Beyond the Standard Model, including Neutrinos

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Outline

- General considerations on physics Beyond the SM (BSM)
	- The SM challenge to New Physics
	- Why Beyond
	- The SM as an effective theory
- Neutrino masses and BSM
	- High scale origin of neutrino masses
	- Low scale origin of neutrino masses
	- Neutrino properties and their implications for model building

The SM challenge to New Physics

but first, a boring preliminary: L and R fermions

A Dirac spinor is not "elementary"

$$
\Psi(x) = \begin{pmatrix} \Psi_1(x) \\ \Psi_2(x) \\ \Psi_3(x) \\ \Psi_4(x) \end{pmatrix} \rightarrow \begin{pmatrix} L^{\dagger - 1} \\ \dfrac{L^{\dagger - 1}}{L} \\ \dfrac{L^{\dagger - 2}}{L} \end{pmatrix} \begin{pmatrix} \Psi_1(x) \\ \Psi_2(x) \\ \Psi_3(x) \\ \Psi_4(x) \end{pmatrix} \qquad (\det L = 1)
$$
\n
$$
\Psi_R(x) = \begin{pmatrix} \Psi_1(x) \\ \Psi_2(x) \\ \Psi_2(x) \\ 0 \\ 0 \end{pmatrix} \qquad \Psi_L(x) = \begin{pmatrix} 0 \\ 0 \\ \Psi_3(x) \\ \Psi_4(x) \end{pmatrix} \qquad \Psi = \Psi_L + \Psi_R
$$

inequivalent, conjugated representations of SO(3,1)

Left- and Right-handed

Most general gauge transformation can mix all L

The SM fermions in a BSM perspective

• d_{iL} u_{iL} e_{iL} ν_{e_iL} $\overline{d_{iR}}$ $\overline{u_{iR}}$ $\overline{e_{iR}}$ + h.c.

- We will also denote the L-handed dofs as follows
	- ψ_L \rightarrow ψ (L-handed)
	- $\cdot \overline{\psi_{\mathsf{R}}} \rightarrow \psi^{\text{c}}$ (L-handed)

• d_i u_i e_i v_{e_i} d_i^c u_i^c e_i^c + h.c.

$$
\Psi = \begin{pmatrix} \epsilon \psi_c^* \\ \psi \end{pmatrix} \quad \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad \qquad \frac{\Psi_L \leftrightarrow \psi}{\Psi_R \leftrightarrow \psi^c}
$$

$$
\bullet \quad \Psi_L = \begin{pmatrix} 0 \\ \psi \end{pmatrix}, \overline{\Psi_R} = \begin{pmatrix} 0 \\ \epsilon \, \psi_c \end{pmatrix}
$$

$$
\begin{array}{ll}\n\bullet & \Psi_L \to \begin{pmatrix} 0 & \\ & L \end{pmatrix} \Psi_L \\
\hline\n\overline{\Psi}_R \to \begin{pmatrix} 0 & \\ & L^{T-1} \end{pmatrix} \overline{\Psi}_R\n\end{array}
$$

$$
\psi\to L\psi
$$

$$
\psi^c \to L \psi^c
$$

There exists a unique way of combining two elementary $\mathbf{\Theta}$ fermions in a Lorentz invariant way

$$
\psi_1\psi_2=\psi_2\psi_1=\psi_1^\alpha\epsilon_{\alpha\beta}\psi_2^\beta
$$

Most general (Lorentz) mass term and Yukawa with $\psi_1,...,\psi_n$

$$
\frac{m_{ij}}{2}\psi_i\psi_j + \text{h.c.}
$$

$$
\frac{\lambda_{ij}}{2}\psi_i\psi_j\phi^{(*)} + \text{h.c.}
$$

(only terms allowed by gauge invariance)

The SM challenge to New Physics

The SM challenge to New Physics

$$
\bar{\Psi}_{i}i\gamma^{\mu}D_{\mu}\Psi_{i} - \frac{1}{4}F_{\mu\nu}^{a}F^{a\mu\nu}
$$
gauge few %

$$
\mathcal{L}_{\rm SM}^{\rm ren} = +\lambda_{ij}\Psi_{i}\Psi_{j}H + \text{h.c.}
$$
flavor few %
+|D_{\mu}H|^{2} - V(H) symmetry breaking few 10%

