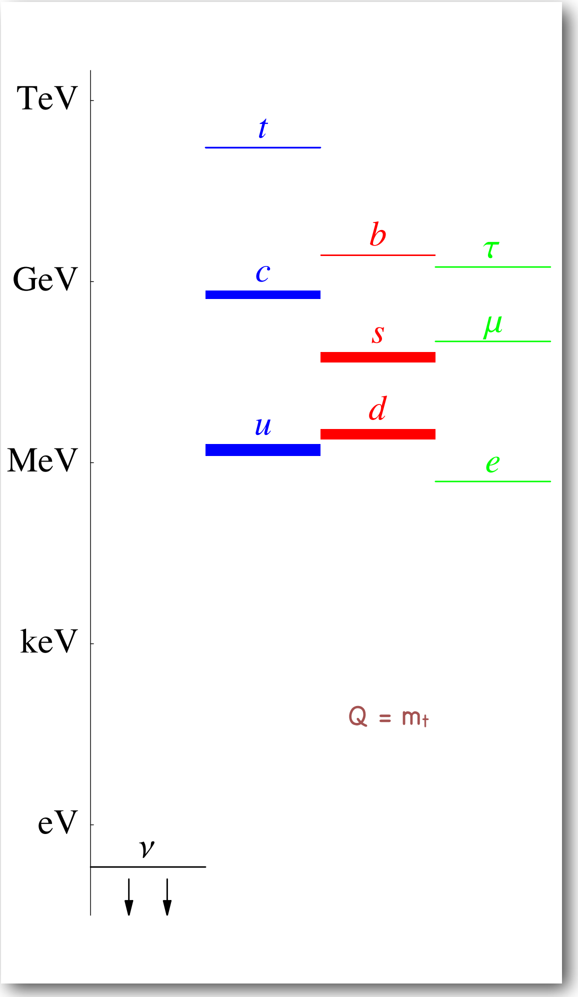
Neutrino masses

Neutrino masses are small

- Neutrino have masses (oscillations)
- Natural scale of fermion masses: $\langle H \rangle \approx 174$ GeV
- Why $m_\nu / \langle H \rangle < 10^{-12}$?
- Must have a different origin than $m_e / \langle H \rangle \approx 0.3 \times 10^{-5}$
  - quantitatively larger hierarchy
  - family independent
  - compelling understanding available



- Reminder: most general mass term with $\psi_1, \ldots, \psi_n$

$$\frac{m_{ij}}{2}\psi_i\psi_j + \text{h.c.} \qquad \text{(only gauge invariant terms)}$$

$$(\psi_1\psi_2 = \psi_2\psi_1 = \psi_1^\alpha \epsilon_{\alpha\beta}\psi_2^\beta)$$

Neutrino masses in QED + QCD

- Elementary L-handed fermions (1 family)

	d	d^c	u	u^c	e	e^c	ν
Q	-1/3	1/3	2/3	-2/3	-1	1	0
$SU(3)_c$	3	3*	3	3*	1	1	1

- Most general invariant mass terms: $m_d d^c d + m_u u^c u + m_e e^c e + \frac{m_\nu}{2} \nu\nu$

- Both charged leptons and neutrinos are equally allowed to get a mass term (although of different type)

Neutrino masses in the SM

- Elementary L-handed fermions

	Q_i	d^c_i	u^c_i	L_i	e^c_i
SU(3)	3	3*	3*	1	1
SU(2)	2	1	1	2	1
Y	1/6	1/3	-2/3	-1/2	1

- Most general invariant mass terms: none

- After EWSB

$$m_d = \lambda_d v$$
$$m_u = \lambda_u v \qquad m_\nu = 0 \qquad \text{a SM success}$$
$$m_e = \lambda_e v$$

- $m_\nu = 0$ in the SM is a nice starting point but

- $m_\nu \neq 0$ needs extra ingredients

- 2 main options:

general compelling understanding of the smallness of m_ν

1. the new ingredients live at $M \gg M_Z$ (example: see-saw)

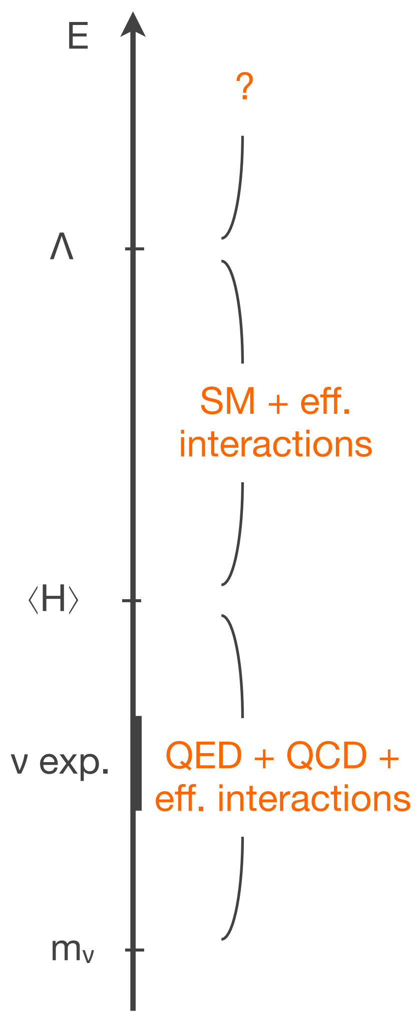
2. the new ingredients live at $M \lesssim M_Z$ (example: Dirac neutrinos)

Option 1: $M \gg M_Z$

Theorem (reminder)

