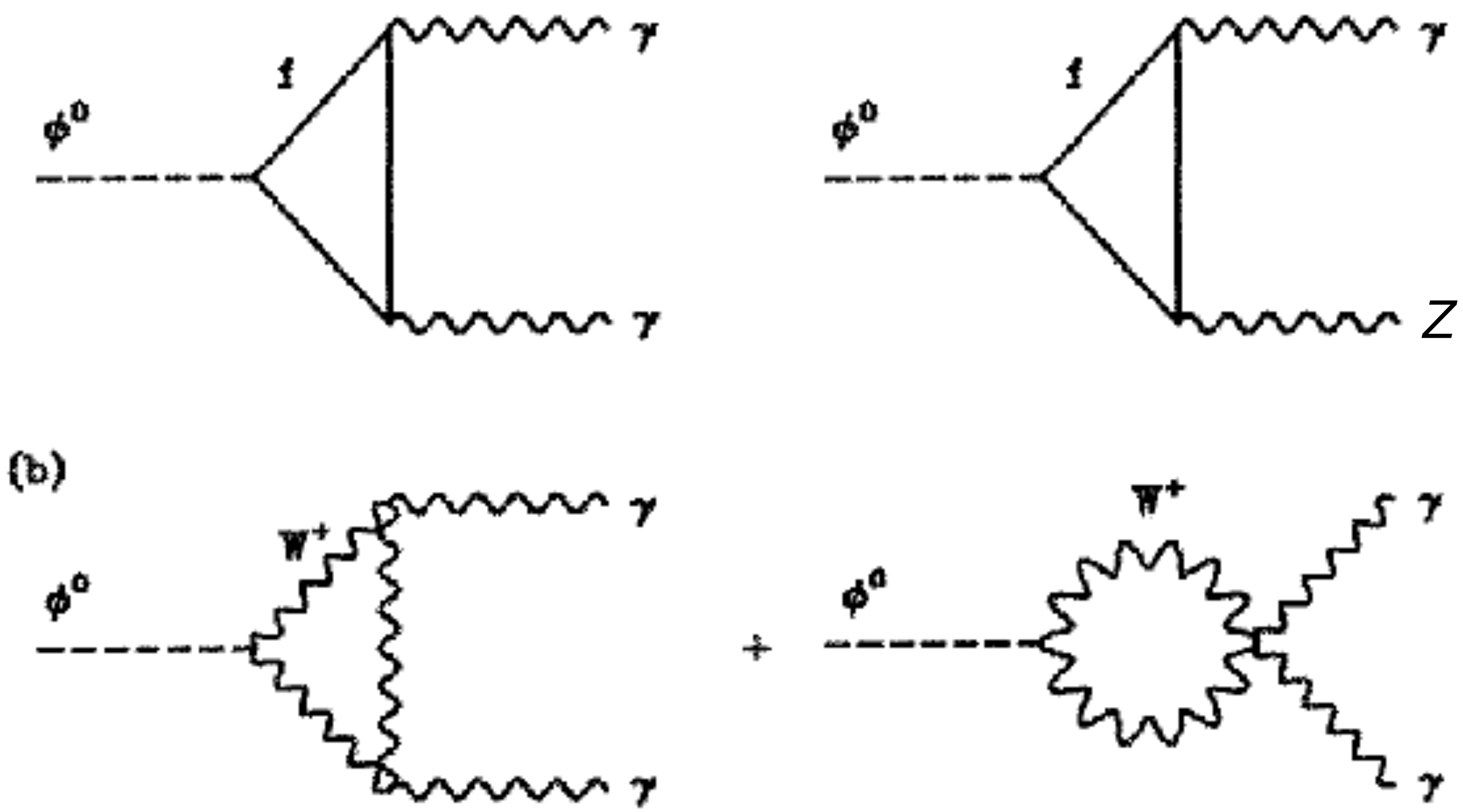
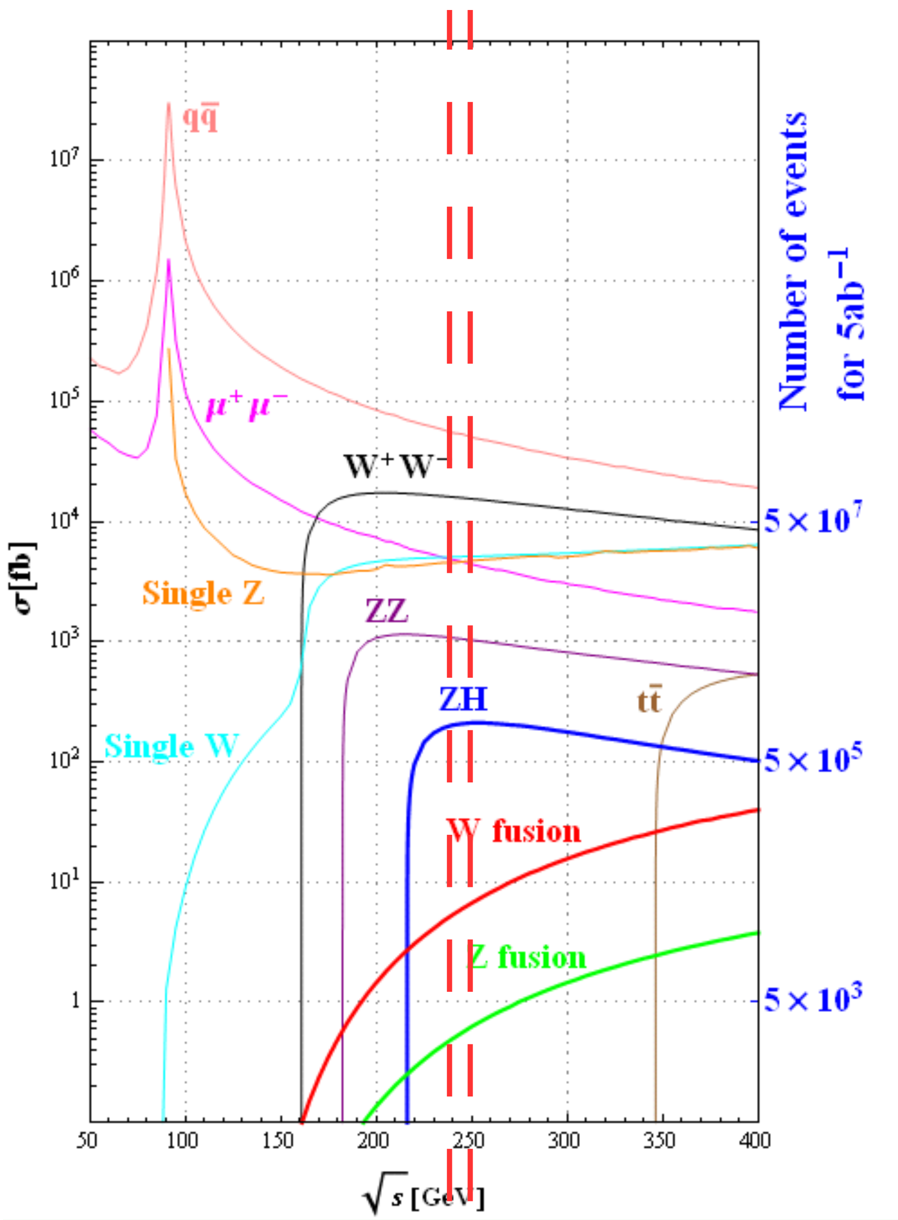
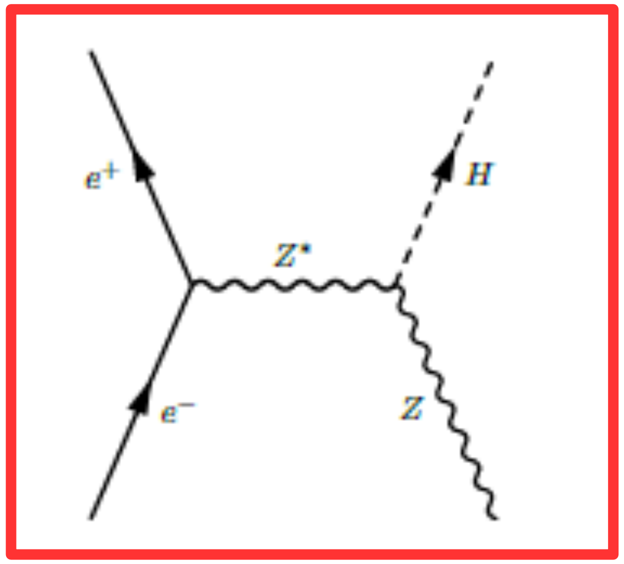
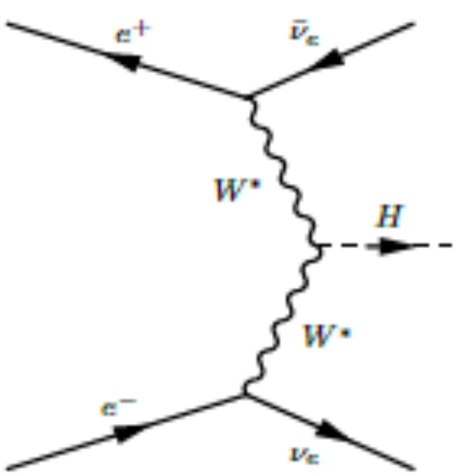
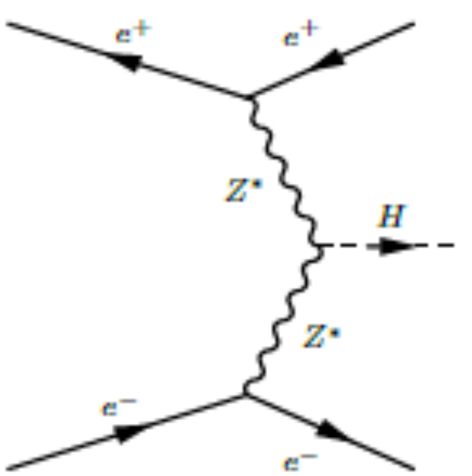


