Heavy flavor physics (II)

Hadronic B Decays

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Hadronic B decays

Background and motivation:

- **Study CP violation – especially direct CP violation**
- **CKM angle measurements**
- **Test of standard model**
- **Signal of new physics—rare decays**
- **… …**

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

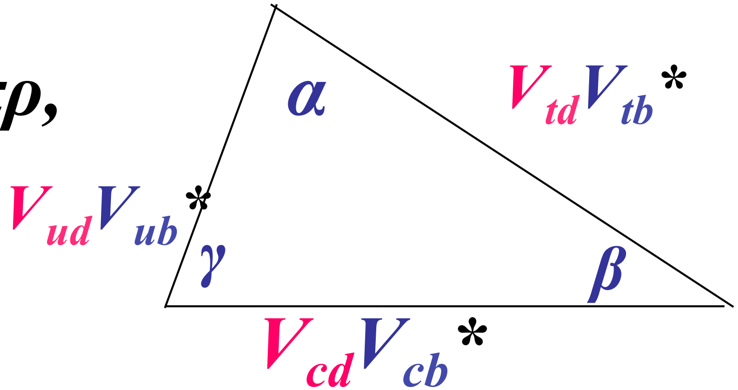
Unitarity $V_{ud}V_{ub}^{*}+V_{cd}V_{cb}^{*}+V_{td}V_{tb}^{*}=0$

The triangles are measured mainly through non-leptonic B (bd) decays

For example: α *by* $B\to\pi\pi$, $\pi\rho$,

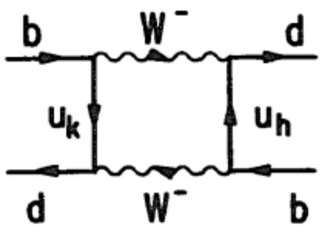
β *by* $B\to J/\psi K$, $K\phi$

γ *by* $B\to DK$, $B_s\to K\rho$,

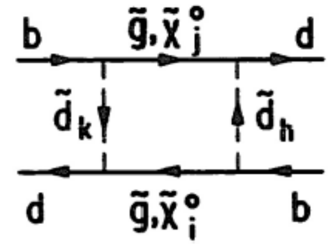


New Physics in FCNC processes

- Mixing

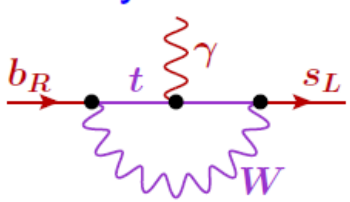


~~OR~~ $\Rightarrow$ AND?

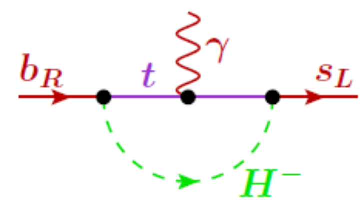


Simple parameterization for each neutral meson: $M_{12} = M_{12}^{\mathrm{SM}}\,(1 + h e^{2i\sigma})$

- Penguin decays



~~OR~~ $\Rightarrow$ AND?



Many operators for $b \to s$ transitions — no simple parameterization of NP

- $V_{td,\,ts}$ only measurable in loops; likely also subleading couplings of new particles

- Isolating modest NP contributions requires many measurements
  Compare NP-independent (tree) with NP-dependent (loop) processes

Hadronic B decays in quark level via the so called 4-quark operators

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{CKM} \sum_i C_i O_i$$

$$O_1 = \bar{u}\gamma^\mu L u \cdot \bar{d}\gamma_\mu L b$$

$$O_2 = \bar{d}\gamma^\mu L u \cdot \bar{u}\gamma_\mu L b$$

$$O_3 = \bar{d}\gamma^\mu L b \cdot \sum_q \bar{q}\gamma_\mu L q$$

$$O_4 = \bar{d}_\alpha\gamma^\mu L b_\beta \cdot \sum_q \bar{q}_\beta\gamma_\mu L q_\alpha$$

$$O_5 = \bar{d}\gamma^\mu L b \cdot \sum_q \bar{q}\gamma_\mu R q$$

$$O_6 = \bar{d}_\alpha\gamma^\mu L b_\beta \cdot \sum_q \bar{q}_\beta\gamma_\mu R q_\alpha$$

$$O_7 = \frac{3}{2}\bar{d}\gamma^\mu L b \cdot \sum_q e_q \bar{q}\gamma_\mu R q$$

$$O_8 = \frac{3}{2}\bar{d}_\alpha\gamma^\mu L b_\beta \cdot \sum_q e_q \bar{q}_\beta\gamma_\mu R q_\alpha$$

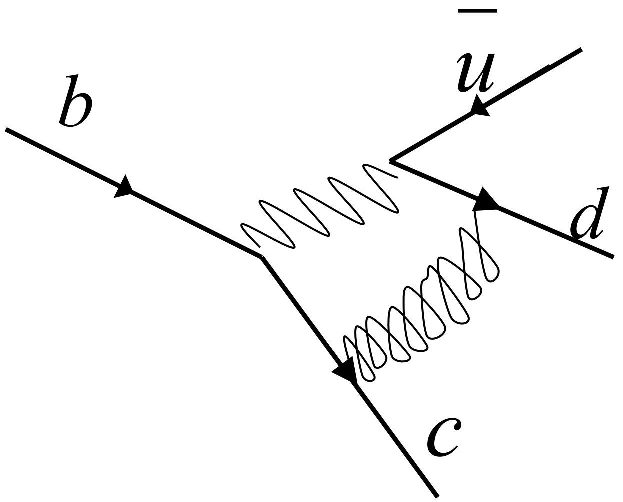
$$O_9 = \frac{3}{2}\bar{d}\gamma^\mu L b \cdot \sum_q e_q \bar{q}\gamma_\mu L q$$

$$O_{10} = \frac{3}{2}\bar{d}_\alpha\gamma^\mu L b_\beta \cdot \sum_q e_q \bar{q}_\beta\gamma_\mu L q_\alpha$$

Tree operators

$$[\bar{c}\gamma^{\mu}(1-\gamma^5)b][\bar{d}\gamma_{\mu}(1-\gamma^5)u] \quad \frac{(ig_2)^2/8}{p^2-M_W{}^2} \quad V_{cb}V_{ud}^*$$

$$=\frac{G_F}{\sqrt{2}} \quad \bar{c}\gamma^{\mu}(1-\gamma^5)b \left[1+\frac{p^2}{M_W^2} + \frac{p^4}{M_W^4} + \ldots\right] \quad \bar{d}\gamma^{\mu}(1-\gamma^5)u$$



$$O_2 = \bar{c}\gamma_{\mu}Lb \cdot \bar{d}\gamma^{\mu}Lu \qquad \propto 1$$

$$O_1 = \bar{c}\gamma^{\mu}Lu \cdot \bar{d}\gamma_{\mu}Lb \qquad \propto \alpha_s$$

$L = 1-\gamma^5$

QCD Penguin operators

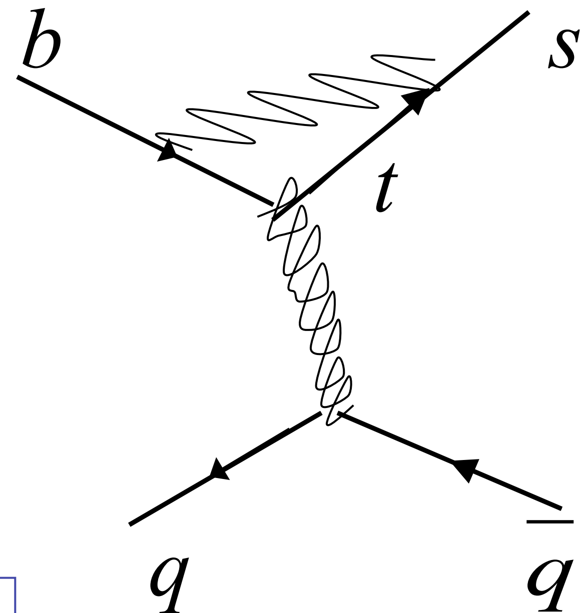
- **Wilson coefficients** $\propto \alpha_s$

$$O_3 = \bar{d}\gamma^{\mu}Lb \cdot \sum_q \bar{q}\gamma_{\mu}Lq$$

$$O_4 = \bar{d}_{\alpha}\gamma^{\mu}Lb_{\beta} \cdot \sum_q \bar{q}_{\beta}\gamma_{\mu}Lq_{\alpha}$$

$$O_5 = \bar{d}\gamma^{\mu}Lb \cdot \sum_q \bar{q}\gamma_{\mu}Rq$$

$$O_6 = \bar{d}_{\alpha}\gamma^{\mu}Lb_{\beta} \cdot \sum_q \bar{q}_{\beta}\gamma_{\mu}Rq_{\alpha}$$



$R = 1 + \gamma^5$

Electroweak penguin operators

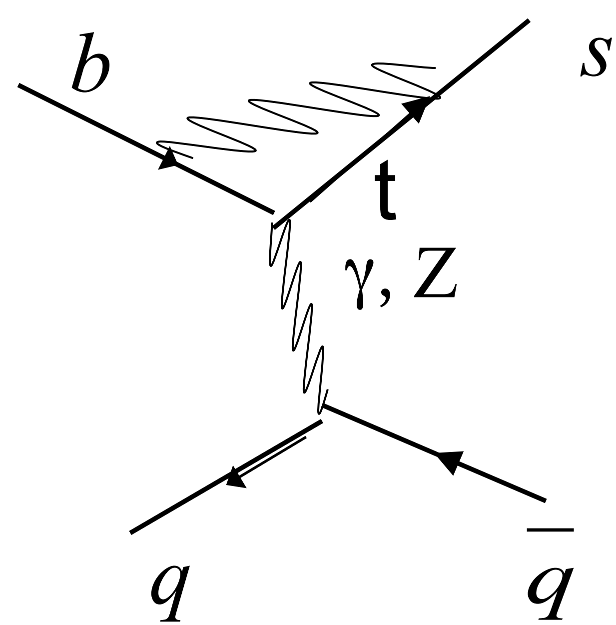
- **Wilson coefficients** $\propto \alpha = 1/137$

$$O_7 = \frac{3}{2}\bar{d}\gamma^{\mu}Lb \cdot \sum_q e_q \bar{q}\gamma_{\mu}Rq$$

$$O_8 = \frac{3}{2}\bar{d}_{\alpha}\gamma^{\mu}Lb_{\beta} \cdot \sum_q e_q \bar{q}_{\beta}\gamma_{\mu}Rq_{\alpha}$$

$$O_9 = \frac{3}{2}\bar{d}\gamma^{\mu}Lb \cdot \sum_q e_q \bar{q}\gamma_{\mu}Lq$$

$$O_{10} = \frac{3}{2}\bar{d}_{\alpha}\gamma^{\mu}Lb_{\beta} \cdot \sum_q e_q \bar{q}_{\beta}\gamma_{\mu}Lq_{\alpha}$$

