NLO calculations

- NLO calculation requires consideration of all diagrams that have an extra factor of α_s
  - real radiation, as we have just discussed
  - virtual diagrams (with loops)
- For virtual diagram, have to integrate over loop momentum
  - but result contains IR singularities (soft and collinear), just as found for tree-level diagrams

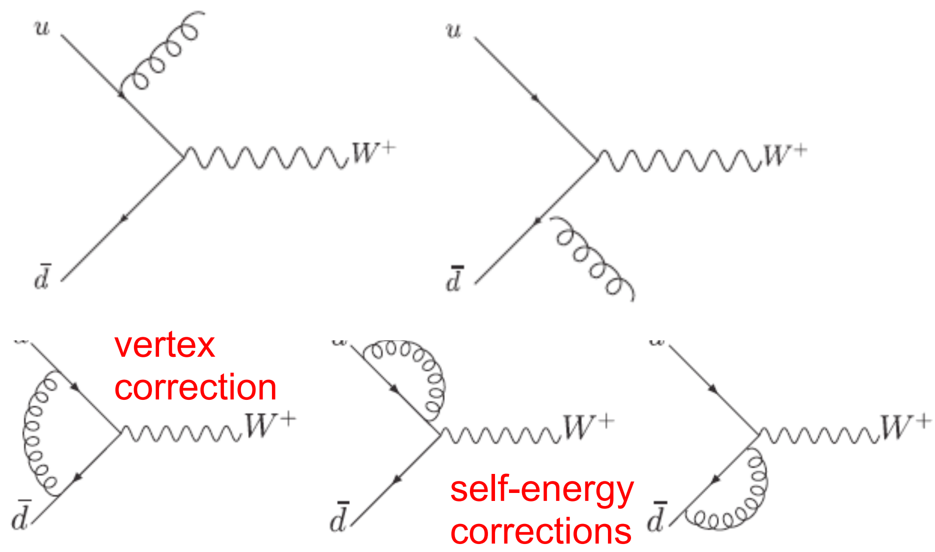
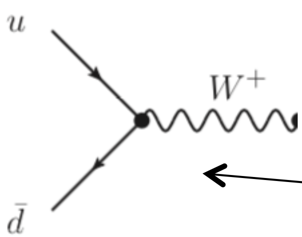


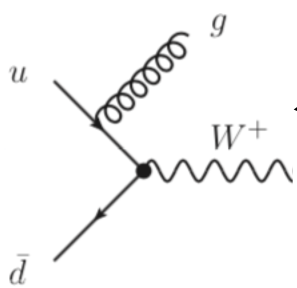
Figure 14. Virtual diagrams included in the next-to-leading order corrections to the Drell–Yan production of a W at hadron colliders.

$O(\alpha_s)$ virtual corrections in NLO cross section arise from interference between tree level and one-loop virtual amplitudes

If we add the real+virtual contributions, we find that the singularities will cancel, for inclusive cross sections. This is shown explicitly for W production in the extra slides. We have to be more clever for differential distributions. See extra slides for more detail.

Observables and orders





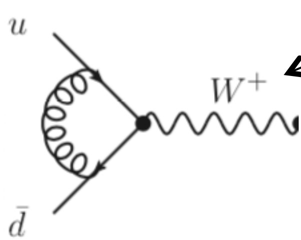
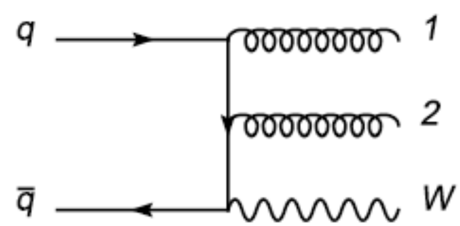


Table 3.1 The order at which various observables related to W production are computed, as a function of the overall power of the strong coupling in the theoretical calculation.

strong coupling order	$\sigma_{\text{tot}},\ d\sigma/dy$	$d\sigma/dp_T(W)$	$d\sigma/d\phi_{jj}$
α_s^0	LO	-	-
α_s^1	NLO	LO	-
α_s^2	NNLO	NLO	LO
α_s^3	N^{3}LO	NNLO	NLO



suppose I want to know $d\sigma/d\phi_{jj}$ to NLO; then I would need the 1 loop correction to the diagram on the left, and the W+3 jets real correction; both of order α_s^3

Power series

Cross section is a power series in α_s

$$d\sigma = \sum_{n=n_0}^{\infty} \alpha_s^n f_n(\ldots)$$

W production

strong coupling order	σ_{tot}, $d\sigma/dy$
α_s^0	LO
α_s^1	NLO
α_s^2	NNLO
α_s^3	N^3LO

For perturbation theory to work, need $\alpha_s \equiv g_s^2/4\pi \ll 1$

Each vertex has g_s in amplitude->g_s^2 is proportional to α_s in cross section

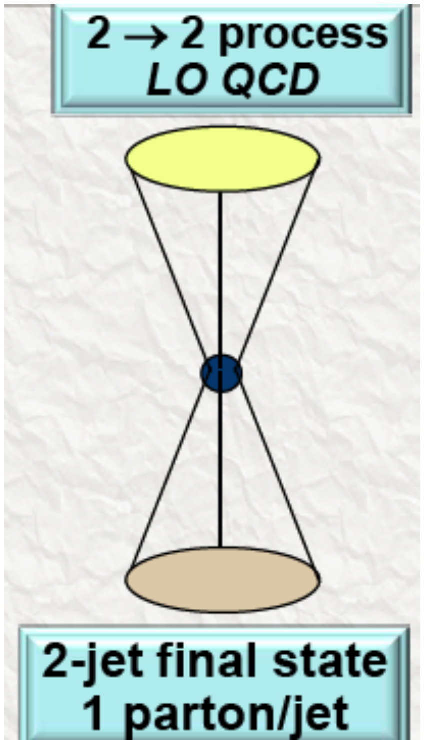
Higher orders->more vertices->more diagrams (n!)->f_n becomes more difficult to calculate

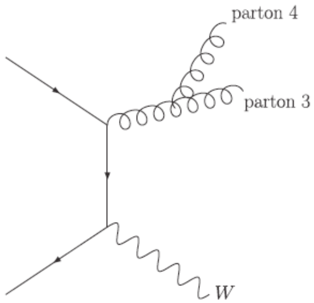
But if $\alpha_s \ll 1$, can truncate series (LO, NLO, NNLO,…)

For W production, NNLO corrections are reasonably small; not true for Higgs production, for example

Brief interlude: jet definitions and algorithms

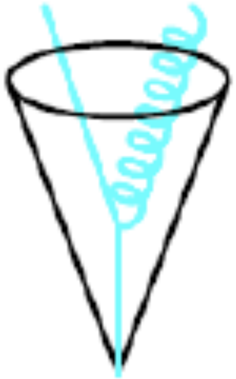
- At (fixed) LO, 1 parton = 1 jet
  - why not more than 1? I have to put a ΔR cut on the separation between two partons; otherwise, there's a collinear divergence. LO parton shower programs effectively put in such a cutoff





$$\log\left(\frac{1}{\Delta R_{34}}\right)$$

- But at NLO, I have to deal with more than 1 parton in a jet, and so now I have to talk about how to cluster those partons
  - i.e. jet algorithms

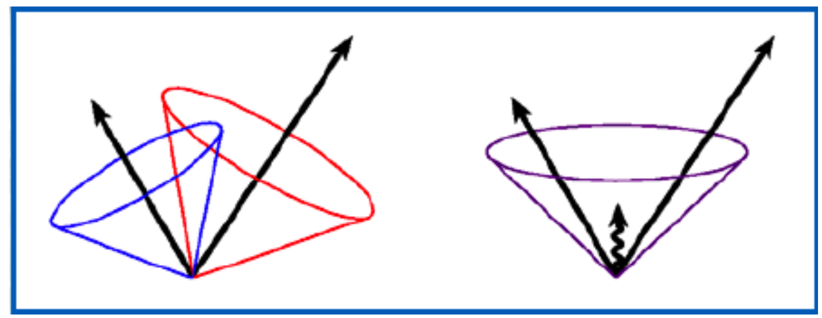


Jet algorithms at NLO

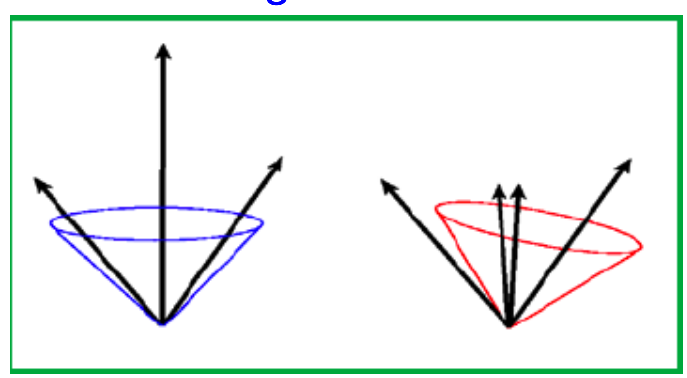
- At NLO (NNLO), there can be two(three) partons in a jet, life becomes more interesting and we have to start talking about jet algorithms to define jets
  - we will see that the addition of the extra parton(s) and virtual terms will cancel the divergence mentioned on the previous slide
- A jet algorithm is based on some measure of localization of the expected collinear spray of particles
- Start with an inclusive list of partons (fixed order), particles (PS shower Monte Carlos, and data)
- End with lists of same for each jet
- …and a list of particles… not in any jet; for example, remnants of the initial hadrons
- Two broad classes of jet algorithms
  - cluster according to proximity in space: cone algorithms ← used almost exclusively at the Tevatron
  - cluster according to proximity in momenta: k_T algorithms ← used almost exclusively at the LHC

What do I want out of a jet algorithm?

- It should be fully specified, including defining in detail any pre-clustering, merging and splitting issues
- It should be simple to implement in an experimental analysis, and should be independent of the structure of the detector
- It should be boost-invariant
- It should be simple to implement in a theoretical calculation
  - it should be defined at any order in perturbation theory
  - it should yield a finite cross section at any order in perturbation theory
  - it should yield a cross section that is relatively insensitive to hadronization effects
- It should be IR safe, i.e. adding a soft gluon should not change the results of the jet clustering

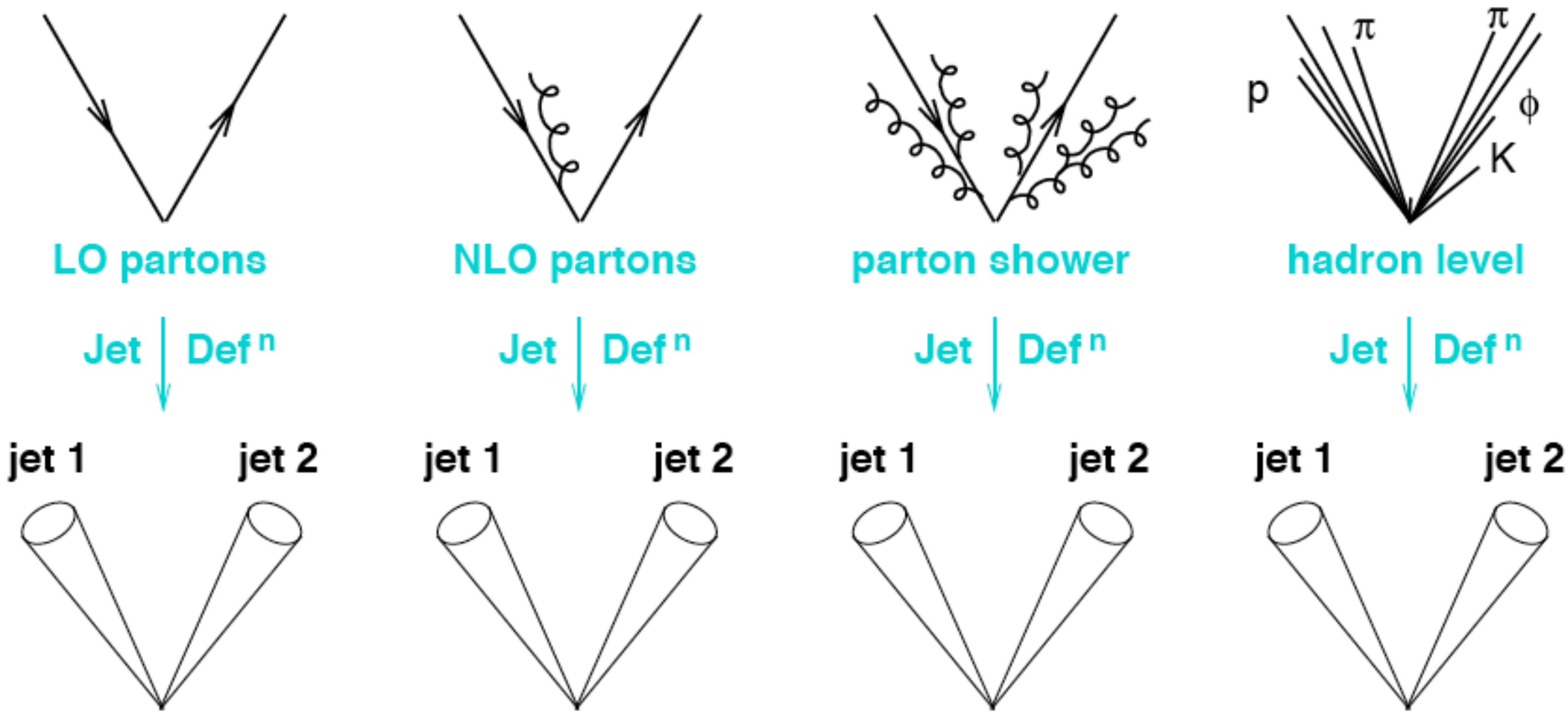


- It should be collinear safe, i.e. splitting one parton into two collinear partons should not change the results of the jet clustering



Jet algorithms

- The algorithm should behave in a similar manner (as much as possible) at the parton, particle and detector levels. Note that differences between levels can unavoidably creep in.



Projection to jets should be resilient to QCD effects

The k_T family of jet algorithms

- p=1
  - the regular k_T jet algorithm
- p=0
  - Cambridge-Aachen algorithm
- p=-1
  - anti-k_T jet algorithm
  - Cacciari, Salam, Soyez '08
  - also P-A Delsart '07 (reverse k_T)
  - soft particles will first cluster with hard particles before clustering among themselves
  - no split/merge
  - leads mostly to constant area hard jets

d=distance measure

$$d_{ij} = \min\left(p_{T,i}^{2p}, p_{T,j}^{2p}\right)\frac{\Delta R_{ij}^2}{D^2}$$

$$d_{ii} = p_{T,i}^{2p}$$

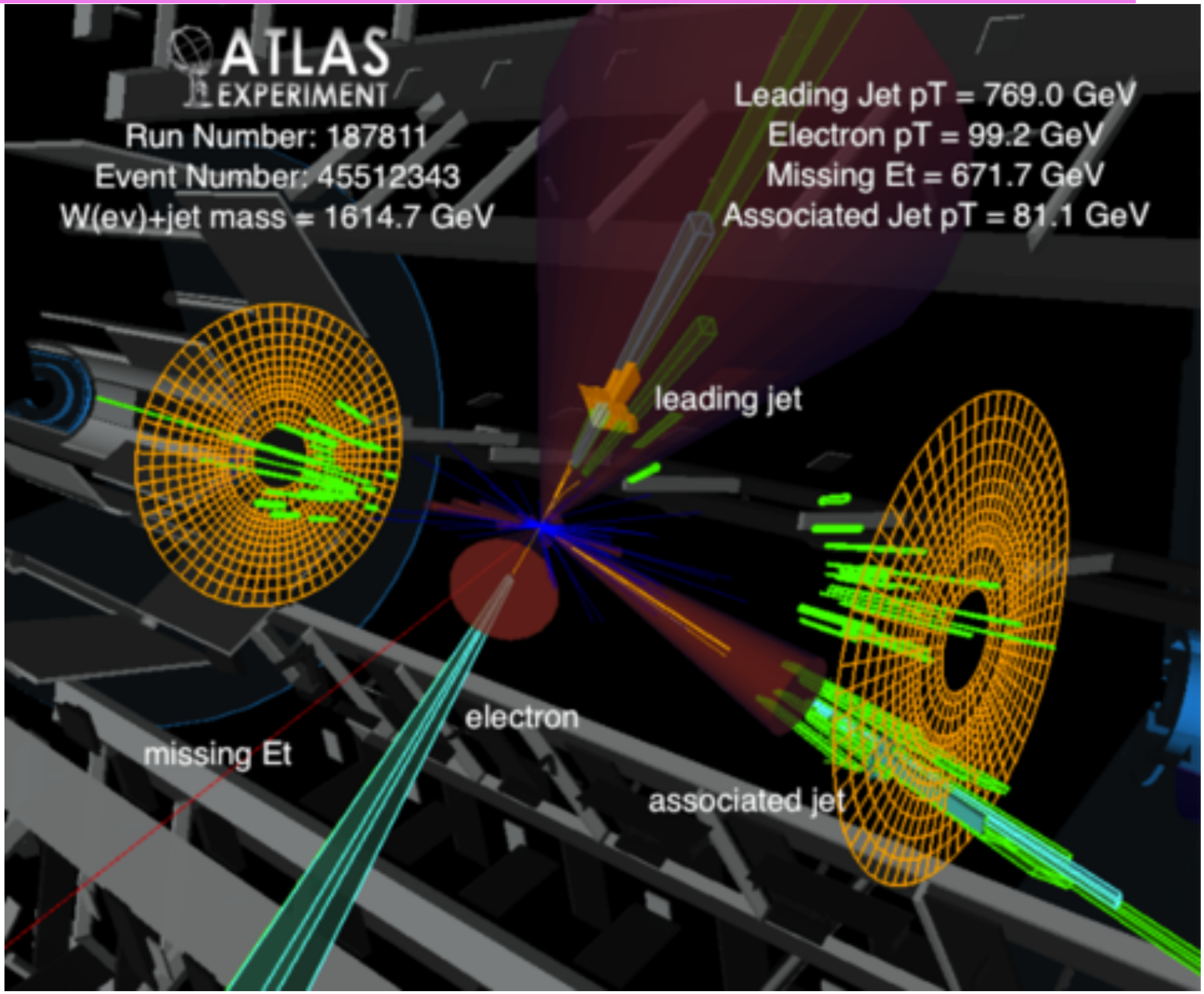
size of jet in Δy-$\Delta\phi$ space

- #1 algorithm for ATLAS, CMS
- Cambridge-Aachen used in some circumstances, e.g. large R jets

ATLAS W + 2 jet event

.with the W boson decaying into an electron and a neutrino

...and the 2 jets defined with the antikT algorithm with R=0.4



Scale choices

- We know that we have two scales, μ_R and μ_F
- We know that they should be associated with the relevant scale in the hard scattering process
    - sometime this scale is evident, like m_W for W production, p_T^{jet} for inclusive jet production
    - but what if I have a process like W+jet(s)
        - there I have both m_W and p_T^{jet}, and these scales can be very different->very different answers
        - for some cases, general scales like H_T may work best
- Often μ_R and μ_F are taken equal to each other, but the physics associated with each is a bit different, so they can be varied separately…as long as the ratio between the two scales is not too large (>2)
- For then, we would introduce a new log into the calculation, the log of the ratio of the two scales
- These logarithms would then have to be re-summed to restore precision to the measurement
- We don’t want to have to do that

sum of transverse momenta of all objects in event

Scale uncertainties

- We try to estimate the uncertainty due to uncalculated higher order terms by varying μ_R,μ_F over some range, typically a factor of 2
- This is normally the best we can do, but we have to keep in mind that higher order corrections can arise from a number of other sources such as Sudakov effects, large color factors, large π^2 terms, the opening of new channels
- These contributions are not estimated by the variation of the scale logarithms and can be larger than the variation

What does the scale dependence for a cross section look like?