* The effect of any high scale [$M \gg M_Z$] physics [responsible for neutrino masses] can be described at low E by effective interactions involving only light [SM] dofs and symmetries (no need to know the microscopic theory and dofs) suppressed by M
* Example: SM interactions can be described at $E \ll M_Z$ by effective Fermi interaction involving only light fermions

Neutrino masses



$$\mathcal{L}^{\text{eff}}_{E \ll \Lambda} = \mathcal{L}^{\text{ren}}_{\text{SM}} + \frac{c_{ij}}{2\Lambda}(h\,l_i)(h\,l_j) + \ldots$$

$$m^{E,D,U}_{ij} = \lambda^{E,D,U}_{ij} v \qquad m^{\nu}_{ij} = c_{ij} v \times \frac{v}{\Lambda}$$

(Majorana)

$$\Lambda \sim 0.5 \times 10^{15}\,\text{GeV}\,c\left(\frac{0.05\,\text{eV}}{m_\nu}\right)$$

- Compelling:
  - An elegant, economical, and model-independent understanding of the smallness of neutrino masses in terms of the heaviness of the scale at which L is violated.
- What makes neutrinos special?
  - They are the only fermions in the SM for which a mass does not arise (after EWSB) from a renormalisable interaction with the Higgs fields. They turn out to be Majorana.
- BUT:
  - Could not ν have a light ν^c partner as all other SM fermions?

(

"Right-handed" neutrinos

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} u^c \\ d^c \end{pmatrix} \quad \begin{pmatrix} \nu \\ e \end{pmatrix} \quad \begin{pmatrix} \nu^c \\ e^c \end{pmatrix}$$

$SU(3)_C \times SU(2)_W \times U(1)_Y$

$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

$SO(10)$

"Right-handed" neutrinos

$$\begin{pmatrix} u \\ d \end{pmatrix} \qquad \begin{matrix} u^c \\ d^c \end{matrix} \qquad \begin{pmatrix} \nu \\ e \end{pmatrix} \qquad \begin{matrix} \nu^c \\ e^c \end{matrix}$$

$SU(3)_C \times SU(2)_W \times U(1)_Y$

$$\lambda_\nu \nu_c L H \to m_\nu = \lambda_\nu v$$

(like the other fermions)

ν_c is a SM singlet and can therefore be heavy

$$\mathcal{L}_{\mathrm{HE}} \supset -\frac{M}{2}\nu^c\nu^c$$

(unlike the other fermions)

Neutrino masses in the SM + ν^c

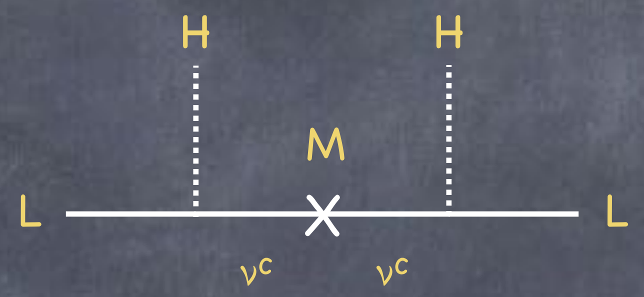
- Elementary L-handed fermions

	Q_i	d^c_i	u^c_i	L_i	e^c_i	ν^c_i
SU(3)	3	3^*	3^*	1	1	1
SU(2)	2	1	1	2	1	1
Y	1/6	1/3	-2/3	-1/2	1	0

- Most general invariant mass terms: $\dfrac{M_{ij}}{2}\nu^c_i \nu^c_j$

See-saw

Integrate out ν^c:



$$\frac{c}{\Lambda}(HL)(HL)$$

$$\frac{c}{\Lambda} = -\lambda^T \frac{1}{M} \lambda$$

$$\boxed{m_\nu = -m_{\mathrm{D}}^T \frac{1}{M} m_{\mathrm{D}}}$$

Majorana

exercise

$$\mathcal{L}(L_i, H, \nu_i^c) = \lambda_{ij}\nu_i^c L_j H + \frac{M_{ij}}{2}\nu_i^c \nu_j^c$$

$$\text{e.o.m. (neglect kinetic term): } \frac{\partial \mathcal{L}}{\partial \nu_i^c} = 0$$

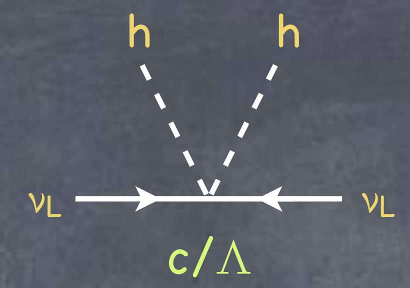
$$\mathcal{L}_{\text{eff}}(L_i, H) = ?$$

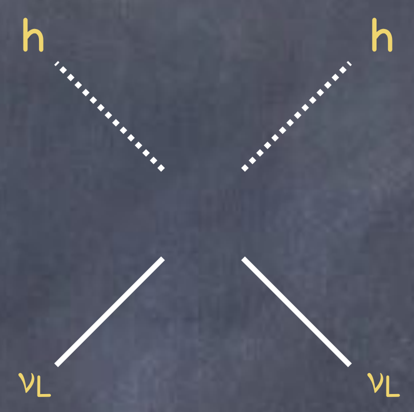
)

Theorem (reminder)