The Production of the Higgs Boson



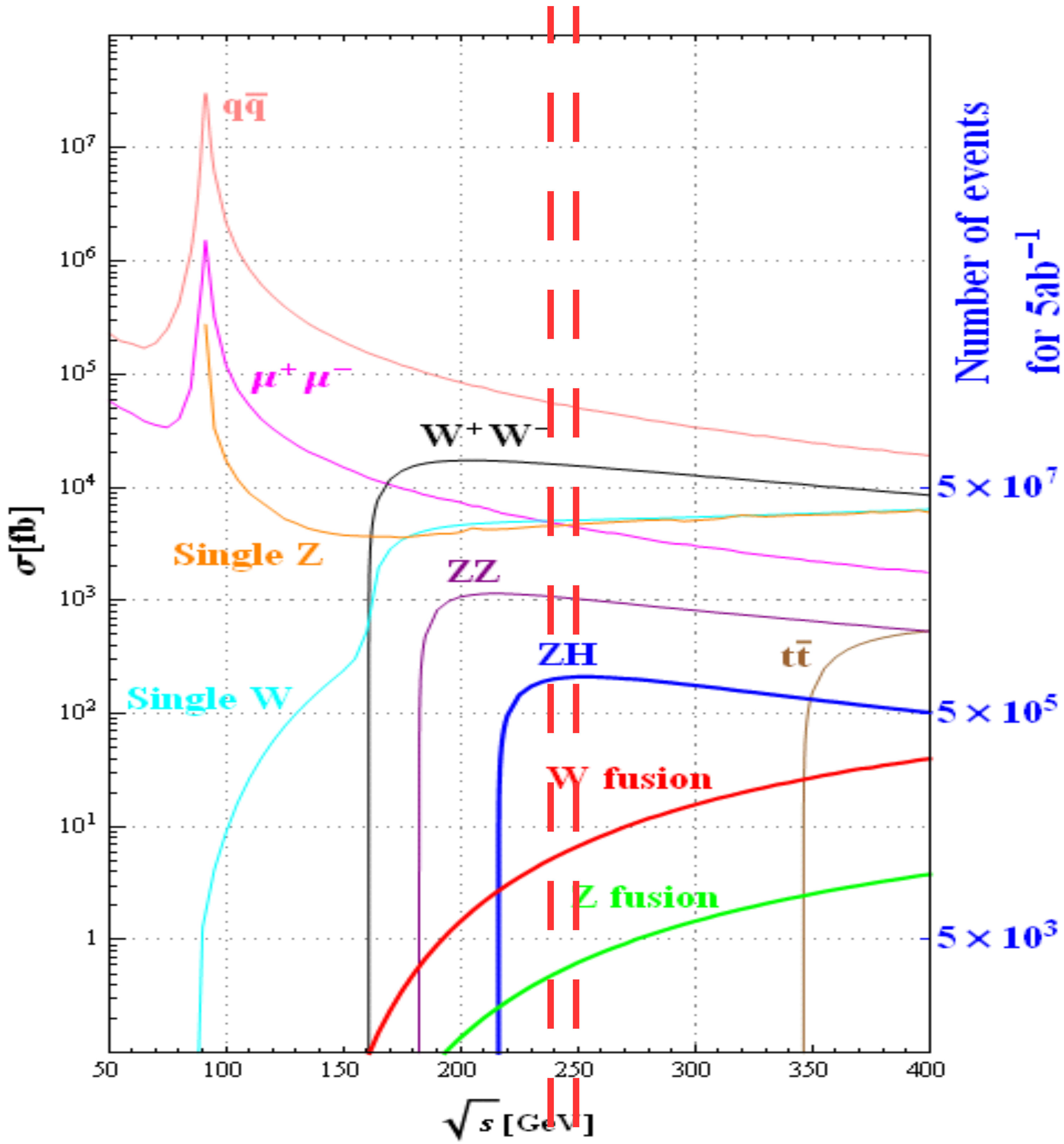






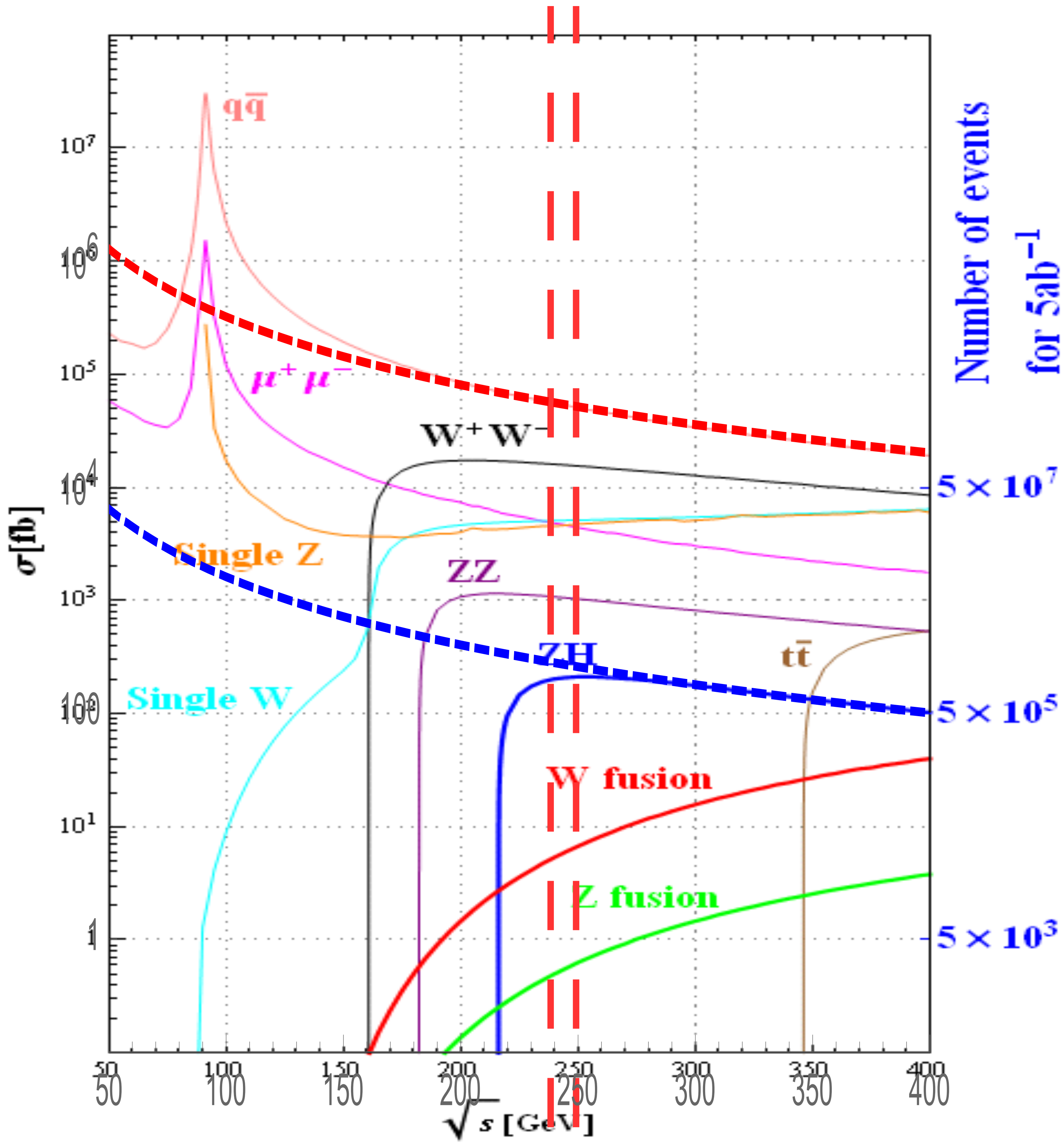
Process	Cross section	Events in 5 ab^{-1}
Higgs boson production, cross section in fb		
$e^+e^- \to ZH$	212	1.06×10^6
$e^+e^- \to \nu\bar{\nu}H$	6.72	3.36×10^4
$e^+e^- \to e^+e^-H$	0.63	3.15×10^3
Total	219	1.10×10^6

The Production of the Higgs Boson



- Thresholds:
  - WW: 2×80=160GeV;
  - ZZ: 2×91=181GeV;
  - Zh: 91+125=216GeV;
  - tt: 2×173=346GeV.

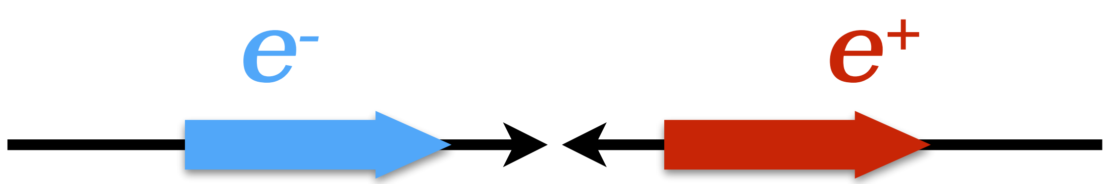
The Production of the Higgs Boson



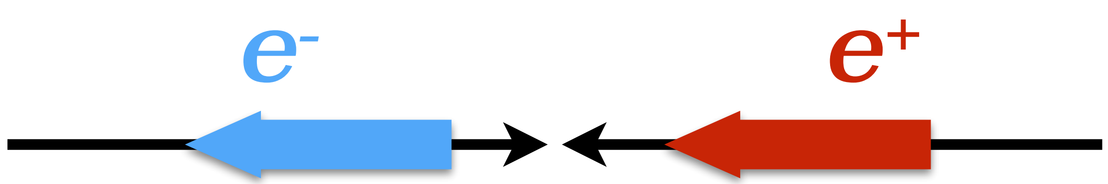
- s-channel processes: s^{-1} suppression in the large s limit from the denominator in the propagator.

