

# Electroweak and Higgs physics

**Shinya KANEMURA**

**兼村 晋哉**



**大阪大学**  
OSAKA UNIVERSITY

# **Plan of the Lectures**

- 1 EW Symmetry Breaking in the Standard Model (SM)**
- 2 Physics of non-Minimal Higgs sectors**
- 3 Higgs as a Probe of New Physics**

# Plan of the Lectures

- 1 EW Symmetry Breaking in the Standard Model (SM)**
- 2 Physics of non-Minimal Higgs sectors**
  - 2-1: Motivation**
  - 2-2: Two Higgs doublet models**
  - 2-3: Other Models**
  - 2-4: Fingerprinting Higgs models**
  - 2-5: Decoupling/Non-decoupling**
  - 2-6: Radiative Corrections to Higgs couplings**
- 3 Higgs as a Probe of New Physics**

# **2 Physics of non-minimal Higgs Sectors**

# **2-1 Motivation**

# Extended Higgs Sector

The “**SM-like**” does not necessarily mean the SM.

Every extended Higgs sector can contain the SM-like Higgs boson ***h*** in its decoupling regime.

# General Extended Higgs models

## Multiplet Structure

$\Phi_{SM} + \text{Singlet}$ ,  $\Phi_{SM} + \text{Doublet}$  (2HDM),  
 $\Phi_{SM} + \text{Triplet}$ , ...

## Additional Symmetry

Discrete or Continuous?

Exact or Softly broken?

## Interaction

Weakly coupled or Strongly Coupled ?

Decoupling or Non-decoupling?

# Multiplet Structure

If the Higgs sector contains more than one scalar bosons, possibility would be

- SM + extra Singlets (NMSSM, B-L Higgs, 1<sup>st</sup> OPT, ...)
- SM + extra Doublets (MSSM, CPV, EW Baryogenesis, 1<sup>st</sup> OPT, Neutrino mass, ...)
- SM + extra Triplets (Type II seesaw, LR models....)
- ....

**Basic data which strongly constrain the shape of extended Higgs sectors**

- Electroweak rho parameter
- Flavor Changing Neutral Current (FCNC)

# EW rho parameter

$$\rho_{\text{exp}} = 1.0004^{+0.0003}_{-0.0004}$$

$$Q = I_3 + Y/2$$

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} = \frac{\sum_i [4T_i(T_i + 1) - Y_i^2] |v_i|^2 c_i}{\sum_i 2Y_i^2 |v_i|^2}$$

$T_i$ : SU(2)<sub>L</sub> isospin

$Y_i$ : hypercharge

$v_i$ : v.e.v.

$c_i$ : 1 for complex representation

1/2 for real representation

$N=1$  SM Higgs doublet  $\Phi$  ( $T=1/2, Y=1$ )  $\rho = 1!$

$N=2$  What kind of (2 field) extended Higgs sector  $\Phi + X(T_X, Y_X)$  can satisfy  $\rho = 1$  ?

We solve the equation

$$4 T_X(T_X+1) = 3 Y_X^2$$



$(T_X, Y_X)$	$X$
(0, 0)	Singlet
(1/2, 1)	Doublet
(3, 4)	Septet
(25/2, 15)	25-plet
....	....

# EW rho parameter

$$\rho_{\text{exp}} = 1.0004^{+0.0003}_{-0.0004}$$

## Possibility

1.  $\rho=1$  at tree SM + doublets ( $\phi$ ) (+ singlets (S)), ...

2.  $\rho \approx 1$  at tree SM + Triplets ( $\Delta$ )

a)  $v_{\Delta} \ll v_{\phi}$

$$\rho_{\text{tree}} = \frac{1 + \frac{2v_{\Delta}^2}{v_{\phi}^2}}{1 + \frac{4v_{\Delta}^2}{v_{\phi}^2}} \simeq 1 - \frac{2v_{\Delta}^2}{v_{\phi}^2}$$

b) Combination of several representations

[ (ex) Georgi-Machasek Model]

$$v_{\Delta} \approx v_{\phi}$$

Multi-doublets (+singlets) seem the most natural choice?