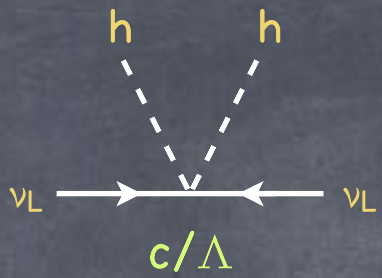
- The effect of any high scale [M » M_Z] physics [responsible for neutrino masses] can be described at low E by effective interactions involving only light [SM] dofs and symmetries (no need to know the microscopic theory and dofs) suppressed by M
- Example: SM interactions can be described at E « M_Z by effective Fermi interaction involving only light fermions
- If the higher E theory is known, the specific form of the NR remnants can be derived
- If the higher E theory is unknown: i) model-independent parameterization of NP, ii) the experimental determination of effective interactions tells us about the microscopic theory
  - e.g.: Fermi Gamow-Teller Sudarshan-Marshak interaction

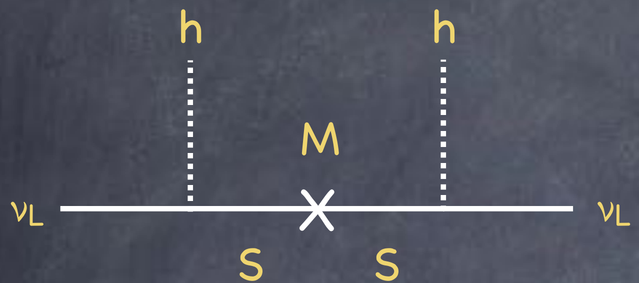
$$\frac{G_F}{\sqrt{2}}\bar{\psi}_1\Gamma^A\psi_2\bar{\psi}_3\Gamma_A\psi_4 \Rightarrow \frac{G_F}{\sqrt{2}}\bar{\psi}_1\gamma^\mu(1-\gamma_5)\psi_2\bar{\psi}_3\gamma_\mu(1-\gamma_5)\psi_4 \Rightarrow \textbf{SM}$$

Possible (tree level) origins of 



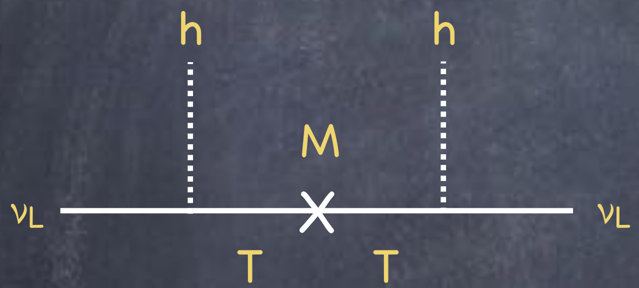
Possible (tree level) origins of





See-saw type I

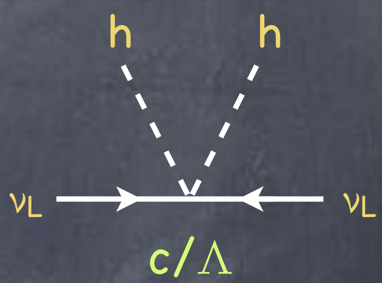
S: SM singlet

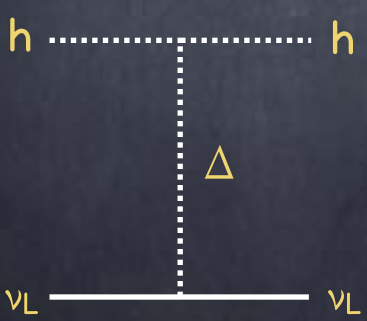


See-saw type III

T: $SU(2)_L$ triplet, $Y = 0$

$E \ll M$ $\Rightarrow$





See-saw type II

T: $SU(2)_L$ triplet, $Y = 1$

Option 2: $M \lesssim M_Z$

Example: Dirac neutrinos

- Lepton number is "exactly" conserved: $h_{ij} = 0$
- Neutrino masses then need an L = -1 neutrino ν^c

$$m^N_{ij}\nu^c_i\nu_j + m^E_{ij}e^c_i e_j + m^D_{ij}d^c_i d_j + m^U_{ij}u^c_i u_j + \text{h.c.}$$

- In the SM:

$$\begin{aligned}\mathcal{L}^{\text{flavor}}_{\text{SM}} &= \lambda^N_{ij}\nu^c_i L_j H + \lambda^E_{ij}e^c_i L_j H^\dagger + \lambda^U_{ij}u^c_i Q_j H + \lambda^D_{ij}d^c_i Q_j H^\dagger + \text{h.c.}\\ &= m^N_{ij}\nu^c_i\nu_j \quad + m^E_{ij}e^c_i e_j \quad + m^U_{ij}u^c_i u_j \quad + m^D_{ij}d^c_i d_j \quad + \text{h.c.} + \ldots\end{aligned}$$

$$m^N_{ij} = \lambda^N_{ij}v \quad m^E_{ij} = \lambda^E_{ij}v \quad m^D_{ij} = \lambda^D_{ij}v \quad m^U_{ij} = \lambda^U_{ij}v$$

- Needs L and $\lambda^N < 10^{-11}$: why?