The Production of the Higgs Boson

- The helicity configuration (in SM, J=1):



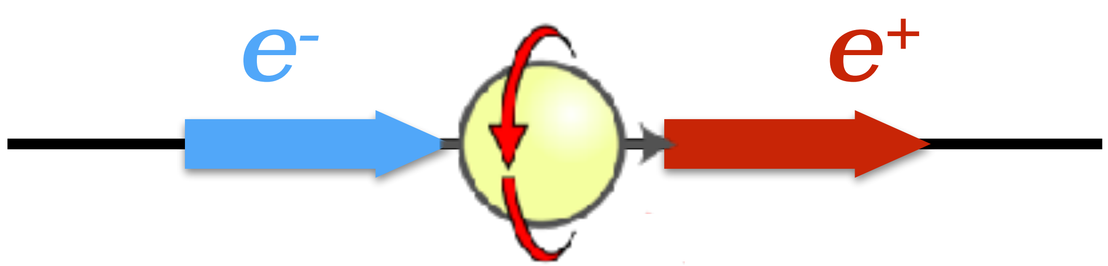
$(+ -)$



$(- +)$

The Production of the Higgs Boson

- The helicity configuration (in SM, J=1).
- The (massless) electron and positron can only couple to the transverse Z*.



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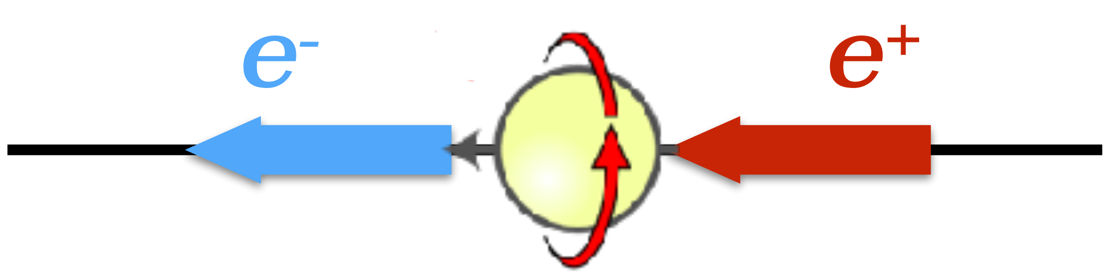
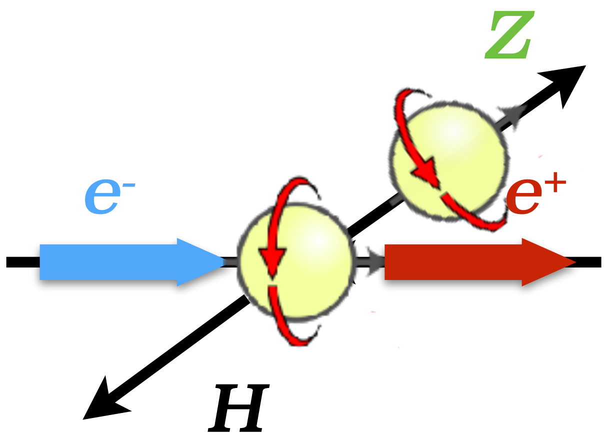


$(- +)$

The Production of the Higgs Boson

- The helicity configuration (in SM, J=1).
- The (massless) electron and positron can only couple to the transverse Z*.
- The Higgs boson couples to both transverse and longitude Z.