# **2-2 Two Higgs doublet models**

# Simplest extension

## 2 Higgs doublet model (2HDM)

$$\Phi_i = \begin{pmatrix} \omega_i^\pm \\ \frac{1}{\sqrt{2}}(v_i + h_i + iz_i) \end{pmatrix} \quad (i = 1, 2)$$

Sharing the VEV  $v = 246 \text{ GeV} = \sqrt{v_1^2 + v_2^2}$   $\tan \beta = \frac{v_2}{v_1}$

Field Mixing  $\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}$   $\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} z \\ A \end{pmatrix}$

$\begin{pmatrix} \omega_1^\pm \\ \omega_2^\pm \end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \omega^\pm \\ H^\pm \end{pmatrix}$

New Particles  $h$   $H$   $A$   $H^\pm$

$\uparrow$   
 $h(125)$

$\underbrace{\hspace{10em}}_{\text{Additional bosons}}$

Other three are unphysical  
Nambu-Goldstone bosons

Deviation in the couplings of  $h(125)$

SM  $hVV$  **1**  $\rightarrow$  2HDM  $hVV$   $\sin(\beta - \alpha)$

# 2 Higgs Doublet Model

$$V_{\text{THDM}} = +m_1^2 |\Phi_1|^2 + m_2^2 |\Phi_2|^2 - \frac{m_3^2}{2} (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) \\ + \frac{\lambda_1}{2} |\Phi_1|^4 + \frac{\lambda_2}{2} |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 \\ + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 + \frac{\lambda_5}{2} \left[ (\Phi_1^\dagger \Phi_2)^2 + (\text{h.c.}) \right]$$

$$\Phi_1 \text{ and } \Phi_2 \Rightarrow h, \quad H, \quad A^0, \quad H^\pm \oplus \text{Goldstone bosons}$$

$\begin{array}{cccc} \uparrow & \uparrow & \uparrow & \text{charged} \\ \text{CPEven} & \text{CPodd} & & \end{array}$

$$m_h^2 = v^2 \left( \lambda_1 \cos^4 \beta + \lambda_2 \sin^4 \beta + \frac{\lambda}{2} \sin^2 2\beta \right) + \mathcal{O}\left(\frac{v^2}{M_{\text{soft}}^2}\right),$$

$$m_H^2 = M_{\text{soft}}^2 + v^2 (\lambda_1 + \lambda_2 - 2\lambda) \sin^2 \beta \cos^2 \beta + \mathcal{O}\left(\frac{v^2}{M_{\text{soft}}^2}\right),$$

$$m_{H^\pm}^2 = M_{\text{soft}}^2 - \frac{\lambda_4 + \lambda_5}{2} v^2,$$

$$m_{A^0}^2 = M_{\text{soft}}^2 - \lambda_5 v^2.$$

$M_{\text{soft}}$ : soft breaking scale

$$\Phi_i = \begin{bmatrix} w_i^+ \\ \frac{1}{\sqrt{2}}(h_i + v_i + i a_i) \end{bmatrix} \quad (i = 1, 2)$$

## Diagonalization

$$\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} H \\ h \end{bmatrix} \quad \begin{bmatrix} z_1^0 \\ z_2^0 \end{bmatrix} = \begin{bmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{bmatrix} \begin{bmatrix} z^0 \\ A^0 \end{bmatrix} \\ \begin{bmatrix} w_1^\pm \\ w_2^\pm \end{bmatrix} = \begin{bmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{bmatrix} \begin{bmatrix} w^\pm \\ H^\pm \end{bmatrix}$$