Low scale lepton number violation

- singlet neutrino mass $M < v$ (EW scale)
- effective description not sound anymore
- $\lambda \approx 10^{-6} (M/v)^{1/2}$

Low-scale origin of L-violation (1)

- TeV-scale see-saw
  - ν^c with M ≈ TeV
  - Probe ν^c through $\lambda\,\nu^c LH$: $m_\nu = -m_D^T \frac{1}{M} m_D, \quad m_D = \lambda\,\langle H\rangle$
  - M ~ TeV $\Rightarrow$ $\lambda = \frac{m_D}{\langle H\rangle} \sim 10^{-6} \left(\frac{m_\nu}{0.05\,\mathrm{eV}}\right)^{1/2} \left(\frac{M}{\mathrm{TeV}}\right)^{1/2}$

    too small for LHC
  - Unless λ » 10^{-7} + cancellations in $m_\nu = -m_D^T \frac{1}{M} m_D$ (2 or more ν^c's)
    - "magical", e.g.: $m_\nu = 0 + \text{corrections}$ if

      $$m^D_{nj} = \alpha_n \beta_j\, m_0, \quad M_R = \mathrm{Diag}(M_1 \ldots M_n), \quad \sum_n \alpha_n^2 M_n = 0$$
    - natural, e.g.:

      $L_e, L_\mu, L_\tau, (\nu_R)_1 \equiv N$ have L = 1, $(\nu_R)_2 \equiv N'$ has L = -1

Low-scale origin of L-violation (2)

- (R_P-violating) supersymmetry
- Supersymmetry does not guarantee (accidental) L (or B) conservation, unlike the SM: $H_d \approx L_i$

$$\begin{aligned} W &= \lambda^U_{ij} u^c_i Q_j H_u + \lambda^D_{ij} d^c_i Q_j H_d + \lambda^E_{ij} e^c_i L_j H_d + \mu H_u H_d \\ &\quad + \lambda''_{ijk} u^c_i d^c_j d^c_k + \lambda'_{ijk} L_i Q_j d^c_k + \lambda_{ijk} L_i L_j e^c_k + \mu_i H_u L_i \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{\rm soft} &= A^U_{ij} \tilde{u}^c_i \tilde{Q}_j H_u + A^D_{ij} \tilde{d}^c_i \tilde{Q}_j H_d + A^E_{ij} \tilde{e}^c_i \tilde{L}_j H_d + B\mu H_u H_d \\ &\quad + A''_{ijk} \tilde{u}^c_i \tilde{d}^c_j \tilde{d}^c_k + A'_{ijk} \tilde{L}_i \tilde{Q}_j \tilde{d}^c_k + A_{ijk} \tilde{L}_i \tilde{L}_j \tilde{e}^c_k + (B\mu)_i H_u \tilde{L}_i \\ &\quad + \tilde{m}^2_Q \tilde{Q}^\dagger \tilde{Q} + (\tilde{m}^2_i H^\dagger_d \tilde{L}_i + \text{h.c.}) + \text{gaugino masses} \end{aligned}$$

- L and B violating terms controlled by $R_P = (-1)^{3(B-L)+2s}$
- A small R_P breaking:
  - induces $(h_{ij}/\Lambda) L_i L_j H H$, with $\Lambda = \tilde{m}$, $h \leftrightarrow$ small R_P breaking
  - makes the LSP unstable (could be any susy partner)

Majorana o Dirac?

Dirac vs Majorana at the particle level

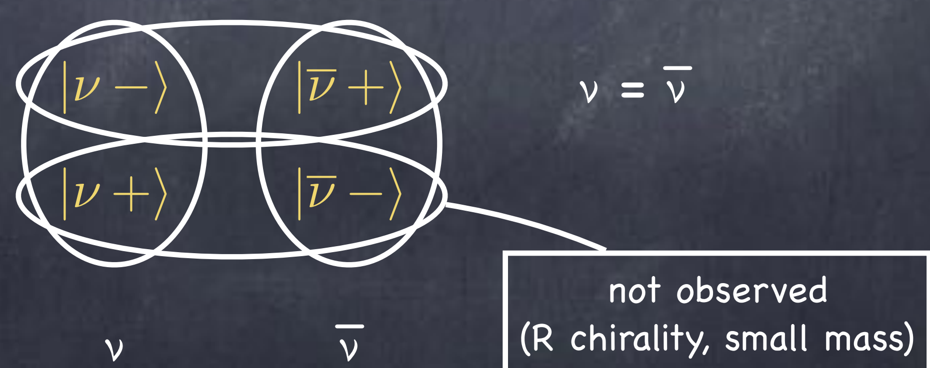
- QFT: each particle p corresponds to an antiparticle $\bar{p}$ (CPT)
- Particle and antiparticle have same mass, spin (opposite helicities if $m = 0$), opposite conserved charges
- CPT does not forbid $p = \bar{p}$; however this cannot happen if the particle is charged or, if $s \neq 0$, massless
- If $Q = 0$ and $m \neq 0$ a fermion can coincide with its antiparticle (Majorana fermion) or be independent of it (Dirac fermion)
- Neutrinos are the only known fermions that can be Majorana or Dirac, all other known fermions are Dirac
- The mass term of a Majorana fermion violates all charges carried by the fermion
- If neutrinos are Majorana, lepton number is violated

Dirac vs Majorana at the particle level

- m = 0
  - the helicity is an invariant of a 1-particle state (1 dof)
  - neutrino particles have 1 dof
  - CC interactions only produce $|\nu -\rangle$ and $|\overline{\nu} +\rangle$ (L chirality)

- m ≠ 0
  - helicity depends on the reference frame, one particle states have 2 dofs corresponding to two opposite helicities

Majorana:
Dirac:



Dirac vs Majorana at the field level

ψ_i "L" fermions

ψ

Most general mass term: $\frac{m_{ij}}{2}\psi_i\psi_j$

$\frac{m}{2}\psi\psi$

m_{ij} symmetric

"Majorana"
breaks any charge of ψ

example: $\psi = \nu_L$
$m \neq 0$ allowed by QED+QCD
breaks lepton number

Dirac vs Majorana at the field level

ψ_i "L" fermions

ψ, ψ^c

Most general mass term: $\frac{m_{ij}}{2}\psi_i\psi_j$

$$\underbrace{\cancel{\frac{m_1}{2}\psi\psi + \frac{m_2}{2}\psi^c\psi^c}}_{\text{“Majorana”}} + \underbrace{m\psi^c\psi}_{\text{“Dirac”}}$$

m_{ij} symmetric

"Majorana"

