Introduction to QCD from an LHC perspective

从大型强子对撞机看量子色动力学

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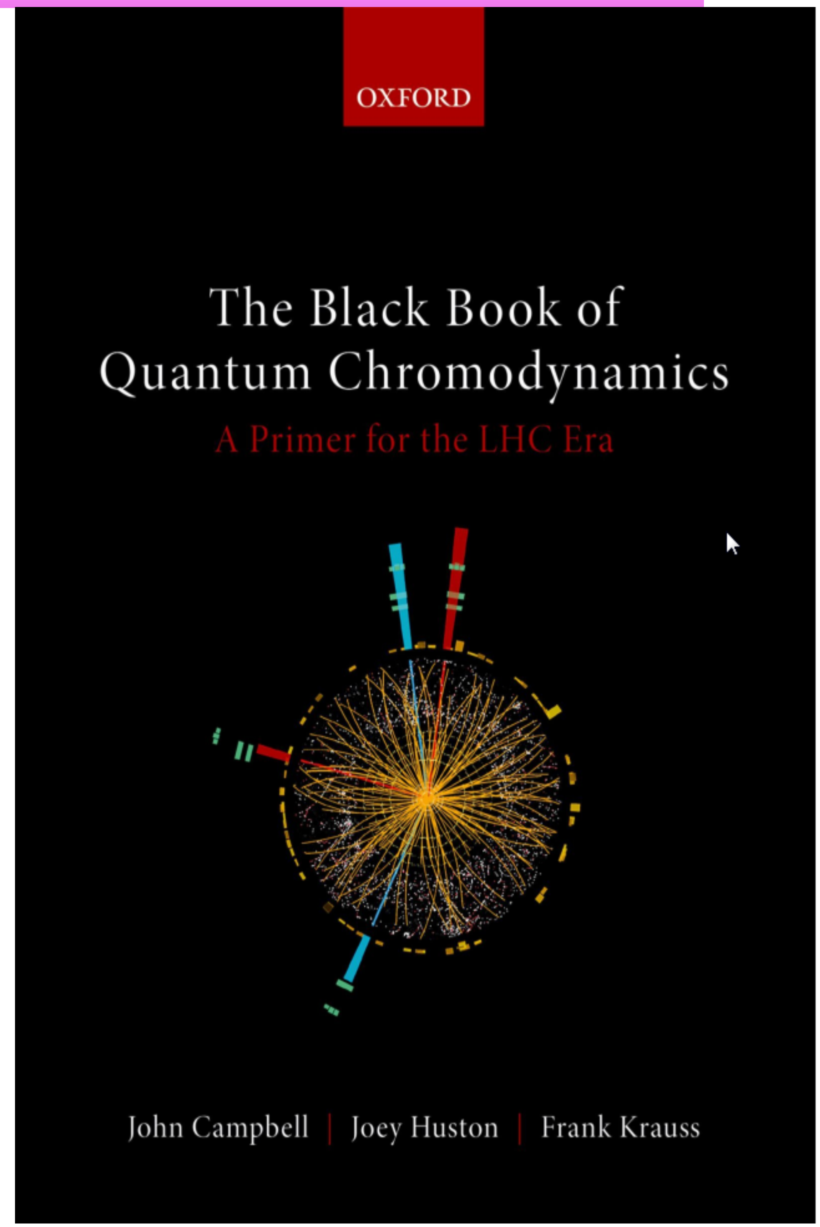
Caveat

- I' m not a theorist

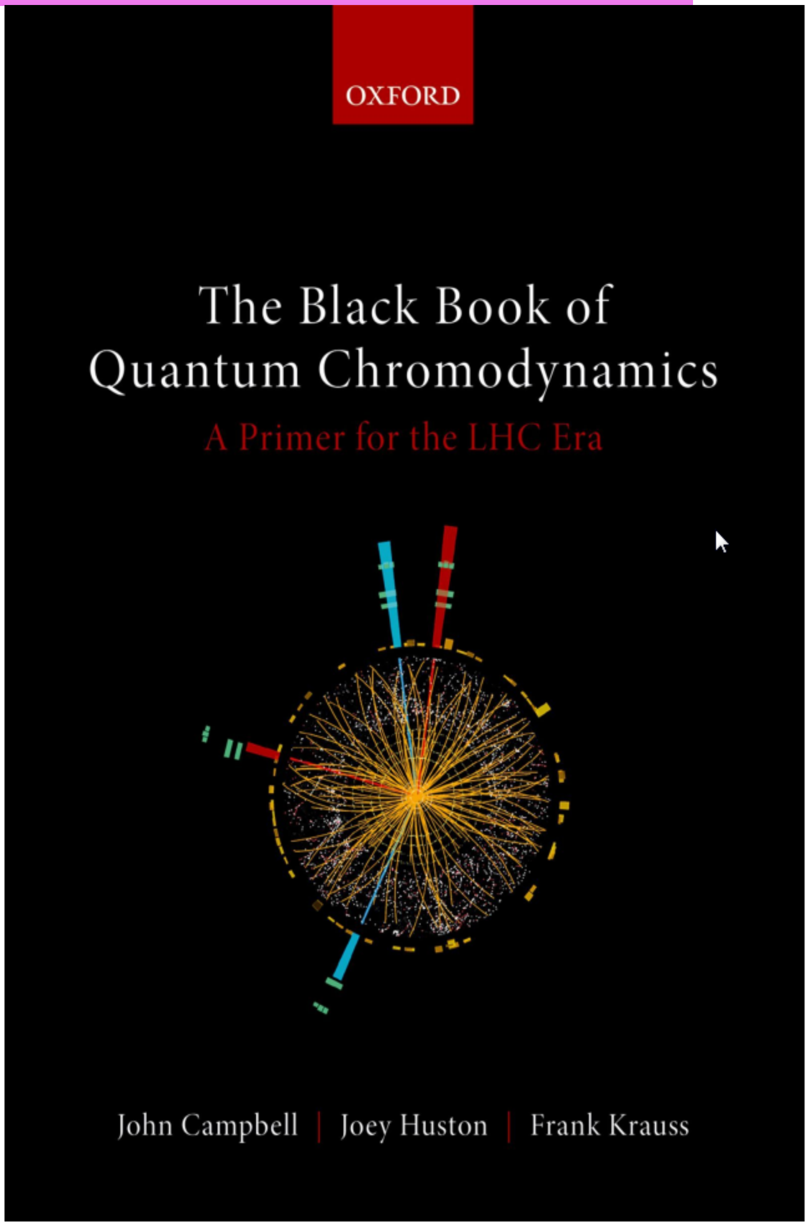
I'm an experimentalist; note the hard-hat

- I don' t even play one on TV
  - although I like the 'Big Bang Theory'
- So my lectures are not going to be in as much technical detail as a theorist would
  - because I probably wouldn' t get the details right
  - and I like the intuitive "rules-of-thumb" approach better
- But I did write a book on QCD
  - my co-authors are theorists and did get the details right
- Some of my notation is from the book, some from a review article I wrote (hep—ph/0611148)



Thanks

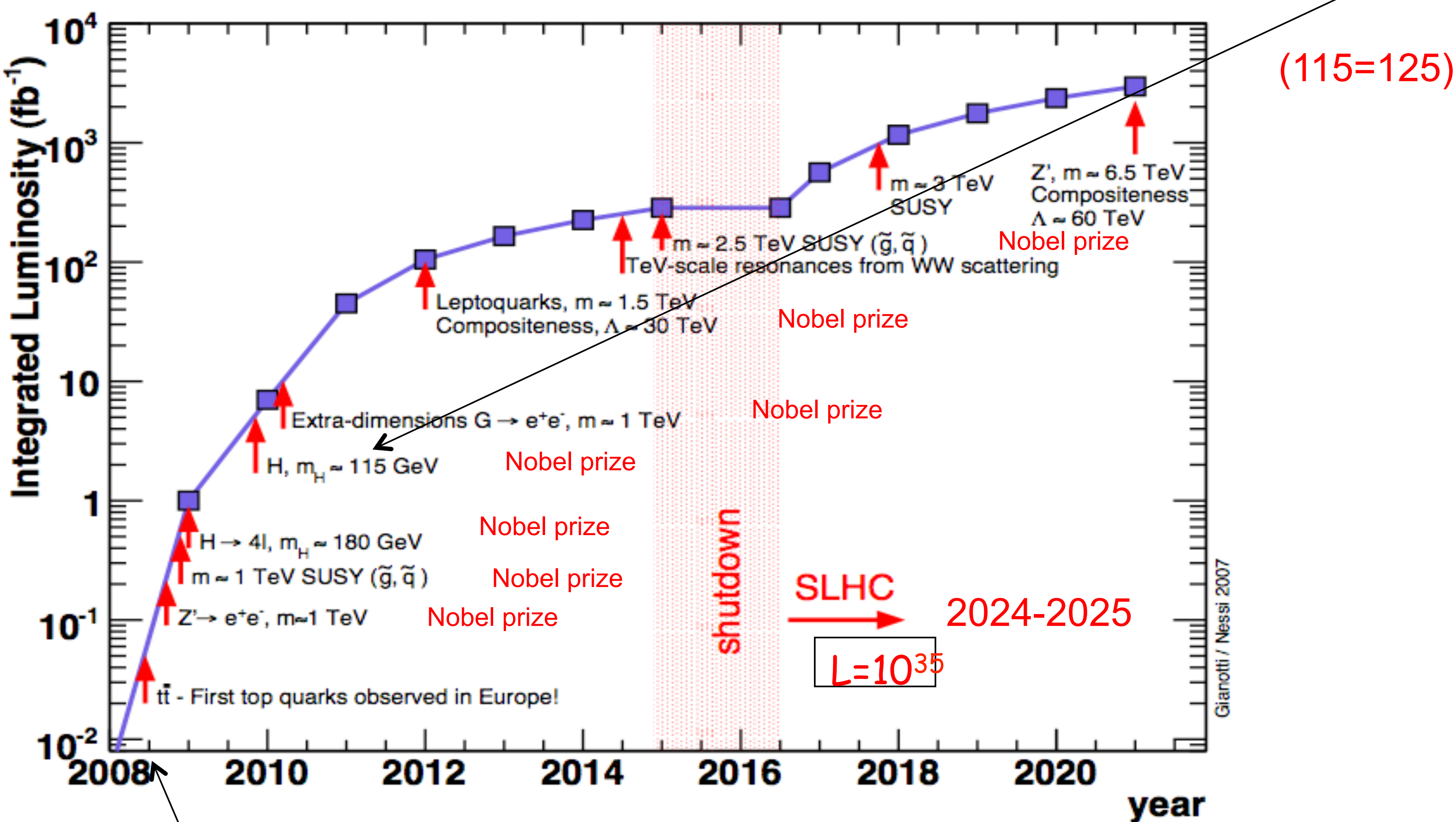
- Thanks to my colleagues John Campbell and Frank Krauss
- Thanks also to G. Ingelmann and K. Ellis from whom I've borrowed a few transparencies



Timeline for LHC discoveries (circa 2006)

LHC vs time: a wild guess ... and wrong so far, but

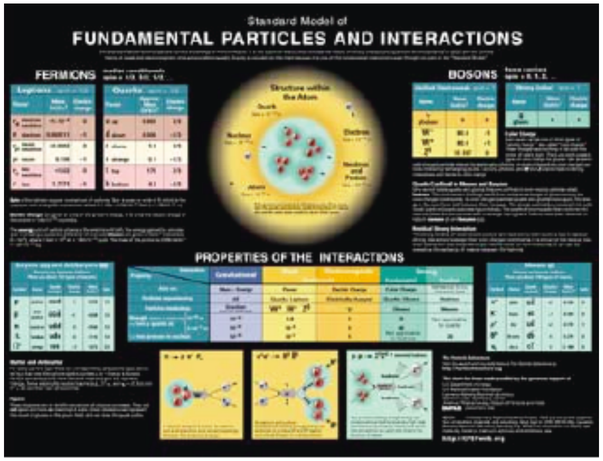
(115=125)