Notable features

• Gauge sector

- chirality
- anomaly cancellation
- electroweak precision tests
- Flavour sector
	- U(3)⁵ and accidental symmetries
	- no tree level FCNC
	- anomalous suppression of loop FCNC
- Higgs sector
	- electric charge conserved
	- custodial symmetry
	- perturbative extrapolation

Gauge sector

 $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$

 $Q_i =$ $\int u_i$ d_i ◆ $L_i =$ $\bigl(\nu_i$ *ei* ◆

- The fermion content is "chiral": no L and R with same G_{SM} quantum numbers
- Equivalently: r irrep on $L \Rightarrow r^*$ is not
- Equivalently: no explicit (G_{SM} symmetric) fermion mass term is allowed
- Extra heavy fermions (M » $\langle H \rangle$) should be "vectorlike"
- A puzzle, a blessing, or what expected?

Another nice property

- Anomaly cancellation
- Is $T_{ijk} = Tr (T_i {T_i, T_k}) = 0$? $T_i = T_A, T_a, Y$

SU(3)³ vectorlike $SU(3)^2 \times SU(2)$ $SU(3)^2 \times U(1)$ $SU(3) \times (not \ SU(3))$ $SU(2)^2 \times U(1)$ $U(1)^3$ grav. anomaly 2*Y^l* + 6*Y^q* + 3*Yu^c* + 3*Yd^c* + *Ye^c* = 0

vectorlike
\n
$$
\text{Tr}(\sigma_a) = 0
$$
\n
$$
2Y_q + Y_{u^c} + Y_{d^c} = 0
$$
\n
$$
)^2 \quad \text{Tr}(\lambda_A) = 0
$$
\n
$$
Y_l + 3Y_q
$$
\n
$$
2Y_l^3 + 6Y_q^3 + 3Y_{u^c}^3 + 3Y_{d^c}^3 + Y_{e^c}^3 = 0
$$
\n
$$
2Y_l + 6Y_q + 3Y_{u^c} + 3Y_{d^c} + Y_{e^c} = 0
$$

(nice, but why??)

Electroweak precision tests (EWPT)

- Accuracy up to the $\%$ level \rightarrow sensitivity to 1loop corrections, which involve
	- g, g', v
	- m_t , $a_s(MZ)$, $\Delta a_{had}(MZ)$
	- \cdot m_h
- and bring together
	- the gauge sector: $g^2/(4\pi)^2$, $g'^2/(4\pi)^2$
	- the flavour sector: $\lambda^2/(4\pi)^2$
	- the EW-breaking sector: $g^2/(4\pi)^2 \log(m_h/M_W)$
- The agreement works because of the relatively low value of mh

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The flavour sector

gauge irreps (vertical) well understood

 $U(3)^5 \times U(1)_H$

Family replication \leftrightarrow the gauge lagrangian cannot tell families \leftrightarrow is U(3)⁵ invariant:

$$
L_i \to U_{ij}^L L_j
$$

\n
$$
e_i^c \to U_{ij}^e e_j^c
$$

\n
$$
U(3)^5 : Q_i \to U_{ij}^Q Q_j \to \mathcal{L}_{\text{SM}}^{\text{gauge}} \to \mathcal{L}_{\text{SM}}^{\text{gauge}}
$$

\n
$$
u_i^c \to U_{ij}^{u^c} u_j^c
$$

\n
$$
d_i^c \to U_{ij}^{d^c} d_j^c
$$

also $U(1): H \to e^{i\alpha} H \Rightarrow \mathcal{L}_{\text{SM}}^{\text{EWSB}} \to \mathcal{L}_{\text{SM}}^{\text{EWSB}}$