"Dirac"
Q(ψ) + Q(ψ^c) = 0
(all charged SM fermions)

(e.g. Dirac neutrino mass term: $m\,\overline{\nu_R}\nu_L$
$\psi^c = \overline{\nu_R} \quad \psi = \nu_L$
needs ν_R
does not break lepton number)

(e.g. electron mass term: $m\,\overline{e_R}e_L$
$\psi^c = \overline{e_R} \quad \psi = e_L$)

Dirac vs Majorana in the interaction

* The difference shows up only in the m ≠ 0 case:
  - Dirac (m = 0)

    $$\overline{\nu_L}|0\rangle = |\nu -\rangle \qquad \nu_L|0\rangle = |\overline{\nu}+\rangle$$

  - Majorana (m = 0)

    $$\overline{\nu_L}|0\rangle = |\nu -\rangle \qquad \nu_L|0\rangle = |\nu +\rangle$$

* In oscillations, once the O(m/E) terms have been neglected:
  - the helicity does not play a role
  - there is no L-violation
  - oscillation formulae are identical for Dirac and Majorana ν's

Dirac vs Majorana (particle content)

- The difference shows up only in the m ≠ 0 case:
  - Dirac (m ≠ 0)

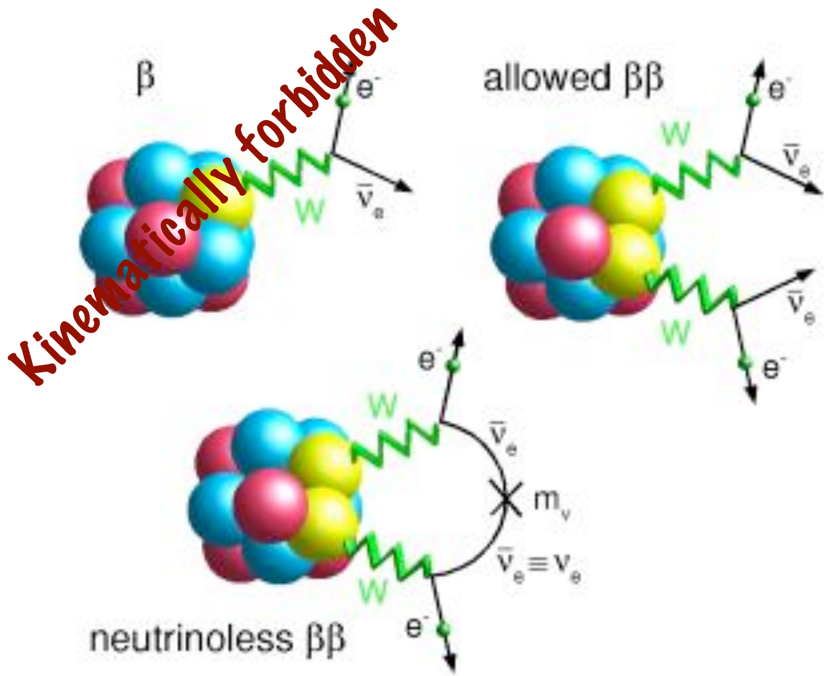
$$\overline{\nu_L}|0\rangle = |\nu\,-\rangle + \mathcal{O}\left(m/E\right)|\nu\,+\rangle \qquad \nu_L|0\rangle = |\overline{\nu}\,+\rangle + \mathcal{O}\left(m/E\right)|\overline{\nu}\,-\rangle$$

  - Majorana (m ≠ 0)

$$\overline{\nu_L}|0\rangle = |\nu\,-\rangle + \mathcal{O}\left(m/E\right)|\nu\,+\rangle \qquad \nu_L|0\rangle = |\nu\,+\rangle + \mathcal{O}\left(m/E\right)|\nu\,-\rangle$$

- In oscillations, once the O(m/E) terms can be neglected:
  - the helicity does not play a role
  - there is no L-violation
  - oscillation formulae are identical for Dirac and Majorana ν's