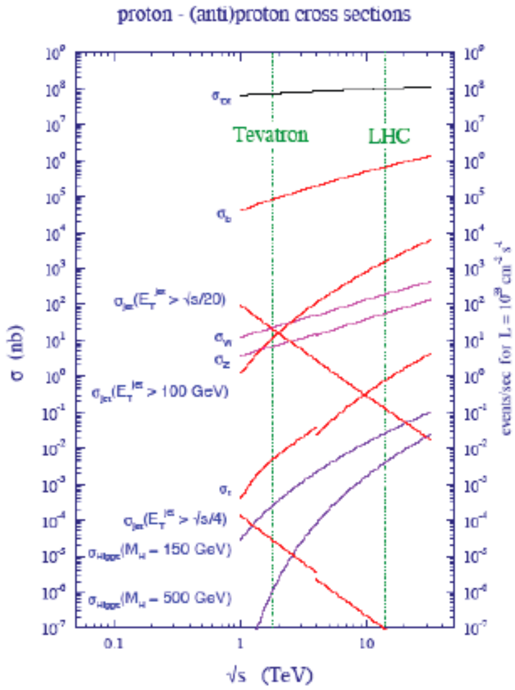


ok, we slipped by a few years

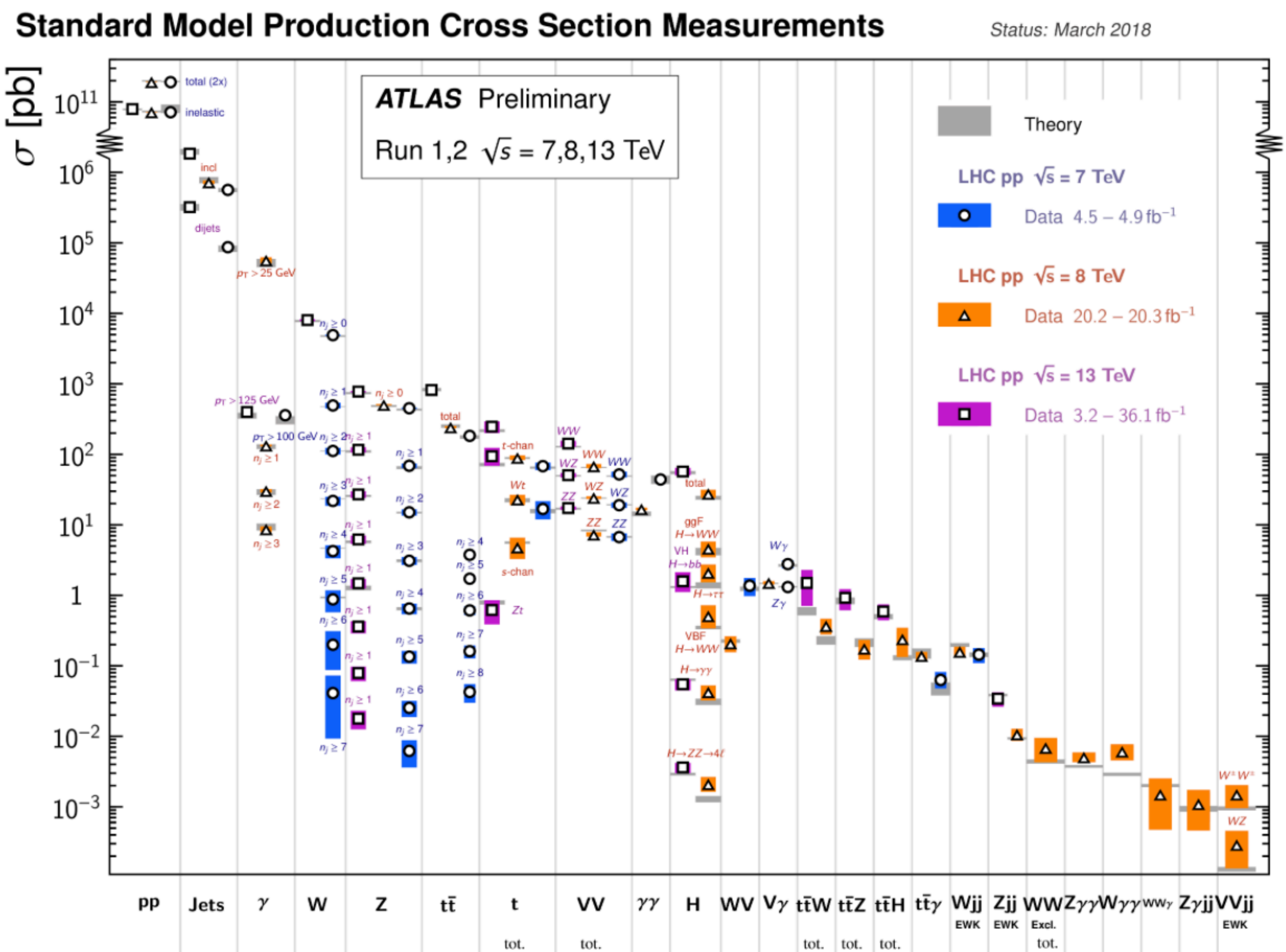
Understanding cross sections at the LHC

- We haven't gone to Stockholm yet to collect our Nobel prizes, but we have had to understand the Standard Model at the LHC
  - in fact, I coined the phrase "Re-discover the Standard Model"
- We're all looking for BSM physics at the LHC
- Before we publish BSM discoveries from the LHC, we had to/are having to make sure that we measure/understand SM cross sections
  - detector and reconstruction algorithms operating properly
  - SM backgrounds to BSM physics correctly taken into account
  - and, in particular, that QCD at the LHC is properly understood





...and we have rediscovered the Standard Model and have measured a lot of (SM) cross sections



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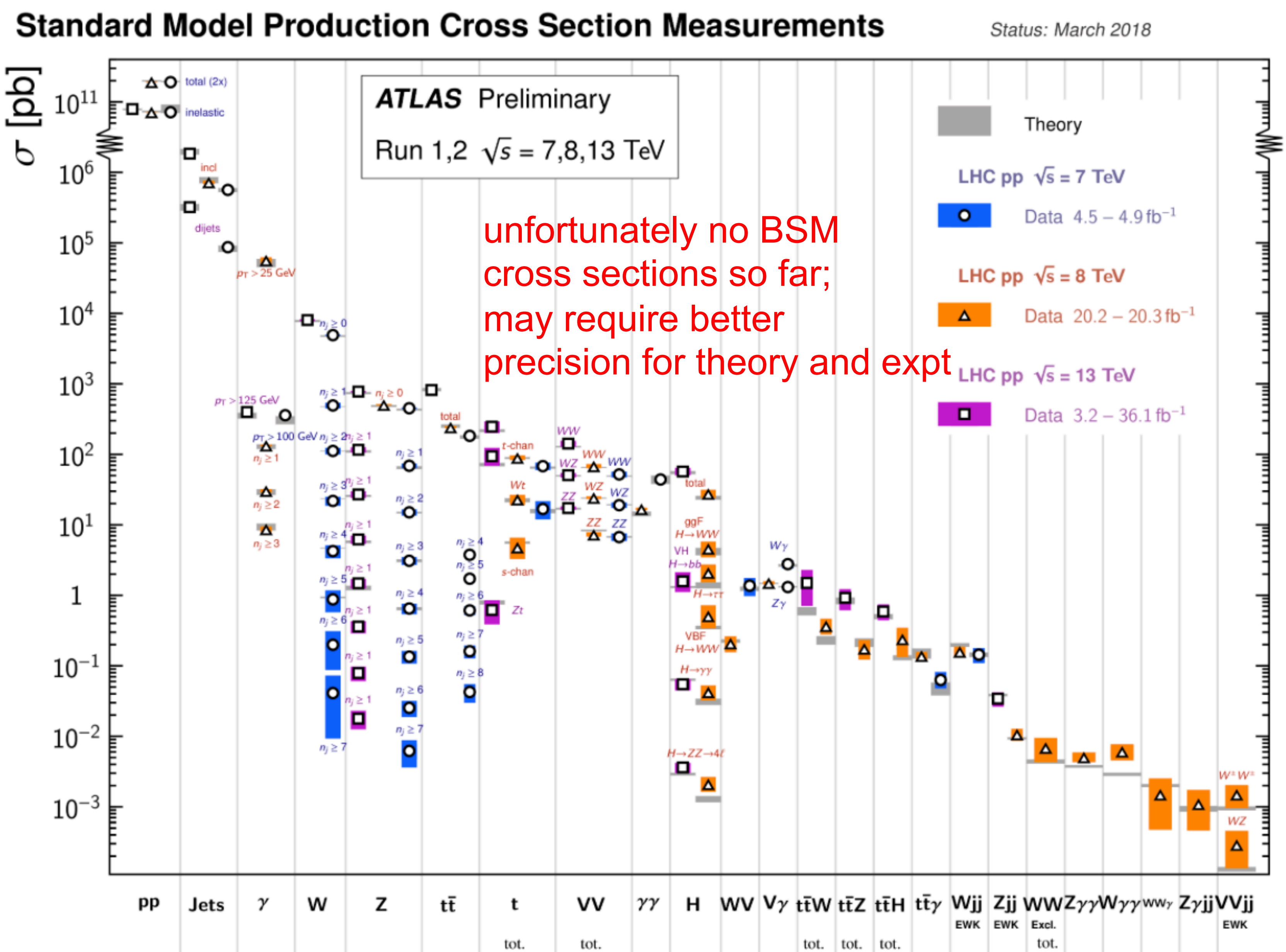


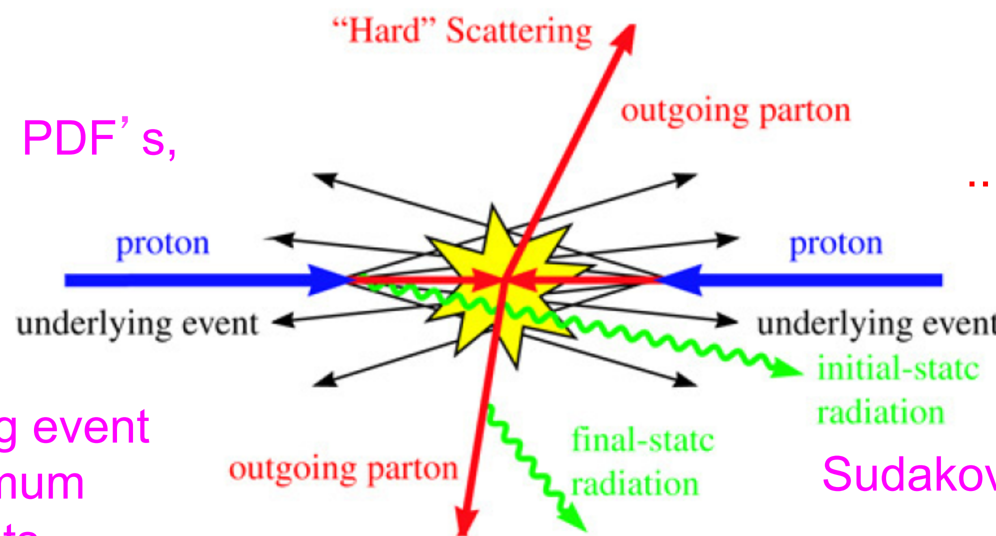
unfortunately no BSM cross sections so far; may require better precision for theory and expt