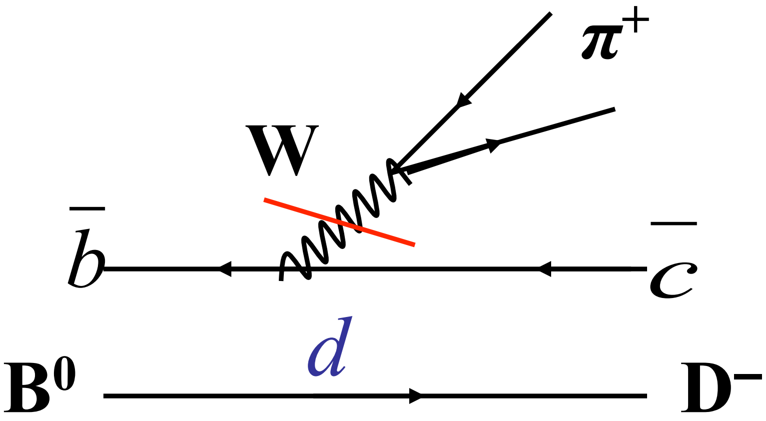


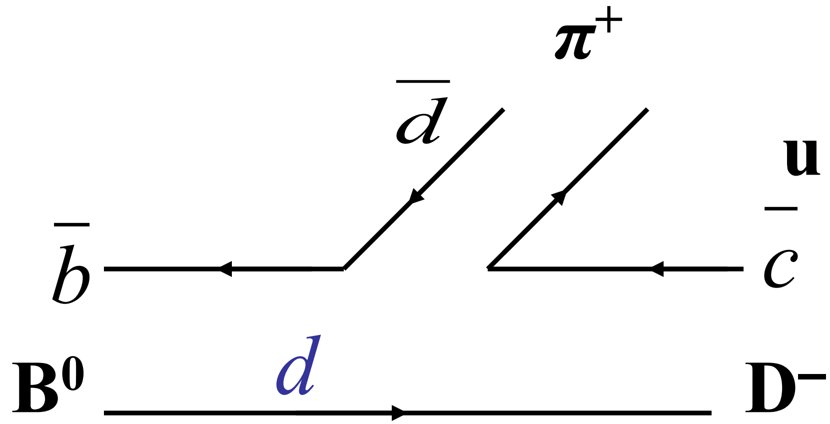
1. Naïve (Generalized) Factorization

Bauer, Stech, Wirbel, Z. Phys. C29, 637 (1985); ibid 34, 103 (1987) ： Semi-leptonic and non-leptonic B→D,J/Ψ,D→K,π

- Chau, Cheng, Sze, Yao, Tseng, Phys. Rev. D43, 2176, (1991); D58, 019902, (1998) (E)
- Ali, Kramer, Lu, Phys. Rev. D58, 094009 (1998) ： Charmless B decays

Two of the 4-quark operators contribute to the $B \to \pi^+ D^-$ decay





$$O_2 = \bar{u}\gamma^\mu L d \cdot \bar{b}\gamma_\mu L c \qquad\qquad O_1 = \bar{u}\gamma^\mu L c \cdot \bar{b}\gamma_\mu L d$$

Color favored **Color suppressed**

$$C_2 \sim 1 \quad > \quad C_1(1/3+S_8) \equiv C_1/N_c \sim -0.2/3$$

Naïve factorization

Two body non-leptonic B decay matrix element factorize to two parts:

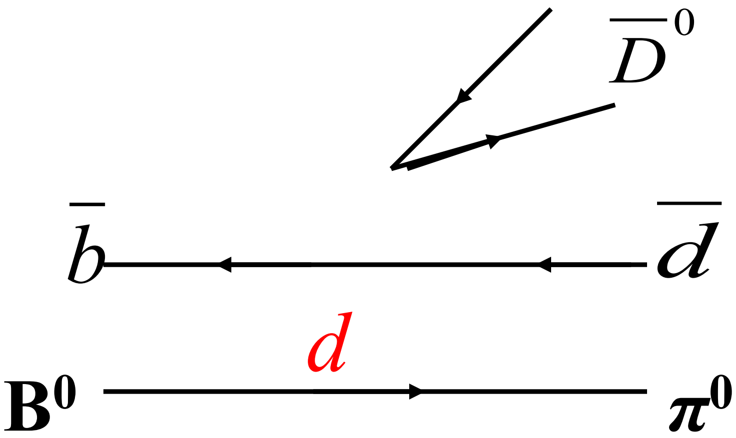
- **4-quark operators with Wilson coefficients: short distance, perturbative calculable**
- **Hadronic parameters: Form factor and decay constant**

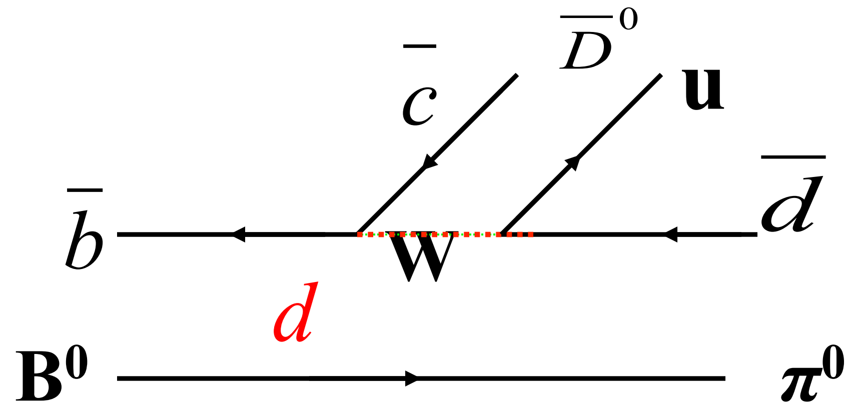
$$< \pi^+ D^- | H_{eff} | B > = a_1 \langle \pi | \bar{u} \gamma^\mu L d | 0 \rangle \langle D | \bar{b} \gamma_\mu L c | B \rangle$$

$$= (C_2 + C_1 / N_c) f_\pi F_0^{B \to D}$$

Class I decay： $\propto a_1 \sim 1$, color favored decays

Class II decays: $B\to\pi^0\overline{D}^0$





$$O_1 = \bar{u}\gamma^{\mu} L c \cdot \bar{b}\gamma_{\mu} L d \qquad O_2 = \bar{u}\gamma^{\mu} L d \cdot \bar{b}\gamma_{\mu} L c$$

Color favored **Color suppressed**

$$C_1 \sim -0.2 \quad \sim \quad C_2(1/3 + s_8) \equiv C_2/N_c \sim +1/3$$

Class II decays: $B\to\pi^0\overline{D}^0$

Two body non-leptonic B decay matrix element factorize to two parts:

$$\left\langle \pi^0 \overline{D}^0 \right| H_{eff} \left| B^0 \right\rangle = a_2 \left\langle D \right| \overline{u}\gamma^\mu L c \left| 0 \right\rangle \left\langle \pi \right| \overline{b}\gamma_\mu L d \left| B \right\rangle$$

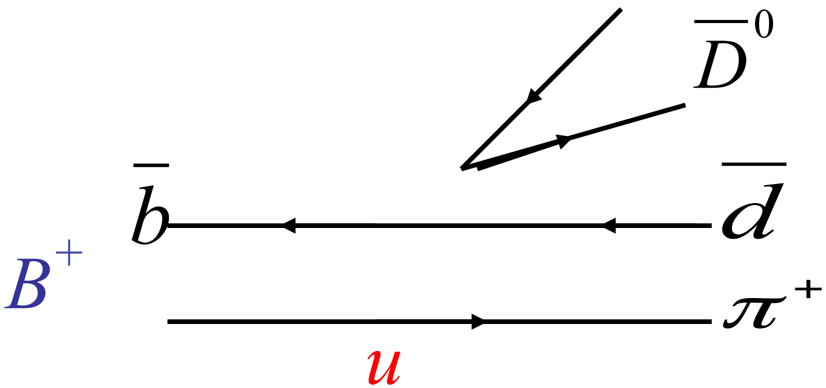
$$= (C_1+C_2/N_c)\, f_D\, F_0^{B\to\pi}$$

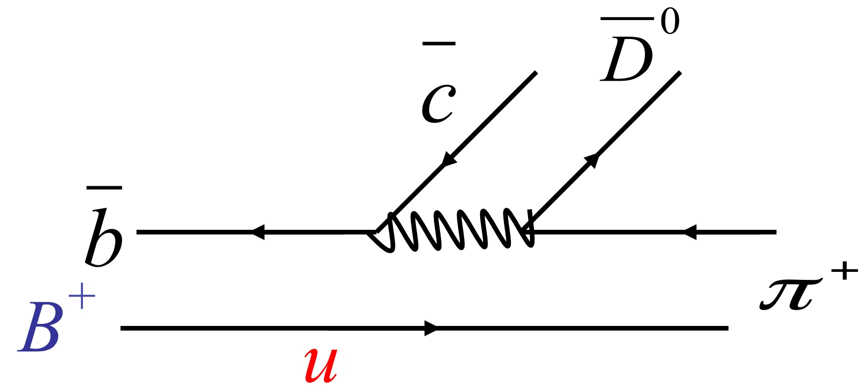
Class II decay： $\propto a_2 = C_1+C_2/N_c$ **small,**

Experimental branching ratios large,

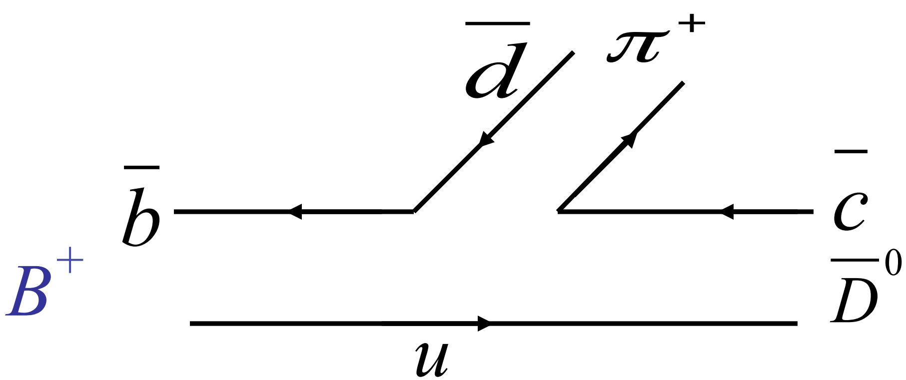
Non-factorizable contribution should be larger than expected

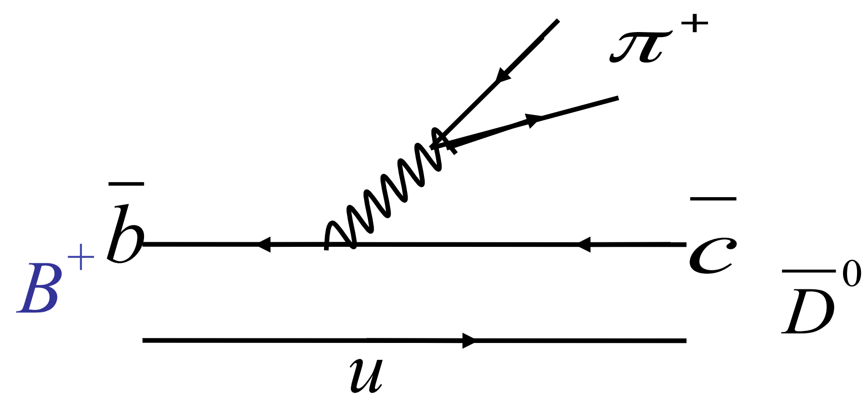
Class III decays: $B\to\pi^+\overline{D}^0$





$$O_1=\bar{u}\gamma^{\mu}Lc\cdot\bar{b}\gamma_{\mu}Ld \qquad O_2=\bar{u}\gamma^{\mu}Ld\cdot\bar{b}\gamma_{\mu}Lc$$