$0\nu 2\beta$ decay



Violates lepton number
Needs Majorana neutrinos

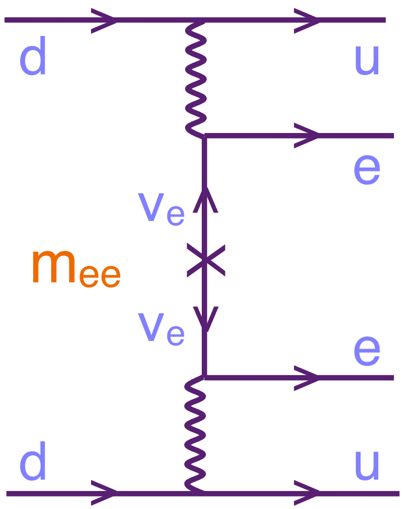
0ν2β decay

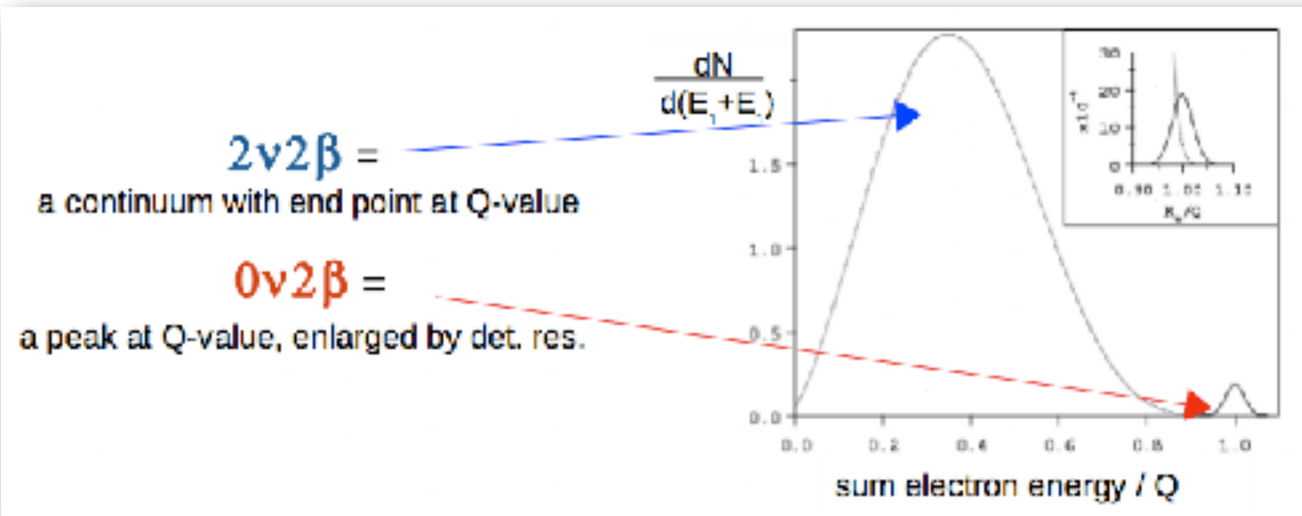
$$(A,Z) \to (A,Z+2) + 2e^-; \quad \text{e.g.: } {}^{76}\text{Ge} \to {}^{76}\text{Se} + 2e^-$$

Depends on
- Phases
- Nuclear matrix elements
- Dirac vs Majorana

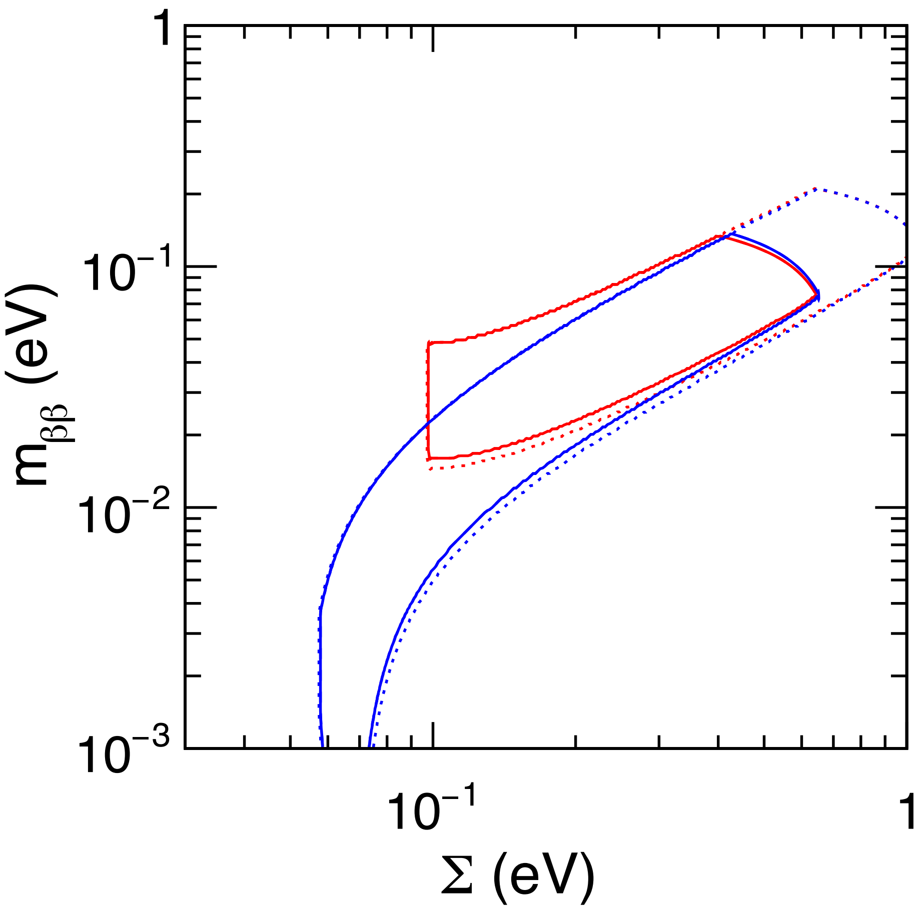
$$\Gamma \propto |m_{ee}|^2 \, \langle Q \rangle^2$$

$$m_{ee} = U_{eh}^2 m_h = c_{13}^2 (m_1 c_{12}^2 + m_2 s_{12}^2 e^{2i\alpha}) + m_3 s_{13}^2 e^{2i\beta'}$$





$$|m_{ee}| < \mathcal{O}(1) \times 0.2\,\text{eV (Heidelberg-Moscow)} \to \mathcal{O}(1) \times 0.01\,\text{eV (Genius)}$$