Understanding cross sections at the LHC

LO, NLO and NNLO calculations
K-factors

PDF' s,

...from Rick Field



underlying event
and minimum
bias events

Sudakov form factors

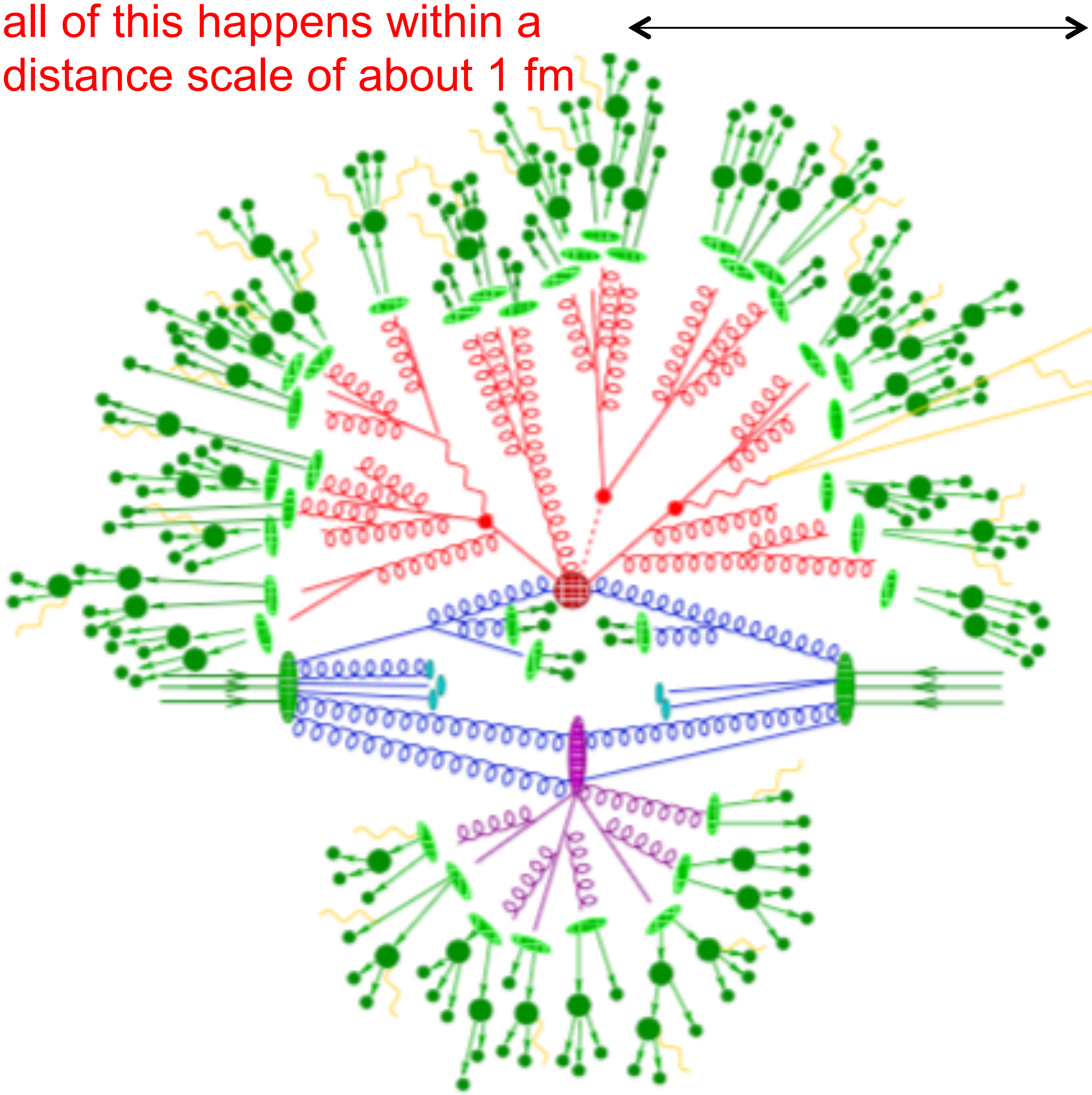
jet algorithms and jet reconstruction

…means understanding QCD

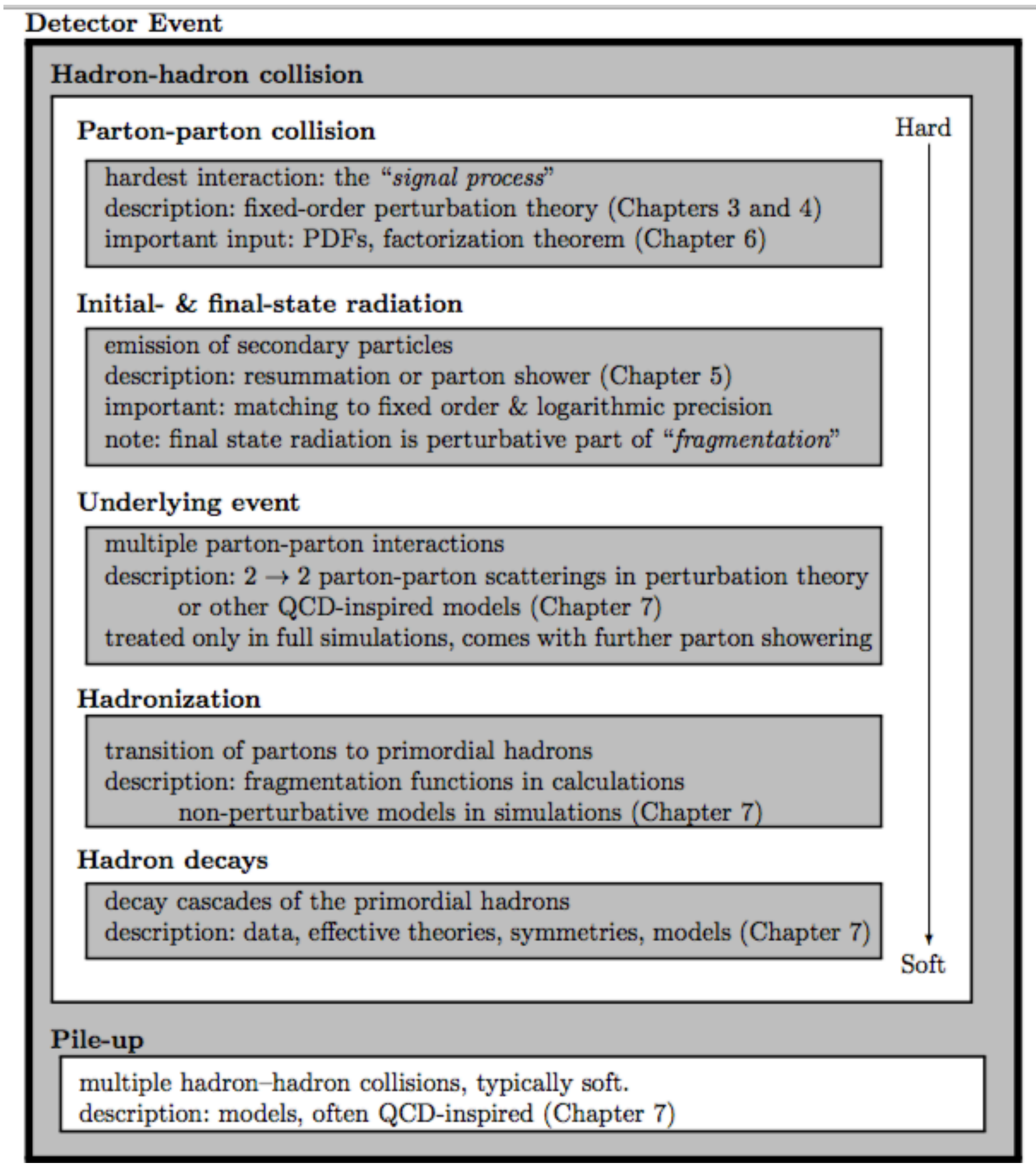
I'll give an introduction to some of these topics.
Will be followed up by other lecturers.

...or, from my co-author Frank Krauss

all of this happens within a distance scale of about 1 fm



substantially more than 1 fm

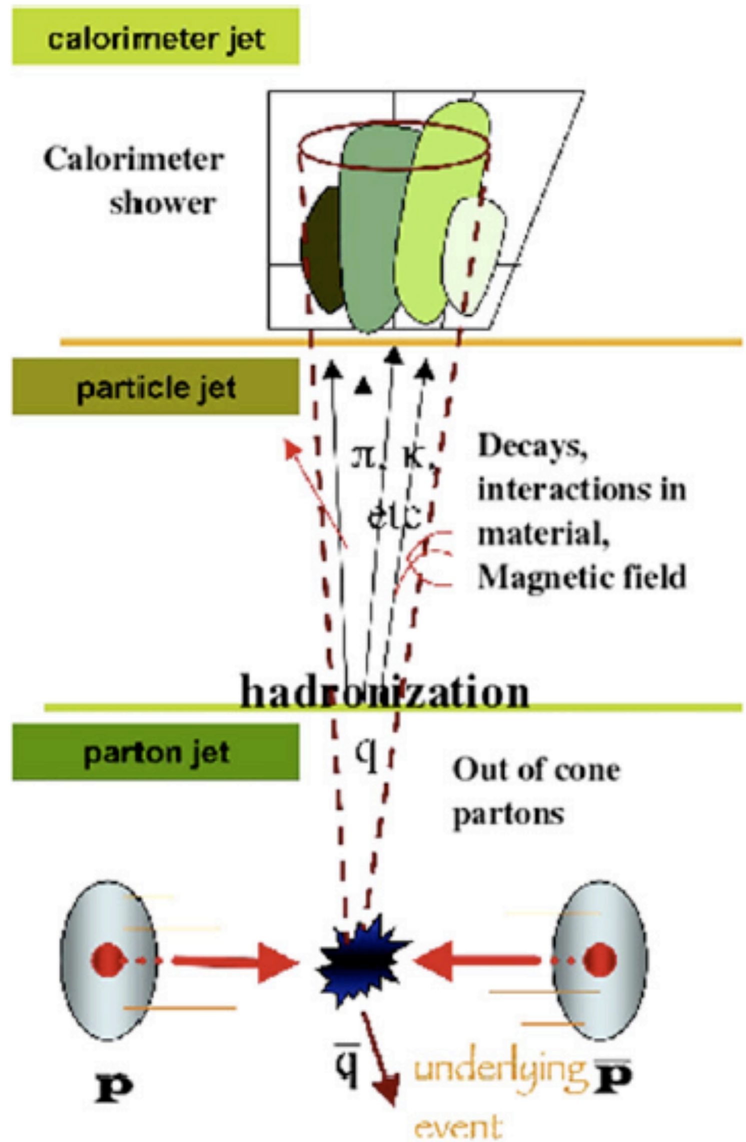


Some definitions (from book)

- The fundamental challenge to interpret experimentally observed final states is that pQCD is most easily applied to the short-distance degrees of freedom, i.e. to quarks and gluons, while the long-distance degrees of freedom seen in the detectors are color-singlet bound states
- The overall scattering process evolves from the incoming long-distance hadrons in the beams, to the short-distance scattering process, to the long-distance outgoing final states
- The separation of these steps is essential both conceptually and calculationally

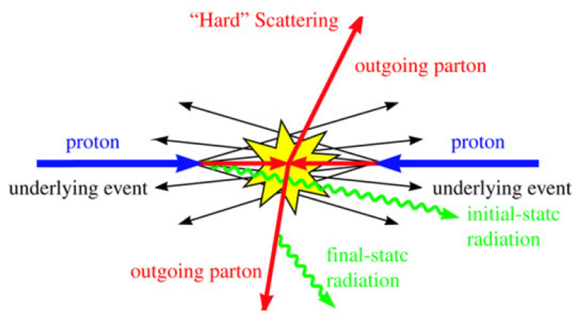
…and a word about jets

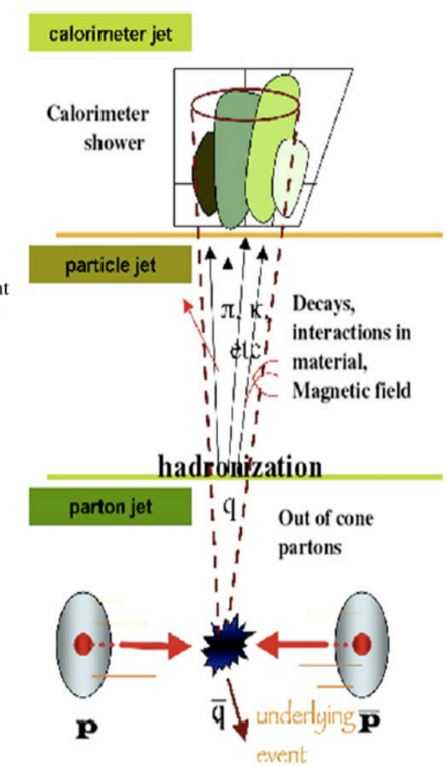
- Most of the interesting physics signatures at the LHC involves final states with jets of hadrons
- A jet is reconstructed from energy depositions in calorimeter cells and/or from charged particle track momenta, and ideally is corrected for detector response and resolution effects so that the resultant 4-vector corresponds to that of the sum of the original hadrons
- The jets can be further corrected , for hadronization effects, back to the parton(s) from which the jet originated,or the theory can be corrected to the hadron level
- The resultant measurements can be compared back to parton shower predictions, or to the short-distance partons described by fixed-order pertubative calculations



…another word about jets

- We pick out from the incident beam particles, the short-distance partons that participate in the hard collision
- The partons selected can emit radiation prior to the short distance scattering leading to initial state radiation*
- The remnants of the original hadrons, with one parton removed, will interact with each other, producing an underlying event
- Next comes the short-distance, large momentum transfer scattering process that may change the character of the scattering partons, and/or produce more partons
  - the cross section for this step is calculated to fixed order in pQCD

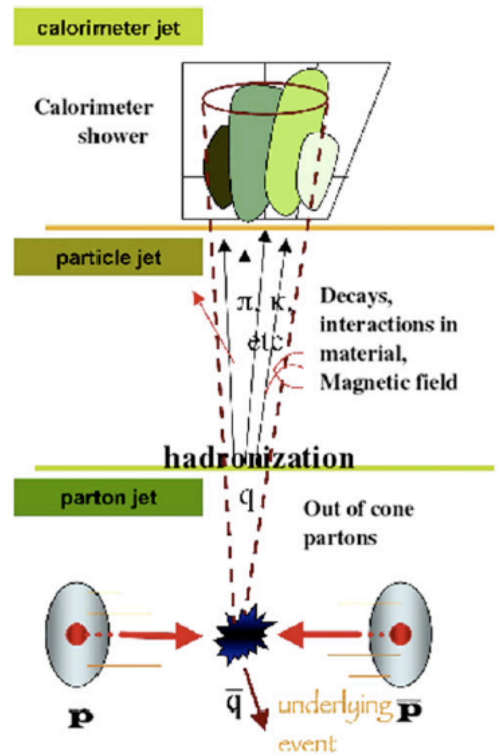


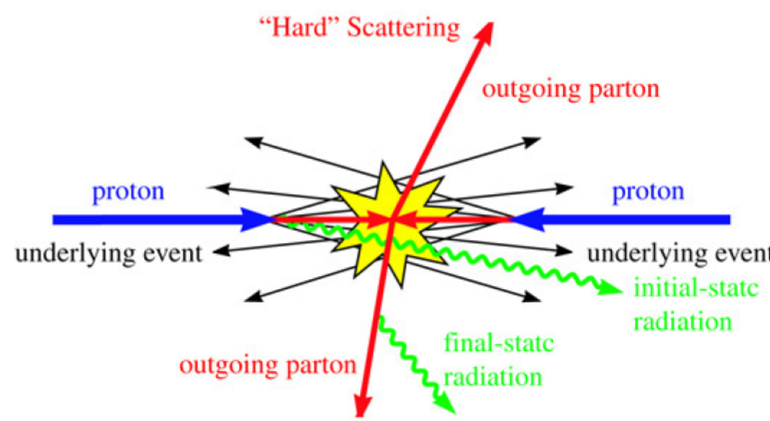


*this is from a Monte Carlo perspective. In performing a fixed order calculation, we actually can't identify whether a gluon is radiated off of the initial state or off of the final state. In fact, there is an interference between the relevant diagrams.

…still another word about jets

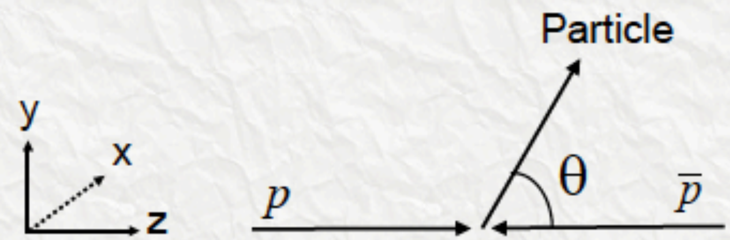
- Then comes another color radiation step, when many new gluons and quark pairs are added to the final state
- The final step in the evolution to the long distance states involves a nonperturbative hadronization process that organizes the colored degrees of freedom
- This non-perturbative hadronization step is accomplished in a model-dependent fashion





Some kinematic definitions

Rapidity (y) and Pseudo-rapidity (η)



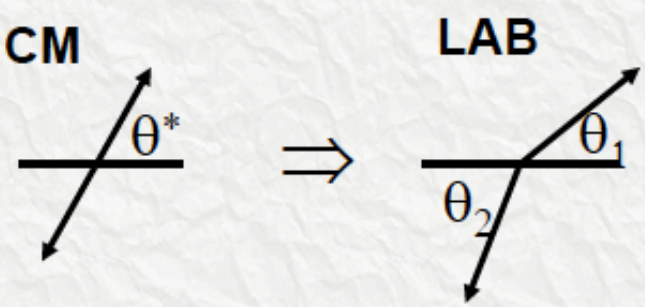
$$y \equiv \frac{1}{2}\ln\frac{E+p_z}{E-p_z} = \frac{1}{2}\ln\frac{1+\beta\cos\theta}{1-\beta\cos\theta}$$

$$\beta\cos\theta = \tanh y \quad \text{where } \beta = p/E$$

In the limit $\beta \to 1$ (or $m << p_T$) then

$$\eta \equiv y\big|_{m=0} = \frac{1}{2}\ln\frac{1+\cos\theta}{1-\cos\theta} = -\ln\tan\frac{\theta}{2}$$

LAB System ≠ parton-parton CM system



Δη and p_T are invariant under longitudinal boosts

Some kinematic definitions

To satisfy listed requirements for jet algorithms, use p_T,y and ϕ to characterize jets