- Here, we're specifically looking at inclusive jet production, but this holds for other collider processes
- Write cross section indicating explicit scale-dependent terms for NLO
- First term (lowest order) in (3) leads to monotonically decreasing behavior as scale increases (the LO piece)
- Second term is negative for $\mu < p_T$, positive for $\mu > p_T$
- Third term is negative for factorization scale $M < p_T$
- Fourth term has same dependence as lowest order term
- Thus, lines one and four give contributions which decrease monotonically with increasing scale while lines two and three start out negative, reach zero when the scales are equal to p_T, and are positive for larger scales
- At NLO, result is a roughly parabolic behavior

Consider a large transverse momentum process such as the single jet inclusive cross section involving only massless partons. Furthermore, in order to simplify the notation, suppose that the transverse momentum is sufficiently large that only the quark distributions need be considered. In the following, a sum over quark flavors is implied. Schematically, one can write the lowest order cross section as

$$E\frac{d^3\sigma}{dp^3} \equiv \sigma = a^2(\mu)\,\hat{\sigma}_B \otimes q(M) \otimes q(M) \tag{1}$$

where $a(\mu) = \alpha_s(\mu)/2\pi$ and the lowest order parton-parton scattering cross section is denoted by $\hat{\sigma}_B$. The renormalization and factorization scales are denoted by μ *and* M, respectively. In addition, various overall factors have been absorbed into the definition of $\hat{\sigma}_B$. The symbol $\otimes$ denotes a convolution defined as

$$f \otimes g = \int_x^1 \frac{dy}{y} f(\frac{x}{y})\, g(y). \tag{2}$$

When one calculates the $\mathcal{O}(\alpha_s^3)$ contributions to the inclusive cross section, the result can be written as

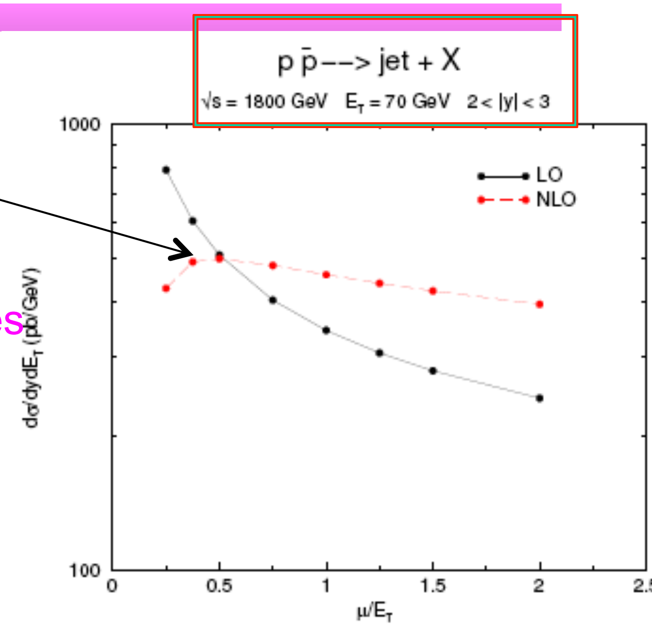
$$\begin{aligned}
&(1) \quad \sigma = a^2(\mu)\,\hat{\sigma}_B \otimes q(M) \otimes q(M)\\
&(2) \quad\quad + 2a^3(\mu)\, b \ln(\mu/p_T)\hat{\sigma}_B \otimes q(M) \otimes q(M)\\
&(3) \quad\quad + 2a^3(\mu)\, \ln(p_T/M) P_{qq} \otimes \hat{\sigma}_B \otimes q(M) \otimes q(M)\\
&(4) \quad\quad + a^3(\mu)\, K \otimes q(M) \otimes q(M).
\end{aligned} \tag{3}$$

In writing Eq. (3), specific logarithms associated with the running coupling and the scale dependence of the parton distributions have been explicitly displayed; the remaining higher order corrections have been collected in the function K in the last line of Eq. (3). The μ

Why does the scale dependence have the shape it does?

- Write cross section indicating explicit scale-dependent terms
- First term (lowest order) in (3) leads to monotonically decreasing behavior as scale increases (the LO piece)
- Second term is negative for $\mu<p_T$, positive for $\mu>p_T$
- Third term is negative for factorization scale $M < p_T$
- Fourth term has same dependence as lowest order term
- Thus, lines one and four give contributions which decrease monotonically with increasing scale while lines two and three start out negative, reach zero when the scales are equal to p_T, and are positive for larger scales
- At NLO, result is a roughly parabolic behavior

Note that NLO=LO for a scale of about $p_T/2$; for other scales NLO>LO, or NLO<LO



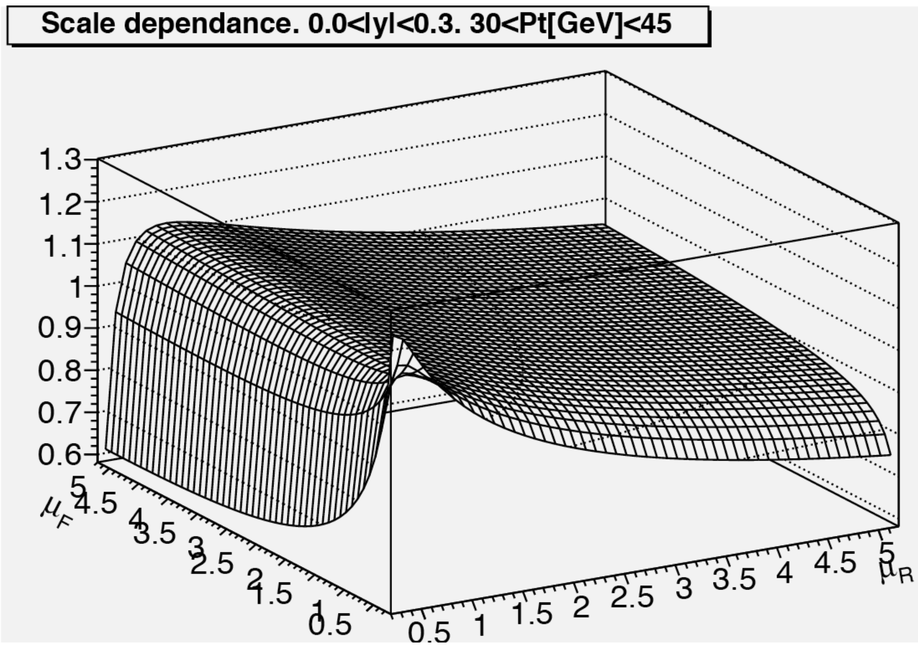
When one calculates the $\mathcal{O}(\alpha_s^3)$ contributions to the inclusive cross section, the result can be written as

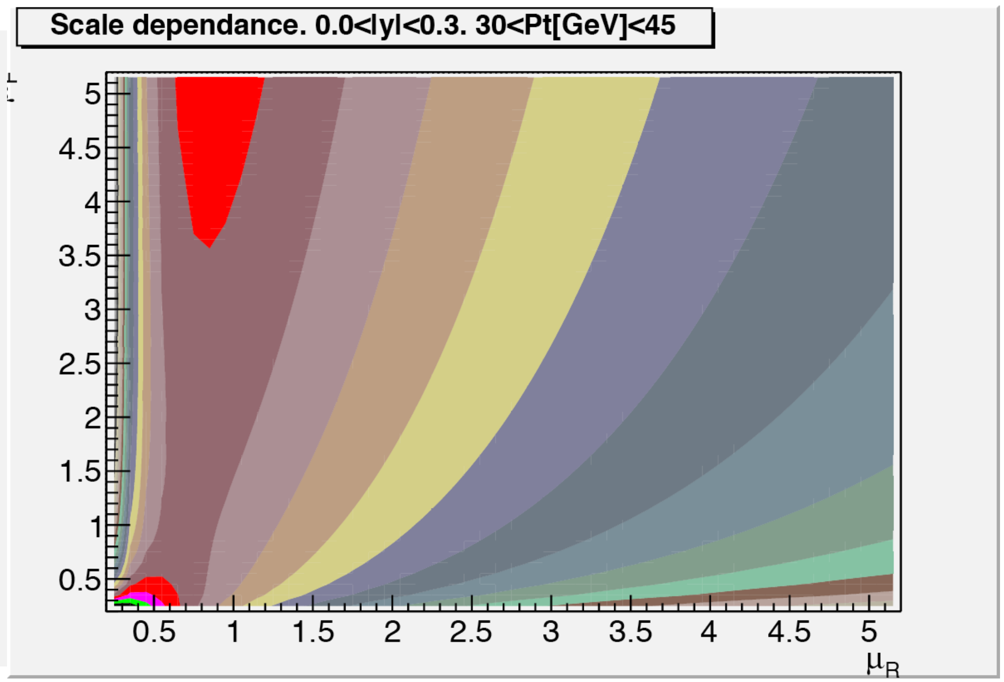
$$
\begin{aligned}
(1)\quad \sigma &= a^2(\mu)\,\hat{\sigma}_B \otimes q(M) \otimes q(M) \\
(2)\quad &+ 2a^3(\mu)\, b \ln(\mu/p_T)\hat{\sigma}_B \otimes q(M) \otimes q(M) \\
(3)\quad &+ 2a^3(\mu)\, \ln(p_T/M) P_{qq} \otimes \hat{\sigma}_B \otimes q(M) \otimes q(M) \\
(4)\quad &+ a^3(\mu)\, K \otimes q(M) \otimes q(M). \qquad (3)
\end{aligned}
$$

In writing Eq. (3), specific logarithms associated with the running coupling and the scale dependence of the parton distributions have been explicitly displayed; the remaining higher order corrections have been collected in the function K in the last line of Eq. (3). The μ

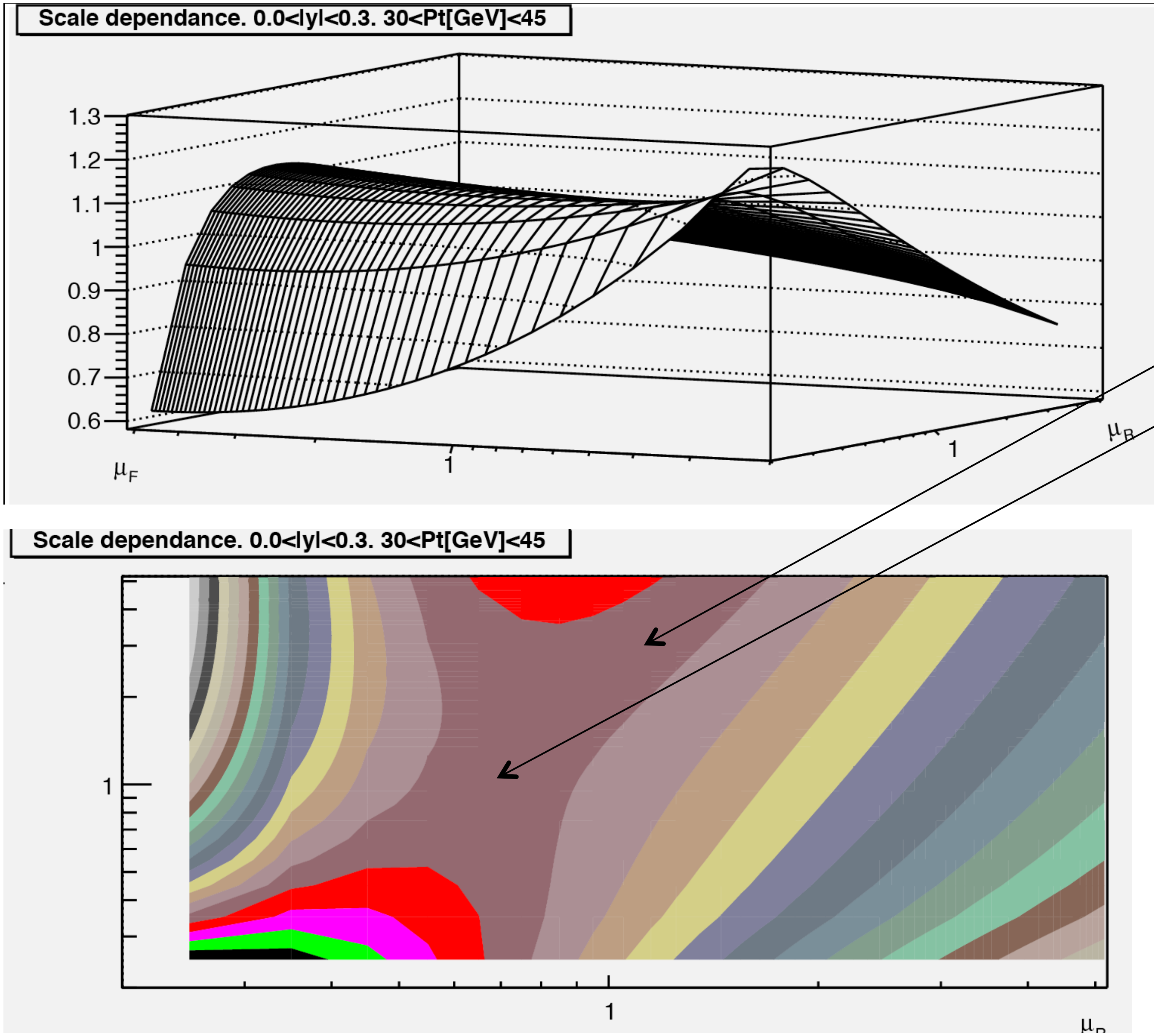
Look at scale dependence in 2-D

Jet production at the LHC



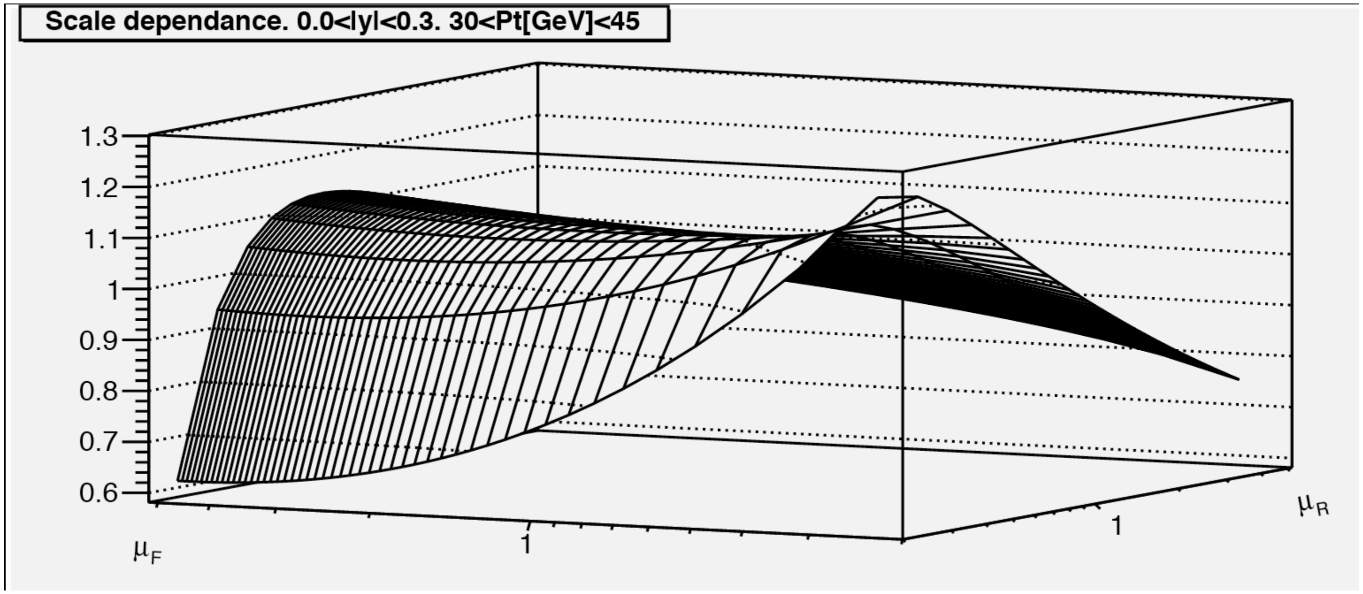


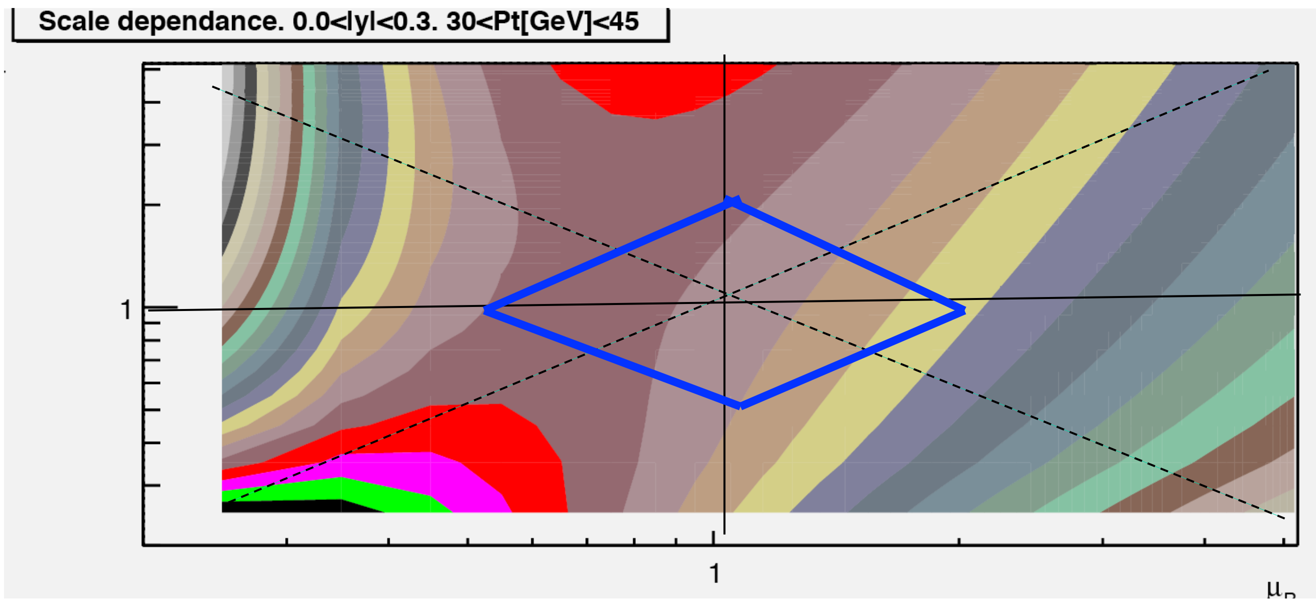
It’s also useful to use a log-log scale



- …since perturbative QCD is logarithmic
- Note that there’s a saddle region, and a saddle point, where locally there is little slope for the cross section with respect to the two scales
- This is kind of the ‘golden point’ and typically around the expected scale (p_T^{jet} in this case)

It's also useful to use a log-log scale



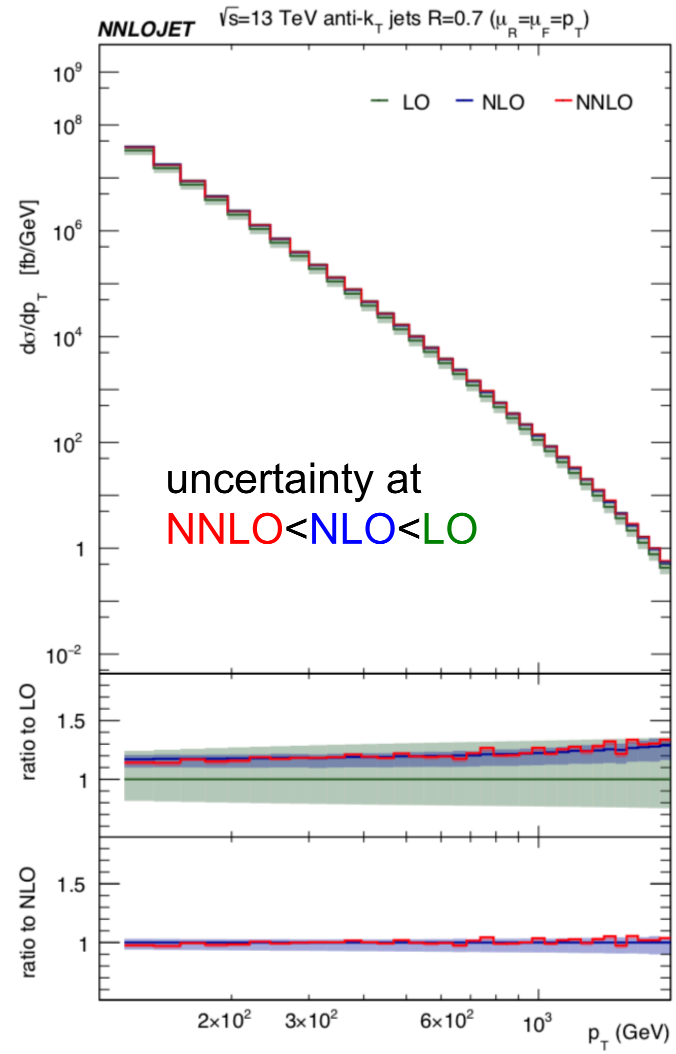


- Choose $p_T j^{et}$ as the central scale
- The scale variation represents an estimate of the uncalculated higher orders
- Typically vary both μ_R and μ_F up and down from their central values to estimate the scale uncertainty
- ...sometimes making sure that the ratio of the two scales is never larger than two, creating the diamond

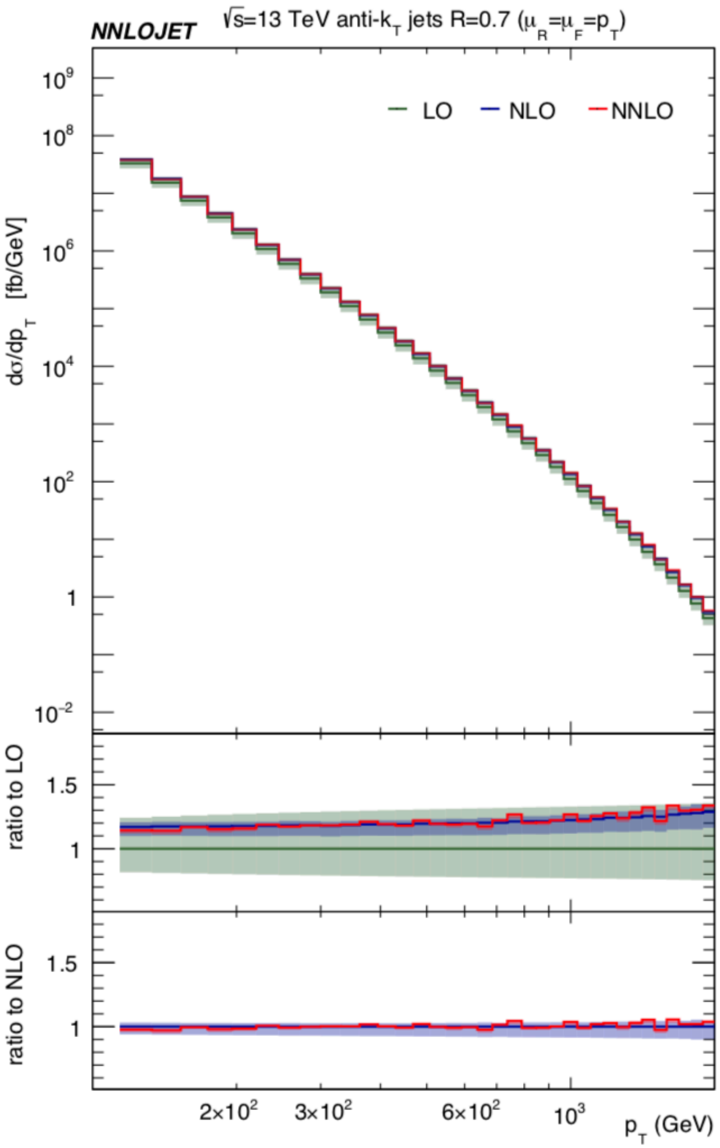
Advantages of higher orders

- Less sensitivity to unphysical input scales, i.e. renormalization and factorization scales
- NLO is first level of prediction where normalization (and sometimes shape) can be taken seriously
- At NNLO can take uncertainties more seriously
- More physics
  - parton merging gives structure in jets
  - more species of incoming partons