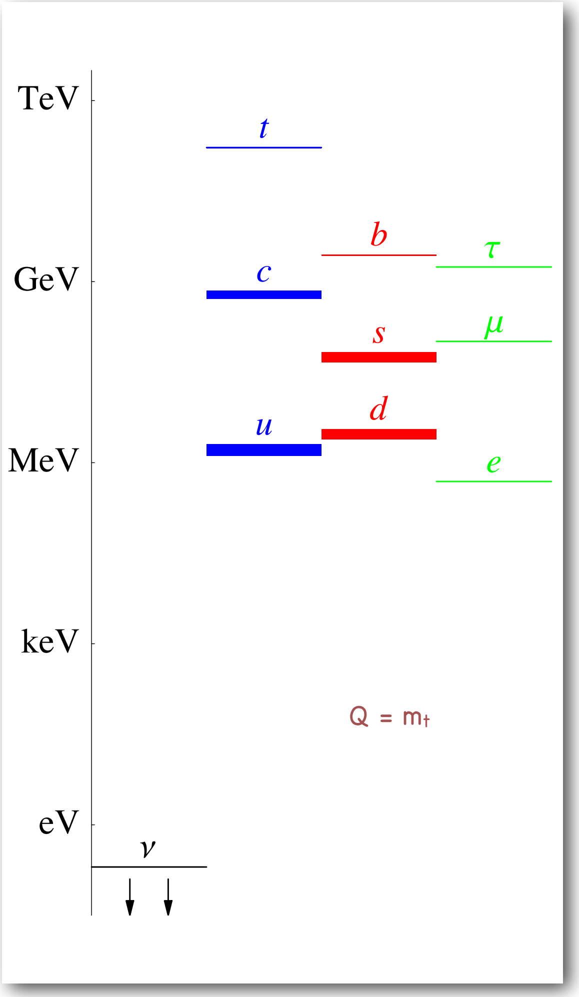
1
10^{-1}
10^{-2}
10^{-3}
$m_{\beta\beta}$ (eV)
10^{-1}
1
Σ (eV)

Neutrino masses are small

What else do we know?

Neutrino have masses (oscillations)

- Natural scale of fermion masses: $\langle H \rangle \approx 174$ GeV
- Why $m_\nu / \langle H \rangle < 10^{-12}$?
- Must have a different origin than $m_e / \langle H \rangle \approx 0.3 \times 10^{-5}$
    - quantitatively larger hierarchy
    - family independent
    - compelling understanding available



Neutrino (lepton) flavour parameters

- Quark sector

$$m_{ij}^{D}\overline{d_{iR}}d_{jL} + m_{ij}^{U}\overline{u_{iR}}u_{jL} = m_{d_i}\overline{d'_{iR}}d'_{iL} + m_{u_i}\overline{u'_{iR}}u'_{iL}$$

$$j^{\mu}_{\mathrm{c,had}} = \overline{u}_{iL}\gamma^{\mu}d_{iL} = V_{ij}\overline{u'_{iL}}\gamma^{\mu}d'_{jL}$$

- Lepton sector, including neutrino masses (Majorana for definitess)

$$\frac{m_{ij}^{\nu}}{2}\nu_{iL}\nu_{jL} + m_{ij}^{E}\overline{e_{iR}}e_{jL} = \frac{m_{\nu_i}}{2}\nu'_{iL}\nu'_{iL} + m_{e_i}\overline{e'_{iR}}e'_{iL}$$

$$j^{\mu}_{\mathrm{c,lep}} = \overline{\nu}_{iL}\gamma^{\mu}e_{iL} = U^{\dagger}_{ij}\overline{\nu'_{iL}}\gamma^{\mu}e'_{jL}$$

Physical flavour parameters in the lepton sector

$$\frac{m_{\nu_i}}{2}\nu_{iL}\nu_{iL} + m_{e_i}\overline{e_{iR}}e_{iL} \qquad j^{\mu\dagger}_{\mathrm{c,lep}} = U_{ij}\bar{e}_{iL}\gamma^\mu\nu_{jL}$$

$$U = \underbrace{\begin{pmatrix} e^{i\gamma_1} & & \\ & e^{i\gamma_2} & \\ & & e^{i\gamma_3} \end{pmatrix}}_{\text{unphysical}} \begin{pmatrix} \text{standard par.} \end{pmatrix} \underbrace{\begin{pmatrix} 1 & & \\ & e^{i\alpha} & \\ & & e^{i\beta} \end{pmatrix}}_{\substack{\text{physical} \\ \text{(Majorana)}}}$$

9 = 3 + 3 + 1 + 2

Physical flavour parameters in the lepton sector

$$\frac{m_{\nu_i}}{2}\nu_{iL}\nu_{iL} + m_{e_i}\overline{e_{iR}}e_{iL} \qquad j^{\mu\dagger}_{\mathrm{c,lep}} = U_{ij}\bar{e}_{iL}\gamma^{\mu}\nu_{jL}$$

$$\boxed{m_e,\ m_\mu,\ m_\tau,\ m_{\nu_1},\ m_{\nu_2},\ m_{\nu_3},\ \theta_{23},\ \theta_{12},\ \theta_{13},\ \delta,\ \alpha,\ \beta}$$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}s^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}s^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