U(3)5

 $\psi_i^\dagger i \sigma^\mu D_\mu \psi_i - \frac{1}{4}$ $F^a_{\mu\nu}F^{a\mu\nu}$ gauge $\mathcal{L}_{\rm SM}^{\rm ren} = +\lambda_{ij}\psi_i\psi_jH + \text{h.c.}$ flavor $+|D_{\mu}H|^2 - V(H)$ symmetry breaking

The flavour (Yukawa) lagrangian is is not $U(3)^5$ invariant (unless $\lambda_{ij}=0$)

 $U(3)^5: q_i \to U_{ij}^q q_j \Rightarrow \lambda_D \to U_{d^c}^T \lambda_D U_Q \qquad \mathcal{L}_{\rm SM}^{\rm SB} \to \mathcal{L}_{\rm SM}^{\rm SB}$ $l_i \rightarrow U^{l}_{ij} l_j$ $e^c_i \rightarrow U^{e^c}_{ij} e^c_j \qquad \lambda_E \rightarrow U^{T}_{e^c} \lambda_E U_L \quad \,\,\mathcal{L}^{\text{gauge}}_{\text{SM}} \rightarrow \mathcal{L}^{\text{gauge}}_{\text{SM}}$ $u_i^c \to U_{ij}^{u^c} u_j^c \longrightarrow \lambda_U \to U_{u^c}^T \lambda_U U_Q \longrightarrow \langle h \rangle \to \langle h \rangle$ $d_i^c \rightarrow U_{ij}^{d^c} d_j^c$

 $\mathcal{L}^{\text{flavor}}_{\text{SM}} = \lambda^E_{ij} e^c_i l_j H^\dagger + \lambda^D_{ij} d^c_i q_j H^\dagger + \lambda^U_{ij} u^c_i q_j H + \text{h.c.}$

Accidental symmetries (ren lagrangian)

The flavour lagrangian breaks $U(3)^5 \times U(1)_H$ to \circ $U(1)_e \times U(1)_\mu \times U(1)_\tau \times U(1)_B \times U(1)_Y$

$$
\lambda_{ij}^E e_i^c L_j H^\dagger \rightarrow \lambda_{e_i} e_i^{c'} L_i' H^\dagger
$$

\n
$$
\lambda_{ij}^D d_i^c Q_j H^\dagger \rightarrow \lambda_{d_i} d_i^{c'} Q_i' H^\dagger
$$

\n
$$
\lambda_{ij}^U u_i^c Q_j H \rightarrow \lambda_{u_i} V_{ij} u_i^{c'} Q_i' H
$$

 \bullet L_e L_μ L_τ individual lepton numbers (also L = L_e + L_μ + L_τ total) B Baryon number

- Welcome that they arise as accidental symmetries \circ
	- prediction of the SM, not by hand $\boldsymbol{\varnothing}$
	- broken by non perturbative effects $\boldsymbol{\vartheta}$
	- allows SM extensions in which they are broken (e.g. GUTs, see-saw) $\boldsymbol{\varnothing}$ (effects can be parameterised by non-renormalisable operators)
	- allows such extensions to be tested (e.g. nu masses, proton decay) $\boldsymbol{\varnothing}$
	- property not necessarily shared by SM extensions $\boldsymbol{\varnothing}$

No tree level FCNC

Fermion masses: $H =$ $\overline{1}$ ⇤ 0 $v +$ *h* $\overline{\sqrt{2}}$ ⇥ (unitarity gauge)

 $\mathcal{L}^{\text{flavor}}_{\text{SM}} = \lambda^E_{ij} e^c_i L_j H^\dagger + \lambda^D_{ij} d^c_i Q_j H^\dagger + \lambda^U_{ij} u^c_i Q_j H + \text{h.c.}$ $\begin{array}{lll} \displaystyle{=m_{ij}^E e_i^c e_j}\quad &\displaystyle{+m_{ij}^D d_i^c d_j}\quad &\displaystyle{+m_{ij}^U u_i^c u_j}\quad +\mathrm{h.c.}+\dots \end{array}$

In terms of mass eigenstates:

 $j_{\rm c, had}^{\mu} = \overline{u}_i \sigma^{\mu} d_i = V_{ij} \overline{u}_i^{\prime} \sigma^{\mu} d_j^{\prime}$ $j_{\rm n, had}^{\mu} = (j_{\rm n, had}^{\mu})^{\prime}$ $j_{em,\mathrm{had}}^{\mu} = (j_{em,\mathrm{had}}^{\mu})^{\prime}$

 $V = U_u U_d^\intercal \Big]$ Cabibbo Kobayashi Maskawa (CKM) matrix

Anomalously small loop-induced FCNC

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 $\times \frac{g^4}{(4\pi)^2}$

 M_W^2

Expect:

- K^0 $\overline{K^0}$ oscillations
- Instead: 106 smaller $\boldsymbol{\vartheta}$
- Because of peculiar flavour structure of the SM, or $\boldsymbol{\sigma}$ approximate U(2)5 symmetry of SM lagrangian
- Challenge for new physics at TeV scale Ø

Approximate flavour symmetry

- $\sqrt{2}$ ⇥ The flavour lagrangian is approximately $\mathsf{U}(2)^5$ 00 0 $\boldsymbol{\varnothing}$ flavour symmetric (exactly symmetric in the limit $\lambda=$ 00 0 ⇤ ⌅ $0 \quad 0 \quad \lambda_{33}$ which also implies $V = 1₃$)
- This (or equivalently the smallness of $\lambda_{1,2}$ and V_{ij} i≠j) is the $\boldsymbol{\mathcal{O}}$ origin of the anomalously small FCNC processes in the SM (and the origin of the flavour problem)

Anomalously small loop-induced FCNC

~

Because of the approximate $U(2)^5$ $\boldsymbol{\varnothing}$

$$
\frac{1}{M_W^2} \times \frac{g^4}{(4\pi)^2} \times \epsilon
$$
\n
$$
\epsilon = 0 \quad \text{in the U(2)}^5 \text{ limit}
$$
\n
$$
\epsilon \sim 10^{-6} \quad \text{experiment}
$$

$$
\bullet \quad \left(\epsilon = (V_{su_i}^{\dagger} V_{u_i d})(V_{su_j}^{\dagger} V_{u_j d}) f\left(\frac{m_{u_i}^2}{M_W^2}, \frac{m_{u_j}^2}{M_W^2}\right) \right)
$$
\n
$$
i = 3: f = O(1), \ |V_{td}V_{ts}| \ll 1
$$
\n
$$
i = 1, 2: \ |V_{td}V_{is}| = O(1), \ f \ll 1 \ \Big)
$$

allenge for new physics at TeV

Same for CP-violating effects $\boldsymbol{\vartheta}$

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- electroweak precision tests

• Flavour sector

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• Higgs sector

- electric charge conserved
- custodial symmetry
- perturbative extrapolation

The Higgs sector

Most general gauge invariant ren. lagrangian for H:

 $\mathcal{L}_H = (D_\mu H)^{\dagger} (D^\mu H) - V (H^{\dagger} H)$ $V(H^{\dagger}H) = \mu^{2}H^{\dagger}H + \lambda_{H}(H^{\dagger}H)^{2}$

 Ω λ H > 0

- φ μ^2 < 0 \Rightarrow <H> \neq 0 \Rightarrow electroweak symmetry breaking
- ϕ ($\mu^2 > 0 \Rightarrow$ still electroweak symmetry breaking, but at $\Lambda \approx m_{\pi}$)

QED unbroken

Fix the Higgs quantum numbers from fermion masses. Then the electric charge is automatically conserved

$$
\langle H \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}, v > 0, v^2 = \frac{|\mu^2|}{2\lambda_H} \approx (174 \,\text{GeV})^2 \qquad m_H^2 = 4 \,\lambda_H (v^2) \, v^2
$$

$$
T = aY + b_aT_a
$$
, *a*, *b*_a real, $T_a = \frac{\sigma}{2}$, $Y = \frac{1}{2}$

$$
0 = T \langle H \rangle = \frac{v}{2} \left(\frac{b_1 - ib_2}{a - b_3} \right) \Rightarrow T \propto Q
$$

3 broken generators \leftrightarrow 3 massive vectors \leftrightarrow 3 unphysical \circ Goldstone bosons \leftrightarrow 1 real physical Higgs particle