Transverse Energy/Momentum

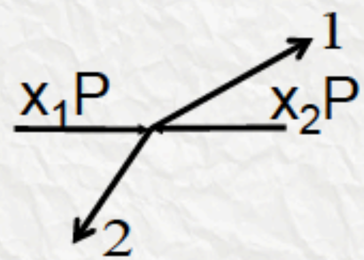
$$E_T^2 \equiv p_x^2 + p_y^2 + m^2 = p_T^2 + m^2 = E^2 - p_z^2$$

$$p_z = E \tanh y$$
$$E = E_T \cosh y$$
$$p_z = E_T \sinh y$$

$$p_T \equiv p \sin\theta \xrightarrow[m \to 0]{} E_T$$

Invariant Mass

$$M_{12}^2 \equiv (p_1^\mu + p_2^\mu)(p_{1\mu} + p_{2\mu})$$
$$= m_1^2 + m_2^2 + 2(E_1 E_2 - \boldsymbol{p}_1 \cdot \boldsymbol{p}_2)$$
$$\xrightarrow[m_1, m_2 \to 0]{} 2E_{T1}E_{T2}(\cosh\Delta\eta - \cos\Delta\phi)$$



Partonic Momentum Fractions

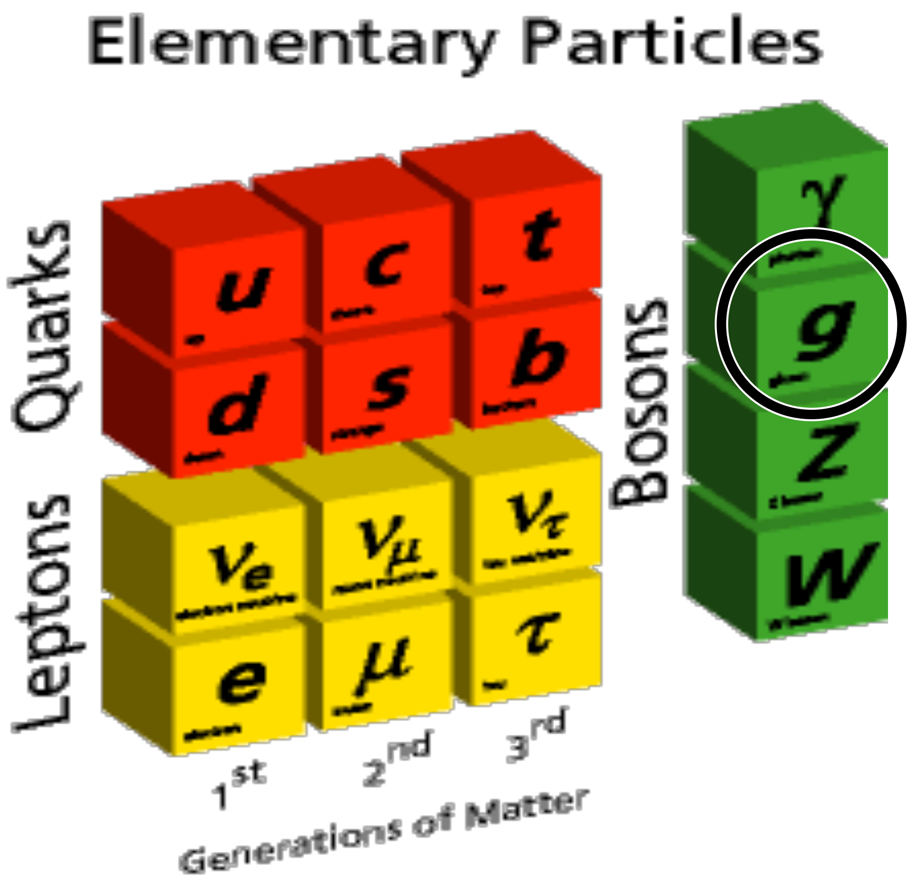
$$x_1 = \left(e^{\eta_1} + e^{\eta_2}\right)E_T \big/ \sqrt{s}$$
$$x_2 = \left(e^{-\eta_1} + e^{-\eta_2}\right)E_T \big/ \sqrt{s}$$

$$\text{Parton CM (energy)}^2 \rightarrow \hat{s} = x_a x_b s$$

$$x_T \equiv 2E_T \big/ \sqrt{s} = x_{1,2}(\eta_{1,2} = 0)$$

$$0 < x_1, x_2 < 1$$
$$x_T^2 < x_1 x_2 < 1$$

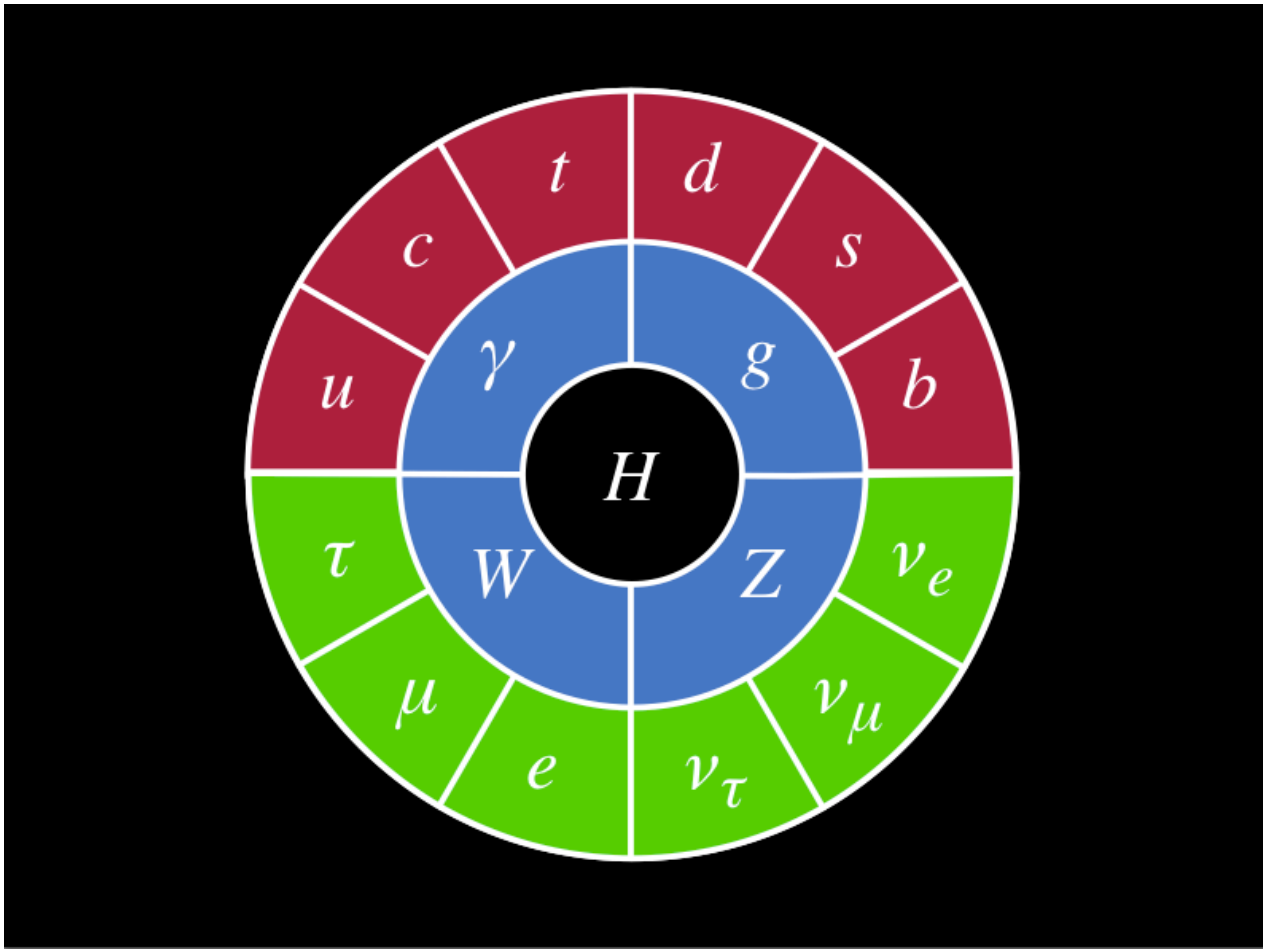
Back to the Standard Model



The Standard Model has been extremely successful, although admittedly incomplete.

In these lectures, we're most interested in QCD and thus the force carrier of the strong force (the gluon) and its interaction with quarks (and with itself).

Oops, we left one Standard Model Particle out



The Standard Model has been extremely successful, although admittedly incomplete.

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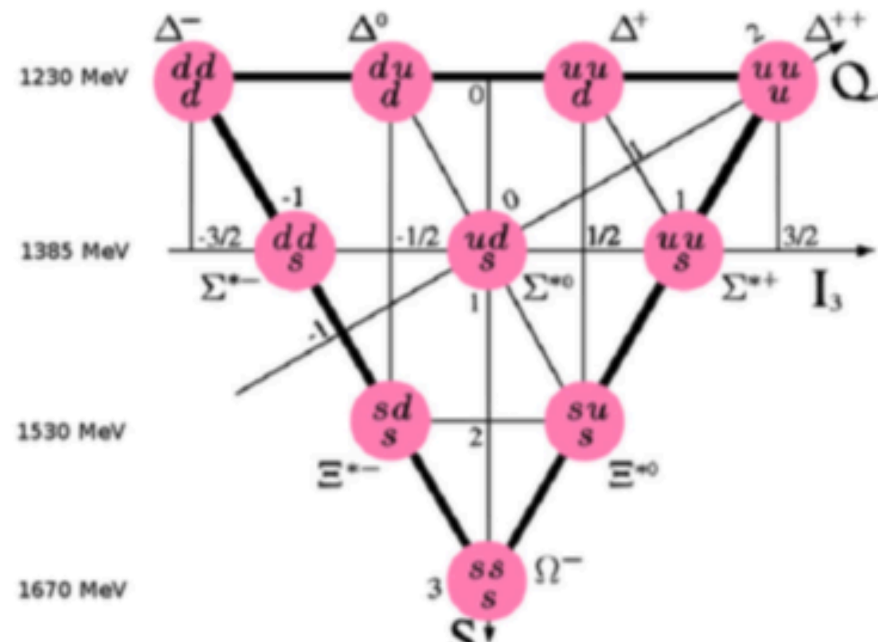
Start with the evidence for existence of the color degree of freedom

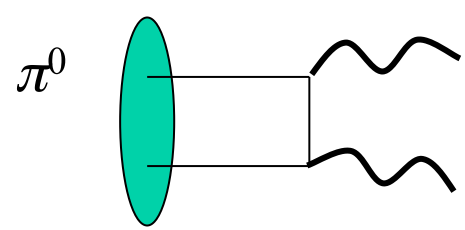
SU(3) of color: evidence

$\Delta^{++} = u\uparrow u\uparrow u\uparrow$

- symmetric in flavor, space, spin
- Fermi-Dirac statistics requires totally anti-symmetric wave function
- introduce new degree of freedom, color
  - red
  - blue
  - green
  - …so r+b+g = white



- $\Gamma(\pi^o \to \gamma\gamma)$ = 7.7+/-0.6 eV
  - theory agrees only if there is an extra factor of 3 (colors) introduced