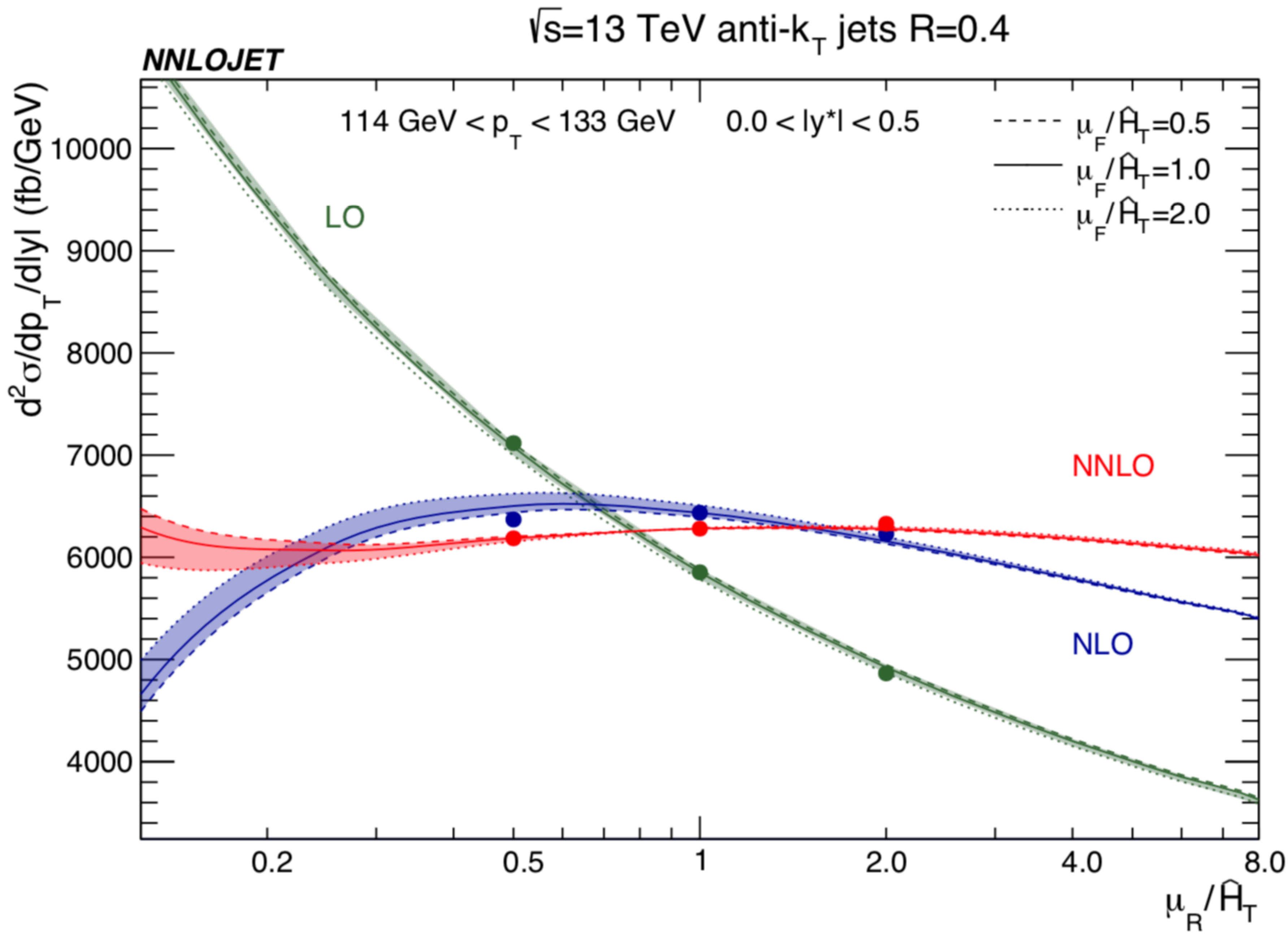
consider inclusive jet prod at LO, NLO, NNLO



More scale terms in NNLO expression



$$
\begin{aligned}
&\sigma(\mu_R,\mu_F,\alpha_s(\mu_R),L_R,L_F) = \\
&\Big(\frac{\alpha_s(\mu_R)}{2\pi}\Big)^2 \hat\sigma^{(0)}_{ij}\otimes f_i(\mu_F)\otimes f_j(\mu_F) \\
&+\Big(\frac{\alpha_s(\mu_R)}{2\pi}\Big)^3 \hat\sigma^{(1)}_{ij}\otimes f_i(\mu_F)\otimes f_j(\mu_F) \\
&\quad +L_R\Big(\frac{\alpha_s(\mu_R)}{2\pi}\Big)^3 2\beta_0\,\hat\sigma^{(0)}_{ij}\otimes f_i(\mu_F)\otimes f_j(\mu_F) \\
&\quad +L_F\Big(\frac{\alpha_s(\mu_R)}{2\pi}\Big)^3 \Big[-\hat\sigma^{(0)}_{ij}\otimes f_i(\mu_F)\otimes\Big(P^{(0)}_{jk}\otimes f_k(\mu_F)\Big) \\
&\qquad\qquad -\hat\sigma^{(0)}_{ij}\otimes\Big(P^{(0)}_{ik}\otimes f_k(\mu_F)\Big)\otimes f_j(\mu_F)\Big] \\
&+\Big(\frac{\alpha_s(\mu_R)}{2\pi}\Big)^4 \hat\sigma^{(2)}_{ij}\otimes f_i(\mu_F)\otimes f_j(\mu_F) \\
&\quad +L_R\Big(\frac{\alpha_s(\mu_R)}{2\pi}\Big)^4 \Big(3\,\beta_0\,\hat\sigma^{(1)}_{ij}+2\,\beta_1\,\hat\sigma^{(0)}_{ij}\Big)\otimes f_i(\mu_F)\otimes f_j(\mu_F) \\
&\quad +L_R^2\Big(\frac{\alpha_s(\mu_R)}{2\pi}\Big)^4 3\,\beta_0^2\,\hat\sigma^{(0)}_{ij}\otimes f_i(\mu_F)\otimes f_j(\mu_F) \\
&\quad +L_F\Big(\frac{\alpha_s(\mu_R)}{2\pi}\Big)^4 \Big[-\hat\sigma^{(1)}_{ij}\otimes f_i(\mu_F)\otimes\Big(P^{(0)}_{jk}\otimes f_k(\mu_F)\Big) \\
&\qquad\qquad -\hat\sigma^{(1)}_{ij}\otimes\Big(P^{(0)}_{ik}\otimes f_k(\mu_F)\Big)\otimes f_j(\mu_F) \\
&\qquad\qquad -\hat\sigma^{(0)}_{ij}\otimes f_i(\mu_F)\otimes\Big(P^{(1)}_{jk}\otimes f_k(\mu_F)\Big) \\
&\qquad\qquad -\hat\sigma^{(0)}_{ij}\otimes\Big(P^{(1)}_{ik}\otimes f_k(\mu_F)\Big)\otimes f_j(\mu_F)\Big] \\
&\quad +L_F^2\Big(\frac{\alpha_s(\mu_R)}{2\pi}\Big)^4 \Big[\hat\sigma^{(0)}_{ij}\otimes\Big(P^{(0)}_{ik}\otimes f_k(\mu_F)\Big)\otimes\Big(P^{(0)}_{jl}\otimes f_l(\mu_F)\Big) \\
&\qquad\qquad +\frac{1}{2}\hat\sigma^{(0)}_{ij}\otimes f_i(\mu_F)\otimes\Big(P^{(0)}_{jk}\otimes P^{(0)}_{kl}\otimes f_l(\mu_F \\
&\qquad\qquad +\frac{1}{2}\hat\sigma^{(0)}_{ij}\otimes\Big(P^{(0)}_{ik}\otimes P^{(0)}_{kl}\otimes f_l(\mu_F)\Big)\otimes f_j(\mu \\
&\qquad\qquad +\frac{1}{2}\beta_0\,\hat\sigma^{(0)}_{ij}\otimes f_i(\mu_F)\otimes\Big(P^{(0)}_{jk}\otimes f_k(\mu_F)\Big) \\
&\qquad\qquad +\frac{1}{2}\beta_0\,\hat\sigma^{(0)}_{ij}\otimes\Big(P^{(0)}_{ik}\otimes f_k(\mu_F)\Big)\otimes f_j(\mu_F)\Big] \\
&\quad +L_F L_R\Big(\frac{\alpha_s(\mu_R)}{2\pi}\Big)^4 \Big[-3\,\beta_0\,\hat\sigma^{(0)}_{ij}\otimes f_i(\mu_F)\otimes\Big(P^{(0)}_{jk}\otimes f_k(\mu_F)\Big) \\
&\qquad\qquad -3\,\beta_0\,\hat\sigma^{(0)}_{ij}\otimes\Big(P^{(0)}_{ik}\otimes f_k(\mu_F)\Big)\otimes f_j(\mu_F)
\end{aligned}
$$

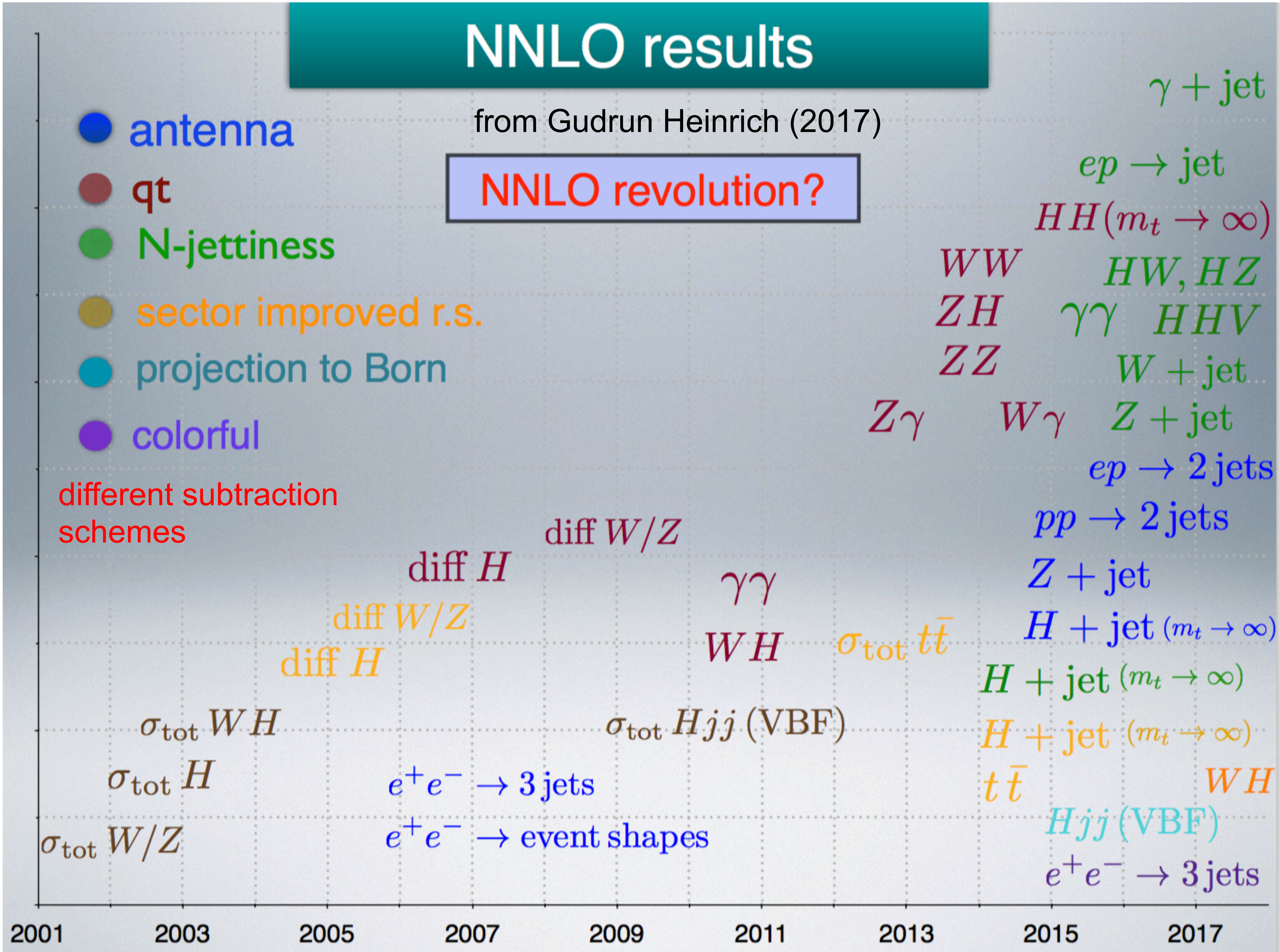


√s=13 TeV anti-k_T jets R=0.4
NNLOJET
114 GeV < p_T < 133 GeV
0.0 < |y*| < 0.5
μ_F/Ĥ_T=0.5
μ_F/Ĥ_T=1.0
μ_F/Ĥ_T=2.0
LO
NLO
NNLO
d²σ/dp_T/d|y| (fb/GeV)
μ_R/Ĥ_T
10000
9000
8000
7000
6000
5000
4000
0.2
0.5
1
2.0
4.0
8.0

State of the art for NLO/NNLO

Relative order	2->1	2->2	2->3	2->4	2->5	2->6
1	LO					
α_s	NLO	LO				
α_s^2	NNLO	NLO	LO			
α_s^3		NNLO	NLO	LO		
α_s^4			*NNLO*	NLO	LO	
α_s^5					NLO	LO
α_s^6						NLO

- LO: well under control, even for multiparticle final states
- NLO: well understood for 2->1, 2->2, 2->3, 2->4 (W/Z+3 jets, ttbb,WWbb,tttt,…); 2->5 (W+4 jets) and even 2->6 (W+5 jets)
- NNLO: we're in the middle of a NNLO revolution, with 2->2 processes having been calculated (W/Z/γ/Higgs+1 jet, dijet) with the frontier being 2->3 (W/Z/γ/Higgs+1 jets, 3 jets,etc) at NNLO


NNLO results
from Gudrun Heinrich (2017)
NNLO revolution?
antenna
qt
N-jettiness
sector improved r.s.
projection to Born
colorful
different subtraction schemes
$\gamma + \text{jet}$
$ep \to \text{jet}$
$HH(m_t \to \infty)$
WW
HW, HZ
ZH
$\gamma\gamma$
HHV
ZZ
$W + \text{jet}$
$Z\gamma$
$W\gamma$
$Z + \text{jet}$
$ep \to 2\,\text{jets}$
$pp \to 2\,\text{jets}$
diff W/Z
diff H
$\gamma\gamma$
$Z + \text{jet}$
diff W/Z
$H + \text{jet}\ (m_t \to \infty)$
WH
$\sigma_{\text{tot}}\ t\bar{t}$
diff H
$H + \text{jet}\ (m_t \to \infty)$
$\sigma_{\text{tot}}\ WH$
$\sigma_{\text{tot}}\ Hjj\ (\text{VBF})$
$H + \text{jet}\ (m_t \to \infty)$
$\sigma_{\text{tot}}\ H$
$e^+e^- \to 3\,\text{jets}$
$t\bar{t}$
WH
$Hjj\ (\text{VBF})$
$\sigma_{\text{tot}}\ W/Z$
$e^+e^- \to \text{event shapes}$
$e^+e^- \to 3\,\text{jets}$
2001
2003
2005
2007
2009
2011
2013
2015
2017

Size of higher order corrections

- Some rules-of-thumb
- NLO and (NNLO) corrections are larger for processes in which there is a great deal of color annihilation
  - gg->Higgs
  - gg->γγ
  - these gg initial states want to radiate like crazy
- NLO corrections decrease as more final-state legs are added (K=NLO/LO)
  - *K*(gg->Higgs + 2 jets) < *K*(gg->Higgs + 1 jet) < *K*(gg->Higgs)
  - unless can access new initial state gluon channel
- Can we generalize for uncalculated HO processes?

Casimir for biggest color representation final state can be in

Simplistic rule

$C_{i1} + C_{i2} - C_{f,max}$

L. Dixon

Casimir color factors for initial state

All-orders approaches

- Rather than systematically calculating to higher and higher orders in the perturbative expansion, can also use a number of all-orders approaches
- In resummation, dominant contributions from each order in perturbation theory are singled out and resummed by use of an evolution equation
- Near boundaries of phase space, fixed order calculations break down due to large logarithmic corrections, and these contributions can become important. Resummation takes them into account.

- Consider W production
  - one large logarithm associated with production of vector boson close to threshold
  - takes form of

$$\frac{\alpha_s^n \log^{2n-1}(1-z)}{1-z}$$

  - where

$$z = \frac{Q^2}{\hat{s}} - 1$$

  - other large logarithm is associated with recoil of vector boson at very small p_T
  - logarithms appear as $\alpha_s^n \log^{2n-1}(Q^2/p_T^2)$

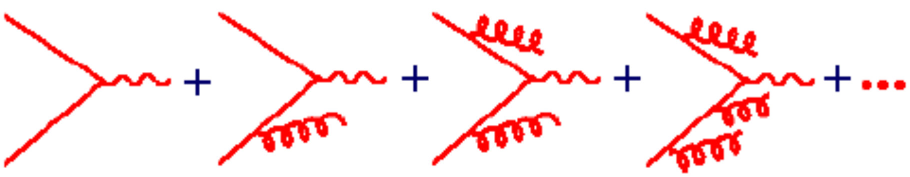
In both cases there is a restriction of phase space for gluon emission and thus the logs become large and are crucial for an accurate prediction

All-orders approaches

- See discussion in extra slides re: adding gluons on to the W + 1 jet process
  - each gluon added yields an additional factor of α_s and two new logarithms

$$d\sigma = \sigma_0(W + 1\ \text{jet})\left[1 + \alpha_S(c_{12}L^2 + c_{11}L + c_{10}) + \alpha_S{}^2(c_{24}L^4 + c_{23}L^3 + c_{22}L^2 + c_{21}L + c_{20}) + \cdots\right]$$

- q_T resummation is resumming the effects of logs of $Q^2/p_T{}^2$



- note that q_T resummation does not change the size of the cross section; it just modifies the p_T distribution of the W

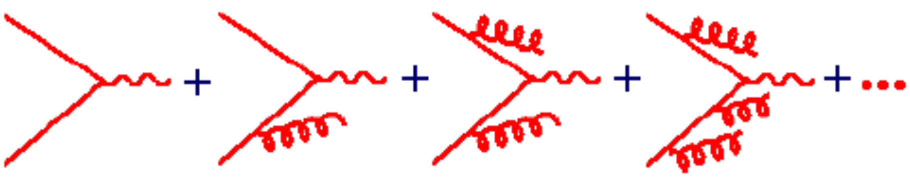
- Size of L depends on kinematic distribution/cuts being considered
- Coefficients c_{ij} depend on color factors
- Thus, addition of each gluon results in additional factor of α_s times logarithms
- In many (typically exclusive) cases, the logs can be large, leading to an enhanced probability for gluon emission to occur
- For most inclusive cases, logs are small and α_s counting may be valid estimator for production of additional jets
- For completely inclusive cross sections, the logs vanish

All-orders approaches

- See discussion in extra slides re: adding gluons on to the W + 1 jet process
  - each gluon added yields an additional factor of α_s and two new logarithms

$$d\sigma = \sigma_0(W+1\ \mathrm{jet})\left[1+\alpha_S(c_{12}L^2+c_{11}L+c_{10}) + \alpha_S{}^2(c_{24}L^4+c_{23}L^3+c_{22}L^2+c_{21}L+c_{20})+\cdots\right]$$

- q_T resummation is resumming the effects of logs of $Q^2/p_T{}^2$



- note that q_T resummation does not change the size of the cross section; it just modifies the p_T distribution of the W

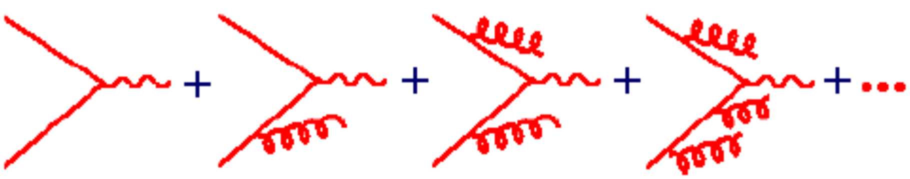
- These are the leading logs (LL) (highest power of log for each power of α_s)
- These are the next-to-leading logs (NLL) (next highest power of log…)
  - …and so on
- We know the structure of the LL's, NLL's, NNLL's
- But we don't know the c_{ij} factors until we do the finite order calculation
- LO gives us the LL
- NLO gives us the NLL
  - …and so on
- The accuracy of the resummation improves with the addition for further higher order information
- A resummation program like ResBos has NNLL accuracy

All-orders approaches

- Remember the expression we had after adding gluons on to the W + 1 jet process
  - each gluon added yields an additional factor of α_s and two new logarithms

$$d\sigma = \sigma_0(W + 1\ \mathrm{jet})\left[1 + \alpha_S(c_{12}L^2 + c_{11}L + c_{10}) + \alpha_S{}^2(c_{24}L^4 + c_{23}L^3 + c_{22}L^2 + c_{21}L + c_{20}) + \cdots\right]$$

- q_T resummation is resumming the effects of logs of $Q^2/p_T{}^2$



- note that q_T resummation does not change the size of the cross section; it just modifies the p_T distribution of the W

- Expression for W boson transverse momentum in which leading logarithms have been resummed to all orders is given by

$$\frac{d\sigma}{dp_T^2} = \sigma \frac{d}{dp_T^2} \exp\left(-\frac{\alpha_S C_F}{2\pi} \log^2 M_W^2/p_T^2\right)$$

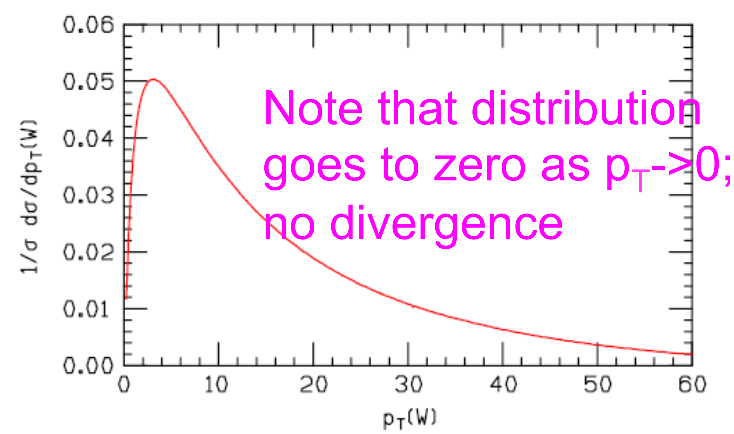


Figure 20. The resummed (leading log) *W* boson transverse momentum distribution.