Class III decays: $B\to\pi^+\overline{D}^0$

Charged B$^{\pm}$decay，Both a_1 and a_2 contribute

$$\left\langle \pi^+\overline{D}^0\middle|H_{eff}\middle|B^+\right\rangle = a_2\left\langle D\middle|\bar{u}\gamma^\mu Lc\middle|0\right\rangle\left\langle \pi\middle|\bar{b}\gamma_\mu Ld\middle|B\right\rangle$$

$$+a_1\left\langle \pi\middle|\bar{u}\gamma^\mu Ld\middle|0\right\rangle\left\langle D\middle|\bar{b}\gamma_\mu Lc\middle|B\right\rangle$$

$$= a_2 f_D F_0^{B\to\pi} + a_1 f_\pi F_0^{B\to D}$$

Class III decay amplitude: $\propto a_1+r\,a_2$

It is similar to class I decays

Generalized factorization

- **class I decays:**

$$M\,(B^0 \to \pi^+ D^-) \;\propto\; C_2 + C_1/N_c^{eff} \;=\; a_1$$

- **class II decays:**

$$M\,(B^0 \to \pi^0 \overline{D^0}) \;\propto\; C_1 + C_2/N_c^{eff} \;=\; a_2$$

- **class III decays:**

$$M\,(B^+ \to \pi^+ \overline{D^0}) \;\propto\; (C_2 + C_1)\,(1+/N_c^{eff}\,)$$

$$= a_1 + r\,a_2$$

Generalized factorization

- $N_c^{eff} \neq 3$ to include non-factorization contributions
- $N_c^{eff} = 2$ can explain many B decay branching ratios, such as $B^0 \to \pi^+ D^-$, $B^+ \to \pi^+ D^0$
- i.e. for class I, III and IV decays FA work well, where the dominant contributions are from FA
- While for other decays, it does not work well, for example Brs of $B^0 \to \pi^0 D^0$ decay is much smaller than experimental value

Ali, Kramer, CDL, PRD58, 094009 (1998)

Brs. of $B \to D\pi$ decays calculated in Factorization approach ($\times 10^{-4}$)

Modes	Fac. A	Exp.
$D^{*+}\pi^-$	29	27.6±2.1
$D^+\pi^-$	30	30±4
$D^{*0}\pi^0$	1.0	1.7±0.5
$D^0\pi^0$	0.7	2.9±0.5
$D^{*0}\pi^+$	48	46±4
$D^0\pi^+$	48	53±5

$a_1=1.08$

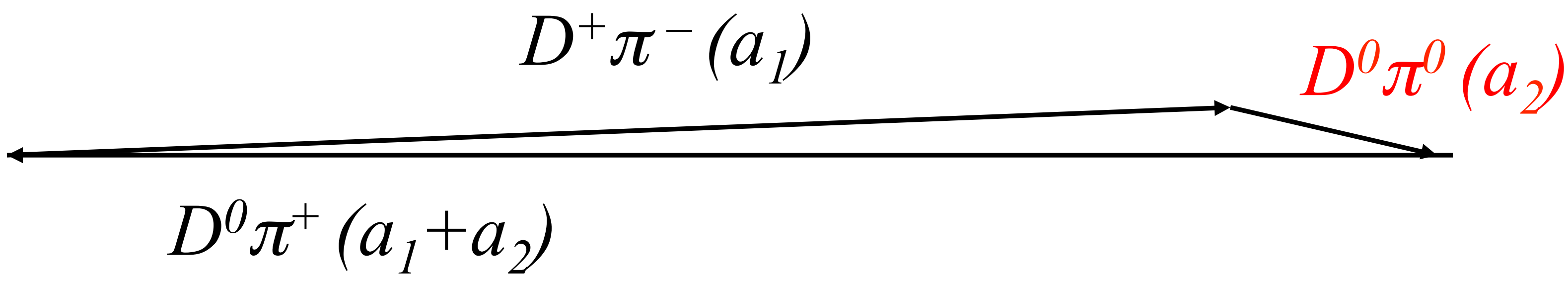
$a_2=0.21$

M. Neubert, B.Stech, **hep-ph/9705292**

Isospin triangle

Later experiments found much larger

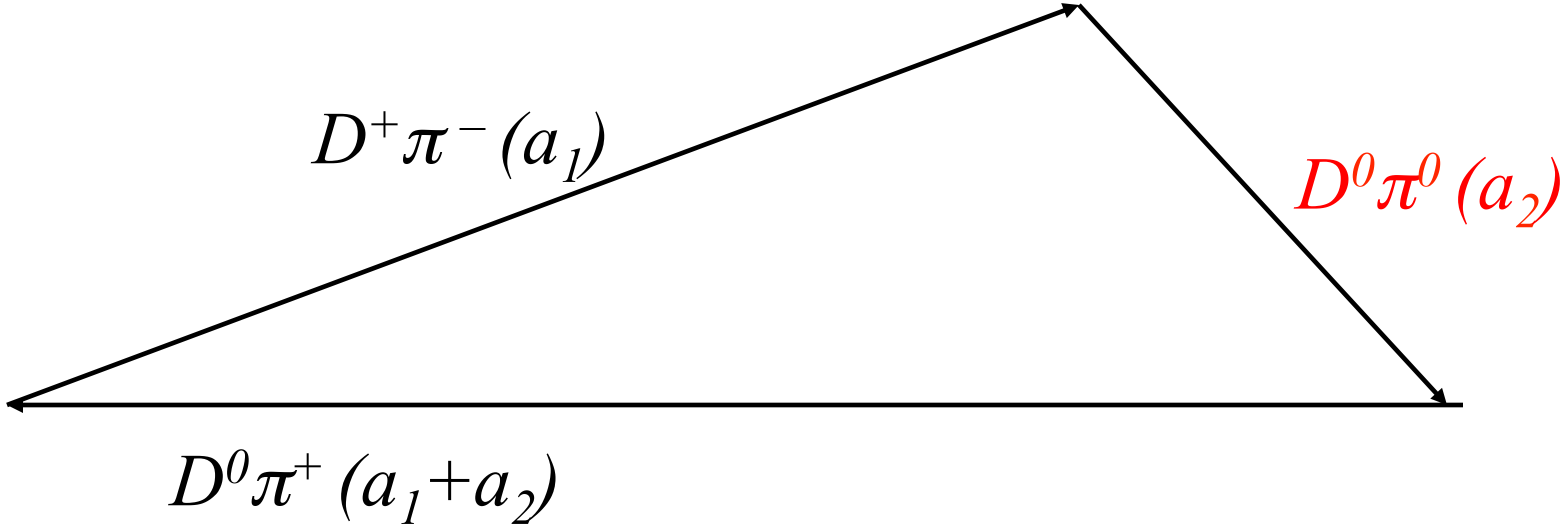
$B^0 \rightarrow D^0$ pi^0 branching ratios



Isospin triangle

Later experiments found much larger $B^0 \to D^0 \pi^0$ branching ratios.

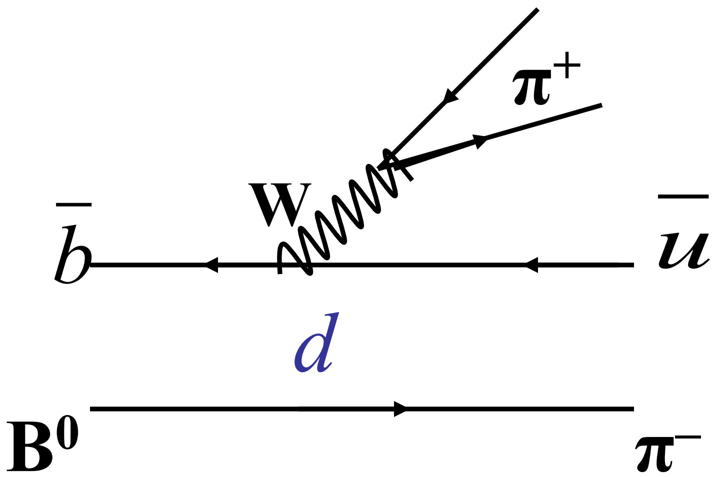
We need a large a_2 and large relative strong phase between a_1 and a_2

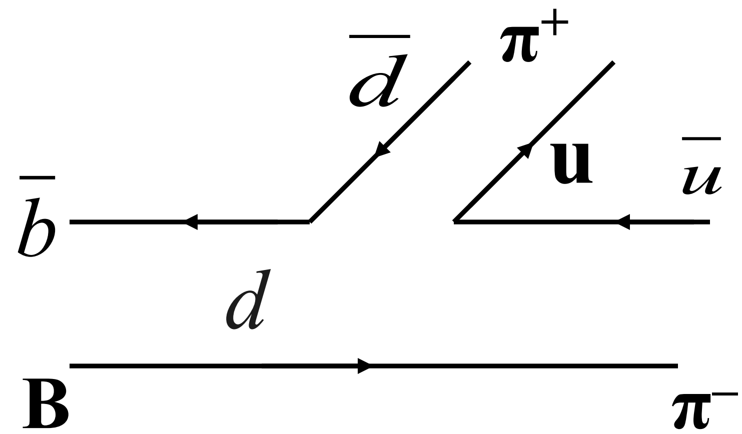


Charmless B decays

- **B → pi pi, pi K etc.**

$B \longrightarrow \pi^+\pi^-$ decay, also belong to class I decays



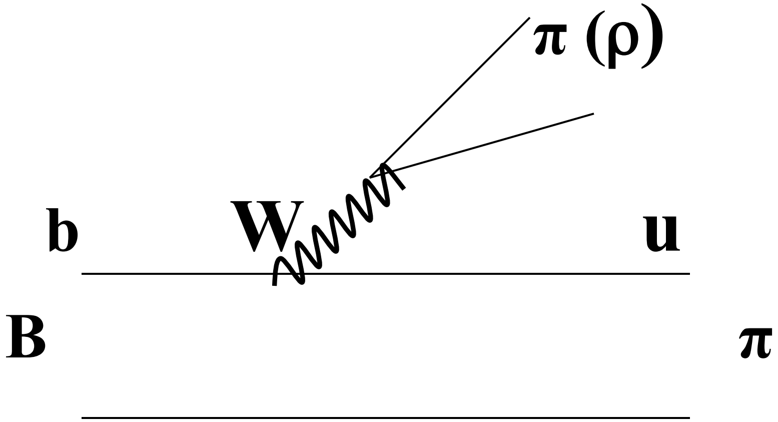


$$O_2 = \bar{u}\gamma^{\mu} L d \cdot \bar{b}\gamma_{\mu} L c \qquad O_1 = \bar{u}\gamma^{\mu} L c \cdot \bar{b}\gamma_{\mu} L d$$