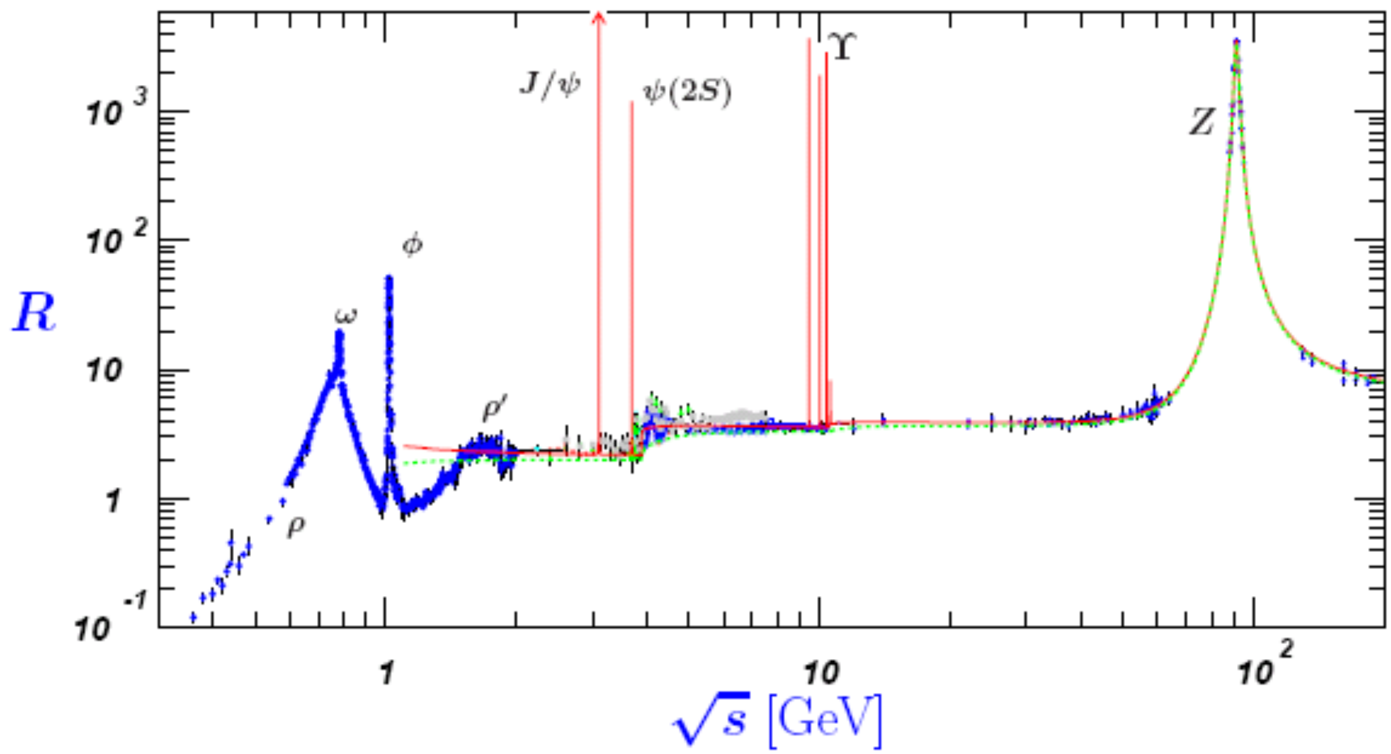


SU(3) of color: evidence

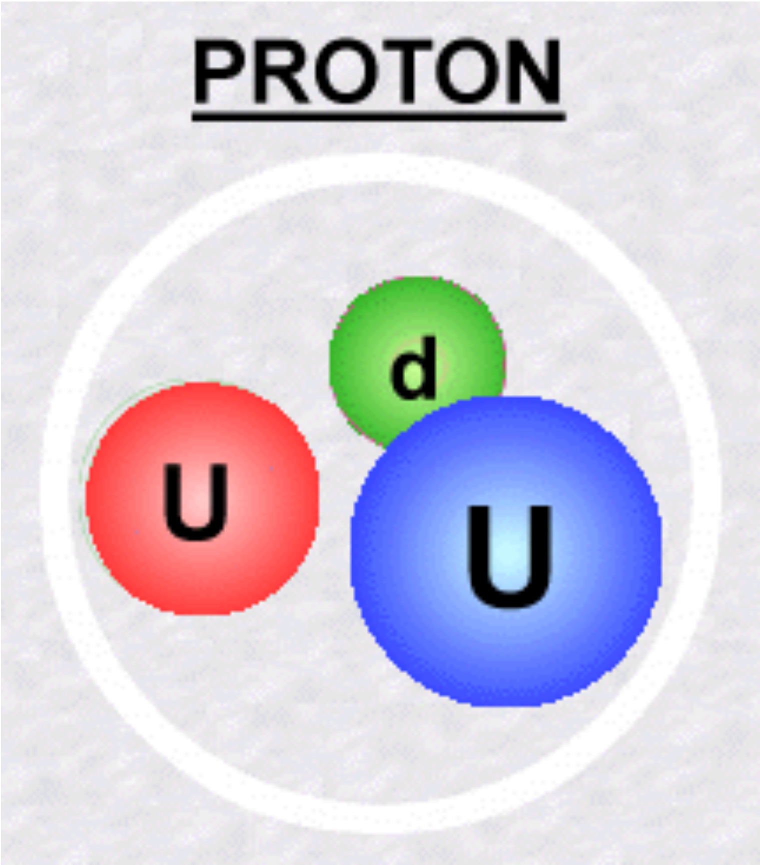
$$R = \frac{\sigma(e^+e^- \to q\bar{q} \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\sum_q e_q^2$$

- with factor of 3 from color fits data

- Plus many checks from later detailed QCD tests

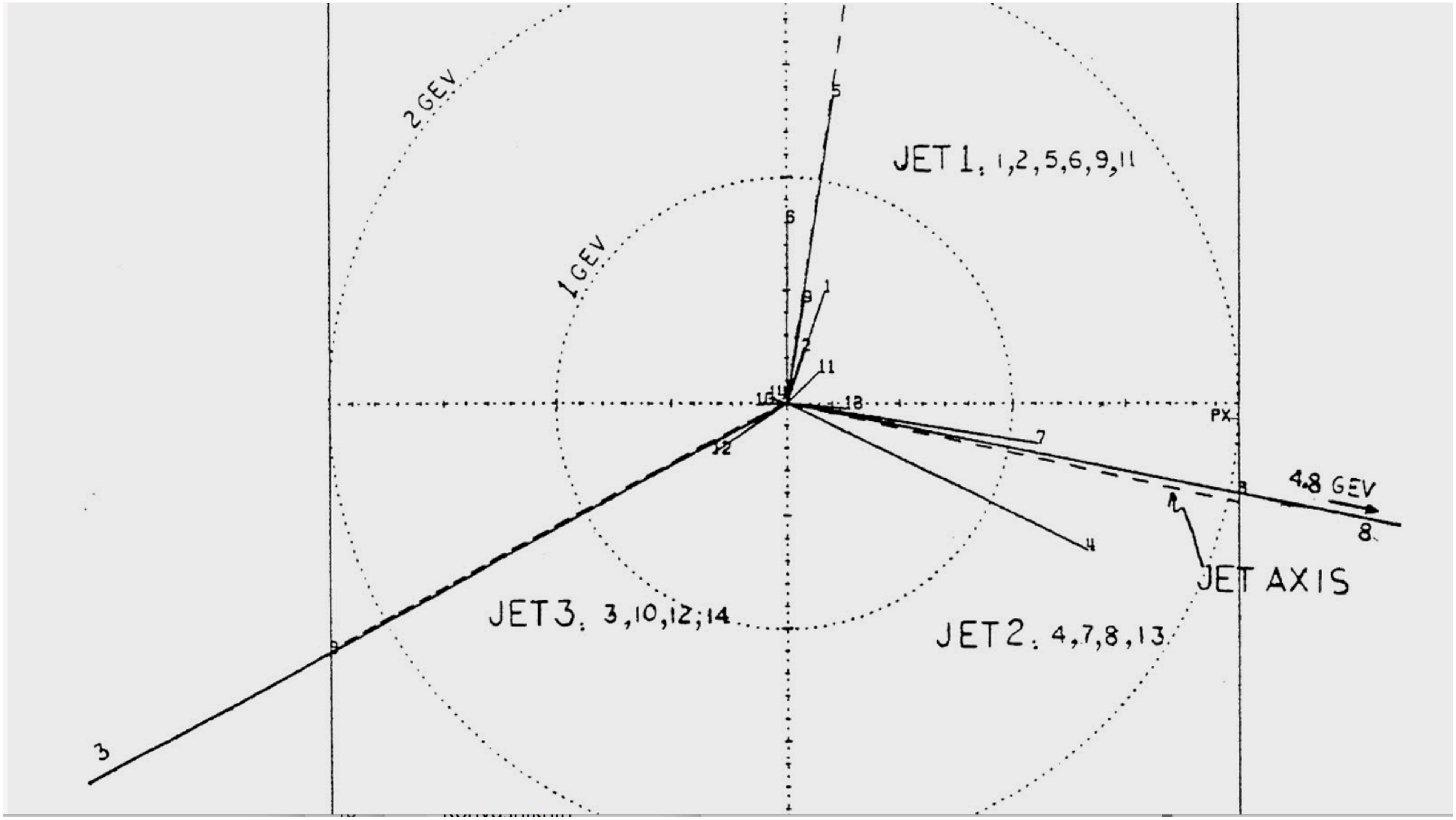


so a proton is...



The gluon itself was discovered in 1979

...at the DESY accelerator in the process e^+e^- -> q-qbar gluon (3 jets in final state)



Gauge=standard of measure/calibration

- Local symmetry -> convention can be decided independently at every space/time point
- Recipe for local gauge symmetry
  - global invariance (gauge symmetry) under a transformation
  - change to local (space-time) dependent transformation ->destroys invariance
  - add new field(s) with transformation properties that compensates and restores the invariance
    - Lagrangian with local gauge invariance and interactions
- Electromagnetism: global ->local charge symmetry
  - invariance restored by introducing vector potential A (magnetic field)
- Strong interaction: global ->local color symmetry
  - invariance under local color transformations restored by gluon field

Construct the QCD Lagrangian

- Quarks come in 3 colors i: $\psi^q == q^i$ (red, blue, green)
  - in fundamental representation of QCD
- They interact with 8 gluons: G^a (red+$\overline{\text{blue}}$, blue+$\overline{\text{green}}$,...)
  - in the adjoint representation of QCD: color+anti-color
- Have to add gauge-fixing term
  - Fadeev-Popov ghosts

more freedom for gluons in Lagrangian than for physical gluon; result is independent of gauge

$$\mathcal{L}_{QCD} = \sum_q \bar{q}^i \left(i \not{D}_{ij} - m_q \delta_{ij}\right) q^j - \frac{1}{4} G^a_{\mu\nu} G^{a,\,\mu\nu} + \mathcal{L}_{g.f.}$$

- Gauge-covariant derivative through 8 Gell-Mann matrices: T_{ij}^a

$$D^\mu_{ij} = \partial^\mu \delta_{ij} - i g_s G^{a,\,\mu} T^a_{ij}$$

- Gauge-kinetic term through structure constants f^{abc} giving tensors

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu$$

3rd term is the non-Abelian term (QCD≠QED)

Construct the QCD Lagrangian

$$\mathcal{L}_{QCD} = \sum_q \bar{q}^i \left(i \not{D}_{ij} - m_q \delta_{ij} \right) q^j - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} + \mathcal{L}_{g.f.}$$

describes the interactions of spin ½ quarks with mass m, and massless spin 1 gluons

field strength tensor derived from gluon field A

Color algebra

- Algebra

$$\left[T^a,\, T^b\right] = i f^{abc}\, T^c$$

generators in adjoint representation, describes self-coupling of gluons

- Normalization of generators

$$\mathrm{Tr}\left[T^a T^b\right] = T^a_{ii} T^b_{ii} = T_R\, \delta^{ab},$$

  - where $T_R=1/2$

- Product

$$\sum_a T^a_{ij} T^a_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{kj} - \frac{1}{N}\,\delta_{ij}\delta_{kl}\right)$$

  - for SU(N), i.e. N==3 for QCD

...continuing

- In particular, for self-energy, etc

$$\sum_a T^a_{ij} T^a_{jl} = \frac{1}{2}\frac{N^2-1}{N}\delta_{il} = C_F \delta_{il}$$

  - where Casimir operator of fundamental representation

    $C_F = \frac{N^2-1}{2N}$ **N=3; C_F=4/3**

  - is the color charge of the quark

- Similarly, from the expression for the structure constants

$$f^{abc} = -2i\,\mathrm{Tr}\left[\left[T^a,\ T^b\right] T^c\right]$$

- One finds

$$\sum_{a,b} f^{abc} f^{abd} = C_A \delta^{cd}$$

  - where the Casimir operator of the adjoint representation C_A=N is the the color charge of the gluon **N=3; C_A=3**

- Fundamental, adjoint representations?
- Let’s ask google

touqra #1 Mar 28, 2007

I find it awkward that quarks are in fundamental representation of SU(3) while gluons are in adjoint representation of SU(3). Is there a reason as to why this is the case? Why aren't they in the same representation or in the current specific representation?

marlon #2 Mar 29, 2007

Why is this awkward ? Good question ! Basically you are asking why there are 8 gluons and not 9.

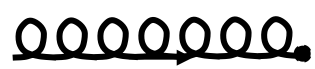
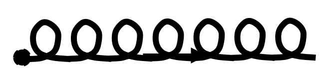
Quarks interact with each other through gluons. Indeed, colour charges (ie quarks) interact via the exchange of colour. Thus, a gluon is able to change the quark's colour quantum number.

Now, if gluons were NOT in the adjoint representation, but any other fundamental SU(3) representation, there would be 9 of them because these representations are 9 dimensional. So why are there 8 gluons ?

One gluon is special : if you make a linear combination of (red-antired + blue-antiblue + green-antigreen)/sqrt(3), you just made a gluon that CANNOT change the colour of a quark. Do you see why ? This gluon is a singlet state and does not respect the definition of a gluon (ie a force carrier that can change the colour of quarks). So, in total we have not 9 but 8 gluons ! To describe these 8 gluons, we need an 8 dimensional space : this is the adjoint space representation of SU (3).

Here is more : http://math.ucr.edu/home/baez/physics/ParticleAndNuclear/gluons.html

Feynman rules of QCD: external, on-shell particles

object ⇒	diagram ⇒	in amplitude
initial quark		$u_f^{\alpha}(p,s)$
final quark		$\bar{u}_f^{\alpha}(p,s)$
initial anti-quark		$v_f^{\alpha}(p,s)$
final anti-quark		$\bar{v}_f^{\alpha}(p,s)$
initial gluon		ε^{μ}
final gluon		$\varepsilon^{*\mu}$

u,v = spinor wave fcn, ε^{μ} = gluon polarisation vector

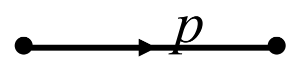
p = 4-momentum (p^{μ} with μ =0,1,2,3 ⇔ E,p_x,p_y,p_z)

s = spin, f = quark flavour, α = 1,2,3 for quark colour

Feynman rules of QCD: internal, off-shell particles

object ⇒ diagram ⇒ in amplitude

quark propagator

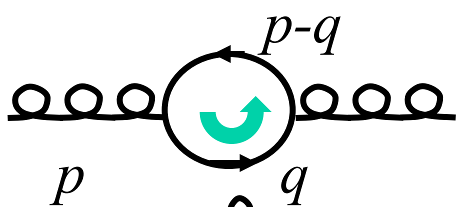


$$i\delta^{\alpha\beta}\frac{1}{\not{p}-m_f+i\varepsilon}=i\delta^{\alpha\beta}\frac{\not{p}+m_f}{p^2-m_f^2+i\varepsilon}$$

gluon propagator

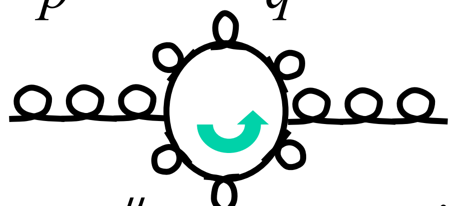
$$-i\delta^{ab}\left[\frac{g_{\mu\nu}}{p^2+i\varepsilon}-\frac{(1-\lambda)p_\mu p^\nu}{(p^2+i\varepsilon)^2}\right]$$

quark loop



$$-\int_0^\infty\frac{d^4q}{(2\pi)^4}\cdots$$

gluon loop



$$+\int_0^\infty\frac{d^4q}{(2\pi)^4}\cdots$$

p,q = 4-momenta, $\not{p}=p^\mu\gamma_\mu$, γ_μ = Dirac matrices, $g_{\mu\nu}$ = metric tensor