Custodial symmetry

$$
\bullet \quad \rho \equiv \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = 1 \text{ (tree level)}
$$

Not guaranteed by gauge invariance or breaking pattern

Peculiar of EW breaking by a doublet (dominant contribution from triplets ruled out)

 \circ Exact in the limit $g' = 0$, $\lambda_U = \lambda_D$

$\rho \approx 1 \leftrightarrow$ (approximate) custodial SU(2)

 $\rho = 1$ if in the g' = 0 limit W^{1,2,3} have equal mass

- \bullet I.e. if a SO(3) ≈ SU(2) symmetry rotates the real fields W^{1,2,3}
- The custodial symmetry in the Higgs sector: the Higgs lagrangian is accidentally SO(4) symmetric, as $|H|^2 = h_{1R}^2 + h_{1I}^2 + h_{2R}^2 + h_{2I}^2$ SO(4) is spontaneously broken to SO(3) by $\langle h_{2R} \rangle \neq 0$
- The custodial symmetry in the fermion sector: $SO(4) \approx SU(2)_{L} \times SU(2)_{R}$, where $SU(2)_{R}$ acts on the righthanded fields
- The symmetry is exact in the limit g' = 0, λ_U = λ_D \rightarrow loop $\boldsymbol{\varnothing}$ corrections to $\rho = 1$

Light Higgs + SM couplings = perturbativity

- $A(W_LW_L \rightarrow W_LW_L) = \Sigma_l$ a_l A_l, a_l = partial wave amplitude
- Unitarity bound: $|a_0| \leq 1$ $\boldsymbol{\sigma}$
- *s* Tree level, no Higgs: $a_0 \sim \frac{3}{16\pi n^2}$, s = (p₁+p₂)², v ≈ 174 GeV \bullet $16\pi v^2$

- Unitarity bound saturated at $s \approx (1.2 \text{ TeV})^2$ $\boldsymbol{\vartheta}$
- Bad behaviour of a_0 due to the longitudinal part of the W \bullet propagator ∼ p_⊬p_v/(M_W)², cancelled by Higgs exchange

(perturbative extrapolation possible up to M_{Pl})

Buttazzo et al

(perturbative extrapolation possible up to M_{PI})

Why Beyond?

Some reasons to go beyond the SM

- Experimental "problems" of the SM
	- Gravity
	- Dark matter
	- Baryon asymmetry
	- Neutrino masses
- Experimental "hints" of physics beyond the SM
	- Quantum number unification
- Theoretical puzzles of the SM
	- $\langle H \rangle \ll M_{\rm Pl}$
	- Family replication
	- Small Yukawa couplings, pattern of masses and mixings
	- Gauge group, no anomaly, charge quantization, quantum numbers
- Theoretical problems of the SM
	- Higgs mass naturalness problem
	- Cosmological constant problem
	- Strong CP problem
	- Landau poles

Experimental "problems" of the SM

- Gravity
- Dark matter
- Baryon asymmetry
- Neutrino masses

Experimental "hints" of physics beyond the SM

• Quantum number unification

p-decay bounds: $M \gg m_H$

an accident?

Theoretical puzzles of the SM

- \bullet $\langle H \rangle \triangleleft M_{\text{Pl}}$
- Family replication
- Small Yukawa couplings, masses and mixings

Theoretical problems of the SM

- Landau poles
- Strong CP problem

 $\theta G_{\mu\nu}\tilde{G}^{\mu\nu} \quad D=4$

- Naturalness problem
- Cosmological constant problem

 $\alpha Q_{\text{max}}^2 H^{\dagger} H$ $D = 2$

$$
\beta Q_{\text{max}}^4 \sqrt{g} \quad D = 0
$$

General approach to physics beyond the EW scale: The SM as an effective theory

Reminder

$$
\bullet \quad S = \int d^4x \, \mathcal{L}(x) \text{ basic object, } [\mathcal{L}] = 4, \ \mathcal{L}(x) = \sum_i c_i \mathcal{O}_i(x)
$$