Color favored **color suppressed**

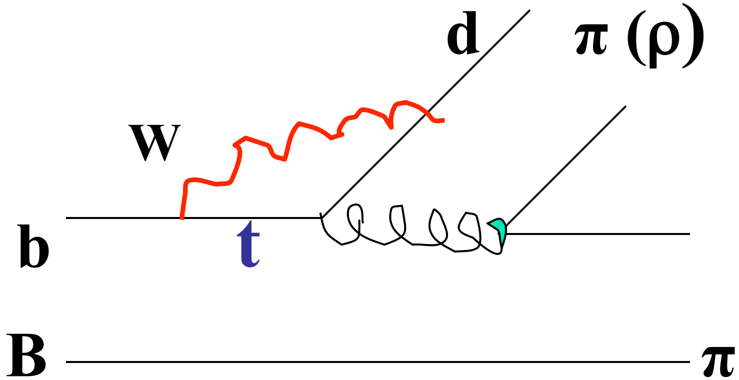
$C_2 \sim 1 \quad > \quad C_1/3 \sim -0.2/3$

B→ππ,πρ,πω decays with penguin contributions (origin of CP violation)



O_1, O_2

tree $\propto V_{ub} V_{ud}^*$



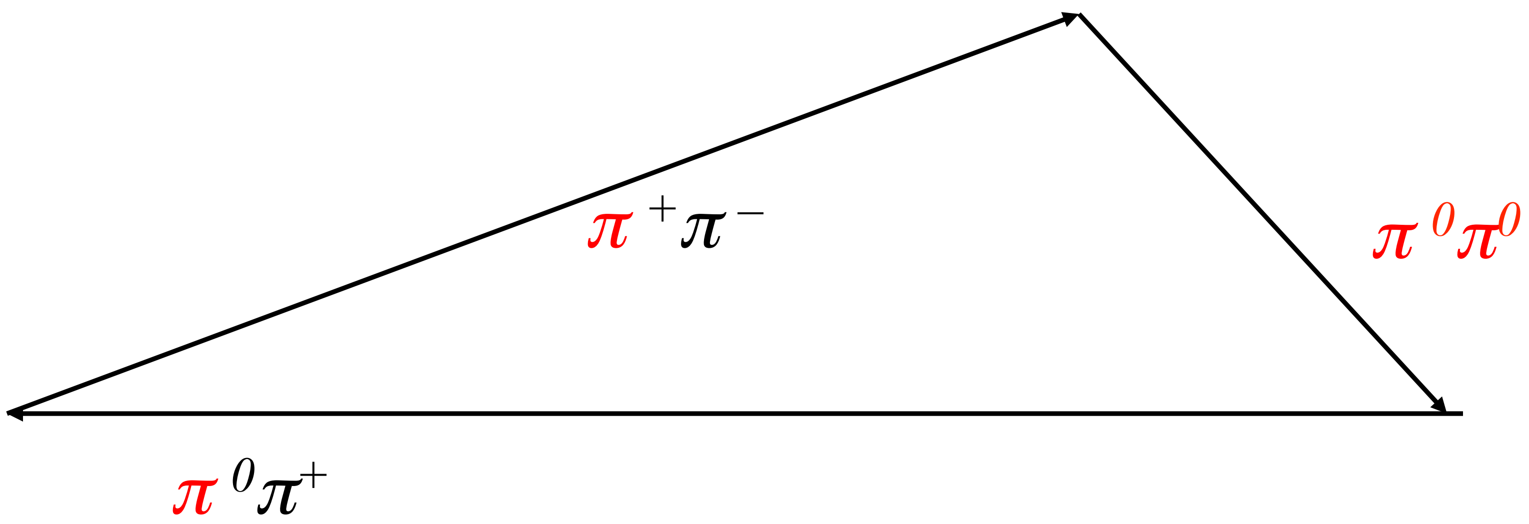
O_3, O_4, O_5, O_6

penguin $\propto V_{tb} V_{td}^*$

Isospin triangle

Later experiments found much larger $B^0 \to \pi^0 \pi^0$ branching ratios than expected

Here We need also a large a_2 and large relative strong phase between a_1 and a_2



Pure penguin decay $B^+ \to \pi^+ K^0$

$$M(B^+ \to \pi^+ K^0) = -i\frac{G_F}{\sqrt{2}} f_K F_0^{B\to\pi}(m_B{}^2 - m_\pi{}^2)$$

$$\times V_{tb}^* V_{ts}\left[a_4 - \frac{1}{2}a_{10} + \left(a_6 - \frac{1}{2}a_8\right)R_5\right]$$

Class IV

There are 4 B→πK channels, proportional to B→π or B→K form factor:

$B^+ \to \pi^+ K^0$ $B^+ \to \pi^0 K^+$ $B^0 \to K^+ \pi^-$ $B^0 \to K^0 \pi^0$

Some decays with tree contribution, to give direct CP asymmetry by interference

Other penguin decays B→φπ、 φK

Class V

$$M(B^+ \to \pi^+\phi) = -\sqrt{2}G_F f_\phi F_1^{B\to\pi} m_\phi(\varepsilon\cdot p_\pi)$$
$$\times V_{tb}^* V_{td}\left[a_3 + a_5 - \frac{1}{2}(a_7 + a_9)\right]$$

a3,a5 < a4

$$M(B^+ \to K^+\phi) = -\sqrt{2}G_F f_\phi F_1^{B\to K} m_\phi(\varepsilon\cdot p_K)$$
$$\times V_{tb}^* V_{ts}\left[a_3 + a_4 + a_5 - \frac{1}{2}(a_7 + a_9 + a_{10})\right]$$

Shortcomings of Naive FA

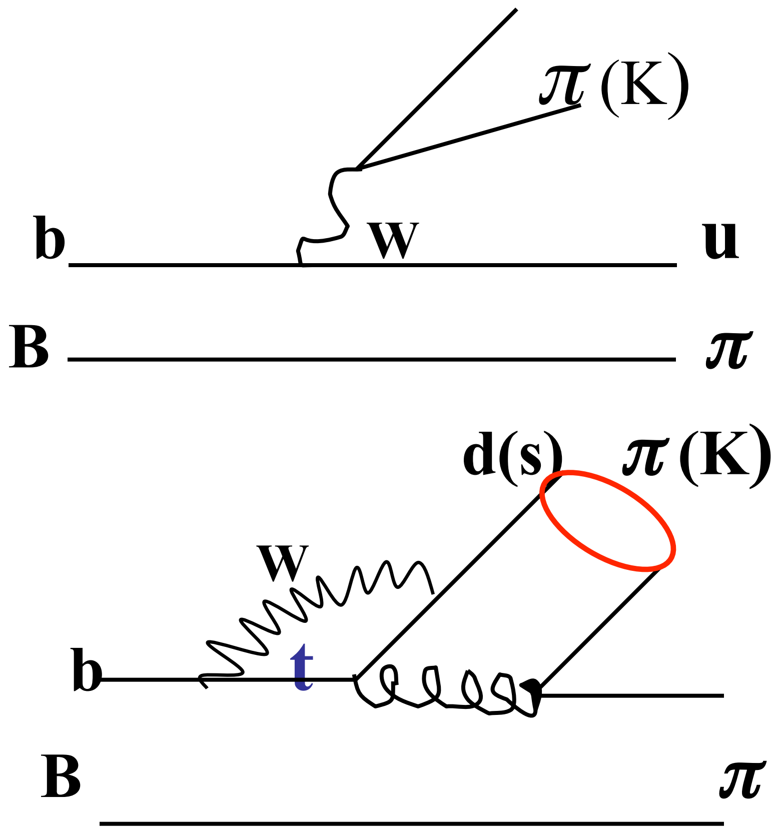
- Non-predictable of the non-factorizable contributions
- Form factors need input from experiments or theoretical calculations

  —most important theoretical uncertainty
- Annihilation type diagram not calculable
- Strong phase too small, and not quantitatively calculable

  — Direct CP asymmetry not predictable
- Final state interaction not predictable

Rich physics in hadronic B decays

CP violation, FCNC, sensitive to new physics contribution…



The standard model describes interactions amongst quarks and leptons

In experiments, we can only observe hadrons

How can we test the standard model without solving QCD?

Perturbative calculations

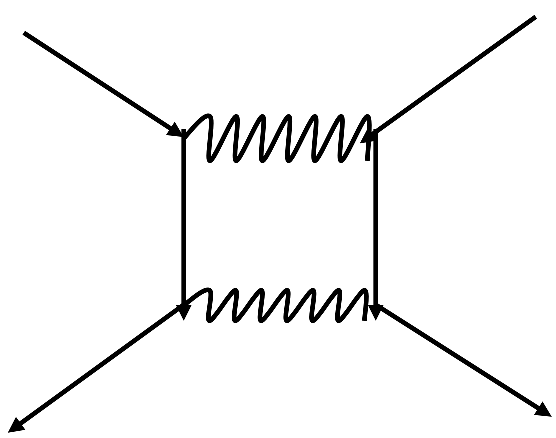
- In principle, all hadronic physics should be calculated by QCD
- In fact, you can always use QCD to calculate any process,

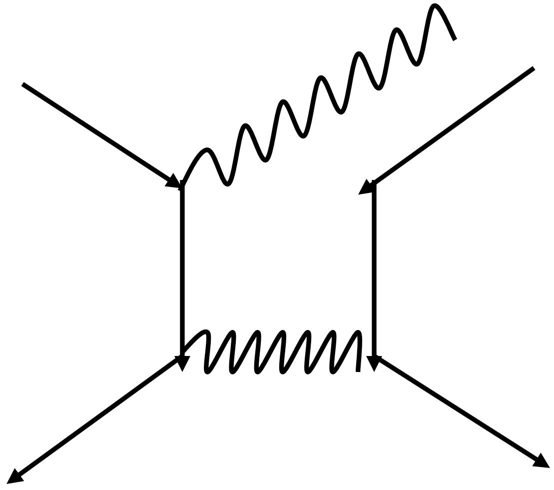
provided you can renormalize the infinities and do all order calculations.