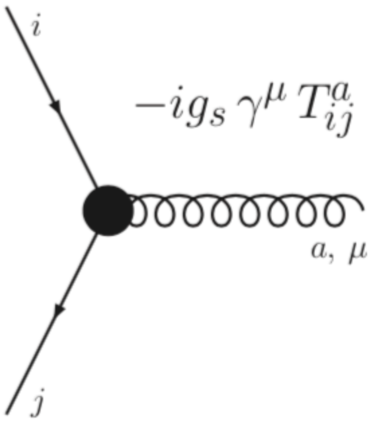
m_f = mass of quark flavour f

ε = infinitesimal to handle poles for p on-shell

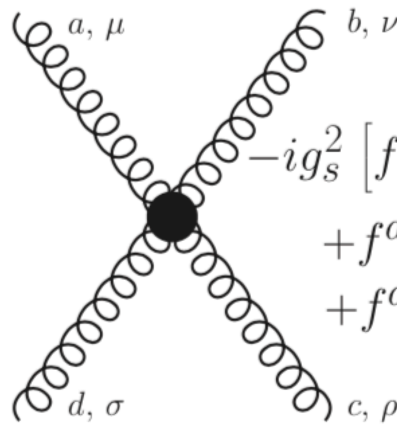
$SU(3)_{colour}$: $a=1,2\ldots8$ for 'gluon', $\alpha,\beta=1,2,3$ for 'quark'

λ = gauge fixing parameter, Feynman: $\lambda=1$ → simple g-propagator

Feynman rules: vertices

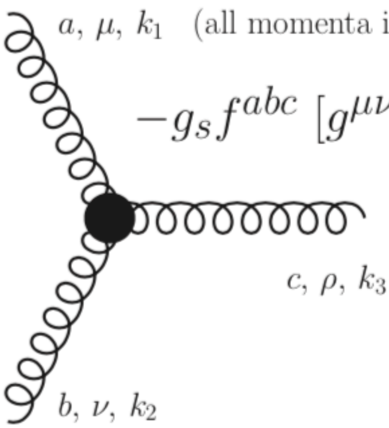


$$-ig_s\,\gamma^\mu\,T^a_{ij}$$



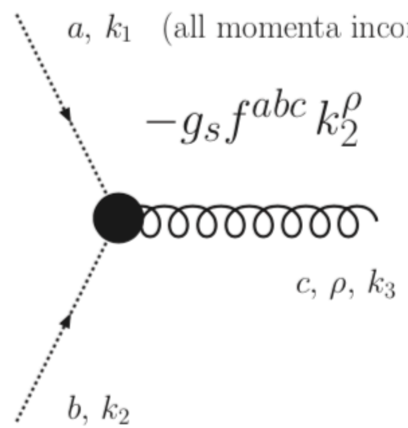
(all momenta incoming)

$$-ig_s^2\left[f^{abe}f^{cde}\,(g^{\mu\rho}g^{\nu\sigma}-g^{\mu\sigma}g^{\nu\rho}) + f^{ace}f^{bde}\,(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\sigma}g^{\nu\rho}) + f^{ade}f^{bce}\,(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\rho}g^{\nu\sigma})\right]$$



(all momenta incoming)

$$-g_s f^{abc}\,[g^{\mu\nu}(k_1-k_2)^\rho + \text{cycl.}]$$



(all momenta incoming)

$$-g_s f^{abc}\,k_2^\rho$$

pQCD 101 - Use QCD Lagrangian to Correct the Parton Model

- Naïve QCD Feynman diagrams exhibit infinities at nearly every turn, as they must in a conformal theory with no "bare" dimensionful scales (ignore quark masses for now).***

First consider life in the Ultra-Violet – short distance/times or large momenta (the Renormalization Group at work):

- The UV singularities mean that the theory

➢ does *not* specify the strength of the coupling in terms of the "bare" coupling in the Lagrangian

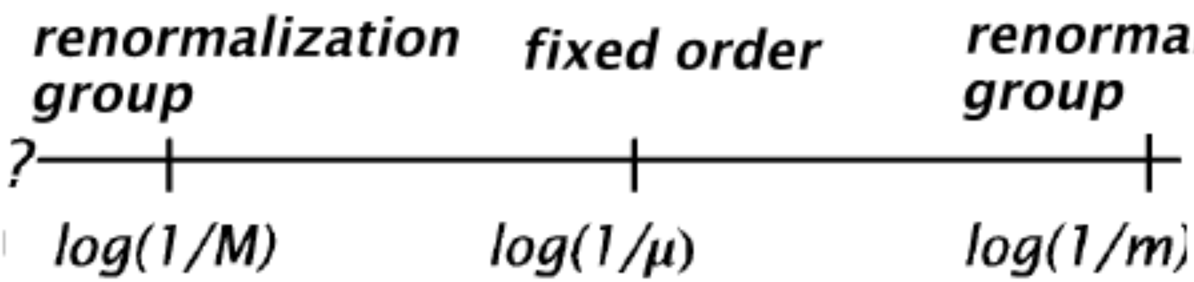
➢ does specify how the coupling varies with scale [$\alpha_s(\mu)$ measures the "charge inside" a sphere of radius $1/\mu$]

*** **Typical of any renormalizable gauge field theory. This is one reason why String theorists want to study something else! We will not discuss the issue of choice of gauge. Typically axial gauges ($\hat{n} \cdot \vec{A} = 0$) yield diagrams that are more parton-model-like, so-called physical gauges.**

Consider a range of distance/time scales – 1/μ

- use the renormalization group below some (distance) scale 1/m (perhaps down to a GUT scale 1/M where theory changes?) to sum large logarithms ln[M/μ]

- use fixed order perturbation theory around the physical scale 1/μ ~ 1/Q (at hadronic scale 1/m things become non-perturbative, above the scale M the theory may change)



Short distance Long distance

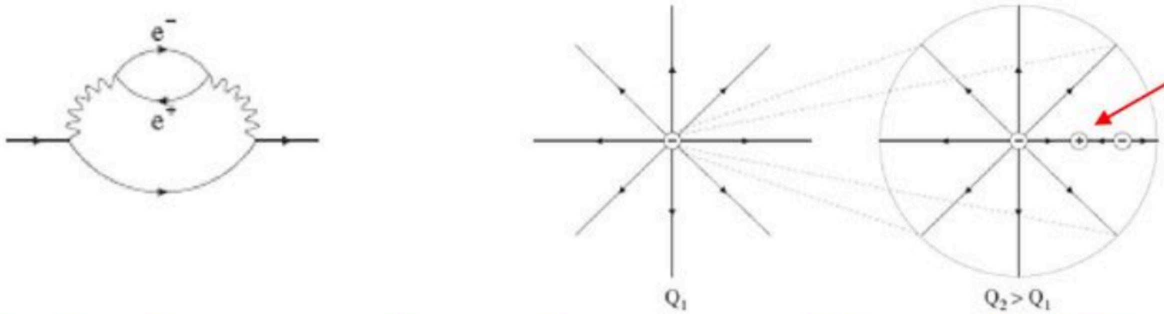
new physics?? perturbative non-perturbative

α and α_s

- The coupling constants for QCD (α_s) and for electromagnetism (α) are not constant, but instead change with the hardness of the interaction
- But in a different way for electromagnetic interactions
- Which we can understand intuitively

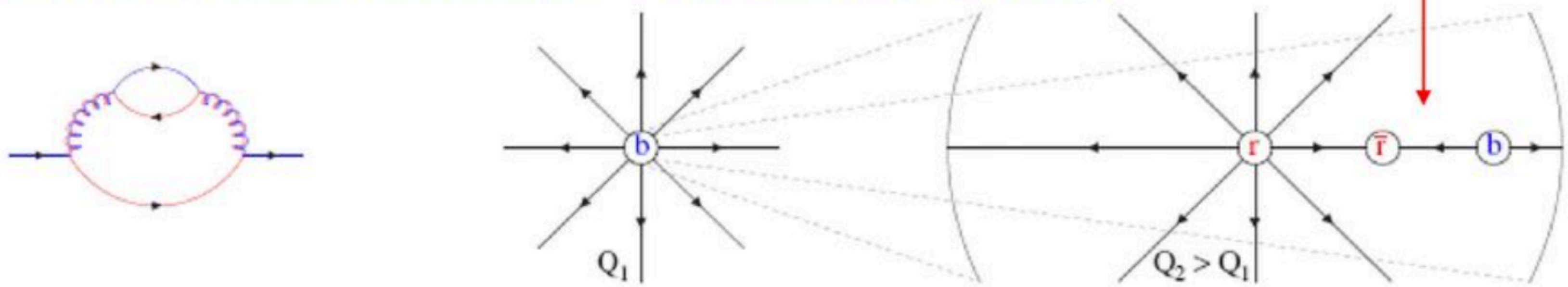
Running couplings: QED vs QCD

QED: Quantum fluctuations polarise vacuum and *screen* electron charge at large dist.

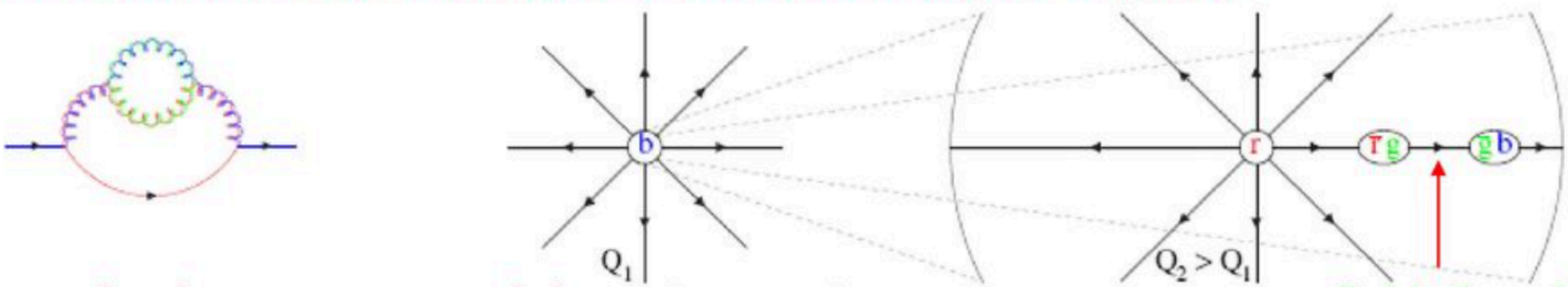


opposite field direction
$\rightarrow$ weakens field

QCD: Quark vacuum fluctuation $\rightarrow$ screening (as in QED)



QCD: Gluon vacuum fluctuation $\rightarrow$ anti-screening (not in QED)



Gluons dominate $\rightarrow$ net result is *anti-screening*

same field direction
$\rightarrow$ strengthens field

G. Ingelman

QED and QCD coupling constants

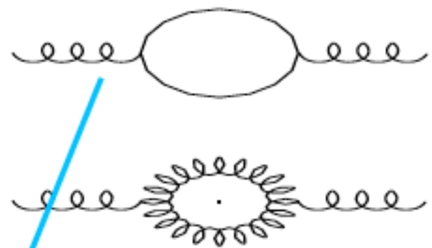
QED:

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 + \frac{\beta_0}{2\pi}\alpha(\mu^2)\log\frac{Q^2}{\mu^2}}$$

$$\beta_0 = \frac{2}{3}$$

Low resolution:
charge is screened
by ee-pairs
High resolution :
charge is big

QCD:



$$\beta_0 = \frac{2}{3} T_F N_F - \frac{11}{3} C_A = \frac{2 N_F - 11 N_C}{3}$$

generated by $q\bar{q}$ fluctuations (as in QED)

gluonic self interaction

N_F number of fermions
N_c number of colours
$T_F = \frac{1}{2}, C_A = N_c$ colour factors

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + b_0\, \alpha_s(\mu^2) \log\frac{Q^2}{\mu^2}}$$

positive term, since $b_0 < 1$

where $b_0 = \frac{-\beta_0}{4\pi}$

$\beta_0 < 0$ for $N_F \leq 16$
$\rightarrow$ anti - screening
charge is spread - out
by gluons, i.e. at infinite
resolution charge is
very small

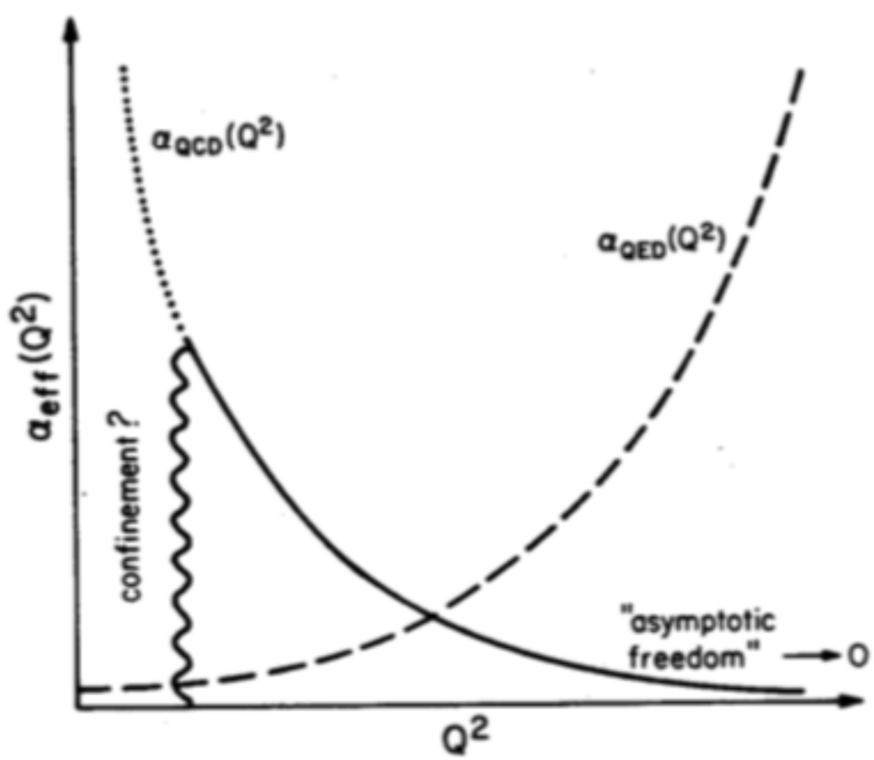
Note there can be different conventions, i.e. β_o can be defined so it's positive, but then $b_o = +\beta_o/4\pi$