You could get the same predictions by using PDFs in which the transverse momentum (k_T) has not been integrated out

p_T distributions

- If we look at average transverse momentum of Drell-Yan pairs as a function of mass, we see that there is an increase that is roughly logarithmic with the mass
  - as expected from the logs that we saw accompanying soft gluon emission
- If we look at the average transverse momentum of Drell-Yan pairs as a function of center-of-mass energy, there is an increase that is roughly logarithmic with the center of mass energy
  - as we expect from the logs resulting from the increase in phase space for gluon emission as the center of mass energy grows

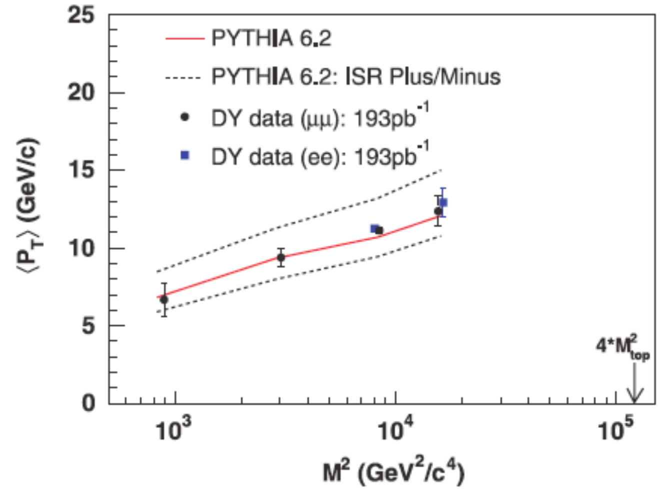
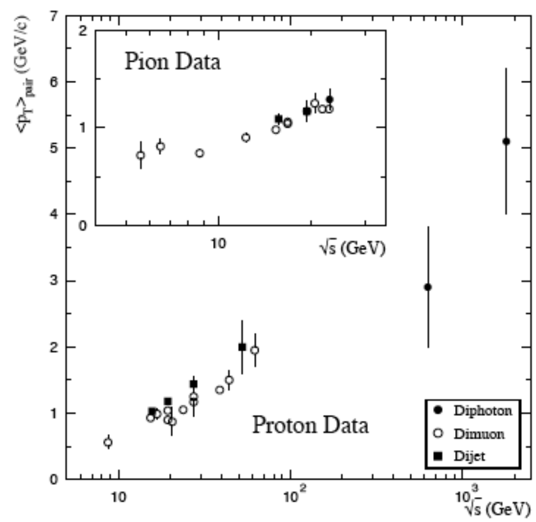


Figure 42. The average transverse momentum for Drell–Yan pairs from CDF in Run 2, along with comparisons to predictions from PYTHIA.

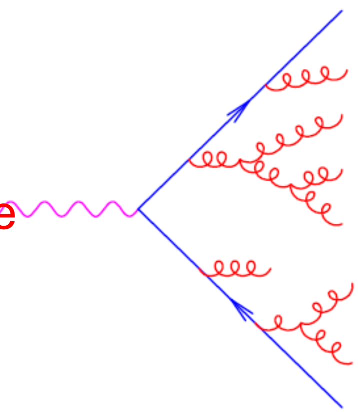


Parton showers

- A different, but related approach for re-summing logarithms, is provided by parton showering
- By the use of the parton showering process, a few partons produced in a hard interaction at a high-energy scale can be related to partons at an energy scale close to Λ_{QCD}.
- At this lower energy scale, a universal non-perturbative model can then be used to provide the transition to hadrons
- Parton showering allows for evolution, using DGLAP formalism, of parton fragmentation function

Parton Cascade

…plus
similar for
initial state

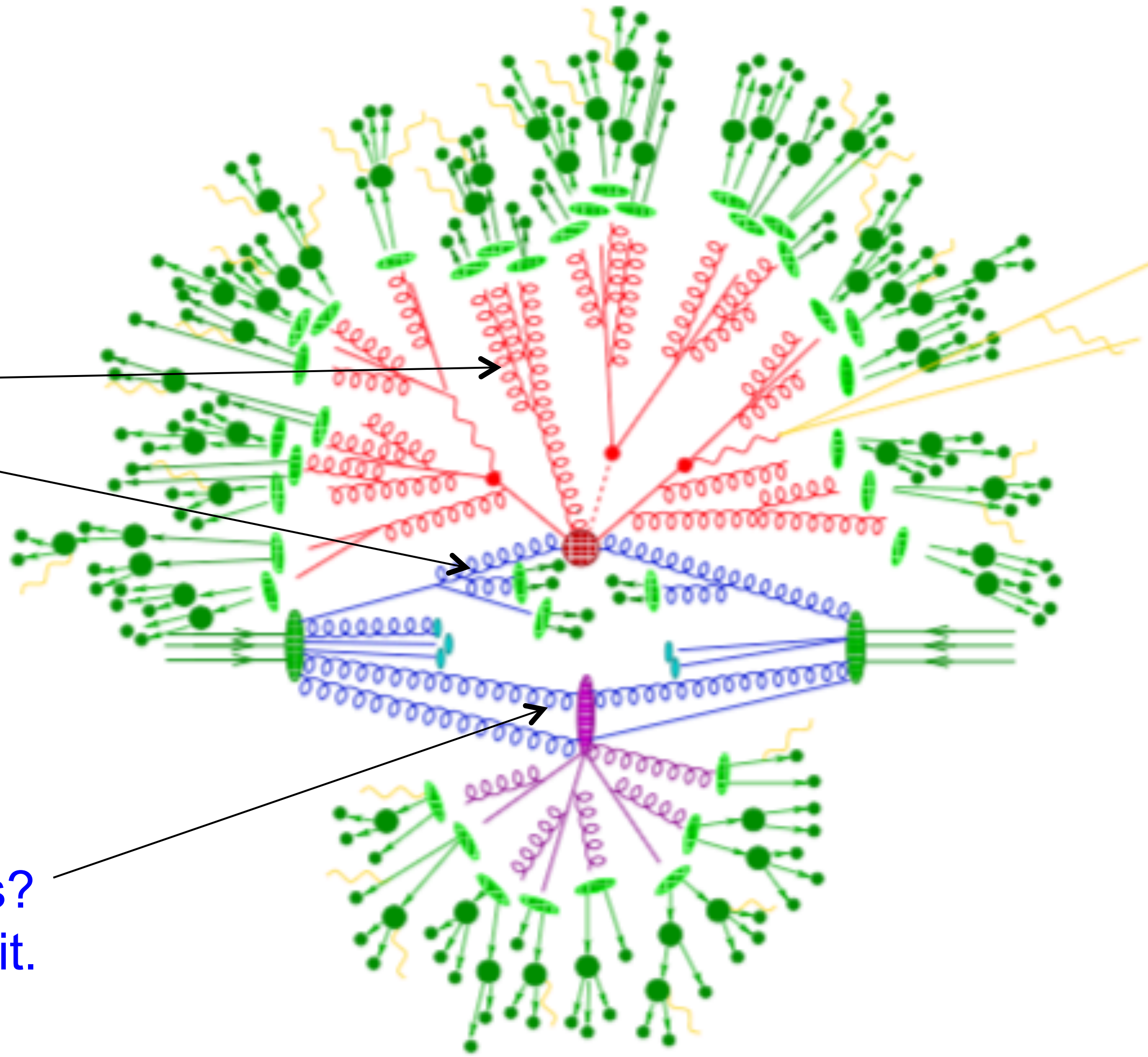


- Due to successive branching, parton cascade or shower develops. Each outgoing line is source of new cascade, until all outgoing lines have stopped branching. At this stage, which depends on cutoff scale t_0, outgoing partons have to be converted into hadrons via a hadronization model.

- Successive values of an evolution variable t, a momentum fraction z and an azimuthal angle ϕ are generated, along with the flavors of the partons emitted during the parton shower

...from our t-shirt

Note that gluon radiation
is coming from both
the final state
and the initial state

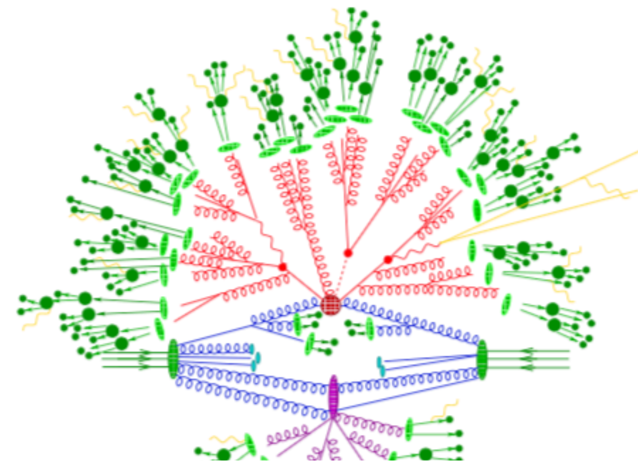


....but what's this?
We'll have to wait.

...from our t-shirt

Consider QCD final state radiation,
from q->qg as an example

Radiation pattern is given by



$$\mathrm{d}w^{q\to qg} = \frac{\alpha_s(k_\perp^2)}{2\pi} C_F \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \frac{\mathrm{d}\omega}{\omega} \left[1 + \left(1 - \frac{\omega}{E}\right)^2\right]$$

$$\overset{\omega = E(1-z)}{=} \frac{\alpha_s(k_\perp^2)}{2\pi} C_F \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \mathrm{d}z \frac{1+z^2}{1-z} = \frac{\alpha_s(k_\perp^2)}{2\pi} C_F \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \mathrm{d}z\, P_{qg}^{(1)}(z)\,.$$

- divergent structures for:

$z \to 1$ (soft divergence)	$\longleftrightarrow$	infrared/soft logarithms
$k_\perp^2 \to 0$ (collinear/mass divergence)	$\longleftrightarrow$	collinear logarithms

- cut regularise with cut-off $k_{\perp,\mathrm{min}} \sim 1\mathrm{GeV} > \Lambda_{\mathrm{QCD}}$

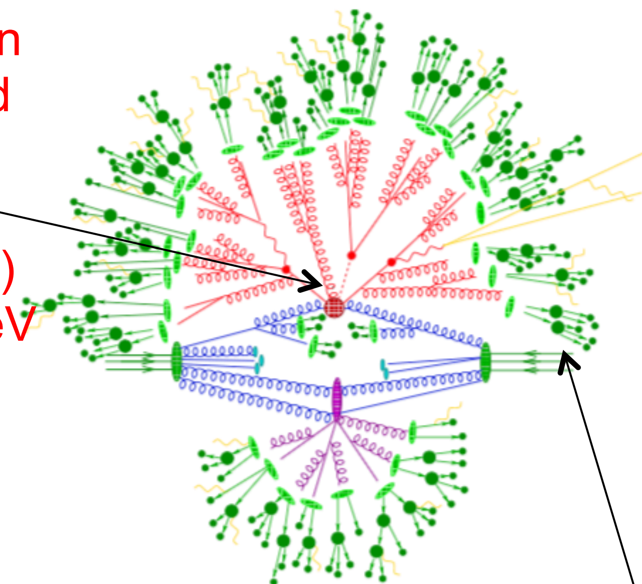
...from our t-shirt

- There are two perturbative regimes
  - a regime of jet production, where $k_T \sim \omega >> k_{Tmin}$
    - emission probabilities scale like $\alpha_s(k_T) << 1$
    - standard fixed-order QCD applies
  - a region of jet evolution where $k_{Tmin} < k_T < \omega$
    - emission probablities scale like $\alpha_s(k_T)\log^2(k_T^2) \sim 1$
    - perturbative parameter is not α_s anymore, but α_s * towers of logarithms
    - LL, NLL, etc
    - parton showers apply

$$
dw^{q\to qg} = \frac{\alpha_s(k_\perp^2)}{2\pi} C_F \frac{dk_\perp^2}{k_\perp^2} \frac{d\omega}{\omega} \left[1 + \left(1 - \frac{\omega}{E}\right)^2\right]
$$

$$
\overset{\omega=E(1-z)}{=} \frac{\alpha_s(k_\perp^2)}{2\pi} C_F \frac{dk_\perp^2}{k_\perp^2} dz \frac{1+z^2}{1-z} = \frac{\alpha_s(k_\perp^2)}{2\pi} C_F \frac{dk_\perp^2}{k_\perp^2} dz\, P_{qg}^{(1)}(z) .
$$

perturbation theory valid for these scales (~100 GeV) t~1/100 GeV

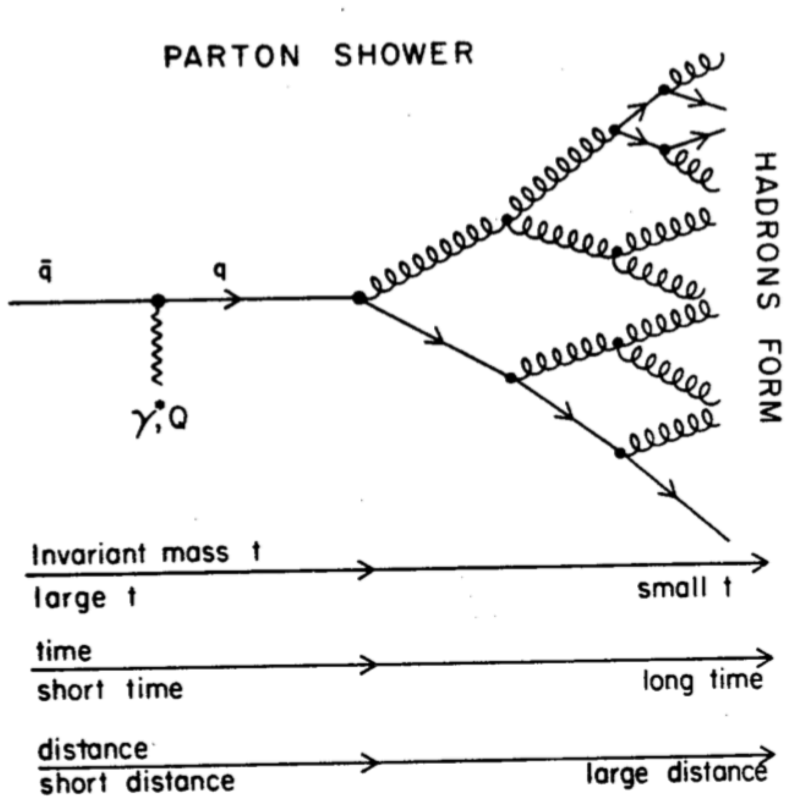


perturbation theory breaks down on scales of about 1 GeV (t~1/GeV)
It's at this point that the partons form hadrons through non-perturbative physics

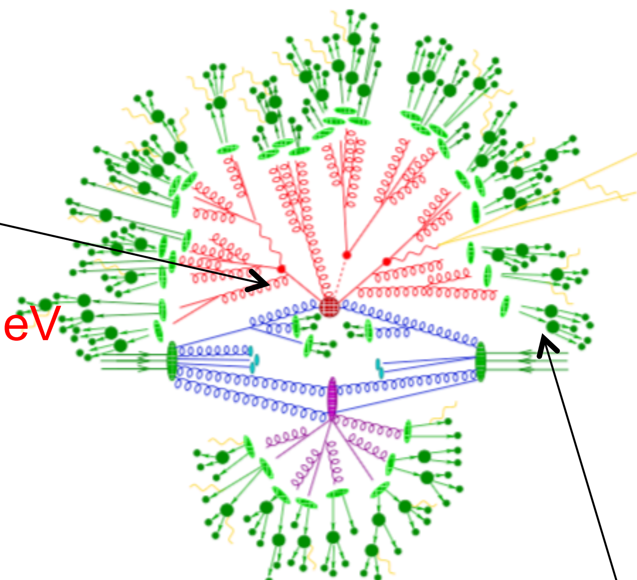
...from our t-shirt

$$\mathrm{d}w^{q\to qg} = \frac{\alpha_{\mathrm{s}}(k_\perp^2)}{2\pi}\, C_F\, \frac{\mathrm{d}k_\perp^2}{k_\perp^2}\, \frac{\mathrm{d}\omega}{\omega} \left[1+\left(1-\frac{\omega}{E}\right)^2\right]$$

$$\overset{\omega=E(1-z)}{=} \frac{\alpha_{\mathrm{s}}(k_\perp^2)}{2\pi}\, C_F\, \frac{\mathrm{d}k_\perp^2}{k_\perp^2}\, \mathrm{d}z\, \frac{1+z^2}{1-z} = \frac{\alpha_{\mathrm{s}}(k_\perp^2)}{2\pi}\, C_F\, \frac{\mathrm{d}k_\perp^2}{k_\perp^2}\, \mathrm{d}z\, P_{qg}^{(1)}(z)\,.$$



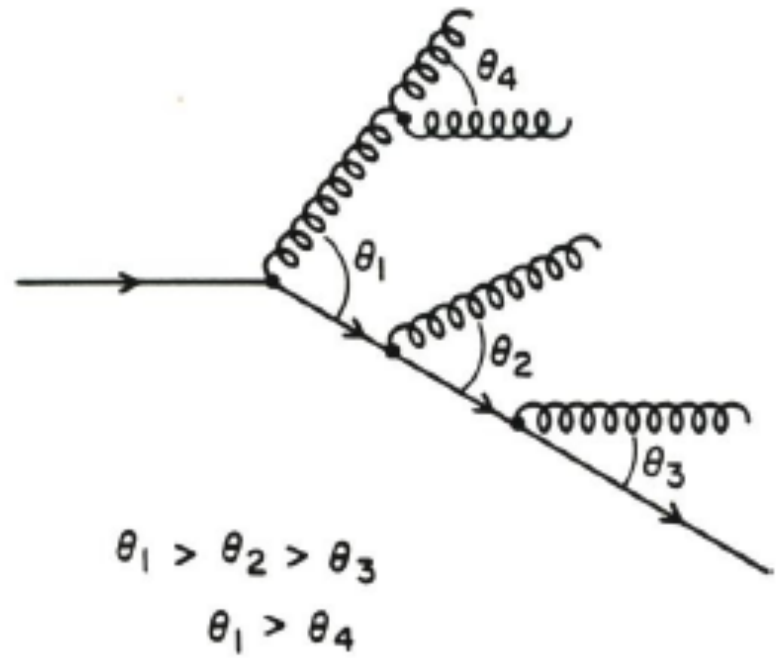
perturbation theory valid for these scales (~100 GeV) time~1/100 GeV



perturbation theory breaks down on scales of about 1 GeV (time~1/GeV)
It's at this point that the partons form hadrons through non-perturbative physics

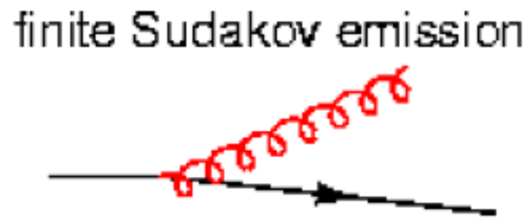
Parton shower evolution

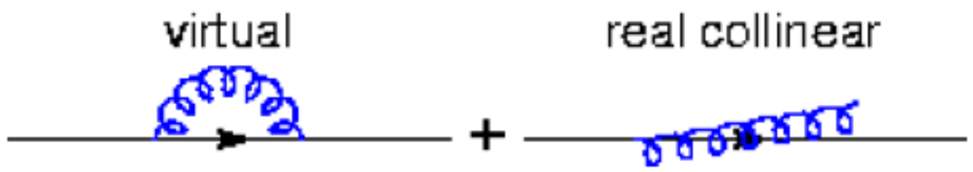
- On average, emitted gluons have decreasing angles with respect to parent parton directions
  - angular ordering, an aspect of color coherence
- The evolution variable *t* can be directly related to θ, the opening angle between the two partons [Herwig], or the square of the transverse momentum between the two partons [Pythia,Sherpa]