$(+-)$

$$\langle Z(\lambda)H|Z^*(m)\rangle = \frac{g m_Z}{c_W} d^1_{m,\lambda}(\theta)\Gamma_{\lambda 0}$$

The Production of the Higgs Boson

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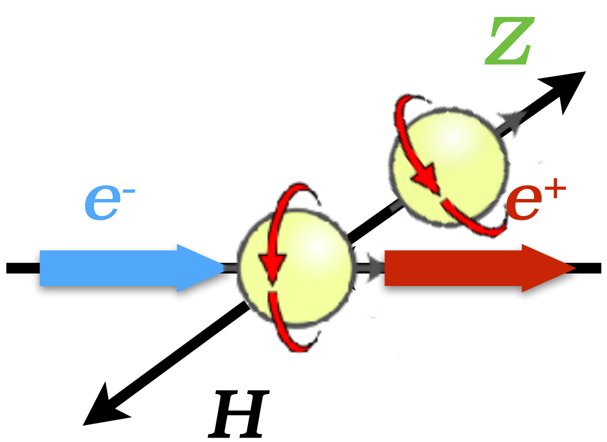

$(+ -)$

$$\langle Z(\lambda) H | Z^*(m) \rangle = \frac{g m_Z}{c_W} d^1_{m,\lambda}(\theta) \Gamma_{\lambda 0}$$

- The longitudinal mode of Z is the Goldstone boson.

$$\begin{pmatrix} \partial_\mu G^+ + ieA_\mu G^+ + \frac{ie\left(c_W^2 - s_W^2\right)}{2 s_W c_W} Z_\mu G^+ + \frac{ie}{2 s_W} W_\mu^+ \left(\sqrt{2} v + H^0 + iG^0\right) \\ \frac{1}{\sqrt{2}} \partial_\mu H^0 + \boxed{\frac{i}{\sqrt{2}} \partial_\mu G^0} + \frac{ie}{\sqrt{2} s_W} W_\mu^- G^+ - \boxed{\frac{ie}{2\sqrt{2} s_W c_W} Z_\mu^0 (\sqrt{2} v} + H^0 + iG^0) \end{pmatrix}$$

The Production of the Higgs Boson

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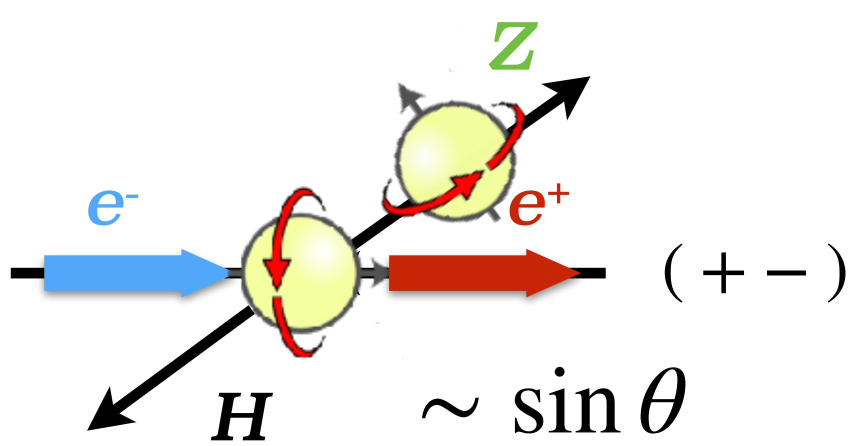

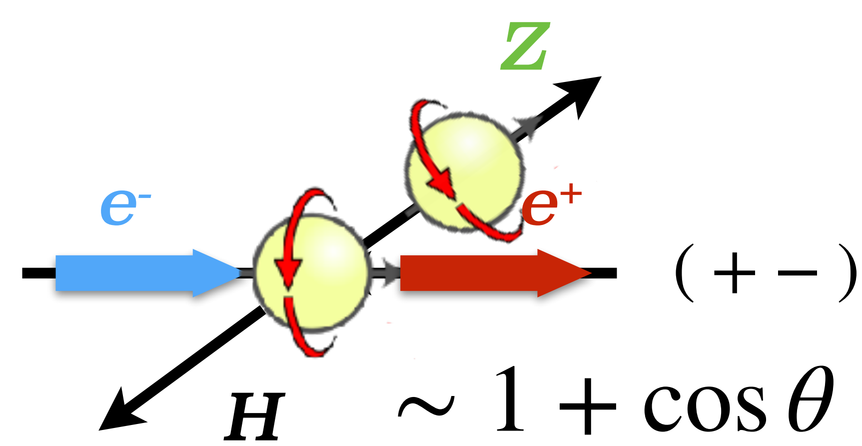

$$\Gamma_{00} = -E_Z/m_Z, \quad \Gamma_{10} = -1$$

The Production of the Higgs Boson

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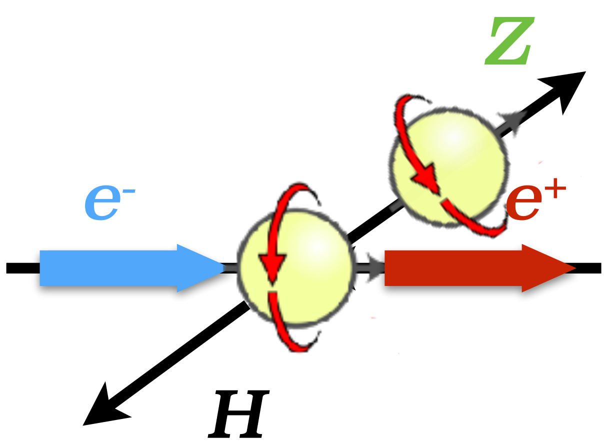


$$(+ -)$$

$$\langle Z(\lambda) H | Z^*(m) \rangle = \frac{g m_Z}{c_W} d^1_{m,\lambda}(\theta) \Gamma_{\lambda 0}$$

- The longitudinal mode of Z is the Goldstone boson.

$$\frac{d\sigma}{d\cos\theta} = \frac{G_F^2 m_Z^6 (v_e^2 + a_e^2)\beta}{32\pi[(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2]} \left[|\Gamma_{00}^2| \sin^2\theta + |\Gamma_{10}|^2 (1 + \cos^2\theta) \right]$$

$$= \frac{G_F^2 m_Z^6 (v_e^2 + a_e^2)}{16\pi[(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2]} \left(1 + \frac{p_Z^2}{2m_Z^2} \sin^2\theta \right) \left[1 - \frac{(m_H + m_Z)^2}{s} \right]^{1/2} \left[1 - \frac{(m_H - m_Z)^2}{s} \right]^{1/2}$$

The Production of the Higgs Boson

- The Electron and positron can be polarized.
- The initial state prepared by the collider should be described by a density matrix.
- The initial state electron and initial state positron is fully disentangled, so the density matrix can be decomposed as a tensor product of the electron density matrix and the positron density matrix.