Dimension D of O(x): Ø

renormalizable renormalizable
theory

 $D = 0$ constant terms, not physical without gravity D = 1 linear terms, reabsorbed by shifting fields (SSB) $D = 2,3$ "relevant" operators [c] = M², M D = 4 "marginal operators" [c] = g D ≥ 5 "irrelevant operators" [c] = 1/MD-4

effective iective
neory

The SM as an effective theory

Effective theories

• $\mathcal{L}(\phi, \Phi)$ ϕ light, Φ heavy

• At scales $E \times \Lambda \sim$ mass of Φ

$$
\mathcal{L}_{\textrm{eff}}(\phi) = \mathcal{L}(\phi,\Phi(\phi)) = \mathcal{L}_{\textrm{ren}}(\phi) + \sum_n \frac{c_n}{\Lambda^n} \mathcal{O}_{4+n}(\phi)
$$

• Only depends on light dofs

The SM as a effective theory

$$
\mathcal{L}_{E\ll\Lambda}^{\text{eff}} = \mathcal{L}_{\text{SM}}^{\text{ren}} + \sum_{n} \frac{c_n}{\Lambda^n} \,\mathcal{O}_{4+n}
$$

- Consistent renormalisation at each order in (E/Λ)
- Low E effects suppressed by (E/Λ)ⁿ
- Allows a general parameterisation of the low scale indirect effects of any heavy new physics in terms of light fields only
- The identification of O_{4+n} would provide a crucial handle on the underlying physics (example: from Fermi theory to SM)
- No clear hint of O_{4+n} from the TeV scale (but hints from higher scales)
- The best chance for indirect NP effects to emerge is if they violate the "accidental symmetries" symmetries of $\mathcal{L}^{\text{ren}}_{\text{SM}}$, also called: L_e, L_u, L_τ B
- NP effects can also emerge if suppressed in $\mathcal{L}^{\text{ren}}_{\text{SM}}$, e.g. if they contribute to
	- Flavour Changing Neutral Current (FCNC) processes
	- CP-violating (CPV) processes
	- Electroweak precision tests (EWPT)

The SM effective lagrangian contains only a single dimension 5 operator, which happens to violate lepton number: the "Weinberg operator"

$$
\mathcal{L}_{E\ll\Lambda}^{\text{eff}} = \mathcal{L}_{\text{SM}}^{\text{ren}} + \frac{c_{ij}}{2\Lambda} (h l_i)(h l_j) + \dots
$$

 $(h l) \equiv \epsilon_{rs} h_r l_s$

Neutrino masses from the Weinberg operator

$$
\mathcal{L}_{E\ll\Lambda}^{\text{eff}} = \mathcal{L}_{\text{SM}}^{\text{ren}} + \frac{c_{ij}}{2\Lambda} (h l_i)(h l_j) + \dots
$$

$$
m_{ij}^{E,D,U} = \lambda_{ij}^{E,D,U} v \quad m_{ij}^{\nu} = c_{ij} v \times \frac{v}{\Lambda}
$$
 (Majorana)

$$
\Lambda \sim 0.5 \times 10^{15} \, {\rm GeV} \, c \left(\frac{0.05 \, {\rm eV}}{m_\nu}\right)
$$

Baryon number violating operators

- Arise at the D=6 level (SUSY: D=5)
- With one family, only 4 possible operators:

 G_1 qqql + G_2 d^cd^cucec + G_3 ucecq^{*}q^{*} + G_4 dcucq^{*}l^{*}

- They all (accidentally) conserve B-L (B-breaking L-conserving at D=9)
- Induce p-decay
- $\cdot \Gamma(p \rightarrow e\pi) = f(G_1, G_2, G_3, G_4)$
- $|G_i| \le (10^{15} \text{ GeV})^{-2}$

Bounds on NR terms

\n- ■ B number e.g.
$$
\frac{c}{\Lambda^2}qqq
$$
 (proton decay) $\Lambda > c^{1/2} 10^{15} \text{ GeV}$ $\begin{bmatrix} \frac{\overline{c}}{4} & \frac{\overline{c}}{2} &$