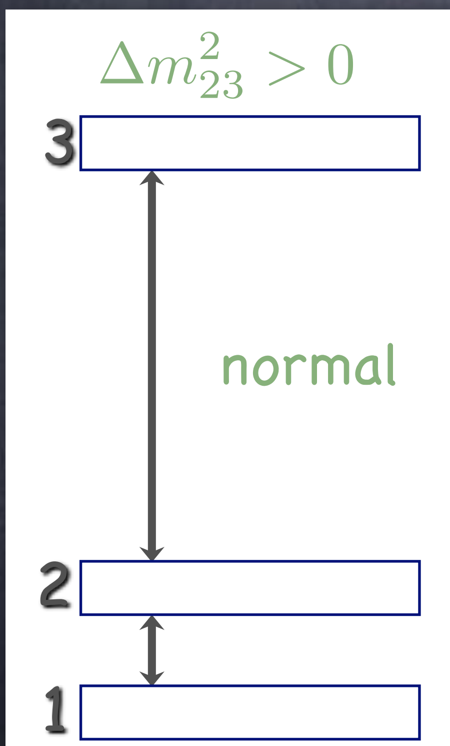
$$0 \le \theta_{23}, \theta_{12}, \theta_{13} \le \frac{\pi}{2}, \quad 0 \le \delta < 2\pi, \quad 0 \le \alpha, \beta < \pi$$

Standard labeling of eigenstates

$0 < \Delta m^2_{12} < |\Delta m^2_{23}|$ uniquely defines the labeling

$\Delta m^2_{12} > 0$ by definition; Δm^2_{23} can have both signs

$$\left(\begin{array}{c} \delta m^2 \equiv \Delta m^2_{12} \\ \Delta m^2 \approx \Delta m^2_{23} \approx \Delta m^2_{13} \end{array} \right)$$



e.g.:

$m_1 < m_2 \ll m_3$

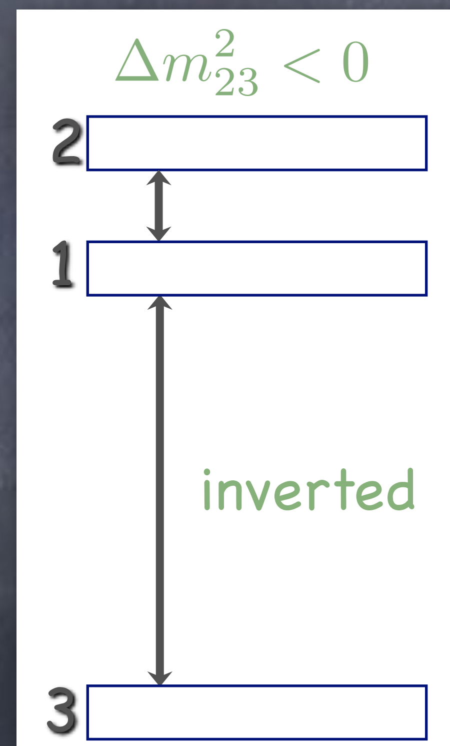
(hierarchical)

$m_1 \approx m_2 \approx m_3$

(degenerate)

$m_1 \approx m_2 < m_3$

(neither)



e.g.:

$m_1 \approx m_2 > m_3$

(inverse hierarchical)

$m_1 \approx m_2 \approx m_3$

(degenerate)

$$\boxed{\Delta m^2_{ij} \equiv m^2_j - m^2_i}$$

Charged sector

$$m_{e,\mu,\tau}$$

Accessible to oscillations

$$\delta m^2$$

$$|\Delta m^2|$$

$$\mathrm{sign}(\Delta m^2)$$

$$\theta_{12}, \theta_{23}, \theta_{13}, \delta$$

Not accessible to oscillations

$$m_{\mathrm{lightest}}$$

$$\alpha$$

$$\beta$$

$$(\Delta m^2_{ij} \equiv m^2_{\nu_j} - m^2_{\nu_i})$$

$$\Delta m^2_{13} = \Delta m^2_{12} + \Delta m^2_{23}$$

$$\delta m^2 \equiv \Delta m^2_{12} \ll |\Delta m^2_{23}| \Rightarrow \Delta m^2_{23} \approx \Delta m^2_{13} \approx \Delta m^2 \equiv (\Delta m^2_{13} + \Delta m^2_{12})/2$$

Accessible
to oscillations

Not accessible
to oscillations

Charged
sector

$m_{e,\mu,\tau}$

Well known

δm^2

$|\Delta m^2|$

$\mathrm{sign}(\Delta m^2)$

$\theta_{12}, \theta_{23}, \theta_{13}, \delta$

Known

Hints

m_{lightest}

α

β

Bounds

$$|\Delta m^2| \approx 2.4 \times 10^{-3}\,\mathrm{eV}^2 \quad \theta_{23} \sim 45^\circ$$

$$\delta m^2 \approx 0.73 \times 10^{-4}\,\mathrm{eV}^2 \quad \theta_{12} \approx 33^\circ$$

$$\theta_{13} \approx 8^\circ$$

$$\mathrm{sign}\,(\Delta m^2) > 0 \quad \text{preferred at } 3\sigma$$

$$\delta \sim 3\pi/2 \,\pm 15\%$$

$|m_{ee}| \equiv |U_{ei}^2 m_{\nu_i}| < \mathcal{O}\,(1) \times 0.2\,\mathrm{eV}$ double-beta decay

$(m^\dagger m)_{ee} = |U_{ei}|^2 m_{\nu_i}^2 < (2\,\mathrm{eV})^2$ beta decay

$\sum m_{\nu_i} \lesssim 0.7\,\mathrm{eV}$ (priors) cosmology

Guidelines for theory:

$$m_{\nu_i} \ll 174\,\mathrm{GeV}$$

$$\theta_{23} \sim 45^\circ (= 45^\circ ?)$$

$$\theta_{12} \approx 33^\circ \neq 45^\circ$$

θ_{13} not so small

$$\delta m^2 / |\Delta m^2| \approx 0.03 \ll 1$$

normal ordering?

large CP-violation?

[Capozzi Marrone Lisi Palazzo arXiv:1804.09678]

Thank you

romanino@sissa.it