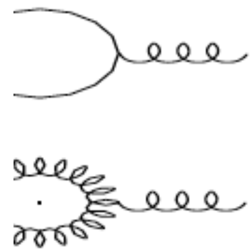
QED and QCD coupling constants

QED:

$$\alpha(Q^2) = \frac{\alpha(}{1 + \frac{\beta_0}{2\pi}\alpha(}$$

Low resolu
charge is s
by ee-pair
High resol
charge is l





$- C_A =$

nicselfinteraction

$_F$ number of fermions

$_c$ number of colours

$= \frac{1}{2}, C_A = N_c$ colour factors

$\alpha_s(Q^2) =$

μ^2

positive term, since $b_0 < 1$

where $b_0 = \frac{-\beta_0}{4\pi}$

→ anti - screening
charge is spread - out
by gluons, i.e. at infinite
resolution charge is
very small

The beta function itself is a series

- as in QED, coupling changes with renormalisation scale μ_R (running coupling)

$$\mu_R^2 \frac{\partial \alpha(\mu_R^2)}{\partial \mu_R^2} = \beta(\alpha)$$

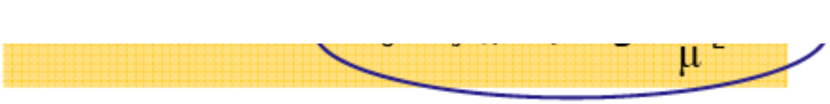
with

$$-\beta(\alpha) = \sum_{n=0}^{\infty} b_n \, \alpha^{2+n} = \frac{\beta_0}{4\pi} \, \alpha_s^2 + \frac{\beta_1}{(4\pi)^2} \alpha_s^3 + \ldots .$$

and

$$\beta_0 = \frac{11}{3} C_A - \frac{4}{3} T_R n_f \, .$$

β_o is only the first term, but its calculation was enough to result in a Nobel prize for Gross, Wilczek and Politzer in 2004, for work done in 1973!



positive term, since $b_0 < 1$

where $b_o = \frac{-\beta_0}{4\pi}$

→ anti - screening

charge is spread - out by gluons, i.e. at infinite resolution charge is very small

the QCD beta function has now been calculated to 5 loops

It's important that the β function is negative

An important component of all QCD cross sections

interaction

f fermions

f colours

N_c colour factors

α_s and Λ

At 1-loop:

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 + b_0\, \alpha(\mu^2) \log \frac{Q^2}{\mu^2}} \qquad \text{with} \quad b_0 = \frac{33 - 2N_F}{12\pi}$$

μ is arbitrary parameter (left-over from renormalisation)

Choose $\mu = \Lambda$: point where effective coupling becomes large

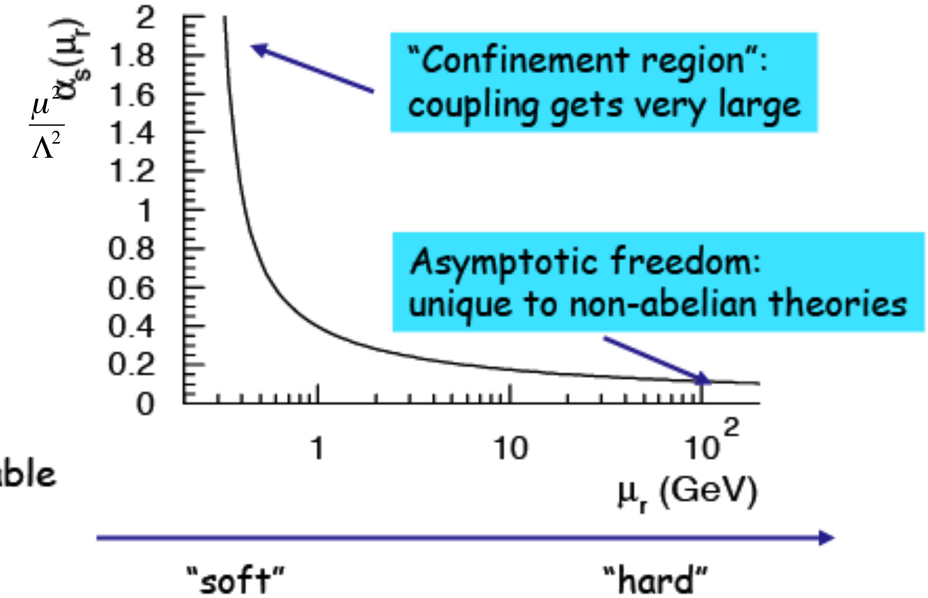
$$\Lambda^2 = \mu^2 \exp(-1/b_0\, \alpha_s(\mu^2)) \quad \text{or} \quad \alpha_s(\mu^2) = \frac{1}{b_0 \log \frac{\mu^2}{\Lambda^2}}$$

Therefore:

$$\alpha_s(Q^2) = \frac{1}{b_0 \log \frac{\mu^2}{\Lambda^2} + b_0 \log \frac{Q^2}{\mu^2}} = \frac{1}{b_0 \log \frac{Q^2}{\Lambda^2}}$$

$Q^2 >> \Lambda^2$: $\alpha_s(Q^2)$ small $\rightarrow$ perturbative QCD applicable

$Q^2 \approx \Lambda^2$: quark and gluons form bound states



Λ is free parameter of theory,
has to be determined by experiment
$\rightarrow$ expected to be of order of hadron mass

QCD explains confinement of colour and allows calculations of hard hadronic processes via perturbative expansion of coupling!

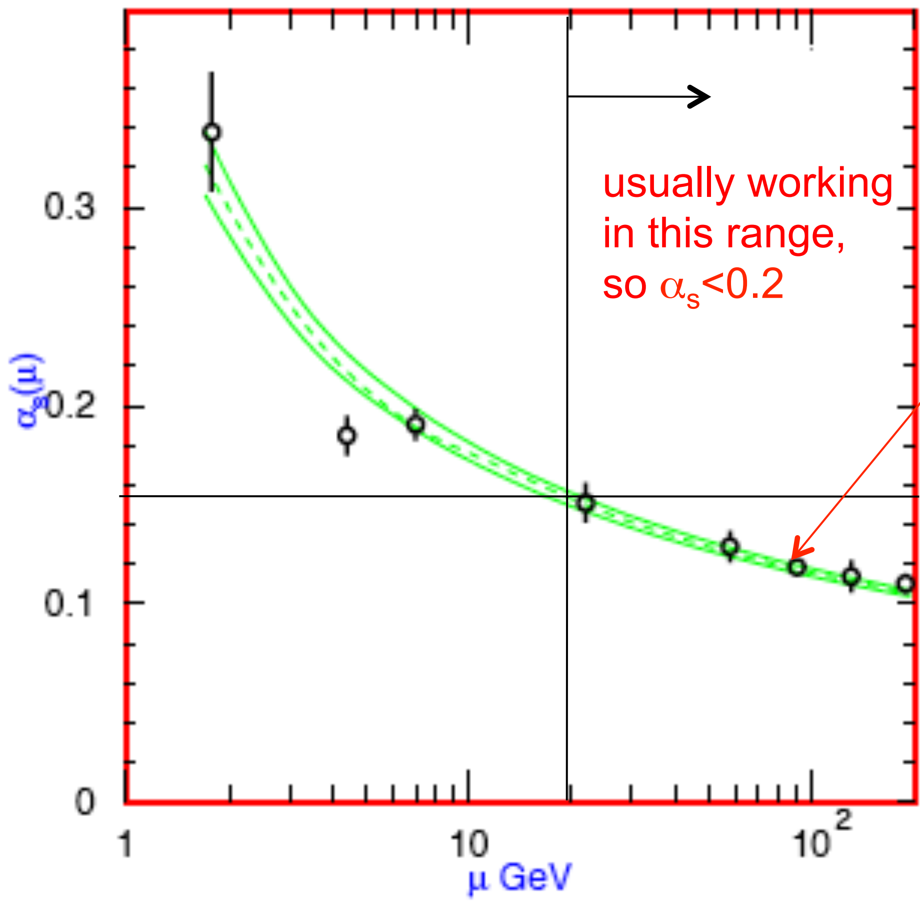
$2\,N_F$

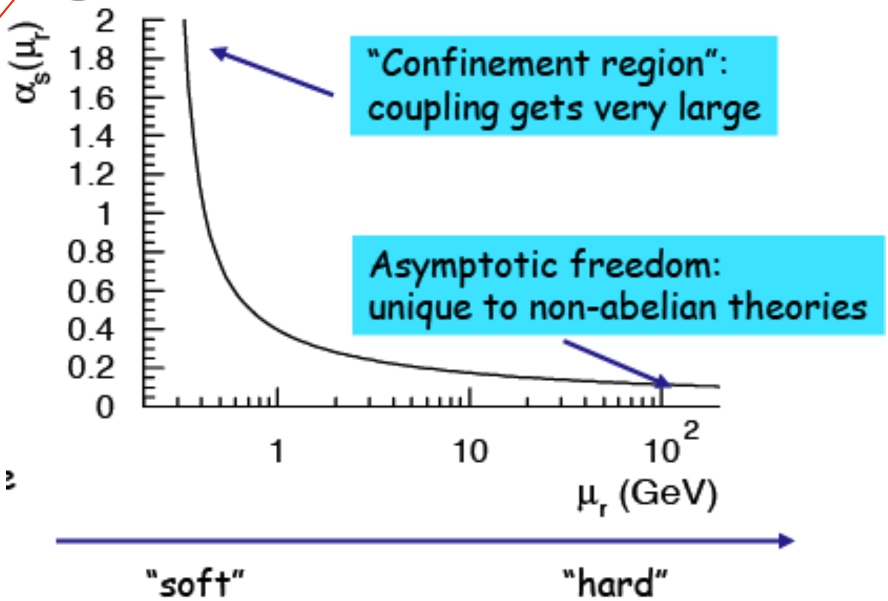
π

n)

slarge

@ scale of m_Z, world average for α_s is 0.118 (NLO) and 0.130 (LO); α_s(NNLO~α_s(NLO)
It's more common now to quote α_s at a scale of m_Z than to quote Λ





Λ is free parameter of theory,
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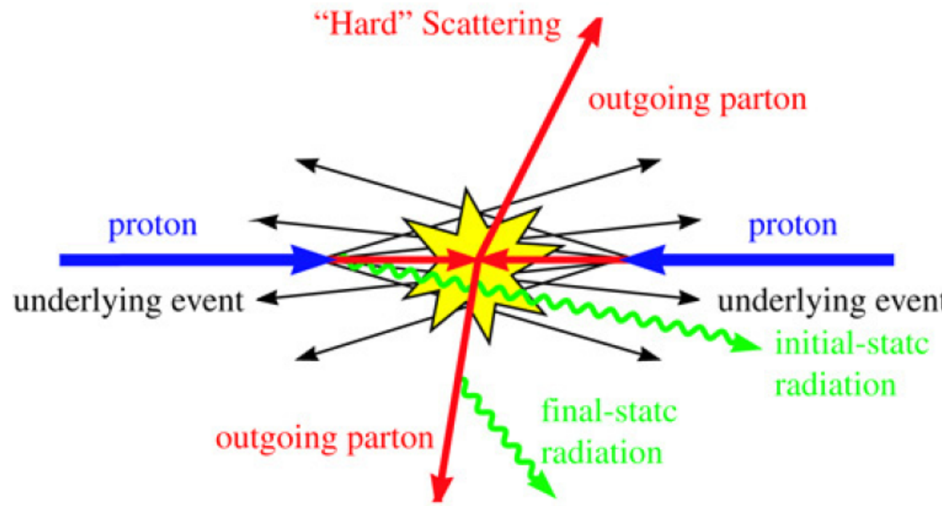
QCD explains confinement of colour and allows calculations of hard hadronic processes via perturbative expansion of coupling !