$$\mathrm{Tr}\rho = 1, \quad \mathrm{Tr}\rho^2 \leqslant 1,$$
$$\rho = \rho_{++}|+\rangle\langle+| + \rho_{+-}|+\rangle\langle-| + \rho_{-+}|-\rangle\langle+| + \rho_{--}|-\rangle\langle-|$$

- The transition probability is

$$\begin{aligned}|\mathcal{M}|^2 &= \mathrm{Tr}(\rho_{e^+}\otimes\rho_{e^-}\mathcal{M}^\dagger\mathcal{M}) \\ &= \sum \rho_{e^+}(\lambda_{e^+}\lambda'_{e^+})\rho_{e^-}(\lambda_{e^-}\lambda'_{e^-})\mathcal{M}^\dagger(\lambda'_{e^+}\lambda'_{e^-})\mathcal{M}(\lambda_{e^+}\lambda_{e^-})\end{aligned}$$

The Production of the Higgs Boson

- Unpolarized beam, pure state, and general case:

$$\rho = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \rho = |\Psi\rangle\langle\Psi|, \quad \rho = \frac{1}{2}\begin{pmatrix} 1+P_L & P_1 - iP_2 \\ P_1 + iP_2 & 1-P_L \end{pmatrix}$$

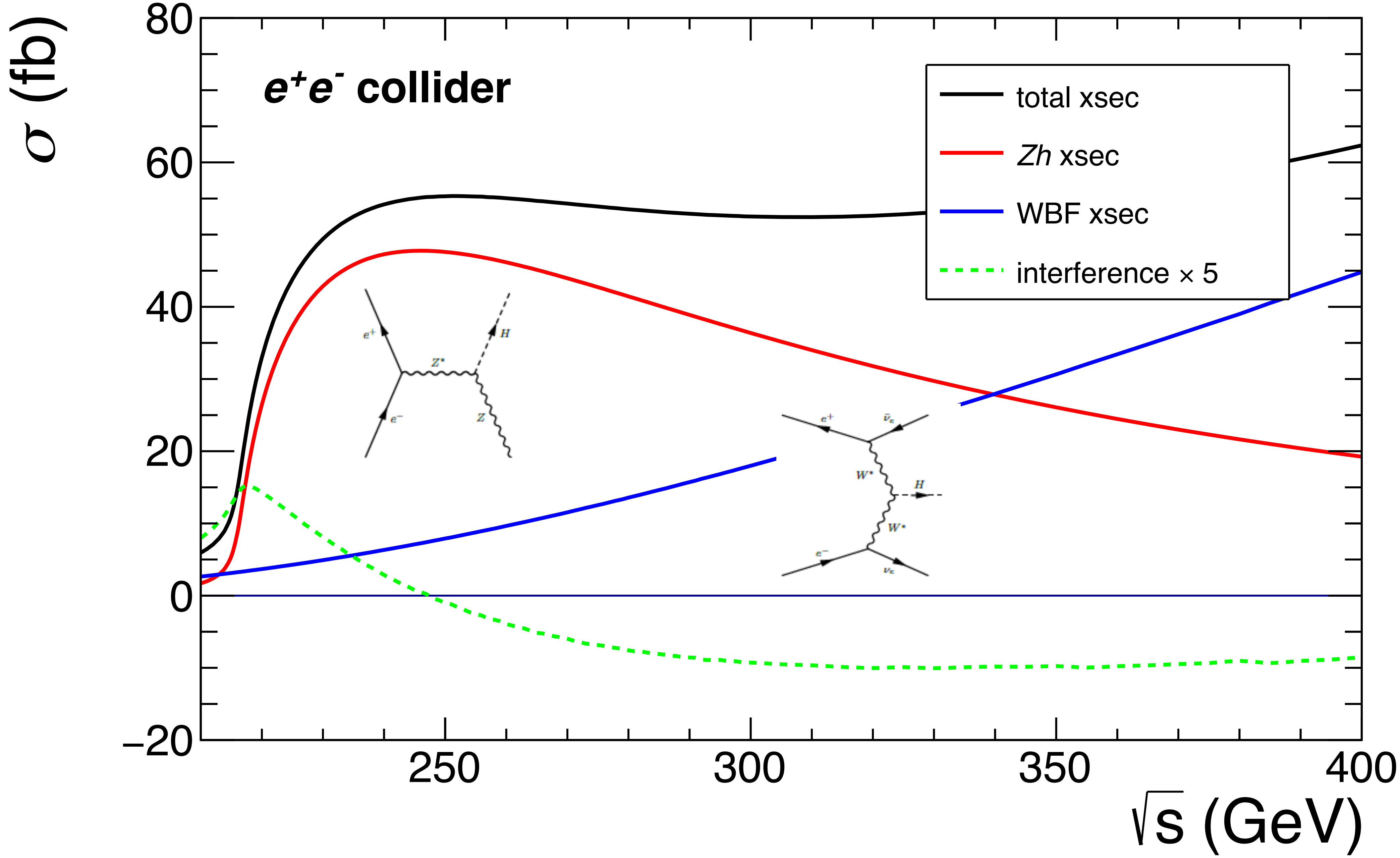
$$\begin{aligned} |\mathcal{M}|^2 = \frac{1}{4}\Big\{ & (1-P_{e^-})(1+P_{e^+})|F_{\mathrm{LR}}|^2 + (1+P_{e^-})(1-P_{e^+})|F_{\mathrm{RL}}|^2 \\ & + (1-P_{e^-})(1-P_{e^+})|F_{\mathrm{LL}}|^2 + (1+P_{e^-})(1+P_{e^+})|F_{\mathrm{RR}}|^2 \\ & - 2P_{e^-}^{\mathrm{T}} P_{e^+}^{\mathrm{T}} \{[\cos(\phi_- - \phi_+)\,\mathrm{Re}\,(F_{\mathrm{RR}} F_{\mathrm{LL}}^*) + \cos(\phi_- + \phi_+ - 2\phi)\,\mathrm{Re}\,(F_{\mathrm{LR}} F_{\mathrm{RL}}^*)] + \ldots \end{aligned}$$

- If the electron and positron are longitudinal polarized, because the left-handed current and the right-handed current interaction are not the same, the total cross section will be changed.

$$\begin{aligned} \sigma_{P_e \; P_e} = \frac{1}{4}\{ & (1+P_{e^-})(1+P_{e^+})\sigma_{\mathrm{RR}} + (1-P_{e^-})(1-P_{e^+})\sigma_{\mathrm{LL}} \\ & + (1+P_{e^-})(1-P_{e^+})\sigma_{\mathrm{RL}} + (1-P_{e^-})(1+P_{e^+})\sigma_{\mathrm{LR}}\} \end{aligned}$$