$$\frac{v_2}{v_1} \equiv \tan \beta$$

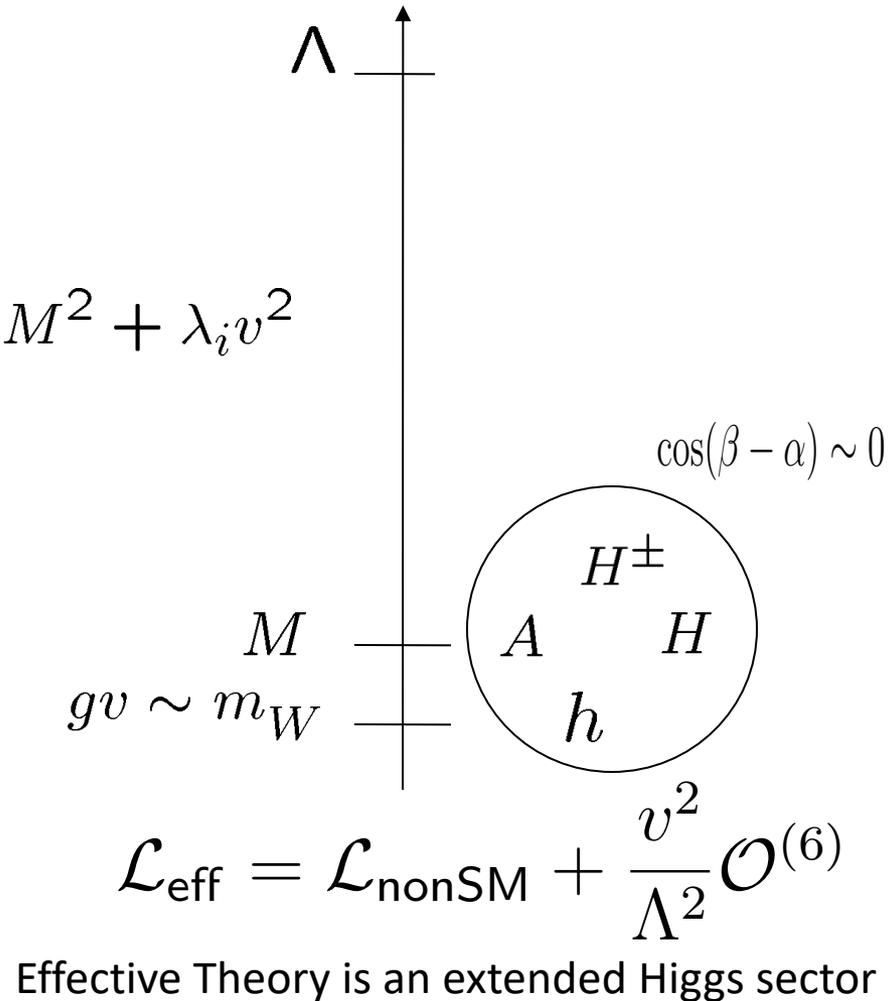
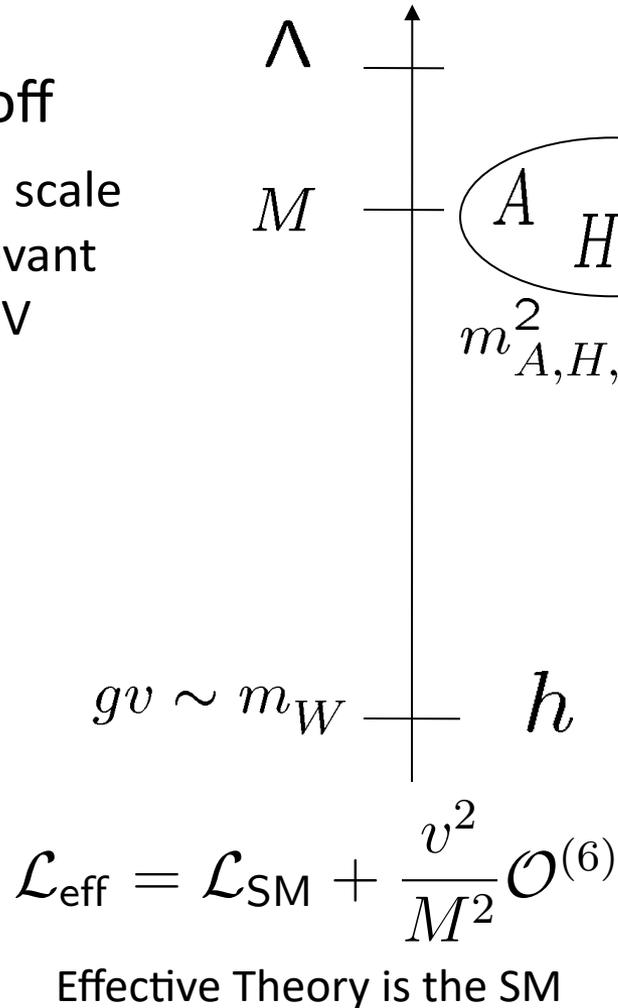
$$M_{\text{soft}} \left( = \frac{m_3}{\sqrt{\cos \beta \sin \beta}} \right):$$

soft-breaking scale  
of the discrete symm.

# Two Possibilities

$\Lambda$ : Cutoff

$M$ : Mass scale  
irrelevant  
to VEV



**Non-decoupling effect**

# Gauge Couplings $hVV$

$$L = g_{hVV} \sin(\beta-\alpha) hVV + g_{HVV} \cos(\beta-\alpha) HVV$$

- Changed by mixing with the other scalars
- Sum-rule for a multi-doublet structure

$$g_{hVV}^2 + g_{HVV}^2 = g_V^2$$

$$\sin^2(\beta-\alpha) < 1 \Leftrightarrow \kappa_V^2 = (g_{hVV}/g_{hVV}^{\text{SM}})^2 < 1$$

$$\frac{g_{hVV}^{\text{THDM}}}{g_{hVV}^{\text{SM}}} = \sin(\beta - \alpha)$$

SM-like case  
 $\sin^2(\beta-\alpha) \approx 1$

- Higgs sector with an exotic representation

$\kappa_V^2 > 1$  is also possible!