- Perturbation calculation means order by order
- Involving loop diagrams
- Therefore divergences unavoidable

Divergences

- Ultraviolet divergences → renormalization
- Infrared divergences ? **Infrared divergence in virtual corrections should be canceled by real emission**
- In exclusive QCD processes → factorization





Factorization can only be proved in power expansion by operator product expansion. To achieve that, we need a hard scale Q

- In the certain order of 1/Q expansion, the hard dynamics characterized by Q factorize from the soft dynamics
- Hard dynamics is process-dependent, but calculable
- Soft dynamics are universal (process-independent) ⇨ predictive power of factorization theorem
- Factorization theorem holds up to all orders in α_s, but to certain power in 1/Q
- In B decays the hard scale Q is just the b quark mass

QCD-methods based on factorization work well for the leading power of $1/m_b$ expansion

collinear QCD Factorization approach
[Beneke, Buchalla, Neubert, Sachrajda, 99']

Perturbative QCD approach based on k_T factorization
[Keum, Li, Sanda, 00'; Lu, Ukai, Yang, 00']

Soft-Collinear Effective Theory
[Bauer, Pirjol, Stewart, 01']

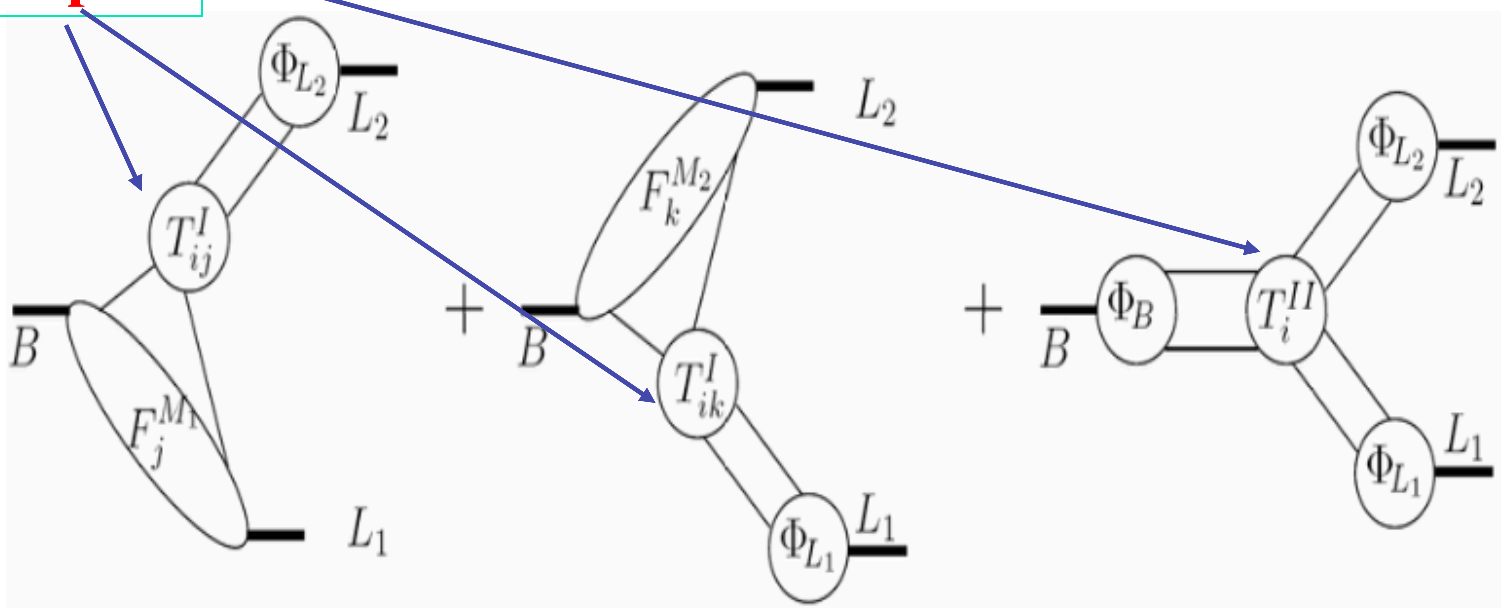
Unavailable for $1/m_b$ power corrections

- Work well for most of charmless B decays, except for ππ, πK puzzle etc.

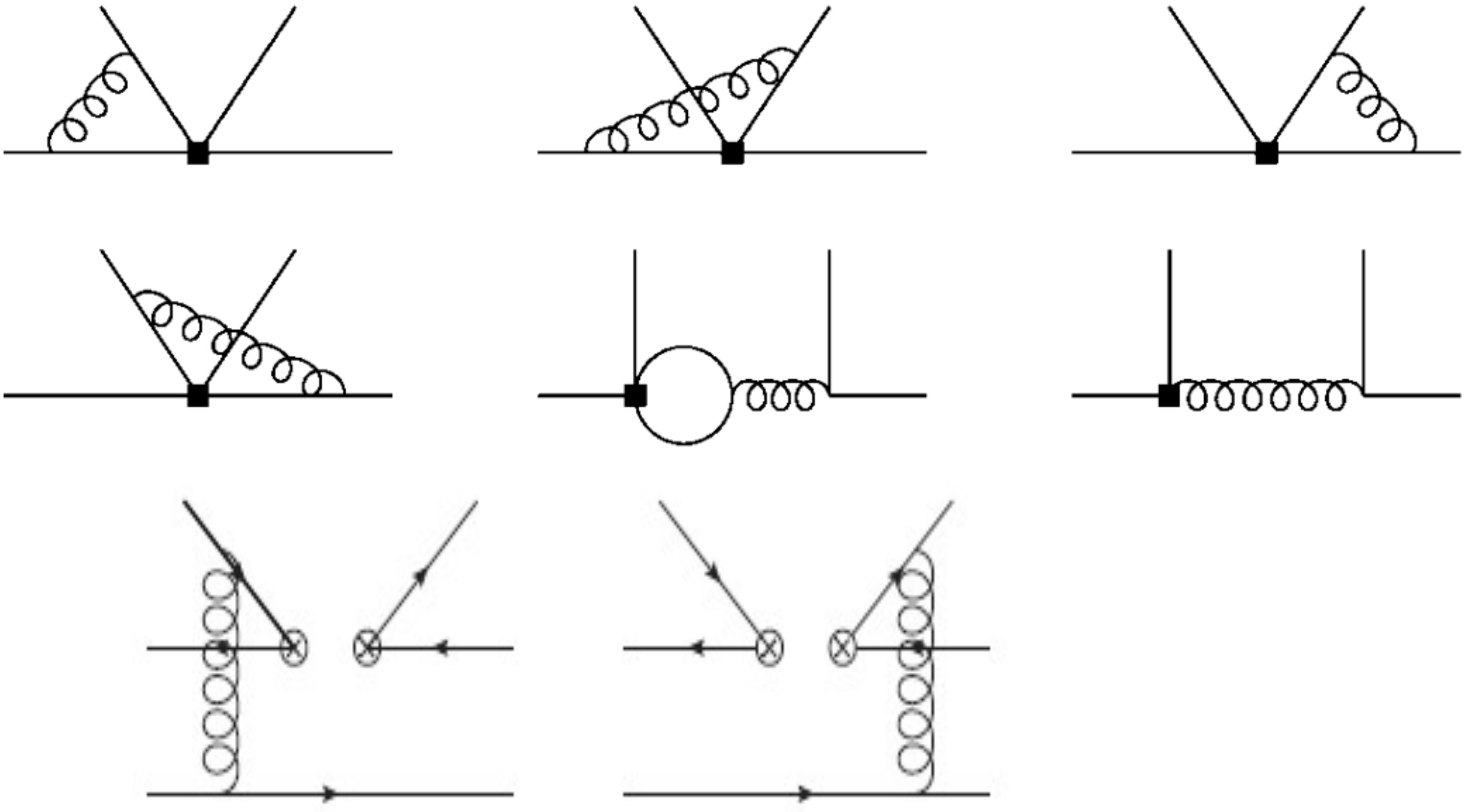
QCD factorization by BBNS: **PRL 83 (1999) 1914; NPB591 (2000) 313**

$$
\begin{aligned}
-\langle L_1 L_2 | Q_i | \bar{B} \rangle \; = \; & \sum_j F_j^{B\to L_1}(m_2^2) \int_0^1 du \, T_{ij}^I(u) \, \Phi_{L_2}(u) \\
& + \sum_k F_k^{B\to L_2}(m_1^2) \int_0^1 dv \, T_{ik}^I(v) \, \Phi_{L_1}(v), \\
& + \int_0^1 d\xi du dv \, T_i^{II}(\xi, u, v) \, \Phi_B(\xi) \, \Phi_{L_1}(v) \, \Phi_{L_2}(u)
\end{aligned}
$$

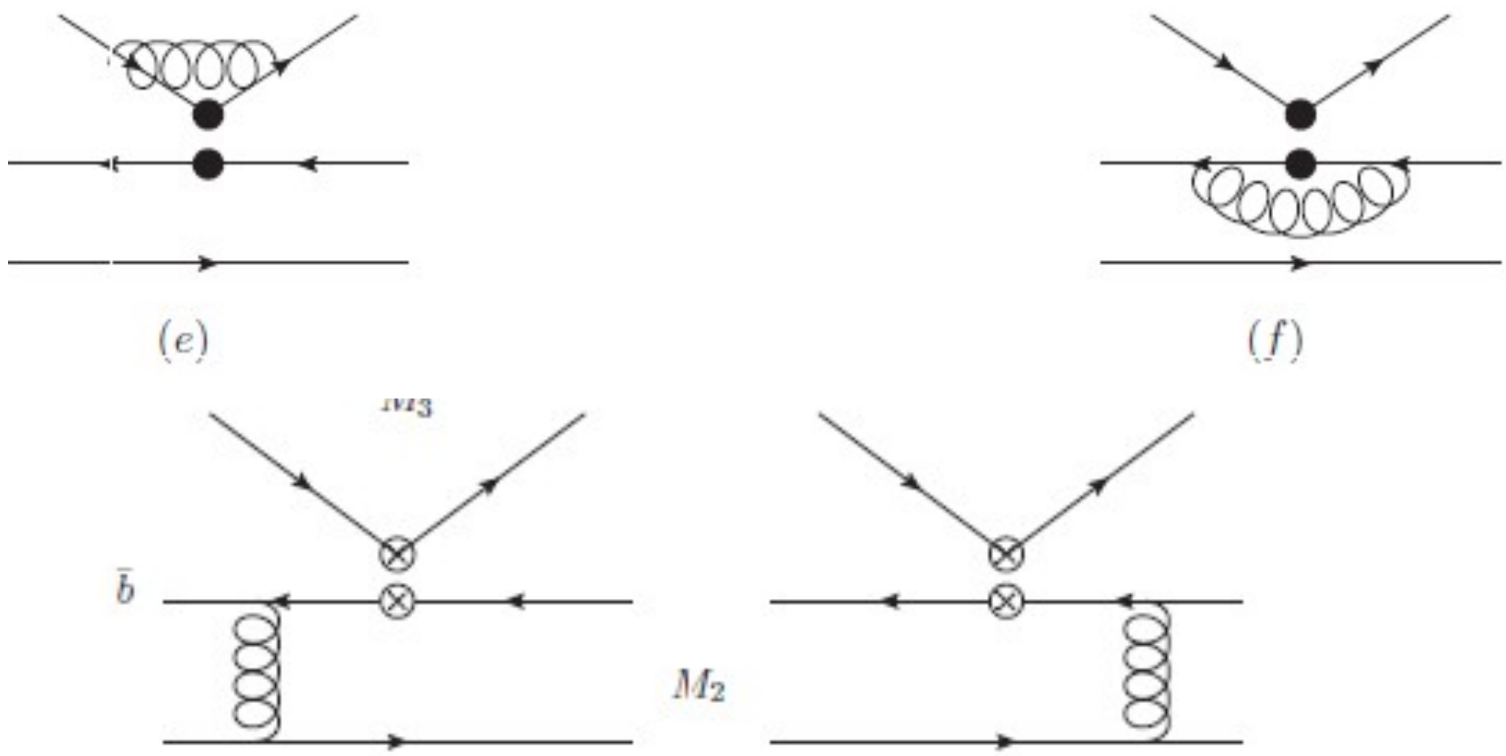
hard part



α_s corrections to the hard part T



The missing diagrams, which contribute to the renormalization of decay constant or form factors