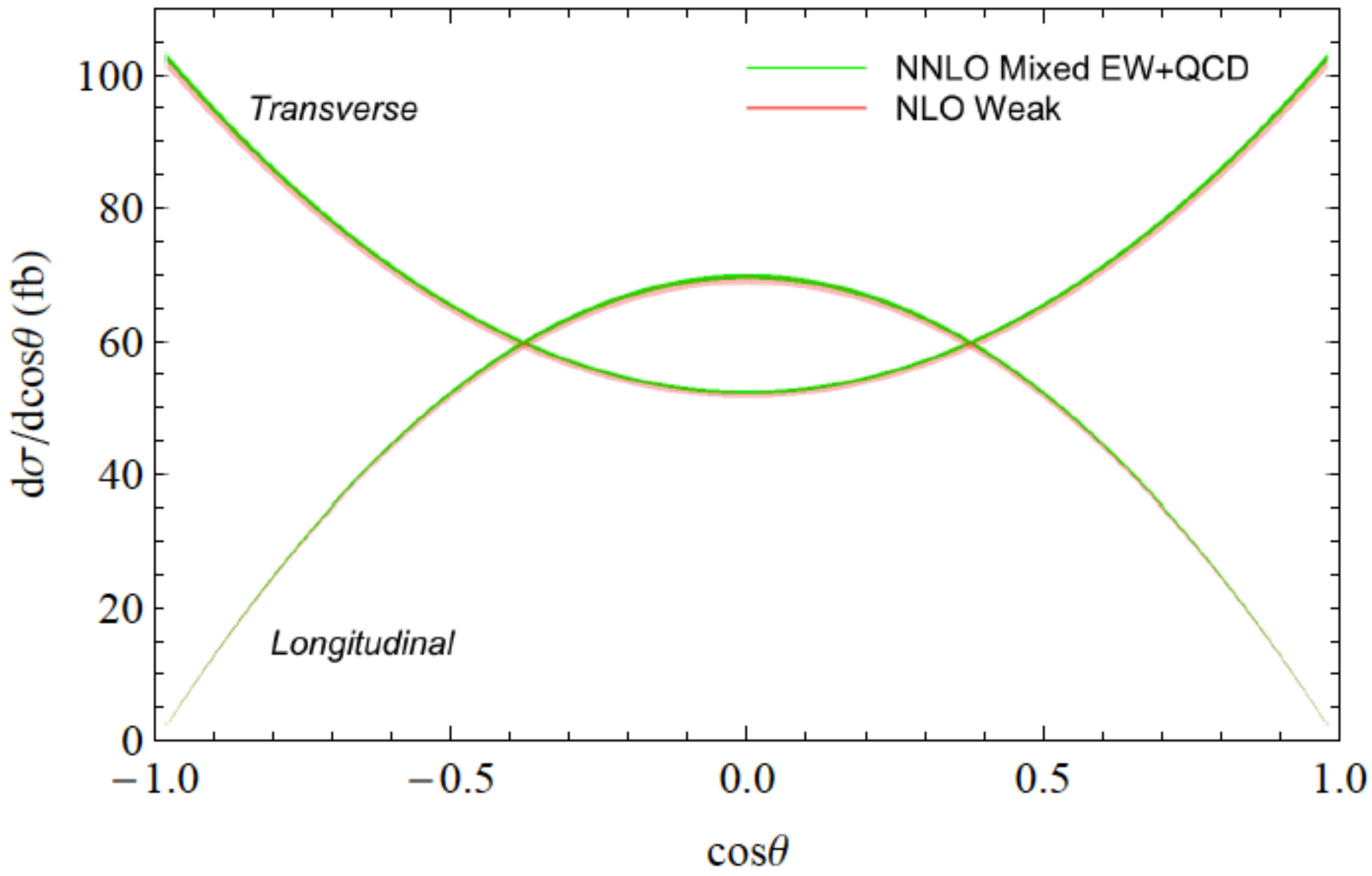
The Production of the Higgs Boson

- Electron positron collider with c.m. E ~240-250GeV: Higgs factory.



The Production of the Higgs Boson

- The theoretical prediction of the total cross section has significant uncertainty.



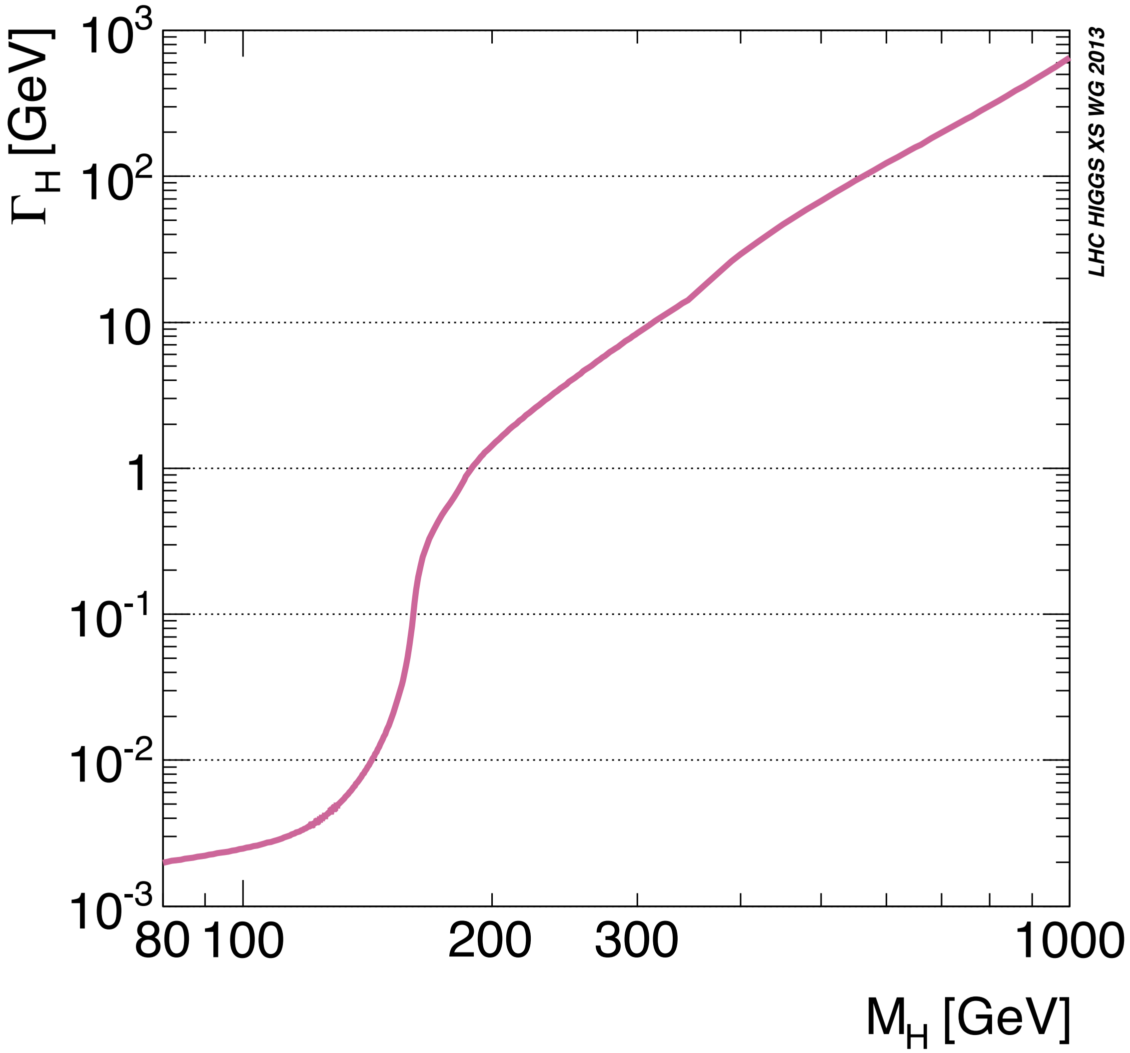
The Production of the Higgs Boson

- The theoretical prediction of the total cross section has significant uncertainty.