Note

- We can only observe emissions above a certain resolution scale
- Below this resolution scale, singularities cancel, leaving a finite remnant
- (some of) the virtual corrections encountered in a full NLO calculation are included by the use of Sudakov suppression between vertices
- So a parton shower Monte Carlo is not purely a fixed order calculation, but has a higher order component as well
- This is a statement that you'll often hear





Sudakov form factors

- Sudakov form factors form the basis for both resummation and parton showering
- We can write an expression for the Sudakov form factor of an initial state parton in the form below, where *t* is the hard scale, t_o is the cutoff scale and *P(z)* is the splitting function

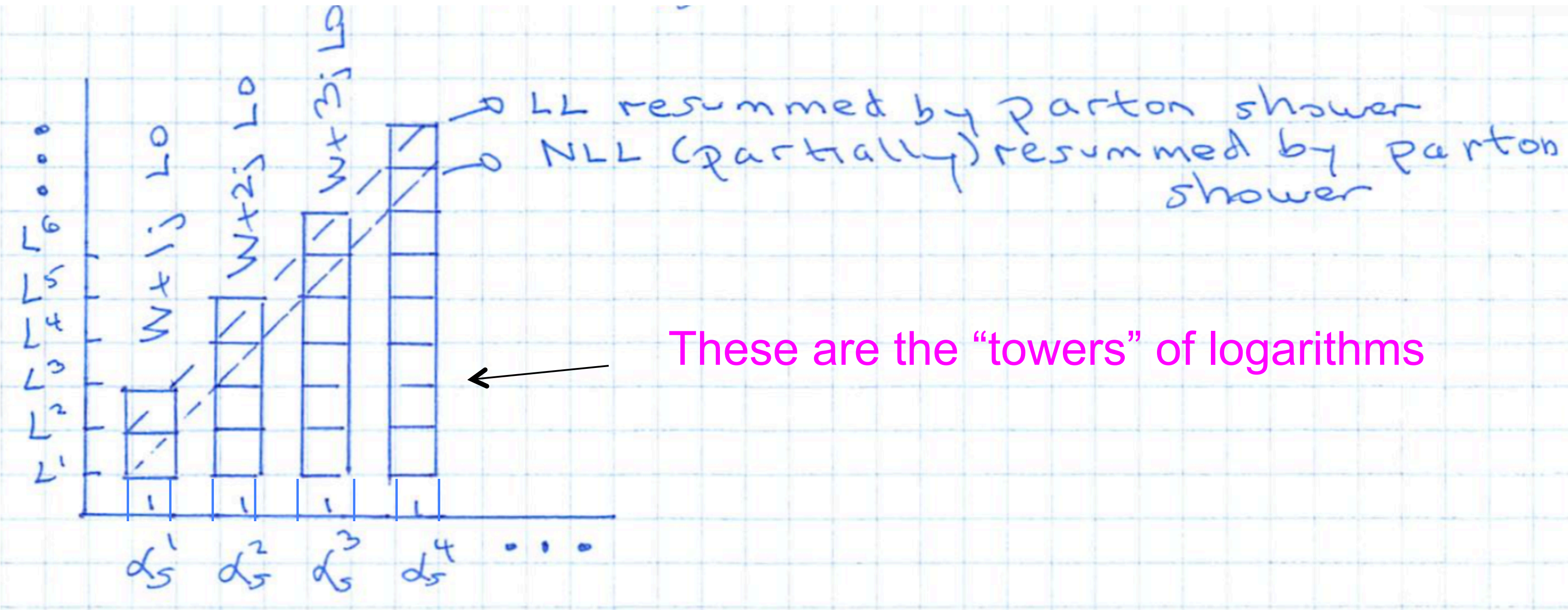
$$\Delta(t) \equiv \exp\left[-\int_{t_0}^{t} \frac{dt'}{t'} \int \frac{dz}{z} \frac{\alpha_S}{2\pi} P(z) \frac{f(x/z,t)}{f(x,t)}\right]$$

- Similar form for the final state but without the pdf weighting
- Sudakov form factor resums all effects of soft and collinear gluon emission (so again the double logs), but does not include non-singular regions that are due to large energy, wide angle gluon emission
- Gives the probability **not** to radiate a gluon greater than some energy
- We can draw explicit (approximate) curves for the Sudakov form factors

- The Sudakov form factor decreases (the probability of radiating increases) as the p_T of the radiated gluon decreases, as the hardness of the interaction increases, or as the x value of the incoming parton decreases (more phase space for gluon radiation)
- The Sudakov form factor is smaller (the probability or radiating is higher) for gluons than for jets, due to the larger color factor of the gluons

Logs, again...

$$\sigma_{W+0j} = a_0 + \alpha_S(a_{12}L^2 + a_{11}L + a_{10})$$
$$+ {\alpha_S}^2(a_{24}L^4 + a_{23}L^3 + a_{22}L^2 + a_{21}L + a_{20}) + \cdots$$
$$\sigma_{W+1j} = \alpha_S(b_{12}L^2 + b_{11}L + b_{10})$$
$$+ {\alpha_S}^2(b_{24}L^4 + b_{23}L^3 + b_{22}L^2 + b_{21}L + b_{20}) + \cdots$$
$$\sigma_{W+2j} = \cdots.$$

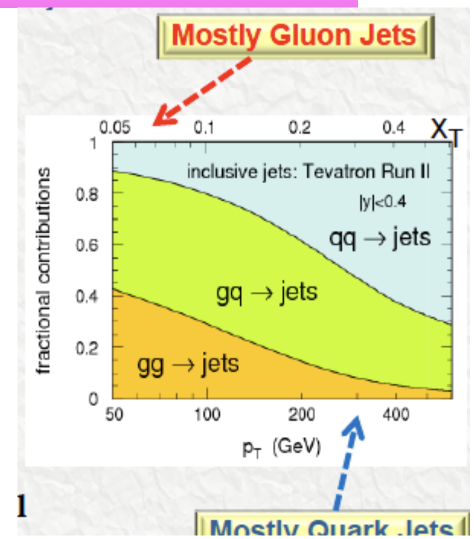


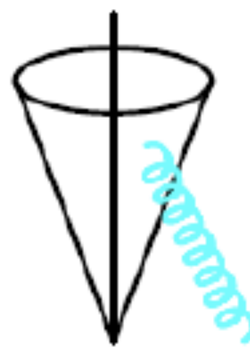
These are the “towers” of logarithms

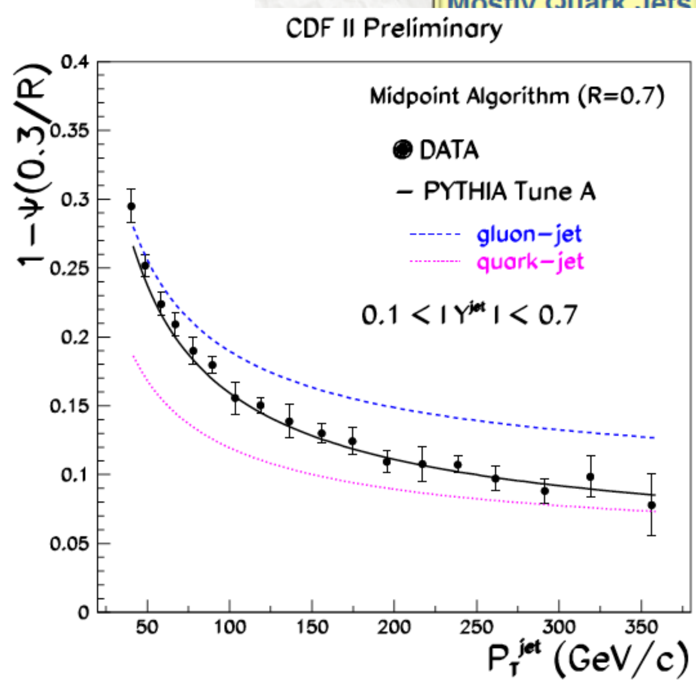
Jet shapes

- The gluon radiation from the parton shower produces the jet shape
- Gluon jets are broader than quark jets, due to the color factors, and thus larger emission probabilities
- Both gluon and quark jets get narrower as the transverse momentum of the jet increases
- Look, for example, at the fraction of jet energy in cone of radius 0.7 that is outside the "core" (0.3) from a Tevatron measurement

at small p_T, jet production dominated by gg and gq scattering due to large gluon distribution at low x







Merging ME and PS approaches

- Parton showers provide an excellent description in regions which are dominated by soft and collinear gluon emission
- Matrix element calculations provide a good description of processes where the partons are energetic and widely separated and also take into account interference effects between amplitudes
    - but do not take into account interference effects in soft and collinear emissions which cannot be resolved, and thus lead to Sudakov suppression of such emissions
- Hey, I know, let's put them together, but we have to be careful not to double-count
    - parton shower producing same event configurations already described by matrix element
    - Les Houches Accord (the first one) allows the ME program to talk to the PS program

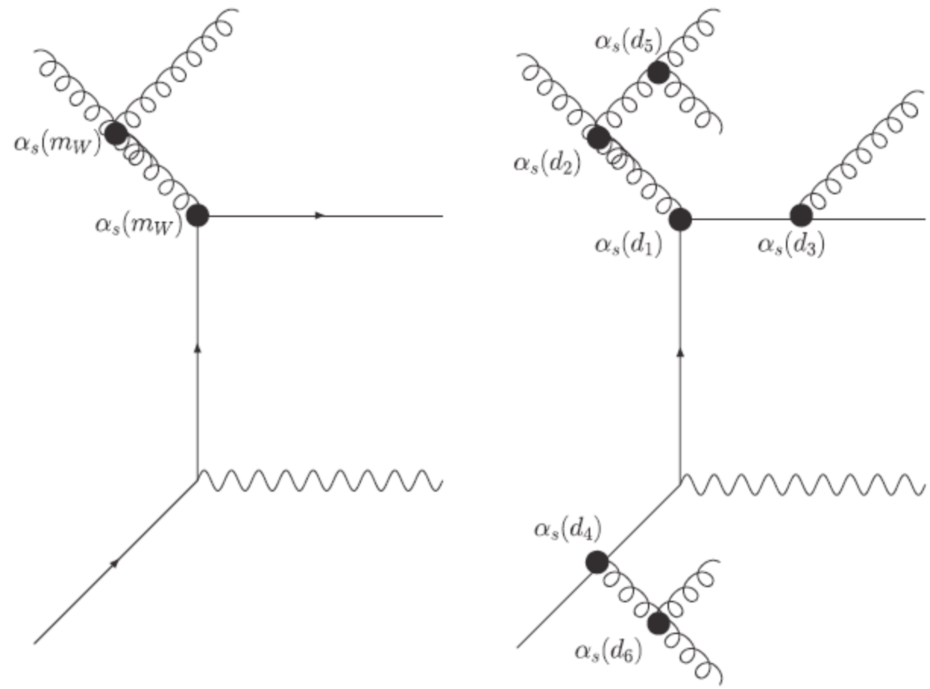
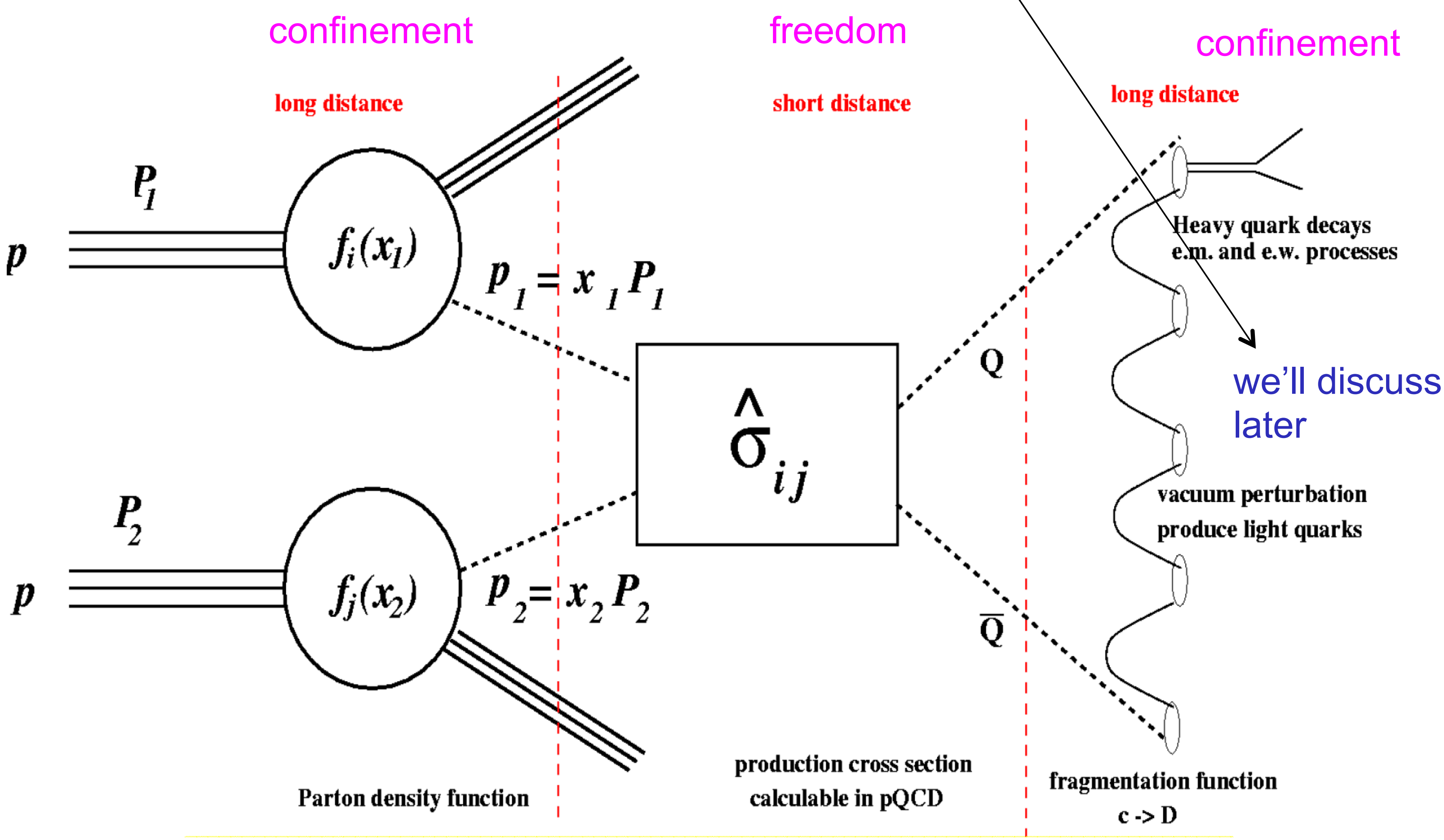


Figure 26. In the NLO formalism, the same scale, proportional to the hardness of the process, is used for each QCD vertex. For the case of the $W+$ 2 jet diagram shown above to the left, a scale related to the mass of the W boson, or to the average transverse momentum of the produced jets, is typically used. The figure to the right shows the results of a simulation using the CKKW formalism. Branchings occur at the vertices with resolution parameters d_i, where $d_1 > d_2 \gg d_{\text{ini}} > d_3 > d_4 > d_5 > d_6$. Branchings at the vertices 1–2 are produced with matrix element information while the branchings at vertices 3–6 are produced by the parton shower.

state of the art: add up to 3 matrix elements at NLO, i.e. H+1,H+2,H+3 jets

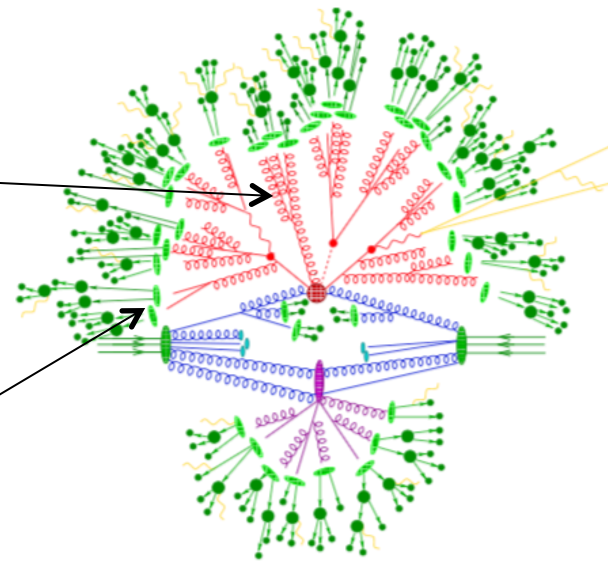
See Sherpa, Powheg, Madgraph5, etc

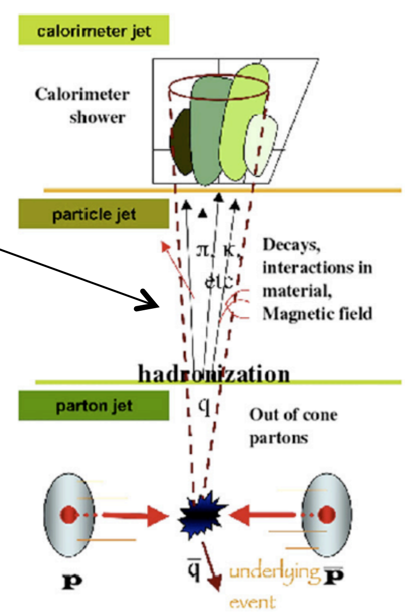
Now is later


confinement
freedom
confinement
long distance
short distance
long distance
P_1
p
$f_i(x_1)$
$p_1 = x_1 P_1$
P_2
p
$f_j(x_2)$
$p_2 = x_2 P_2$
$\hat{\sigma}_{ij}$
Q
$\bar{Q}$
Heavy quark decays
e.m. and e.w. processes
we'll discuss later
vacuum perturbation
produce light quarks
Parton density function
production cross section
calculable in pQCD
fragmentation function
c -> D

Hadronization

- Parton showers in the initial and final state produce a large multiplicity of gluons
- The parton shower evolution variable *t* decreases (for the final state) from a scale similar to the scale of the hard scatter to a scale at which pQCD is no longer applicable (near Λ_{QCD})
- At this point, we must construct models as to how the colored quarks and gluons recombine to form the (colorless) final state hadrons
- The two most popular models are the cluster and string models





Hadronization

- Parton showers in the initial and final state produce a large multiplicity of gluons
- The parton shower evolution variable *t* decreases (for the final state) from a scale similar to the scale of the hard scatter to a scale at which pQCD is no longer applicable (near Λ_{QCD})
- At this point, we must construct models as to how the colored quarks and gluons recombine to form the (colorless) final state hadrons
- The two most popular models are the cluster and string models

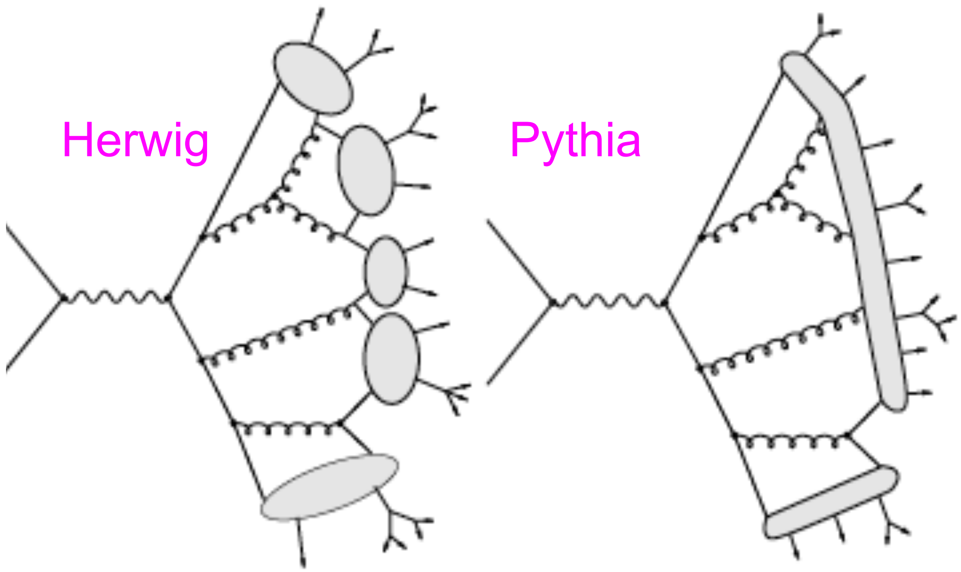


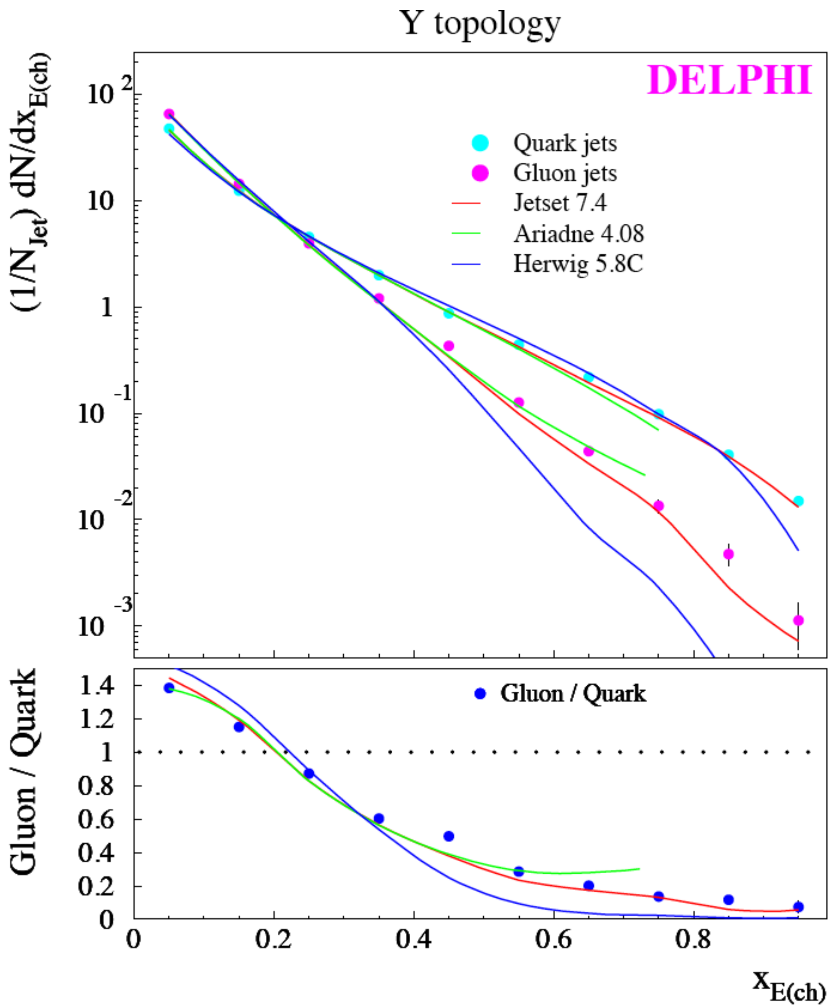
Figure 2: Cluster and string hadronization models.