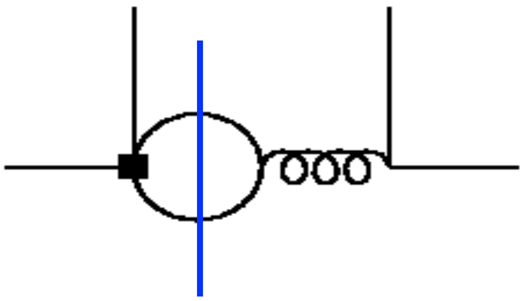
Endpoint divergence appears in these calculations

All strong phases are from perturbative QCD calculations, similar to the FA, which is too small and even wrong sign to the experiments of CP Violation in $B \to \pi\pi(K)$ decays

CP(%)	FA	BBNS	Exp
π^+K^-	+9±3	+5±9	–9.7±1.2
π^0K^+	+8 ± 2	7 ±9	4. 7 ± 2.6
π^+K^0	1. 7± 0. 1	1 ±1	0.9 ±2.5
$\pi^+\pi^-$	–5±3	–6±12	+38±7

Strong phase in QCD factorization

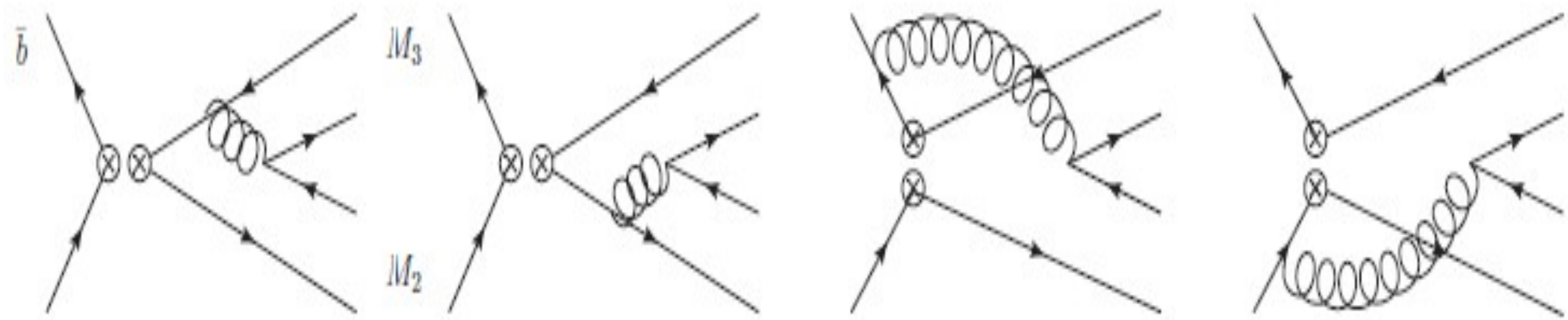
The strong phase of Both QCD factorization and generalized factorization come fromperturbative QCD charm quark loop diagram



It is small, since it is at α_s order

Therefore the CP asymmetry is small

The annihilation type diagrams are important to the source of strong phases



- **However, these diagrams are similar to the form factor diagrams, which have endpoint singularity, not perturbatively calculable.**
- **These divergences are not physical, can only be treated in QCDF as free parameters, which makes CP asymmetry not predictable:**

$$\int_0^1 \frac{dy}{y} \to X_A^{M_1}\,, \qquad \int_0^1 dy\,\frac{\ln y}{y} \to -\frac{1}{2}\,(X_A^{M_1})^2$$

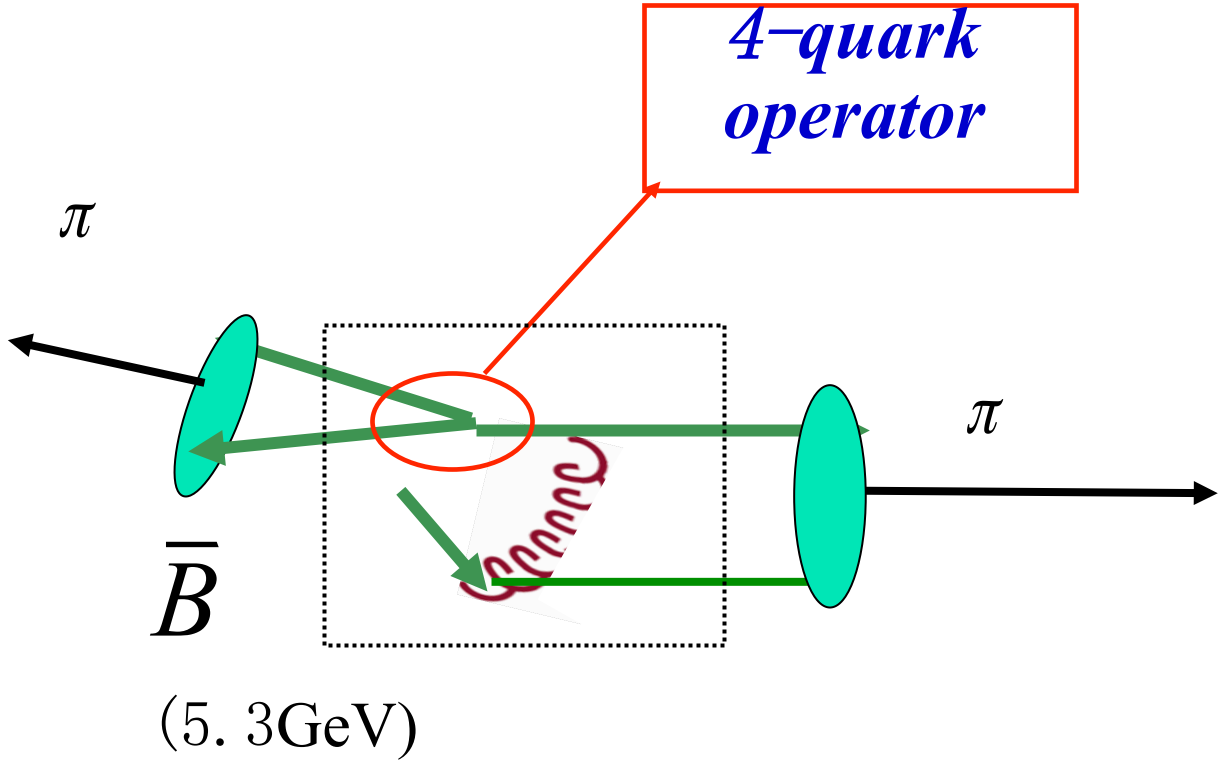
Shortcomings of Naive FA

- Non-predictable of the non-factorizable contributions ✔
- Form factors need input from experiments or theoretical calculations ✗

  —most important theoretical uncertainty
- Annihilation type diagram not calculable ✗
- Strong phase too small, and not quantitatively calculable ✗

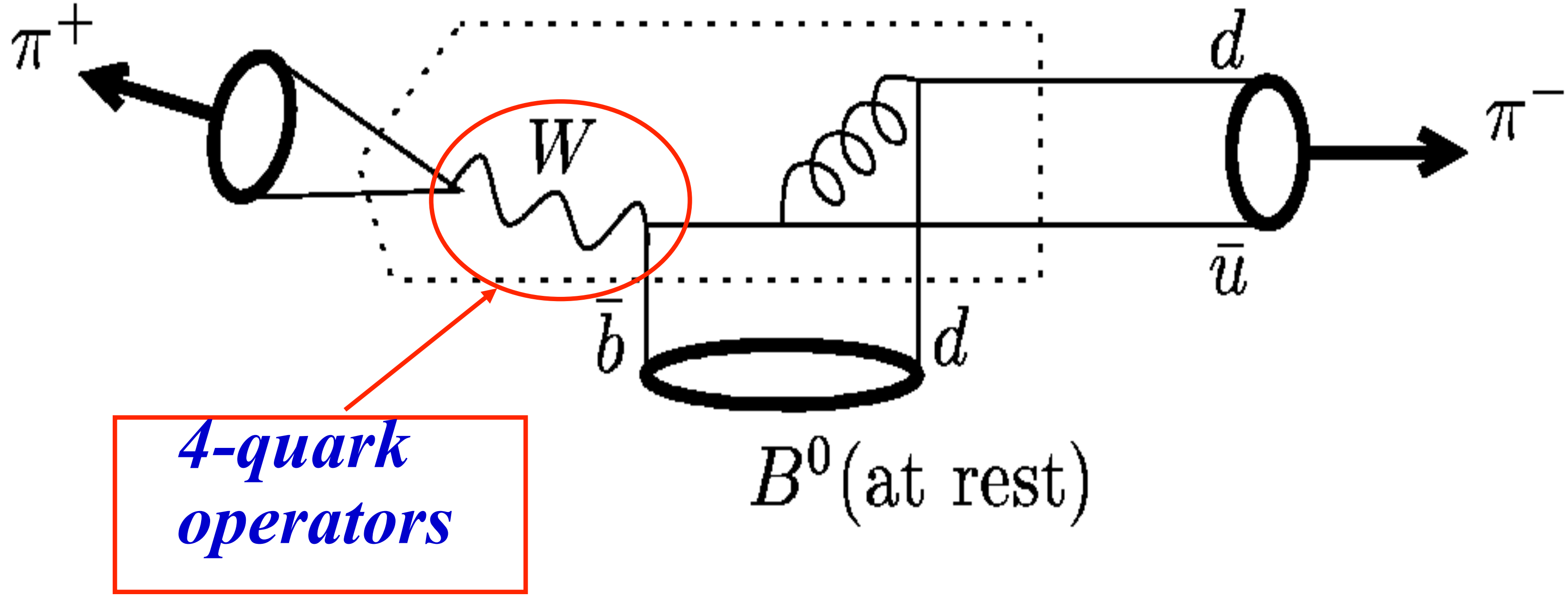
  — Direct CP asymmetry not predictable
- Final state interaction not predictable ✗