$\sqrt{s}$ (GeV)		LO (fb)	NLO Weak (fb)		NNLO mixed electroweak-QCD (fb)			
		$\sigma^{(0)}$	$\sigma^{(\alpha)}$	$\sigma^{(0)}+\sigma^{(\alpha)}$	$\sigma_Z^{(\alpha\alpha_s)}$	$\sigma_\gamma^{(\alpha\alpha_s)}$	$\sigma^{(\alpha\alpha_s)}$	$\sigma^{(0)}+\sigma^{(\alpha)}+\sigma^{(\alpha\alpha_s)}$
240	Total	**223.14**	6.64	229.78	2.42	0.008	2.43	232.21
	L	88.67	3.18	91.86	0.96	0.003	0.97	92.82
	T	134.46	3.46	137.92	1.46	0.005	1.46	139.39
250	Total	**223.12**	6.08	229.20	2.42	0.009	2.42	231.63
	L	94.30	3.31	97.61	1.02	0.004	1.02	98.64
	T	128.82	2.77	131.59	1.40	0.005	1.40	132.99

$\sqrt{s}$	schemes	$\sigma_{\rm LO}$ (fb)	$\sigma_{\rm NLO}$ (fb)	$\sigma_{\rm NNLO}$ (fb)
240	$\alpha(0)$	223.14 ± 0.47	229.78 ± 0.77	$232.21^{+0.75+0.10}_{-0.75-0.21}$
	$\alpha(M_Z)$	252.03 ± 0.60	$228.36^{+0.82}_{-0.81}$	$231.28^{+0.80+0.12}_{-0.79-0.25}$
	G_μ	239.64 ± 0.06	$232.46^{+0.07}_{-0.07}$	$233.29^{+0.07+0.03}_{-0.06-0.07}$
250	$\alpha(0)$	223.12 ± 0.47	229.20 ± 0.77	$231.63^{+0.75+0.12}_{-0.75-0.21}$
	$\alpha(M_Z)$	252.01 ± 0.60	$227.67^{+0.82}_{-0.81}$	$230.58^{+0.80+0.14}_{-0.79-0.25}$
	G_μ	239.62 ± 0.06	231.82 ± 0.07	$232.65^{+0.07+0.04}_{-0.07-0.07}$

The Decay of the Higgs Boson



Degree of freedom of the Final state particle

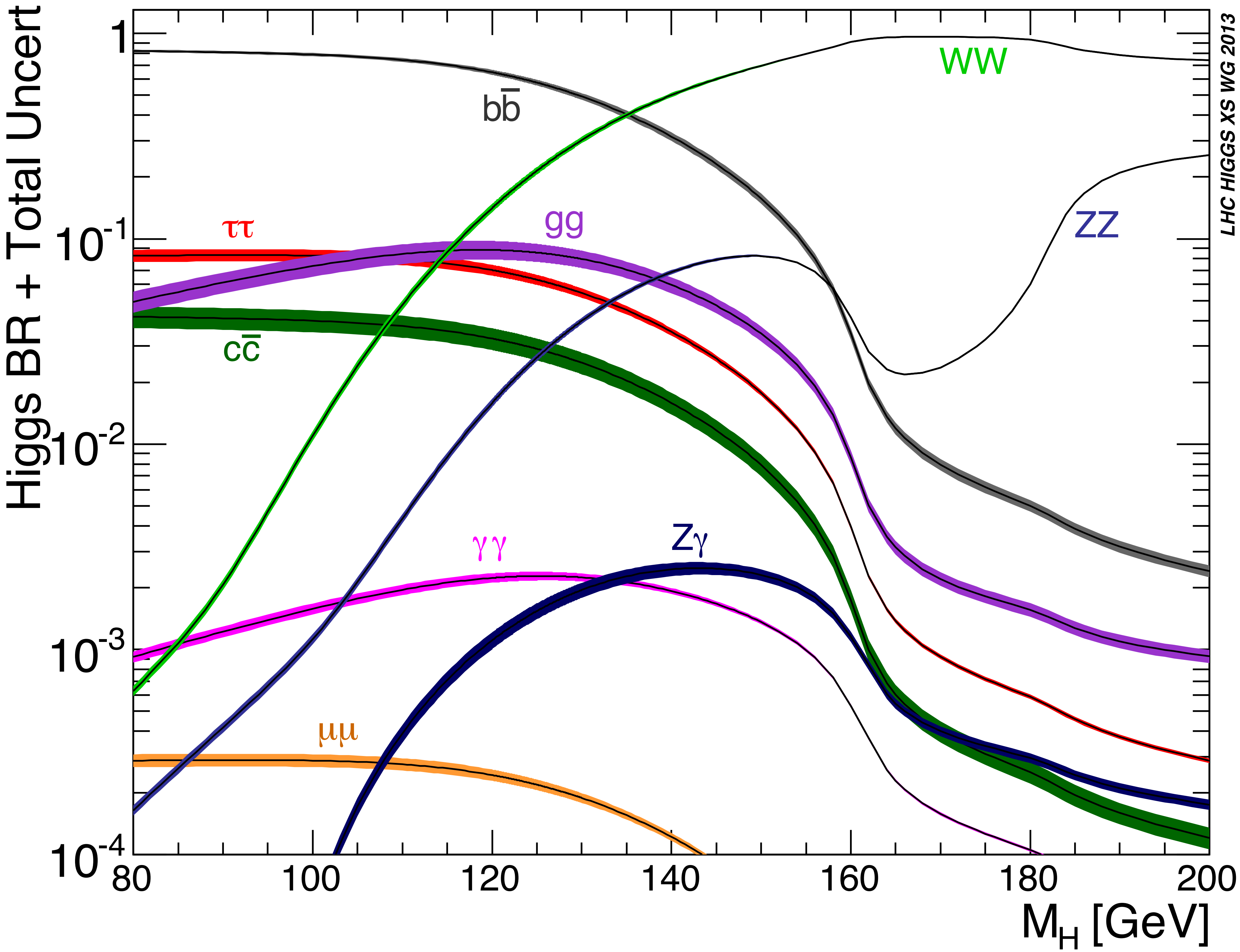
Interaction strength

$$\Gamma_i \sim N_{\rm d.o.f} \times C_i^2 \times m_h \times \lambda_{\rm PS}(n)$$