•In cluster model, there is a non-perturbative splitting of gluons into q-qbar pairs; color-singlet combinations of q-qbar pairs form clusters which isotropically decay into pairs of hadrons

•In string model, relativistic string represents color flux; string breaks up into hadrons via q-qbar production in its intense color field

Fragmentation functions

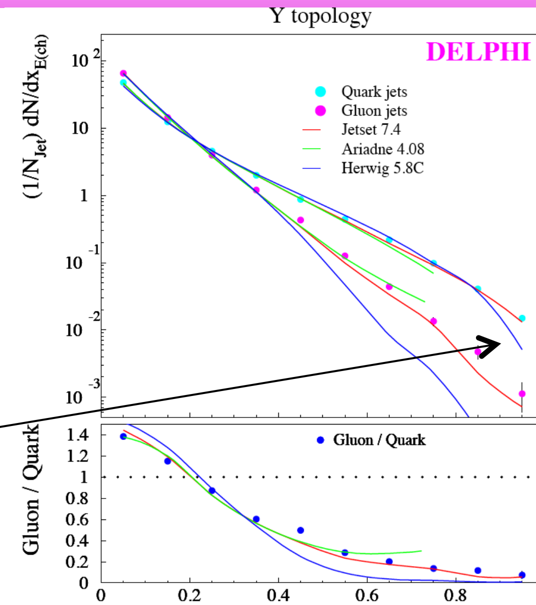
- On a more inclusive note, can define a fragmentation function $D(z,Q^2)$ that describes the probability to find a hadron of momentum fraction z (of the parent parton) at a scale Q
- The parton shower dynamically generates the fragmentation function, but the evolution of the fragmentation function with Q^2 can be calculated in pQCD (just as the evolution of the parton distribution functions can be calculated)
- But, like the PDFs, the value of $D(z,Q_o)$ is not known and must be determined by fits to data
- The data from LEP are the most useful for their determination

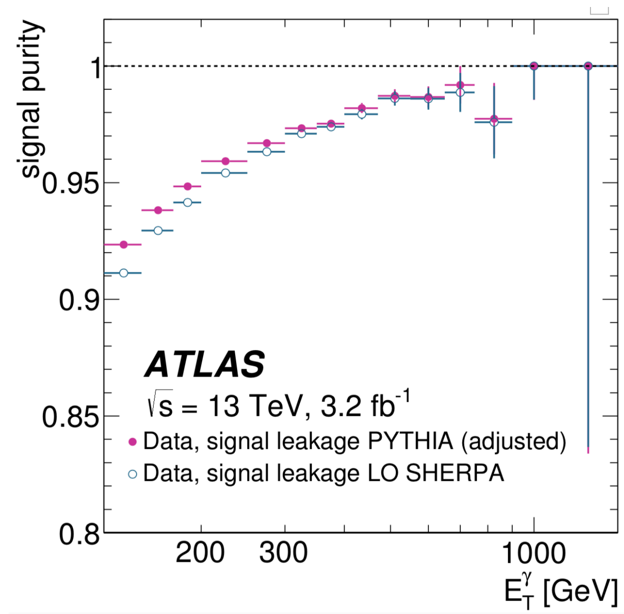


NB: the gluon fragmentation function is much softer; Herwig does not describe the high z gluon fragmentation function well

Jets faking photons

- As we discussed yesterday, jets can fake photons
- But we impose isolation cuts on photon candidates, so only jets in which most of the momentum is concentrated in one high p_T π^o (so z->1) can successfully fake photons
- This becomes increasingly difficult as the p_T of the 'photon' increases (z gets closer to 1), so the true photon fraction increases with the transverse momentum of the photon candidate
- To make matters even more difficult, the fragmentation function at high z decreases with p_T (it evolves)





Some more details

- For outgoing quarks and gluons, have collinear singularities just as for the parton distribution functions
- Fragmentation functions acquire μ dependence just as PDFs did

$$\mu^2 \frac{\partial}{\partial \mu^2} D_i(x,\mu^2) = \sum_j \int_x^1 \frac{dz}{z} \frac{\alpha_s(\mu^2)}{2\pi} D_j\left(\frac{x}{z},\mu^2\right) P_{ji}\left(z,\alpha_s\left(\mu^2\right)\right)$$

  - …just like DGLAP

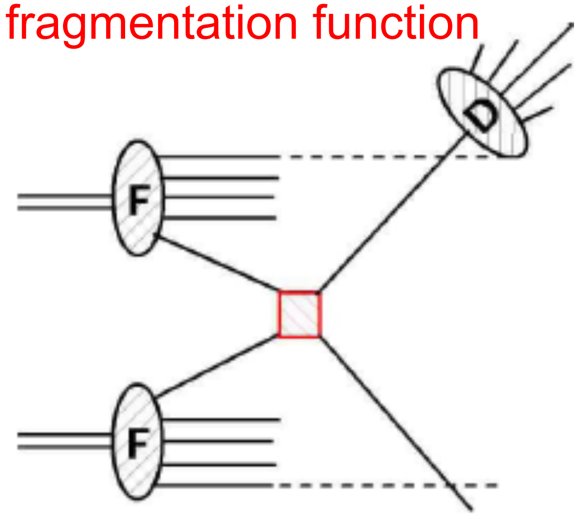
- Can write the single particle cross section as

$$\frac{d\sigma^{\pi}_{pp}}{d\eta dp_T^2} = \iiint f_{a/p}\left(x_a,\mu_F\right) \otimes f_{b/p}\left(x_b,\mu_F\right) \otimes \hat{\sigma}_{ab\to c}\left(p_T,\frac{s}{p_T^2},x_1,x_2,z,\frac{p_T}{\mu_F},\frac{p_T}{\mu}\right) \otimes D_{\pi/c}\left(z,\mu_F\right) \times \left\{1 + \mathrm{O}\left(\frac{m^2}{p_T^2}\right)\right\}$$

- Lowest order splitting functions are identical to those discussed for PDFs

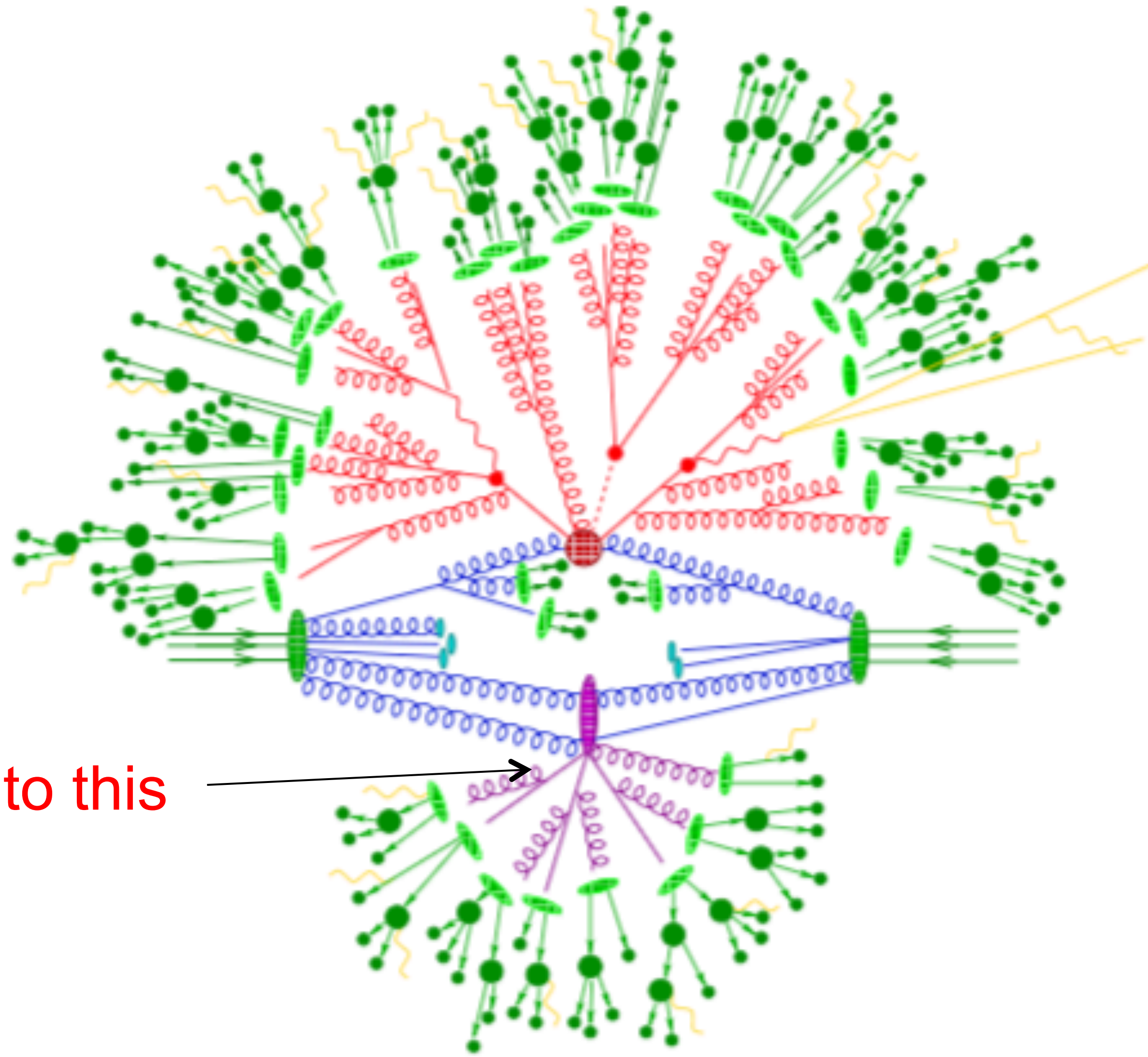
$$P_{ji}\left(z,\alpha_s\left(\mu^2\right)\right) = P_{ji}^{(0)} + \frac{\alpha_s\left(\mu^2\right)}{2\pi} P_{ji}^{(1)}\left(z\right) + \ldots$$

Calculate single particle cross section by convoluting over fragmentation function



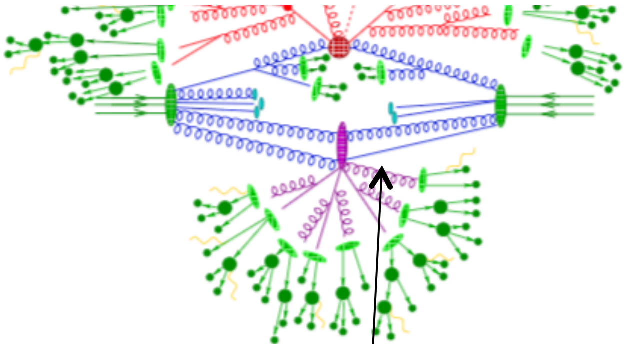
Sum over all fragmentation functions, apply a jet algorithm and voila you have a jet cross section

Multiple parton interactions



Finally, we come to this

Dictionary of Hadron Collider Terminology



EVENT

HADRON-HADRON COLLISION

Primary (Hard) Parton-Parton Scattering

Initial-State Radiation (ISR) = Spacelike Showers associated with Hard Scattering

Underlying Event

Multiple Parton-Parton Interactions: Additional parton-parton collisions (in principle with showers etc) in the same hadron-hadron collision.
= Multiple Perturbative Interactions (MPI)
= Spectator Interactions

Beam Remnants: Left over hadron remnants from the incoming beams. Coloured and hence correllated with the rest of the event

Fragmentation

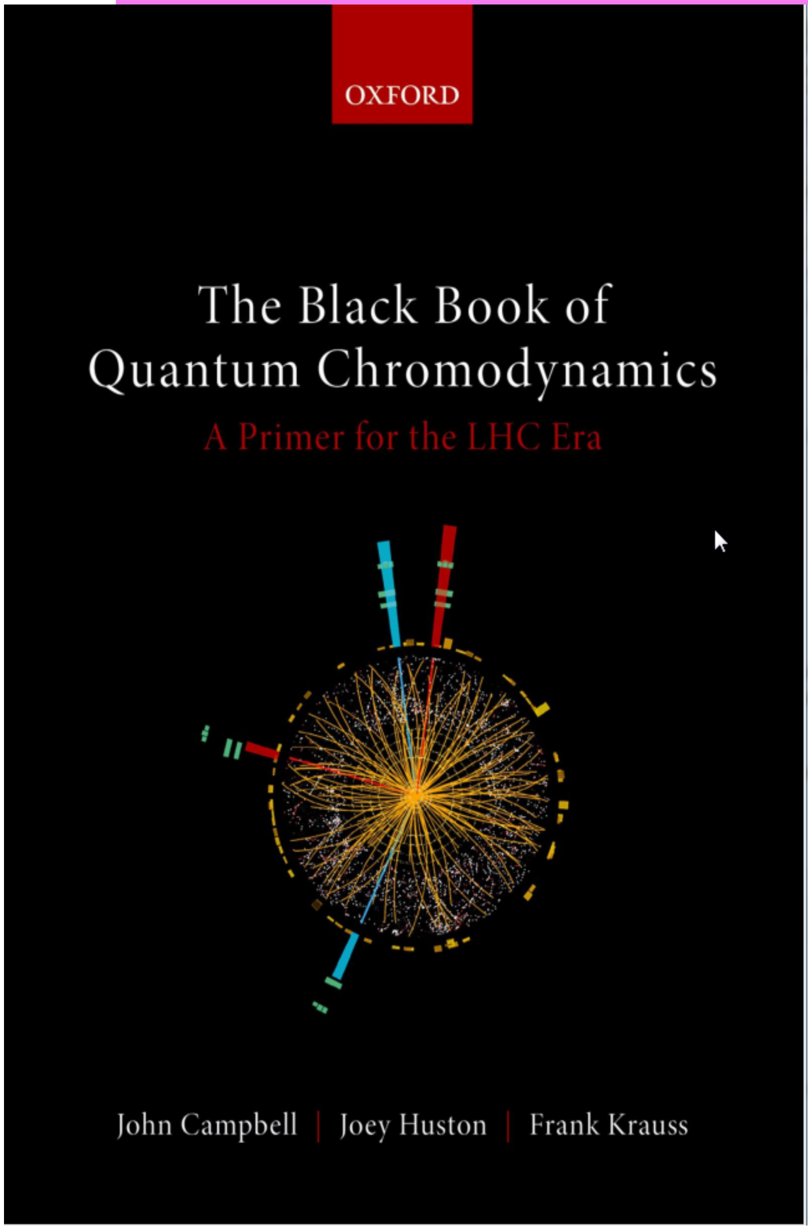
Perturbative: Final-State Radiation (FSR)
- Timelike Showers
- Jet Broadening and Hard Final-State Bremsstrahlung

Non-perturbative: String / Cluster Hadronisation (Colour Reconnections?)

PILE-UP: Additional hadron-hadron collisions recorded as part of the same event.

- In addition to the partons participating in the hard scatter event we are interested in, the other partons are interacting as well, typically at much smaller scales
- Note that they can produce parton showers (which hadronize) as well
- The end result is additional energy in the event that is basically unrelated to the physics we are interested in**->the underlying event**
- This additional energy has to be accounted for when comparisons to predictions are made
    - included in parton shower Monte Carlos; fixed order predictions can be corrected

What's next?



Next season my book will be on the shelves on the Big Bang Theory set. I'm hoping it will be on the coffee table.

If it is, I will make another trip to the set of BBT (and sit again in Sheldon's spot.)

Thanks for listening

- If you have additional questions, I’ll be happy to stay in contact
- My email is huston@pa.msu.edu

Extra slides

Consider gq->Vg

- First write process in terms of all outgoing states, and helicities of particles are indicated by the superscripts. Having only 1 independent helicity configuration simplifies matters for this process

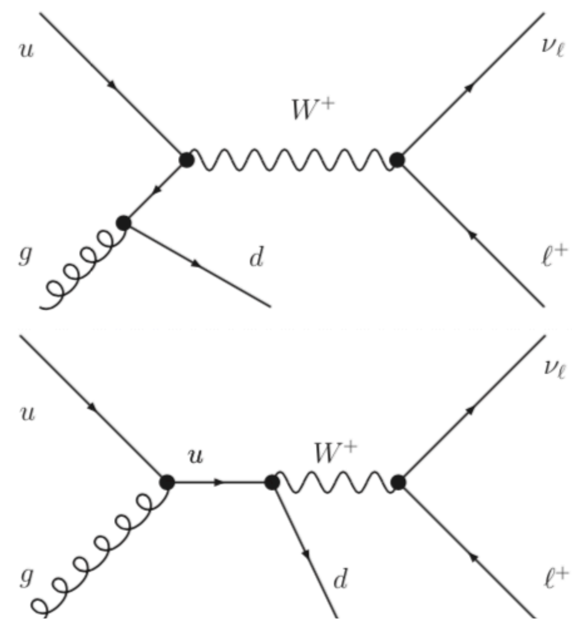
$$0 \to q^+(p_1) + g^+(p_2) + \bar{q}^-(p_3) + \bar{\ell}^-(p_4) + \ell^+(p_5),$$

- Can write the LO amplitude as

$$A^{\mathrm{LO}} = 2e^2 g T^{a_2}_{i_1 i_3} A^{\mathrm{tree}}$$

- Where a_2,i_1 and i_3 are the color indices of the gluon, quark and anti-quark and

$$A^{\mathrm{tree}} = -i \frac{\langle 3\,4\rangle^2}{\langle 1\,2\rangle \langle 2\,3\rangle \langle 4\,5\rangle}.$$



- Can write kinematic terms in matrix element as spinor products

Back to W production to NLO

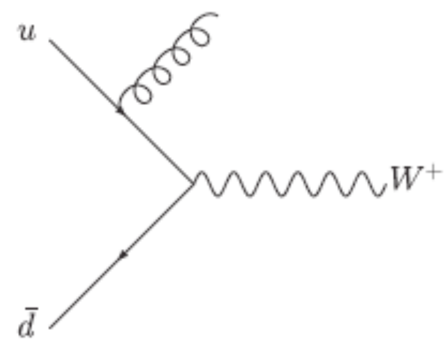
- In 4-dimensions, the contribution of the real diagrams can be written (ignoring diagrams with incoming gluons for simplicity)

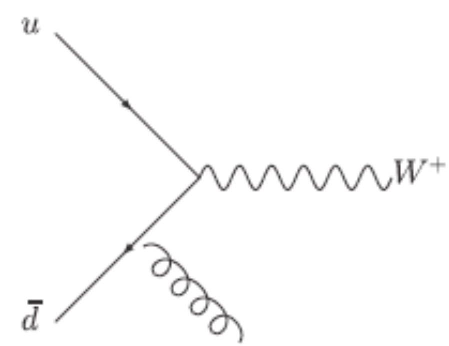
$$\left|M(u\bar{d} \to W^+ g\right|^2 \sim g^2 C_F \left[\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} + \frac{2Q^2\hat{s}}{\hat{u}\hat{t}} \right]$$

$$\sim g^2 C_F \left[\left(\frac{1+z^2}{1-z} \right) \left(\frac{-\hat{s}}{\hat{t}} + \frac{-\hat{s}}{\hat{u}} \right) - 2 \right]$$

  - where

$$z = \frac{Q^2}{s} \text{ and } \hat{s} + \hat{t} + \hat{u} = Q^2$$





- Note that the real diagrams contain collinear singularities, $\hat{u}$->0, $\hat{t}$->0, and soft singularities, z->1

and don't sweat the details; I just want you to see in general terms how a NLO calculation is carried out