Picture of PQCD Approach



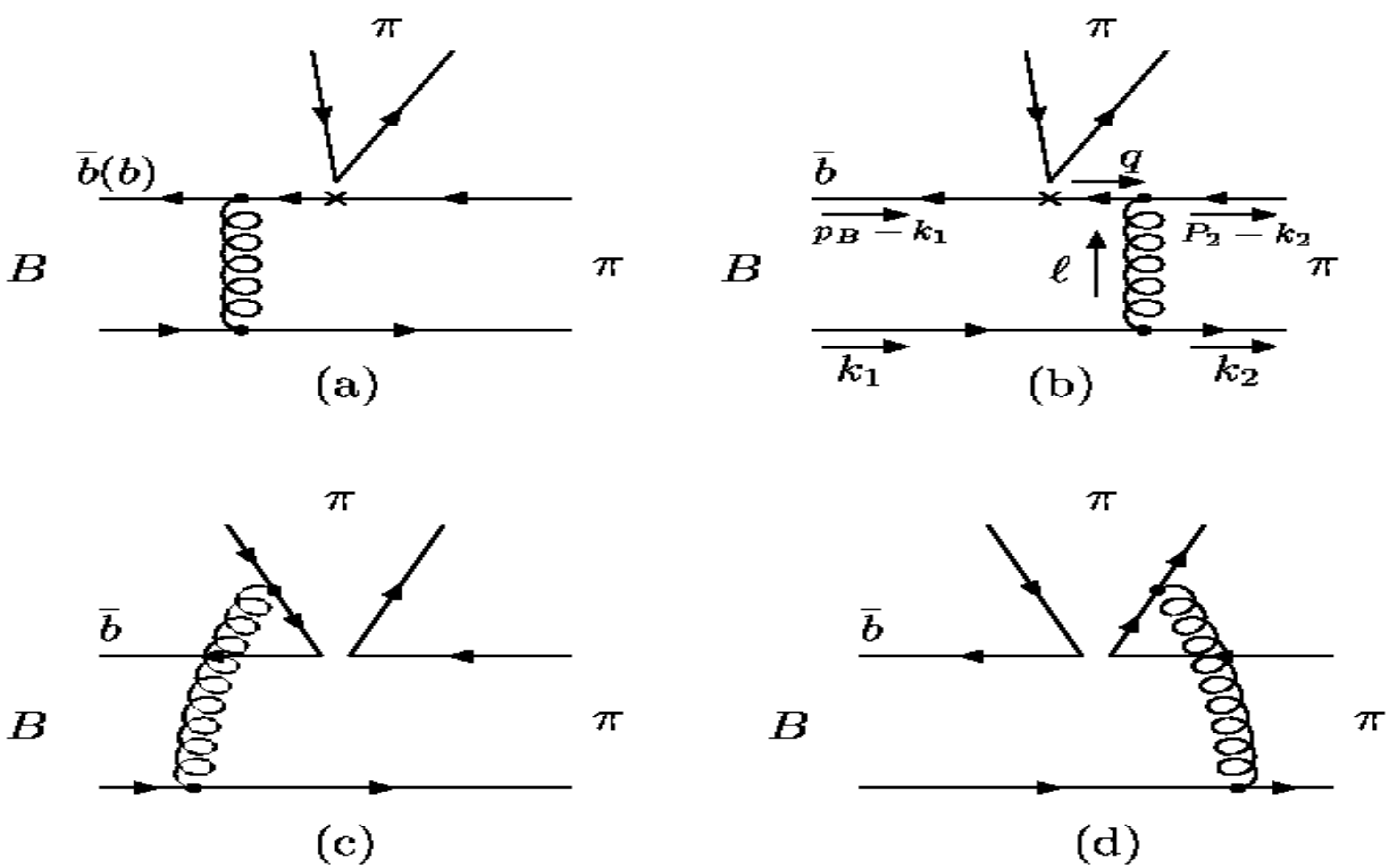
hard interaction inside the dotted line

Picture of PQCD Approach



Inside the dotted square, is the 6-quark interaction, which is perturbative calculable

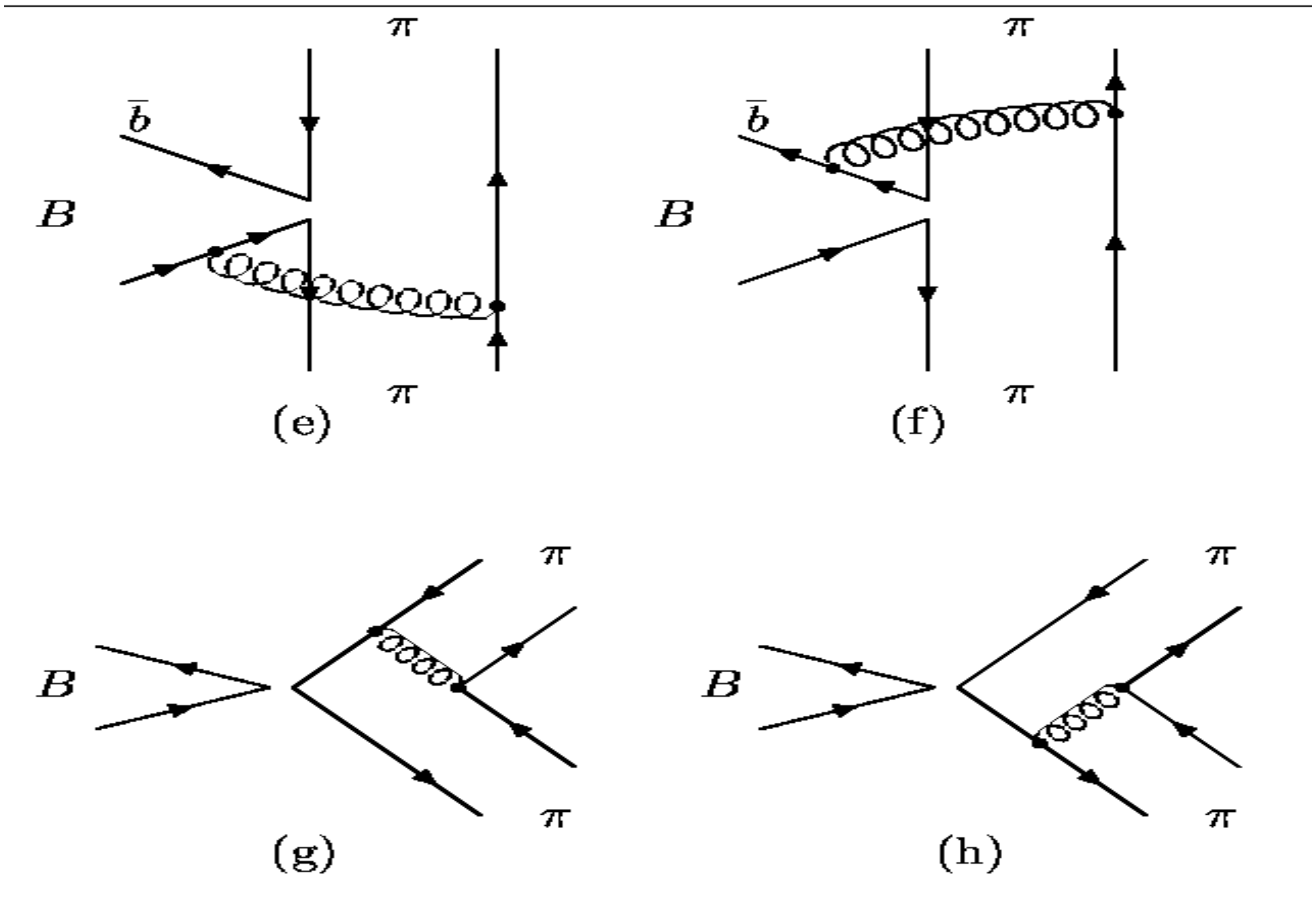
The leading order emission Feynman diagram in PQCD approach



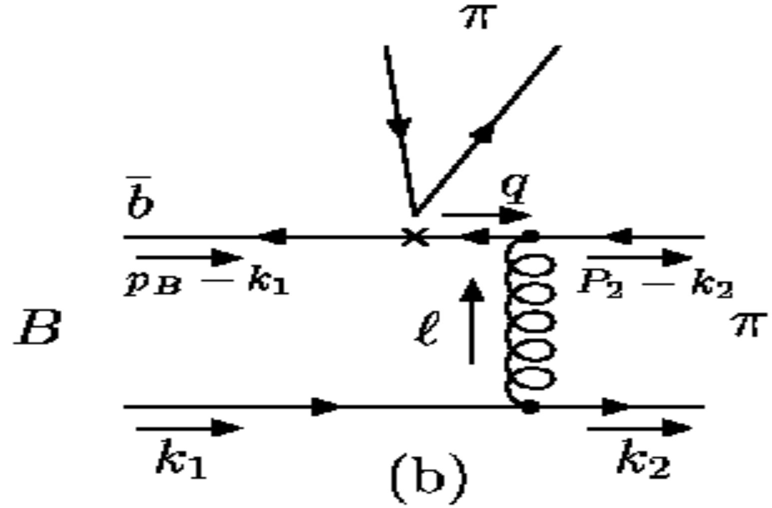
Form factor diagram

Hard scattering diagram

The leading order Annihilation type Feynman diagram in PQCD approach



Endpoint singularity



- **Gluon propagator**

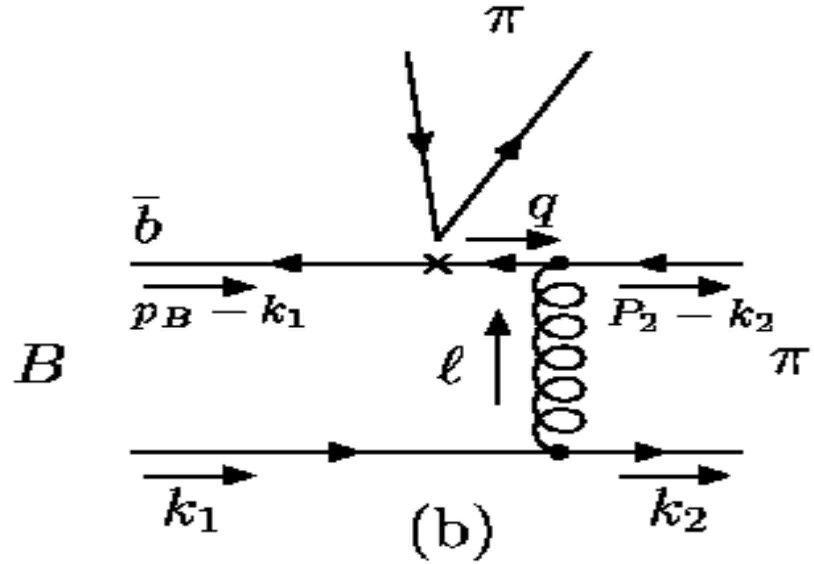
$$\frac{i}{(k_1-k_2)^2}=\frac{i}{-2xym_B^2}$$

- **x,y Integrate from 0→1, that is endpoint singularity**
- **The reason is that, one neglects the transverse momentum of quarks, which is not applicable at endpoint.**
- **If we pick back the transverse momentum, the divergence disappears**

$$\frac{i}{(k_1-k_2)^2}=\frac{i}{-2xym_B^2-(k_1^T-k_2^T)^2}$$

Endpoint singularity

- **It is similar for the quark propagator**

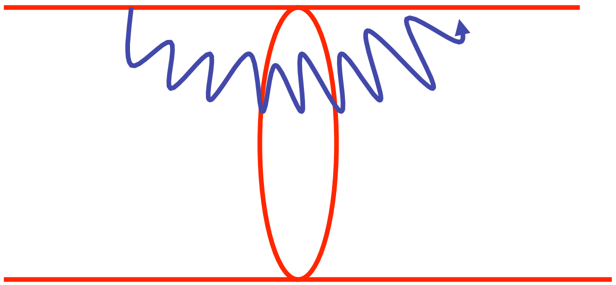


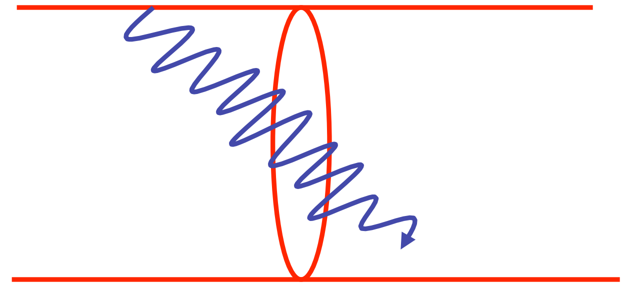
$$\int_0^1 \frac{1}{x} dx = \ln \frac{1}{\varepsilon}$$

$$\int_0^1 \frac{1}{x+k} dx dk = \int dk \left[\ln(x+k) \right]_0^1 = \int dk \left[\ln(1+k) - \ln k \right]$$