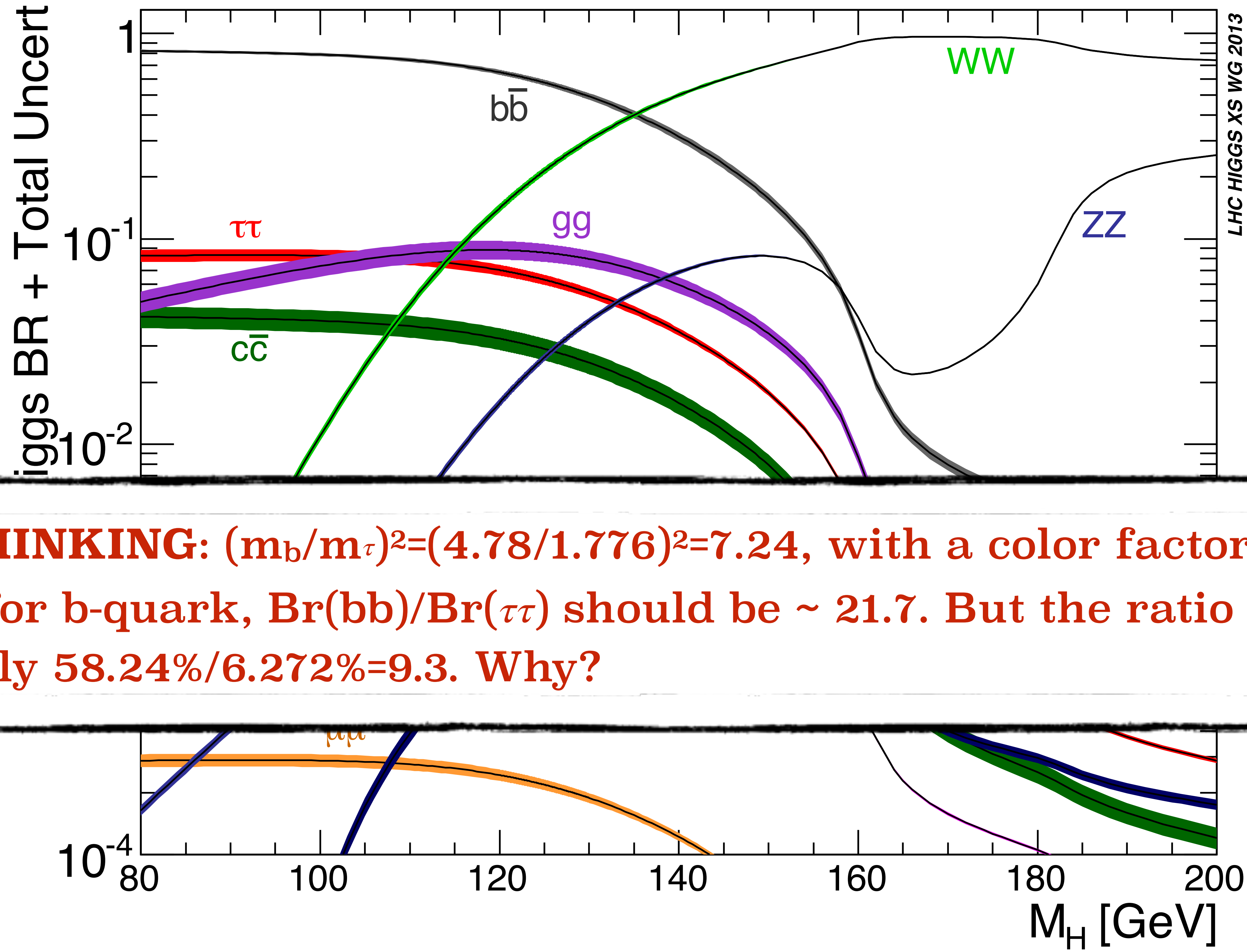
Dimensionless factor from the n-body phase space

	Z	W	t	H
Width (GeV)	2.4952	2.085	1.41	0.00407
M/Γ	36.545	38.55	123	3.07×10^4

The Decay of the Higgs Boson

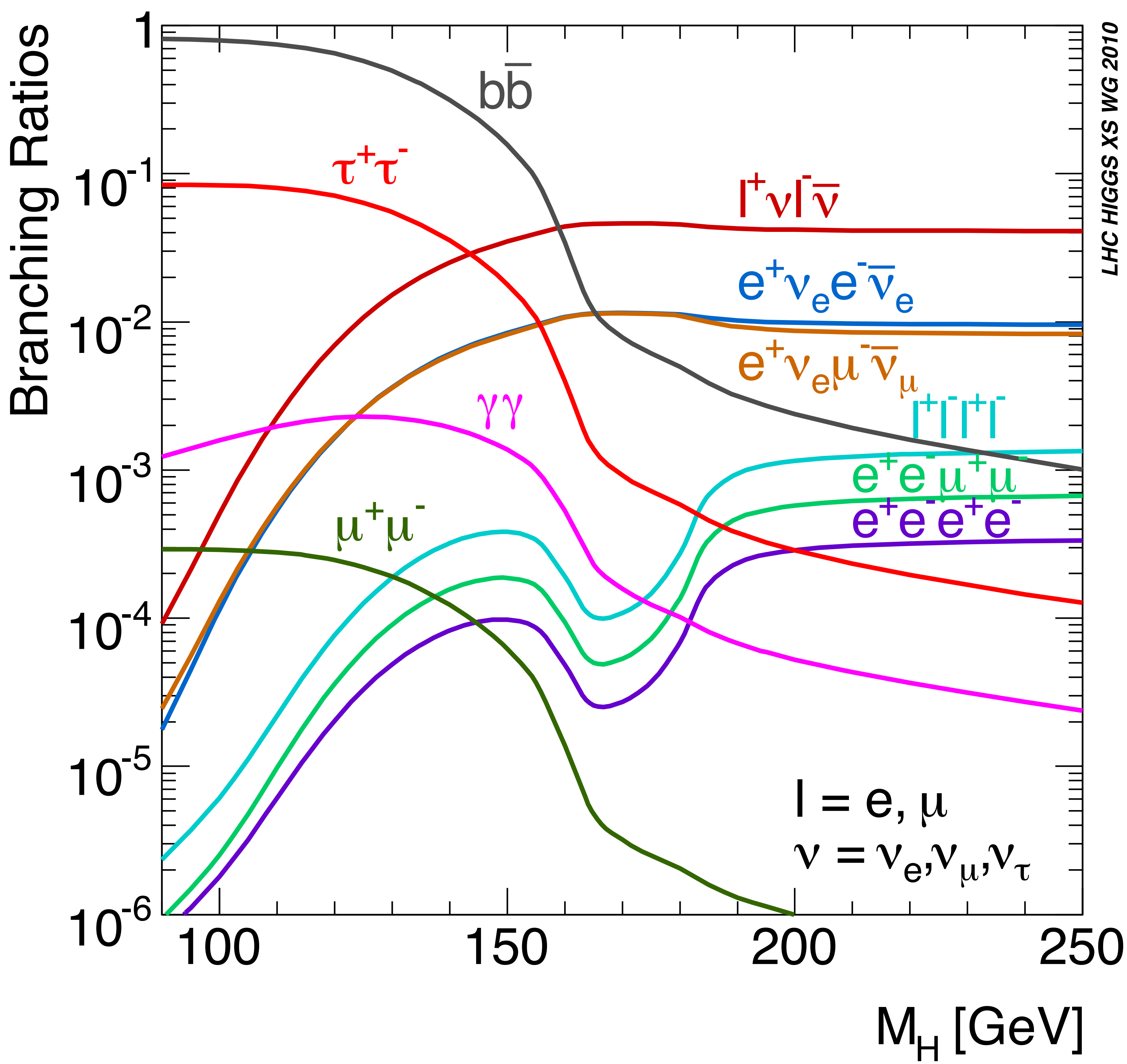


The Decay of the Higgs Boson



THINKING: $(m_b/m_\tau)^2=(4.78/1.776)^2=7.24$, with a color factor of 3 for b-quark, Br(bb)/Br($\tau\tau$) should be ~ 21.7. But the ratio is only 58.24%/6.272%=9.3. Why?

The Decay of the Higgs Boson



The Decay of the Higgs Boson

Channel	Br (%)	# of Events	Var	Rel err (%)
bb	58.24	582400	763	0.1
cc	2.891	28910	170	0.6
$\tau\tau$	6.272	62720	250	0.4
$\mu\mu$	0.02176	218	15	6.9
WW	21.37	213700	462	0.2
ZZ	2.61	26100	162	0.6
gg	8.187	81870	286	0.3
$\gamma\gamma$	0.227	2270	48	2.1
$Z\gamma$	0.153	1530	39	2.5

Next Lecture

- Higgs phenomenology at future lepton colliders.
- A short introduction of the top-quark mass and its measurement at future lepton colliders.