Aside: dimensional regularization

- Suppose we have an integral of the form, typical of the integrals in a NLO calculation

$$I = \int \frac{d^4k}{(2\pi)^4} \frac{1}{\left(k^2 - m^2\right)^2}$$

- We get infinity if we integrate this in 4 dimensions, so go to 4-2ε dimensions

inserted to get the right dimensionality

$$\int \frac{d^4k}{(2\pi)^4} \to (\mu)^{2\varepsilon} \int \frac{d^{4-2\varepsilon}k}{(2\pi)^{4-2\varepsilon}} \to (\mu)^{2\varepsilon} \int \frac{d\Omega_{4-2\varepsilon}}{(2\pi)^{4-2\varepsilon}} \int dk_E k_E^{\,3-2\varepsilon}$$

$$\int \frac{d\Omega_{4-2\varepsilon}}{(2\pi)^{4-2\varepsilon}} = \frac{2}{(4\pi)^{2-\varepsilon}} \frac{1}{\Gamma(2-\varepsilon)}$$

$$(\mu)^{2\varepsilon} \int_0^\infty dk_E \frac{k_E^{\,3-2\varepsilon}}{\left(k_E^{\,2} + m^2\right)^2} = \frac{(\mu)^{2\varepsilon}}{2(m)^{2\varepsilon}} \int_0^1 dz z^{1-\varepsilon} (1-z)^{\varepsilon-1} = \frac{1}{2}\left(\frac{\mu}{m}\right)^{2\varepsilon} \frac{\Gamma(\varepsilon)\Gamma(2-\varepsilon)}{\Gamma(2)}$$

- Using

$$\Gamma(1+z) = z\Gamma(z); \Gamma'(1) = -\gamma_E = -0.5772\ldots$$

Dimensional regularization, continued

- Find

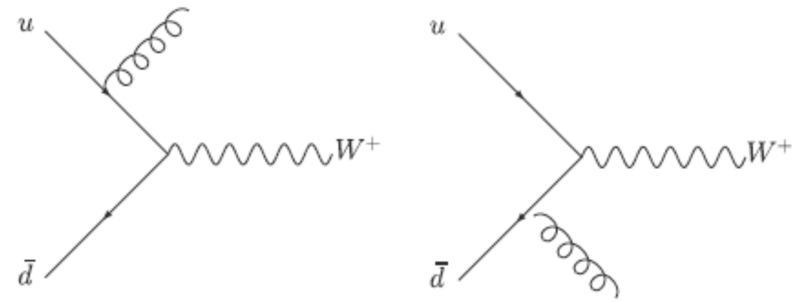
$$I = \frac{\Gamma(\varepsilon)}{(4\pi)^{2-\varepsilon}}\left(\frac{\mu}{m}\right)^{2\varepsilon} \xrightarrow[\varepsilon\text{->}0]{} \frac{1}{(4\pi)^2}\left[+\frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) + 2\ln\left(\frac{\mu}{m}\right) + O(\varepsilon)\right]$$

  - singular bits, plus finite bits as ε->0, plus log singularity as m->0

- Define $\overline{\text{MS}}$ scheme: subtract (absorb) 1/ε pole, γ_E, and ln(4π) bits

Now do the dimension trick for the real part

- Problem: if I work in 4 dimensions, I get divergences
- Solution: working in 4-2ε dimensions, to control the divergences (dimensional reduction)



$$\sigma_{real} = \frac{\alpha_s}{2\pi} C_F \left(\frac{\mu^2}{Q^2}\right)^{\varepsilon} c_\Gamma \left[\left(\frac{2}{\varepsilon^2} + \frac{3}{\varepsilon} - \frac{\pi^2}{3} \right) \delta(1-z) - \frac{2}{\varepsilon} P_{qq}(z) - 2(1-z) + 4(1+z^2) \left[\frac{\ln(1-z)}{1-z} \right]_+ - 2\frac{1+z^2}{1-z} \ln z \right]$$

- with

$$c_\Gamma = \frac{(4\pi)^{\varepsilon}}{\Gamma(1-\varepsilon)}$$

"+ distribution"

$$\left(\frac{\log(1-z)}{1-z}\right)_+ \equiv \lim_{\beta \to 0} \left\{ \frac{\log(1-z)}{1-z} \theta(1-z-\beta) + \frac{1}{2} \log^2(\beta) \delta(1-z-\beta) \right\}$$

We get 1/ε terms from individual soft and collinear singularities

We get 1/ε^2 terms for overlapping IR singularities.

- Note that the divergences are explicit, as we integrated over all of phase space

→ They are not explicit if we make cuts, i.e. on jets. Then we have to use subtraction techniques.

Ditto for the virtual part

$$\sigma_{virt} = \delta(1-z)\left[1 + \frac{\alpha_s}{2\pi} C_F \left(\frac{\mu^2}{Q^2}\right)^{\varepsilon} c'_\Gamma \left(-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} - 6 + \pi^2\right)\right]$$

from soft and collinear bits

- where

$$c'_\Gamma = c_\Gamma + O(\varepsilon^3)$$

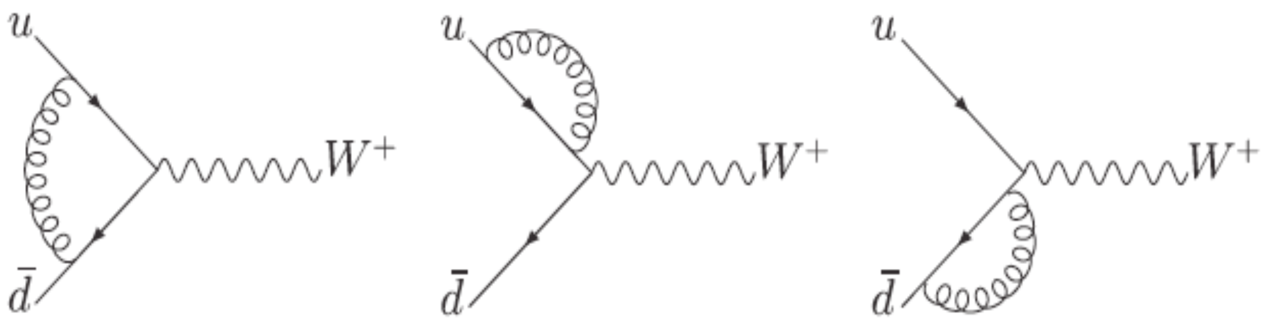


Figure 14. Virtual diagrams included in the next-to-leading order corrections to the Drell–Yan production of a W at hadron colliders.

We also get UV divergences when the loop momenta go off to infinity. The summation of these singularities leads to the running of the strong couplings, i.e. we define the sum of all such contributions (scales $>\mu_{UV}$) as the physical renormalized coupling, α_s.

Now add real and virtual

$$\sigma_{real+virt} = \frac{\alpha_s}{2\pi} C_F \left(\frac{\mu^2}{Q^2}\right)^{\varepsilon} c_\Gamma \left[\left(\frac{2\pi^2}{3} - 6\right)\delta(1-z) - \frac{2}{\varepsilon} P_{qq}(z) - 2(1-z) + 4\left(1+z^2\right)\left[\frac{\ln(1-z)}{1-z}\right]_+ - 2\frac{1+z^2}{1-z}\ln z\right]$$

- Notice that the ε^2 terms cancel
- The divergences that are proportional to the branching probabilities are universal
- We can factorize them into the parton distributions, performing mass factorization by subtracting the counter-term (MSbar scheme)

$$2\frac{\alpha_s}{2\pi} C_F \left[\frac{-c_\Gamma}{\varepsilon} P_{qq}(z) - (1-z) + \delta(1-z)\right]$$

- To get

$$\boxed{\hat{\sigma}_{real+virt} = \frac{\alpha_s}{2\pi} C_F \left[\left(\frac{2\pi^2}{3} - 8\right)\delta(1-z) + 4\left(1+z^2\right)\left[\frac{\ln(1-z)}{1-z}\right]_+ - 2\frac{1+z^2}{1-z}\ln z + 2P_{qq}(z)\ln\frac{Q^2}{\mu^2}\right]}$$

- Plus a similar correction for incoming gluons
- That works for the total cross section, but we need differential distributions for comparisons to data, so we need a general subtraction procedure at NLO, using Monte Carlo techniques

Logarithms

- You can keep applying this argument at higher orders of perturbation theory
- Each gluon that is added yields an additional power of α_s, and via the eikonal factorization outlined, can produce an additional two logarithms (soft and collinear)
- So can write the W + jets cross section as

$$d\sigma = \sigma_0(W + 1\text{ jet})\left[1 + \alpha_S(c_{12}L^2 + c_{11}L + c_{10}) + \alpha_S{}^2(c_{24}L^4 + c_{23}L^3 + c_{22}L^2 + c_{21}L + c_{20}) + \cdots\right]$$

  - where L represents the logarithm controlling the divergence, either soft or collinear (Sudakov logs)
  - note that α_s and L appear together as $\alpha_s L$

- Size of L depends on criteria used to define the jets (min p_T, cone size)
- Coefficients c_{ij} depend on color factors
- Thus, addition of each gluon results in additional factor of α_s times logarithms
- In many (typically exclusive) cases, the logs can be large, leading to an enhanced probability for gluon emission to occur
- For most inclusive cases, logs are small and α_s counting may be valid estimator for production of additional jets
- For completely inclusive cross sections, the logs vanish

Specific example

- Remember we encounter logs whenever an emitted gluon becomes soft and/or collinear

$$[a\,b]\,\mathrm{d}PS_{\text{gluon}} = \frac{1}{E^2}\frac{1}{1-\cos\theta_a}\,E\,\mathrm{d}E\ \mathrm{d}\cos\theta_a$$

- We said the c_{ij} were color factors
- So for emission of parton 5 from parton 1, color factor is C_F
- For emission of parton 4 from parton 3, C_A
- If parton 5 is soft, and collinear with parton 1, and parton 4 is soft, and is collinear with parton 3, have 4 powers of logs

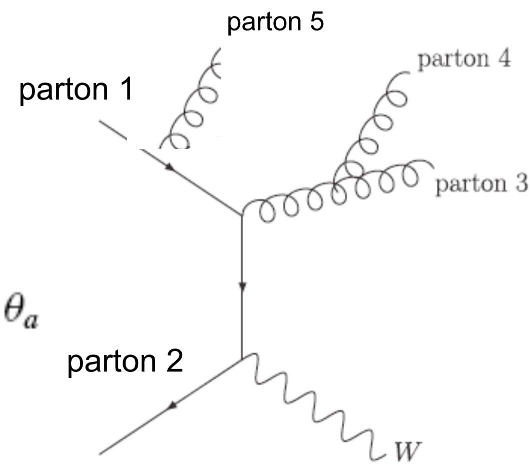


Figure 13. A final-state configuration containing a W and 2 partons. After the jet definition has been applied, either zero, one or two jets may be reconstructed.

not present since have 2 extra gluons, not 1

$$\mathrm{d}\sigma = \sigma_0(W + 1\ \text{jet})\Big[1 + \cancel{\alpha_S(c_{12}L^2 + c_{11}L + c_{10})} + \alpha_S{}^2(c_{24}L^4 + c_{23}L^3 + c_{22}L^2 + c_{21}L + c_{20}) + \cdots\Big]$$

- If one of the partons is not soft or collinear, then only 3 powers of logs
- …and so on
- Factors of 2, π, etc ignored

for W + jets

Re-shuffling

$$d\sigma = \sigma_0(W + 1 \text{ jet})\left[1 + \alpha_S(c_{12}L^2 + c_{11}L + c_{10}) + \alpha_S{}^2(c_{24}L^4 + c_{23}L^3 + c_{22}L^2 + c_{21}L + c_{20}) + \cdots\right]$$

each gluon added has an additional factor of α_s and two additional logs (soft and collinear)
c_{ij} depend on color factors

- re-write the term in brackets as

$$\left[\cdots\right] = 1 + \alpha_S L^2 c_{12} + (\alpha_S L^2)^2 c_{24} + \alpha_S L c_{11}(1 + \alpha_S L^2 c_{23}/c_{11} + \cdots) + \cdots$$
$$= \exp\left[c_{12}\alpha_S L^2 + c_{11}\alpha_S L\right],$$

- Where the infinite series has been resummed into an exponential form
    - first term in expansion is called leading logarithm term, 2nd next-to-leading logarithm, etc
- Now can write out each contribution as a combination of terms in powers of α_s and logarithms

$$\sigma_W = \sigma_{W+0j} + \sigma_{W+1j} + \sigma_{W+2j} + \sigma_{W+3j} + \cdots$$

$$\sigma_{W+0j} = a_0 + \alpha_S(a_{12}L^2 + a_{11}L + a_{10}) + \alpha_S{}^2(a_{24}L^4 + a_{23}L^3 + a_{22}L^2 + a_{21}L + a_{20}) + \cdots$$

$$\sigma_{W+1j} = \alpha_S(b_{12}L^2 + b_{11}L + b_{10}) + \alpha_S{}^2(b_{24}L^4 + b_{23}L^3 + b_{22}L^2 + b_{21}L + b_{20}) + \cdots$$

$$\sigma_{W+2j} = \cdots.$$

as jet definitions change, size of the logs shuffle the contributions from one jet cross section to another, keeping the sum over all contributions the same; for example, as R decreases, L increases, contributions shift towards higher jet multiplicities

Re-shuffling

- Configuration shown to the right can be reconstructed as an event containing up to 2 jets (0,1,2), depending on jet definition and momenta of the partons.
- For a large value of R_{cone}, this is one jet; for a smaller value, it may be two jets
- The matrix elements for this process contain terms proportional to $\alpha_s \log(p_{T3}/p_{T4})$ and as $\log(1/\Delta R_{34})$, so min values for transverse momentum and separation must be imposed
- Suppose that I consider completely inclusive cross sections ($\sigma_{W+>=0\ jets}$)
- Then the logs vanish

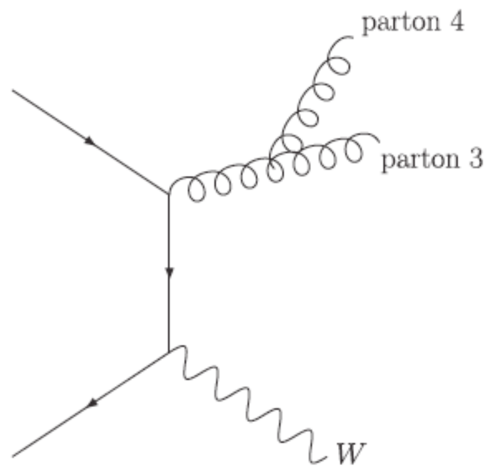
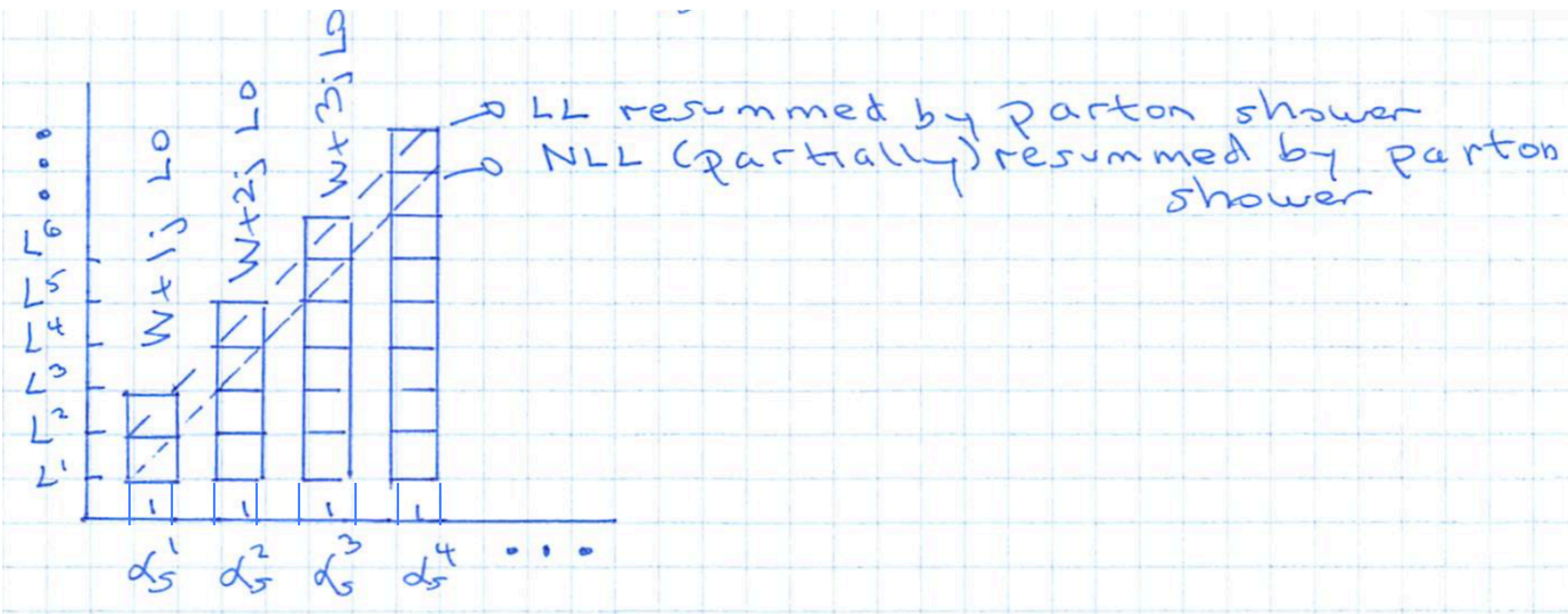


Figure 13. A final-state configuration containing a W and 2 partons. After the jet definition has been applied, either zero, one or two jets may be reconstructed.

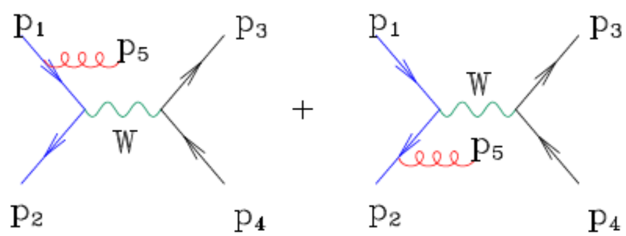
$$
\begin{aligned}
\sigma_{W+0j} &= a_0 + \alpha_S(a_{12}L^2 + a_{11}L + a_{10}) \\
&\quad + \alpha_S{}^2(a_{24}L^4 + a_{23}L^3 + a_{22}L^2 + a_{21}L + a_{20}) + \cdots \\
\sigma_{W+1j} &= \alpha_S(b_{12}L^2 + b_{11}L + b_{10}) \\
&\quad + \alpha_S{}^2(b_{24}L^4 + b_{23}L^3 + b_{22}L^2 + b_{21}L + b_{20}) + \cdots \\
\sigma_{W+2j} &= \cdots .
\end{aligned}
$$

Reviewing

$$\sigma_{W+0j} = a_0 + \alpha_S(a_{12}L^2 + a_{11}L + a_{10})$$
$$+ \alpha_S{}^2(a_{24}L^4 + a_{23}L^3 + a_{22}L^2 + a_{21}L + a_{20}) + \cdots$$
$$\sigma_{W+1j} = \alpha_S(b_{12}L^2 + b_{11}L + b_{10})$$
$$+ \alpha_S{}^2(b_{24}L^4 + b_{23}L^3 + b_{22}L^2 + b_{21}L + b_{20}) + \cdots$$
$$\sigma_{W+2j} = \cdots.$$



Consider matrix element counter-event for W production



real corrections to W production at NLO

- In soft limit (p_5->0), we have

eikonal factor; an approximation to the full matrix element valid when the gluon is soft (we saw this before)

$$\left|M_1(p_1,p_2,p_3,p_4,p_5)\right|^2 = g^2 C_F \frac{p_1 \bullet p_2}{p_1 \bullet p_5\, p_2 \bullet p_5}\left|M_0(p_1,p_2,p_3,p_4)\right|^2$$

- The eikonal factor can be associated with radiation from a given leg by partial fractioning

$$\frac{p_1 \bullet p_2}{p_1 \bullet p_5\, p_2 \bullet p_5} = \left[\frac{p_1 \bullet p_2}{p_1 \bullet p_5 + p_2 \bullet p_5}\right]\left[\frac{1}{p_1 \bullet p_5} + \frac{1}{p_2 \bullet p_5}\right]$$

- Including the collinear contributions, singular as $p_1 \cdot p_5$->0, the matrix element for the counter-event has the structure

$$\left|M_1(p_1,p_2,p_3,p_4,p_5)\right|^2 = \frac{g^2}{x_a p_1 \bullet p_5}\hat{P}_{qq}(x_a)\left|M_0(p_1,p_2,p_3,p_4)\right|^2$$

- where

$$1 - x_a = \frac{p_1 \bullet p_5 + p_2 \bullet p_5}{p_1 \bullet p_2} \qquad \hat{P}_{qq}(x_a) = C_F \frac{1+x^2}{1-x}$$

Making an counter-event

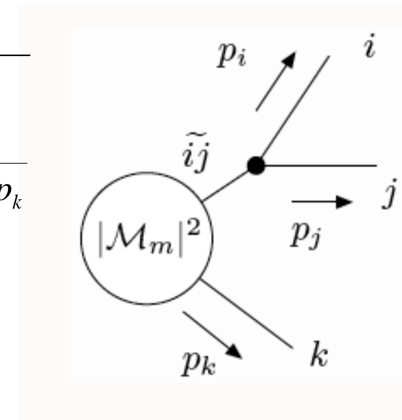
- For event $q(p_1)+\overline{q}(p_2) \to W^+(\nu(p_3)+e^+(p_4))+g(p_5)$
  - with $p_1 + p_2 = \sum_{i=1}^{5} p_i$
- Generate a counter-event $q(x_a p_1)+\overline{q}(p_2) \to W^+(\nu(\tilde{p}_3)+e^+(\tilde{p}_4)$
  - with $x_a p_1 + p_2 = \sum_{i=3}^{4} \tilde{p}_i ; 1 - x_a = \left(p_1 \bullet p_5 + p_2 \bullet p_5\right) / p_1 \bullet p_2$
- Perform a Lorentz transformation on all j final state momenta $\tilde{p}_j = \Lambda^{\mu}_{\nu} p^{n}_{j}, j = 3,4$
  - such that $\tilde{p}^{\mu}_{j} \to p^{\mu}_{j}$
  - for p_5 collinear or soft
- The longitudinal momentum of p_5 is absorbed by re-scaling with x
- The other components of the momentum p_5 are absorbed by the Lorentz transformation
- A lot of transformations done to get the momenta to work out right