Factorization

- Factorization is the key to perturbative QCD
  - the ability to separate the short-distance physics and the long-distance physics
- In the pp collisions at the LHC, the hard scattering cross sections are the result of collisions between a quark or gluon in one proton with a quark or gluon in the other proton
- The remnants of the two protons also undergo collisions, but of a softer nature, described by semi-perturbative or non-perturbative physics

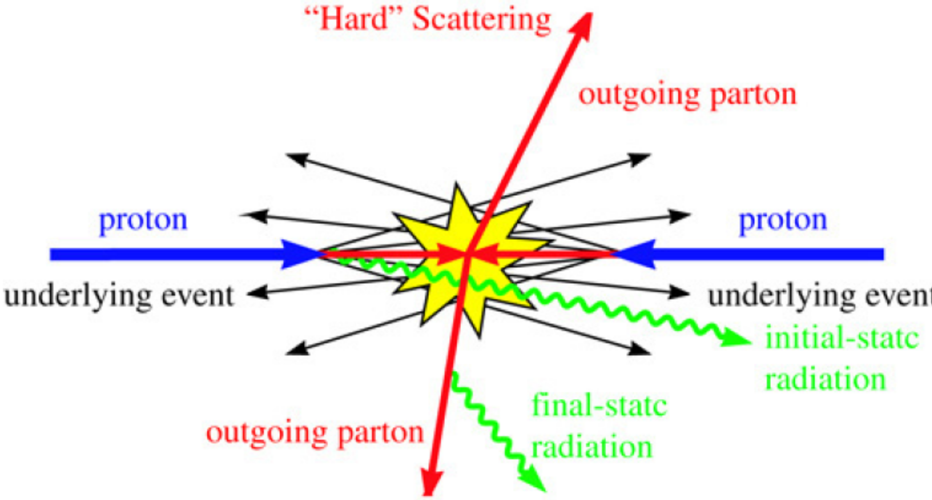


The calculation of hard scattering processes at the LHC requires:
(1)knowledge of the distributions of the quarks and gluons inside the proton, i.e. what fraction of the momentum of the parent proton do they have
->parton distribution functions (pdf' s)
(2) knowledge of the hard scattering cross sections of the quarks and gluons, at LO, NLO, or NNLO in the strong coupling constant α_s

Factorization

- Factorization* is the key to perturbative QCD
    - the ability to separate the short-distance physics and the long-distance physics

*it turns out that factorization is violated at higher orders for certain configurations, but for all practical purposes (including ours), we will assume factorization is good



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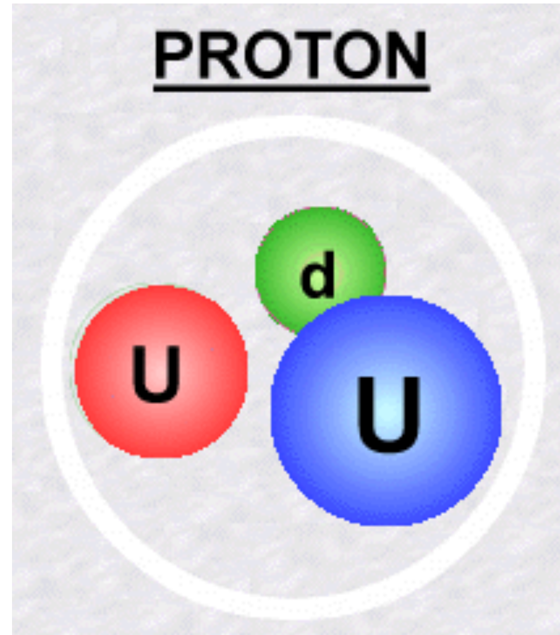
->parton distribution functions (pdf's)
(2) knowledge of the hard scattering cross sections of the quarks and gluons, at LO, NLO, or NNLO in the strong coupling constant α_s

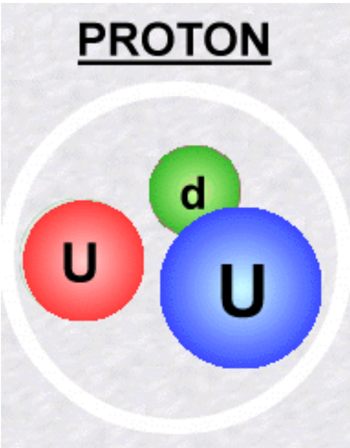
What do we expect for the distribution of partons?

- We know sum rules
- For example, the sum of the momenta of all of the partons inside the proton has to equal the proton's momentum
- If I sum up over all up quarks (and up anti-quarks) in the proton, I will end up with 2
- If I sum up over all down quarks (and down anti-quarks) in the proton, I will end up with 1

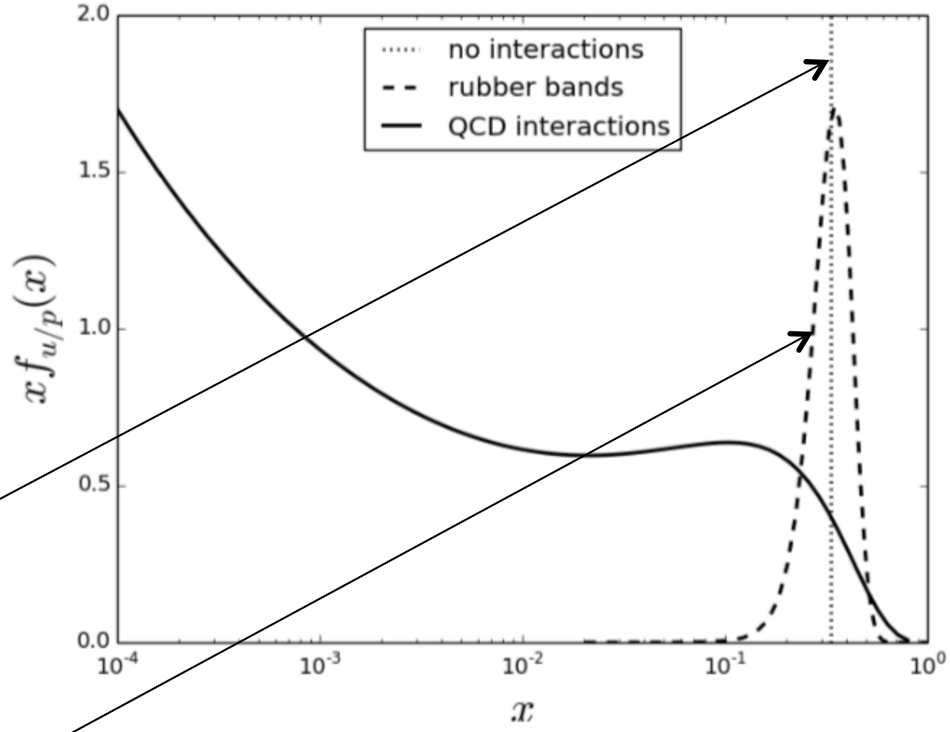


What do we expect for the distribution of partons?

- Simplest Fock state
  - valence quarks only (uud)

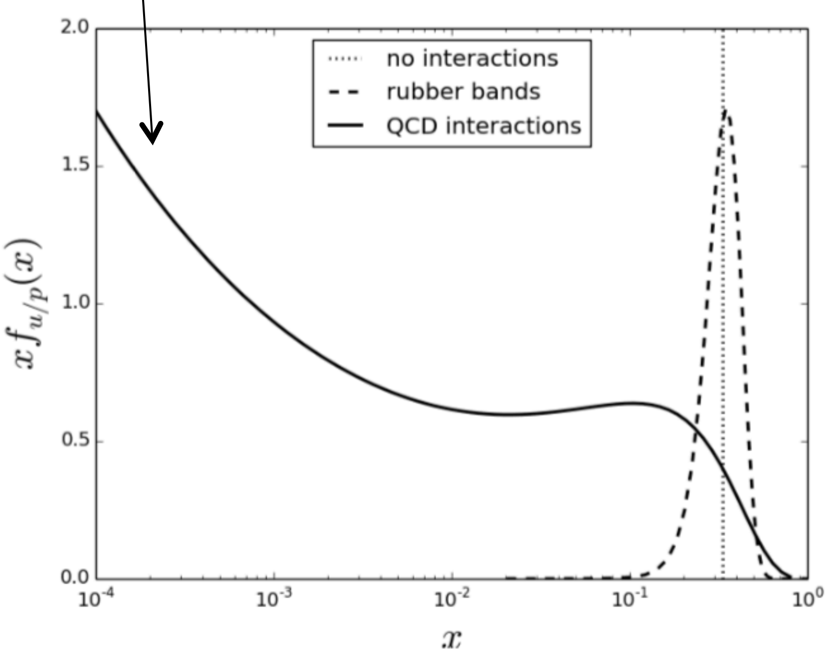


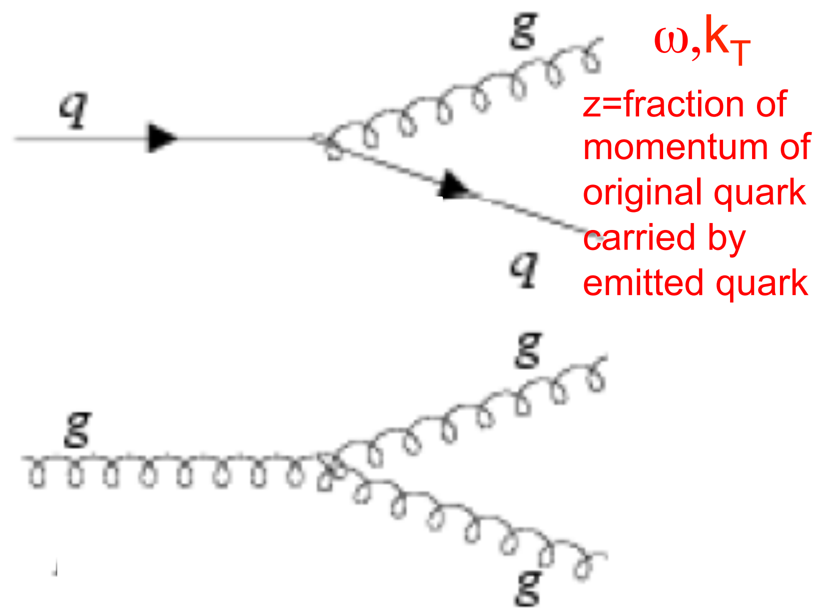
- Naively, no interactions
  - $f_{u,d/p} \sim \delta(x-1/3)$
- Elastic interactions between quarks
  - Gaussian smearing



What do we expect for the distribution of partons?

- Strong interactions develop a sea of soft partons (carrying a small fraction of the parent proton's momentum), depending on the resolution scale





- Consider gluons being emitted off of either quarks, or other gluons; the particle spectrum goes as

$$dn_g^{q,g} = C_{q,g} \cdot \frac{\alpha_s(k_\perp^2)}{\pi} \cdot \frac{d\omega}{\omega} \cdot \frac{dk_\perp^2}{k_\perp^2}$$

- Note the divergence for soft/ collinear gluons being emitted

DGLAP equations

- Parton distributions used in hard-scattering calculations are solutions of DGLAP equations (or in Italy the AP equations)
  - the DGLAP equations determine the Q^2 dependence of the PDF's

DGLAP equations sum leading powers of $[\alpha_s \log\mu^2]^n$ generated by multiple gluon emission in a region of phase space where the gluons are strongly ordered in transverse momentum ($\log \mu >> \log (1/x)$)

For regions in which this ordering is not present (e.g. low x at the LHC), a different type of resummation (BFKL) may be needed

$$\frac{\partial q_i(x,\mu^2)}{\partial \log \mu^2} = \frac{\alpha_S}{2\pi} \int_x^1 \frac{dz}{z} \left\{ P_{q_i q_j}(z,\alpha_S) q_j(\frac{x}{z},\mu^2) + P_{q_i g}(z,\alpha_S)\; g(\frac{x}{z},\mu^2) \right\},$$

$$\frac{\partial g(x,\mu^2)}{\partial \log \mu^2} = \frac{\alpha_S}{2\pi} \int_x^1 \frac{dz}{z} \left\{ P_{g q_j}(z,\alpha_S) q_j(\frac{x}{z},\mu^2) + P_{gg}(z,\alpha_S) g(\frac{x}{z},\mu^2) \right\},$$

  - the splitting functions have perturbative expansions, for use with LO, NLO, NNLO parton distributions

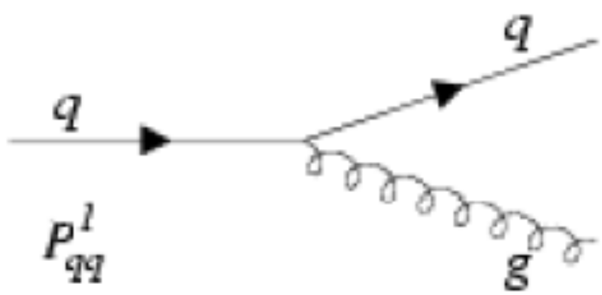
$$P_{ab}(x,\alpha_S) = P_{ab}^{(0)}(x) + \frac{\alpha_S}{2\pi} P_{ab}^{(1)}(x) + \cdots .$$

Altarelli-Parisi splitting functions

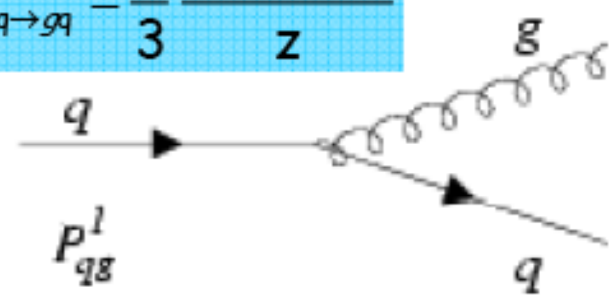
Note that the emitted gluon likes to be soft

Altarelli-Parisi splitting functions:

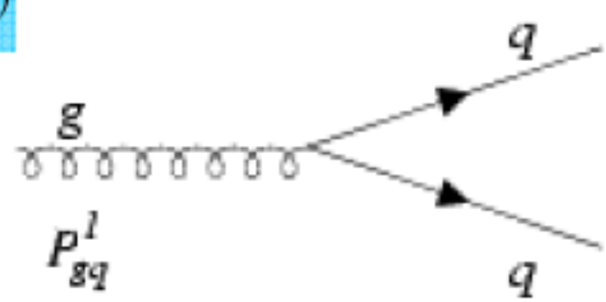
$$P_{q\to qg} = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right)$$



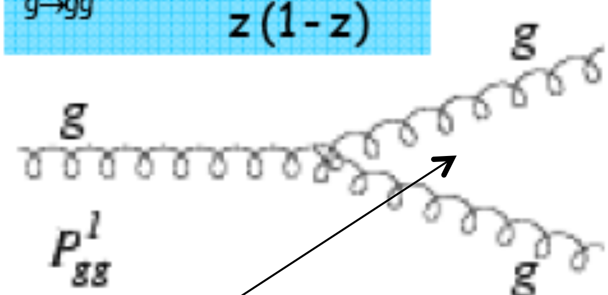
$$P_{q\to gq} = \frac{4}{3}\frac{1+(1-z)^2}{z}$$



$$P_{g\to q\bar{q}} = \frac{n_f^2}{2}\left(z^2+(1-z)^2\right)$$



$$P_{g\to gg} = 3\frac{(1-z(1-z))^2}{z(1-z)}$$



We'll also encounter the A-P splitting functions later, when we discuss parton showering and Sudakov form factors

here the emitted gluon can be soft or hard