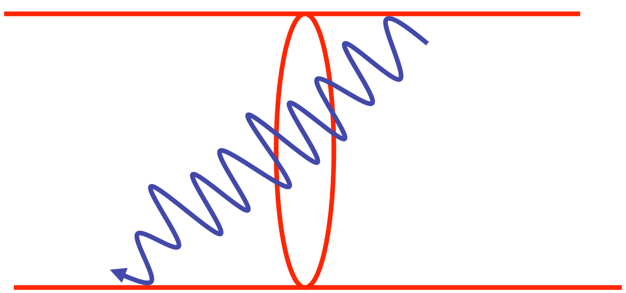
The logarithm divergence disappear if one has an extra dimension

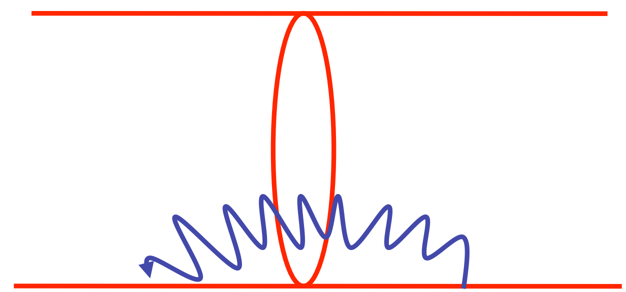
However, with transverse momentum, means one extra scale





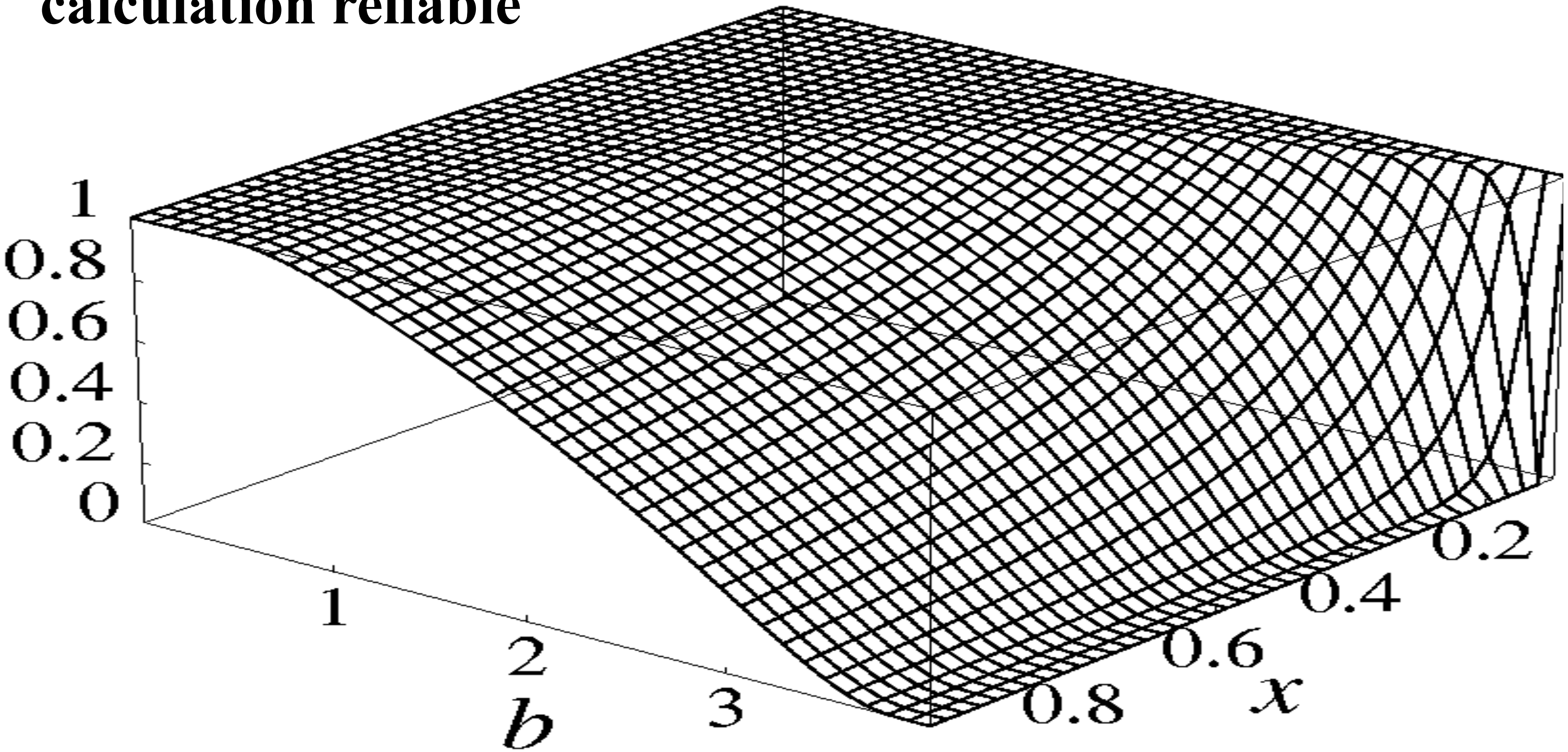
The overlap of Soft and collinear divergence will give double logarithm $ln^2 Pb$, which is too big to spoil the perturbative expansion. We have to use renormalization group equation to produce the so called Sudakov Form factor





Sudakov Form factor exp{–S(x,b)}

This factor exponentially suppresses the contribution at the endpoint (small k_T), makes our perturbative calculation reliable



Branching ratios of $B \to D^{(*)}\pi$ (x 10^{-4})

modes	Fac. A	PQCD	EXP.
$D^{*+}\pi^-$	29	25	27.6±2.1
$D^+\pi^-$	30	27	30±4
$D^{*0}\pi^0$	1.0	2.8	1.7±0.5
$D^0\pi^0$	0.7	2.5	2.9±0.5
$D^{*0}\pi^+$	48	53	46±4
$D^0\pi^+$	48	54	53±5

$a_1=1.08$

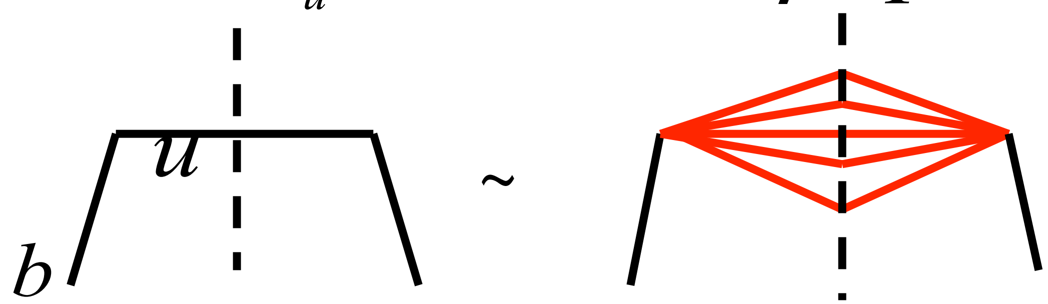
$a_2=0.21$

for Fact. Approach

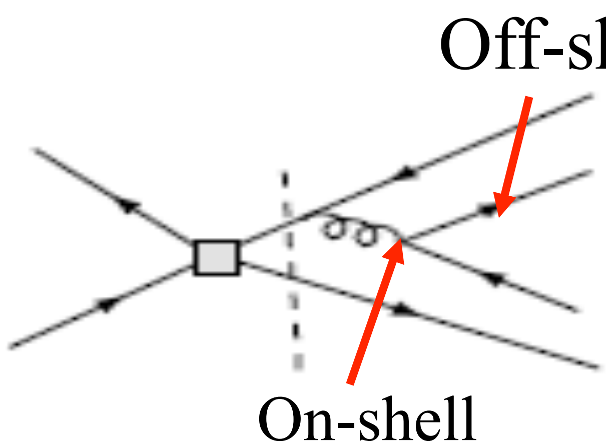
PQCD: Keum et al, PRD**69:094018,2004**

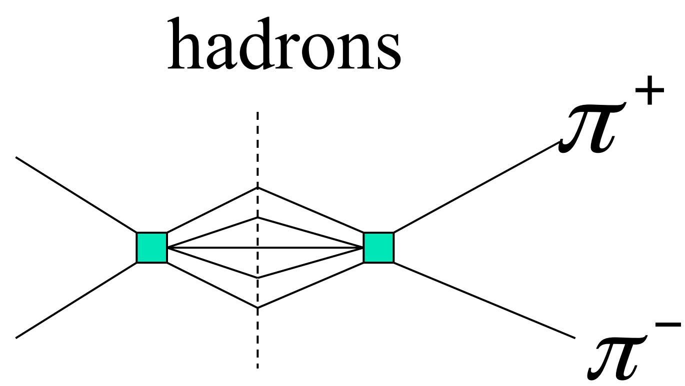
Inclusive Decay and B meson annihilation decay

$B \to X_u l\nu$ $\qquad\qquad$ π, ρ, p, n



Cut quark diagram ~ Sum over final-state hadrons





Large strong phase

CP Violation in $B \to \pi\pi(K)$ (real prediction before exp.)

CP(%)	FA	BBNS	PQCD (2001)	Exp (2004)
$\pi^+ K^-$	+9±3	+5±9	–17±5	–11.5±1.8
$\pi^0 K^+$	+8 ± 2	7 ±9	–13 ±4	+4 ± 4
$\pi^+ K^0$	1. 7± 0. 1	1 ±1	–1. 0±0.5	–2 ±4
$\pi^+ \pi^-$	–5±3	–6±12	+30±10	+37±10

CP Violation in $B \to \pi\pi(K)$

Including large annihilation fixed from exp.

CP(%)	FA	Cheng,HY	PQCD (2001)	Exp
π^+K^-	+9±3	-7.4 ± 5.0	–17±5	–9.7±1.2
π^0K^+	+8 ± 2	0.28±0.10	–13 ±4	4. 7 ± 2.6
π^+K^0	1. 7± 0. 1	4.9 ± 5.9	–1. 0±0.5	0.9 ±2.5
$\pi^+\pi^-$	–5±3	17 ± 1.3	+30±10	+38±7

Summary

- Hadronic B Decays are important in the test of standard model and search for signals of new physics.
- PQCD is useful in calculation of B decays to charmed mesons as well as for light mesons
- The annihilation type diagrams are very important in explaining the K pi puzzle and large direct CP asymmetry found in B decays
- Next-to-leading order perturbative calculations is needed to explain the more and more precise experimental data

Thanks