Example: Final-final CS dipole

- The branching shown can be characterized by Lorentz invariant variables

$$y_{ij,k} = \frac{p_i p_j}{p_i p_j + p_i p_k + p_j p_k}$$

$$\tilde{z}_i = 1 - \tilde{z}_j = \frac{p_i p_k}{p_i p_k + p_j p_k}$$



- The factorized form of the fully differential (m+1) parton cross section that exactly reproduces the corresponding soft and collinear emissions of the real-emission process is

$$d\hat{\sigma}_{m+1} = d\hat{\sigma}_m \sum_{ij} \sum_{k \neq ij} \frac{dy_{ij,k}}{y_{ij,k}} d\hat{z}_i \frac{d\phi_i}{2\pi} \frac{\alpha_s}{2\pi} \frac{1}{N_{ij}^{spec}} \left(1 - y_{ij,k}\right) \left\langle V_{ij,k}\left(\tilde{z}_i, y_{ij,k}\right)\right\rangle$$

1,2 depending on
possible spectators

- The spin-averaged splitting kernels $\langle V_{ij,k} \rangle$ for the branchings q->qg,g->gg, g->qqbar are

$$\left\langle V_{q_i g_j,k}\left(\tilde{z}_i, y_{ij,k}\right)\right\rangle = C_F\left[\frac{2}{1 - \tilde{z}_i + \tilde{z}_i y_{ij,k}} - \left(1 + \tilde{z}_i\right)\right]$$

$$\left\langle V_{g_i g_j,k}\left(\tilde{z}_i, y_{ij,k}\right)\right\rangle = 2C_A\left[\frac{1}{1 - \tilde{z}_i + \tilde{z}_i y_{ij,k}} + \frac{1}{\tilde{z}_i + y_{ij,k} - \tilde{z}_i y_{ij,k}} - 2 + \tilde{z}_i\left(1 - \tilde{z}_i\right)\right]$$

$$\left\langle V_{q_i g_j,k}\left(\tilde{z}_i, y_{ij,k}\right)\right\rangle = T_R\left[1 - 2\tilde{z}_i(1 - \tilde{z}_i)\right]$$

Note that these terms look a lot like parton shower branchings

Merging ME and PS approaches

- A number of techniques to combine, with most popular/correct being CKKW
  - matrix element description used to describe parton branchings at large angle and/or energy
  - parton shower description is used for smaller angle, lower energy emissions
- Division into two regions of phase space provided by a resolution parameter d_{ini}
- Argument of α_s at all of the vertices is taken to be equal to the resolution parameter d_i (showering variable) at which the branching has taken place
- Sudakov form factors are inserted on all of the quark and gluon lines to represent the lack of any emissions with a scale larger than d_{ini} between vertices
  - parton showering is used to produce additional emissions at scales less than d_i
- For typical matching scale, ~10% of the n-jet cross section is produced by parton showering from n-1 parton ME

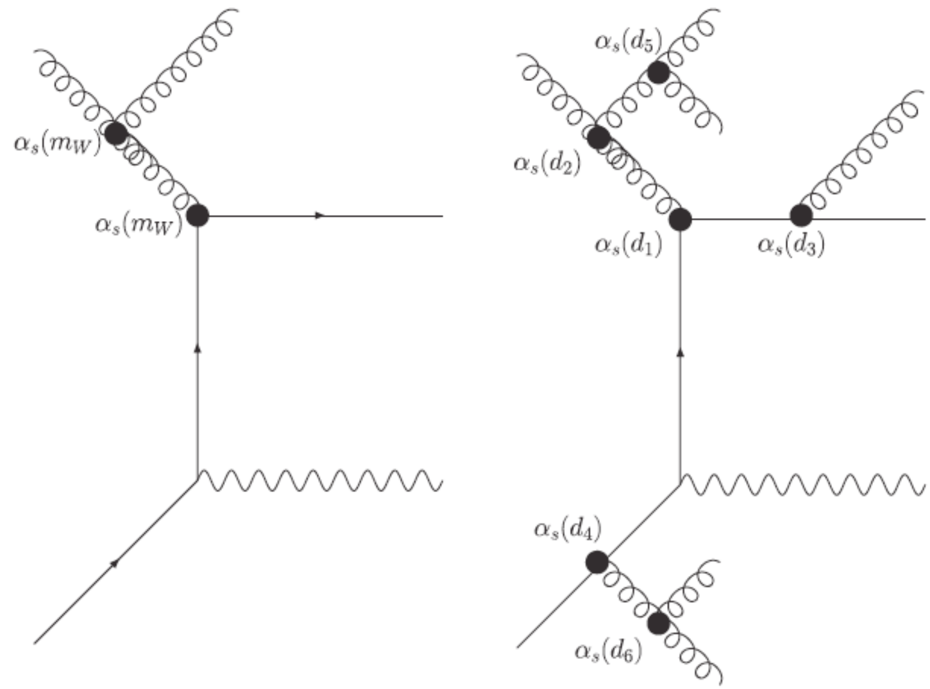
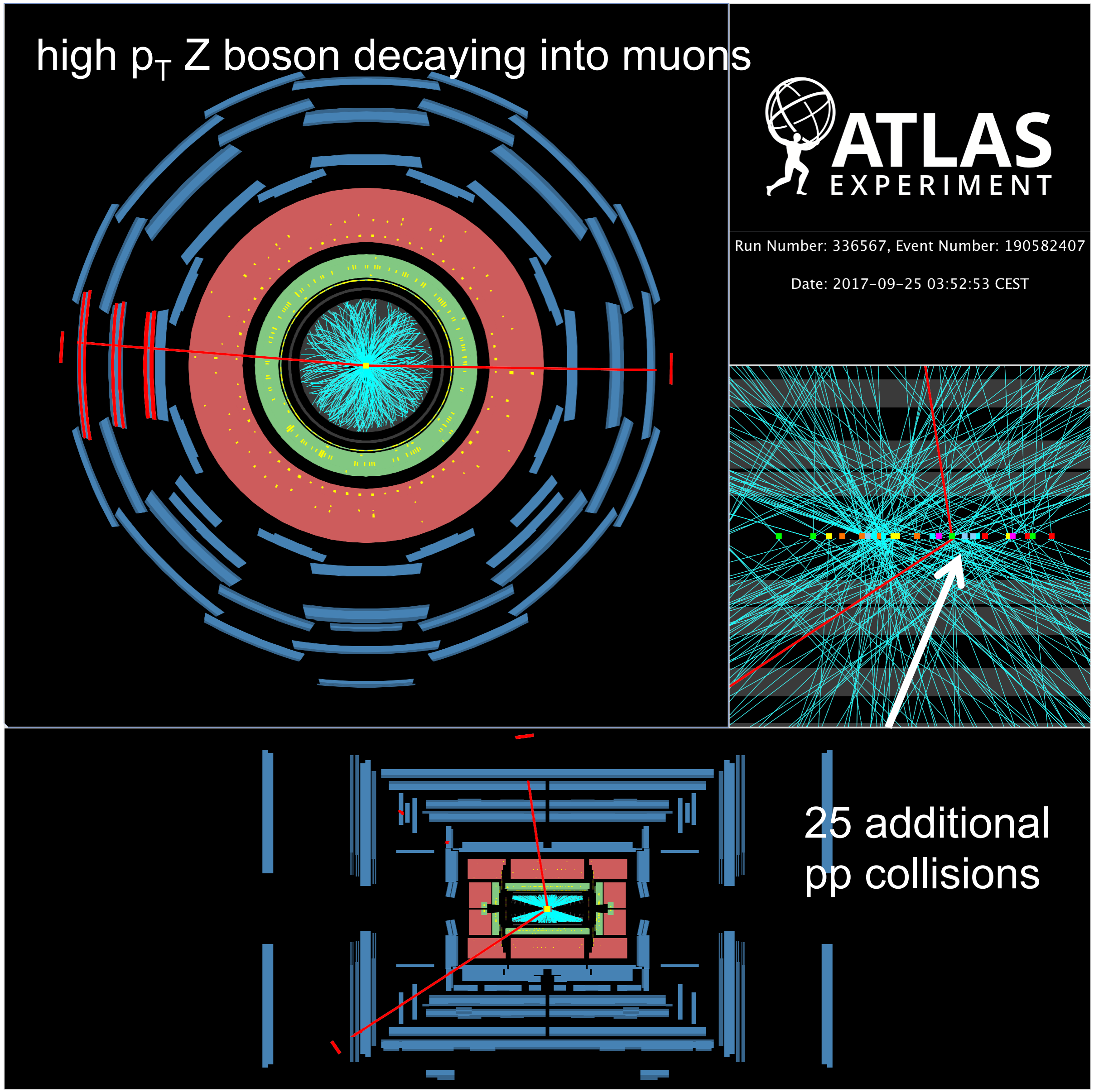


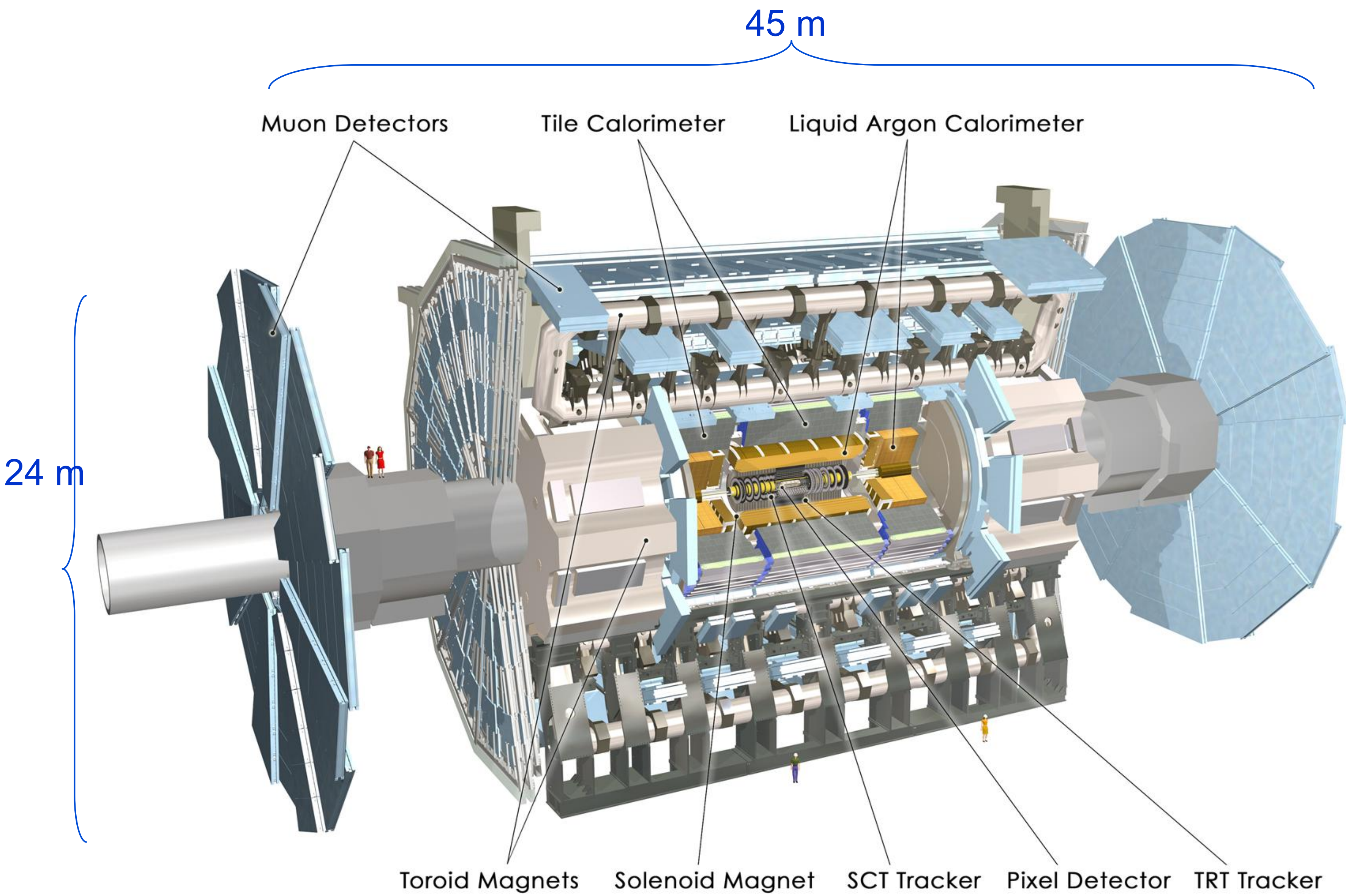
Figure 26. In the NLO formalism, the same scale, proportional to the hardness of the process, is used for each QCD vertex. For the case of the $W+$ 2 jet diagram shown above to the left, a scale related to the mass of the W boson, or to the average transverse momentum of the produced jets, is typically used. The figure to the right shows the results of a simulation using the CKKW formalism. Branchings occur at the vertices with resolution parameters d_i, where $d_1 > d_2 \gg d_{\rm ini} > d_3 > d_4 > d_5 > d_6$. Branchings at the vertices 1–2 are produced with matrix element information while the branchings at vertices 3–6 are produced by the parton shower.

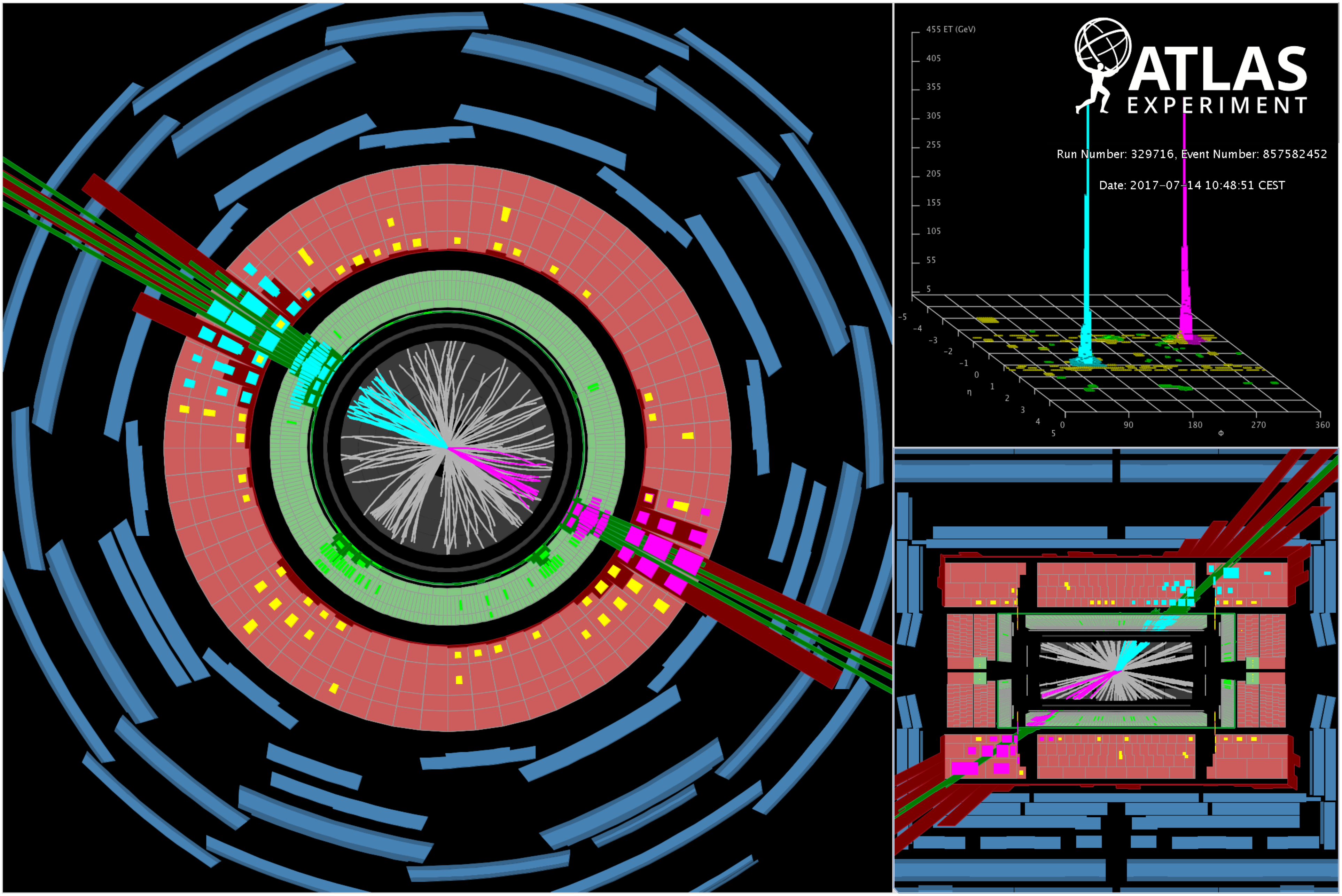
...but wait, there's more (pileup)

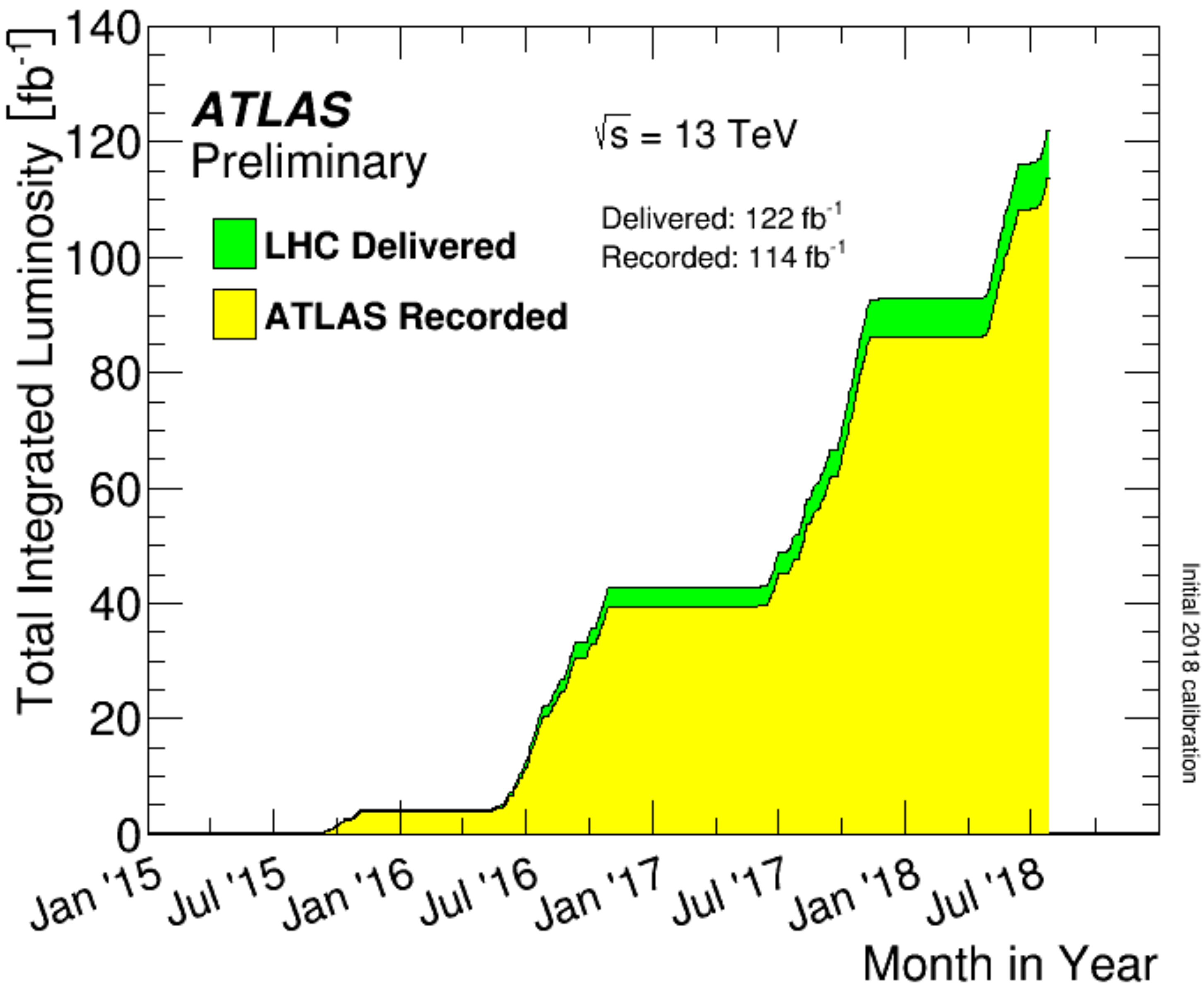
- In order to produce events with low cross sections (such as Higgs boson production), the LHC has to run at high luminosity, i.e. there are many proton-proton collisions in the same beam-crossing as the one which produces the Higgs boson (or other interesting bit of physics)
- The pileup energy is unrelated to the interesting physics and is typically subtracted



Onto the LHC


45 m
24 m
Muon Detectors
Tile Calorimeter
Liquid Argon Calorimeter
Toroid Magnets
Solenoid Magnet
SCT Tracker
Pixel Detector
TRT Tracker


ATLAS
EXPERIMENT
Run Number: 329716, Event Number: 857582452
Date: 2017-07-14 10:48:51 CEST
455 ET (GeV)
405
355
305
255
205
155
105
55
5
η
-5
-4
-3
-2
-1
0
1
2
3
4
5
φ
0
90
180
270
360



LHC luminosity times hadronic cross section equals # events (modulo acceptance factors)