Higgs triplet model  
 Georgi-Machasek model  
 Models with a septet field, ...

# Yukawa Coupling in Extended Higgs Sectors

Multi-Higgs model: **FCNC appears via Higgs mediation**

**2 Higgs doublet models:**

to avoid FCNC, give different charges to  $\Phi_1$  and  $\Phi_2$

Discrete sym.  $\Phi_1 \rightarrow +\Phi_1, \quad \Phi_2 \rightarrow -\Phi_2$

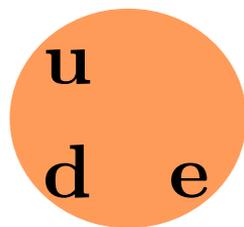
Each quark or lepton couples only one Higgs doublet

**No FCNC at tree level**

**Four Types of Yukawa coupling**

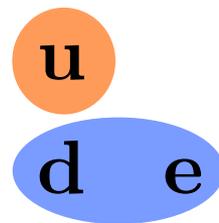
*Barger, Hewett, Phillips*

Classified by  $Z_2$  charge assignment



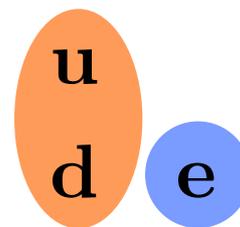
**Type-I**

Neutrinophilic  
Inert



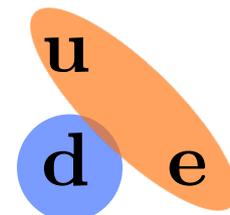
**Type-II**

SUSY



**Type-X**

Radiative Seesaw  
Lepton specific



**Type-Y**

# Type2-2HDM (MSSM) Higgs couplings

$$\text{VEV's: } v_1^2 + v_2^2 = v^2 \simeq (246 \text{ GeV})^2$$

## Higgs mixing

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}$$

$$\tan \beta = \frac{v_2}{v_1}$$

## SM

Gauge coupling:

$$\phi VV \quad (V = Z, W) \Rightarrow$$

$$\begin{array}{cc} hVV & HVV \\ \sin(\beta - \alpha), & \cos(\beta - \alpha) \end{array}$$

Yukawa coupling:

$$\phi b\bar{b} \Rightarrow$$

$$\begin{array}{cc} hb\bar{b} & Hb\bar{b} \\ \frac{\sin \alpha}{\cos \beta}, & \frac{\cos \alpha}{\cos \beta} \end{array}$$

$$\phi t\bar{t} \Rightarrow$$

$$\begin{array}{cc} ht\bar{t} & Ht\bar{t} \\ \frac{\cos \alpha}{\sin \beta}, & \frac{\sin \alpha}{\sin \beta} \end{array}$$

## 2HDM Type2

# SM-like regime

$$\begin{array}{ll} hVV & HVV \\ \sin(\beta - \alpha) & \cos(\beta - \alpha) \end{array}$$

$$\sin(\beta - \alpha) \simeq 1$$

Only the lightest Higgs  $h$  couples to weak gauge bosons

$h$  behaves like the SM Higgs

$$g_{hVV} \rightarrow g_{\phi VV}^{\text{SM}}$$

$$g_{HVV} \rightarrow 0$$

$$y_{htt\bar{t}} \rightarrow y_{\phi t\bar{t}}^{\text{SM}}$$

$$y_{Ht\bar{t}} \rightarrow y_{\phi t\bar{t}}^{\text{SM}} \cot \beta$$

$$y_{hb\bar{b}} \rightarrow y_{\phi b\bar{b}}^{\text{SM}}$$

$$y_{Hb\bar{b}} \rightarrow y_{\phi b\bar{b}}^{\text{SM}} \tan \beta$$

$$y_{h\tau\tau} \rightarrow y_{\phi\tau\tau}^{\text{SM}}$$

$$y_{H\tau\tau} \rightarrow y_{\phi\tau\tau}^{\text{SM}} \tan \beta$$

Type-II 2HDM

# Theoretical Constraints on extended Higgs sectors

- Unitarity bound
- Vacuum Stability bound
- Triviality bound
- Wrong vacuum condition (singlet model)

Many  $\lambda$  couplings  $\rightarrow$  mass prediction changed

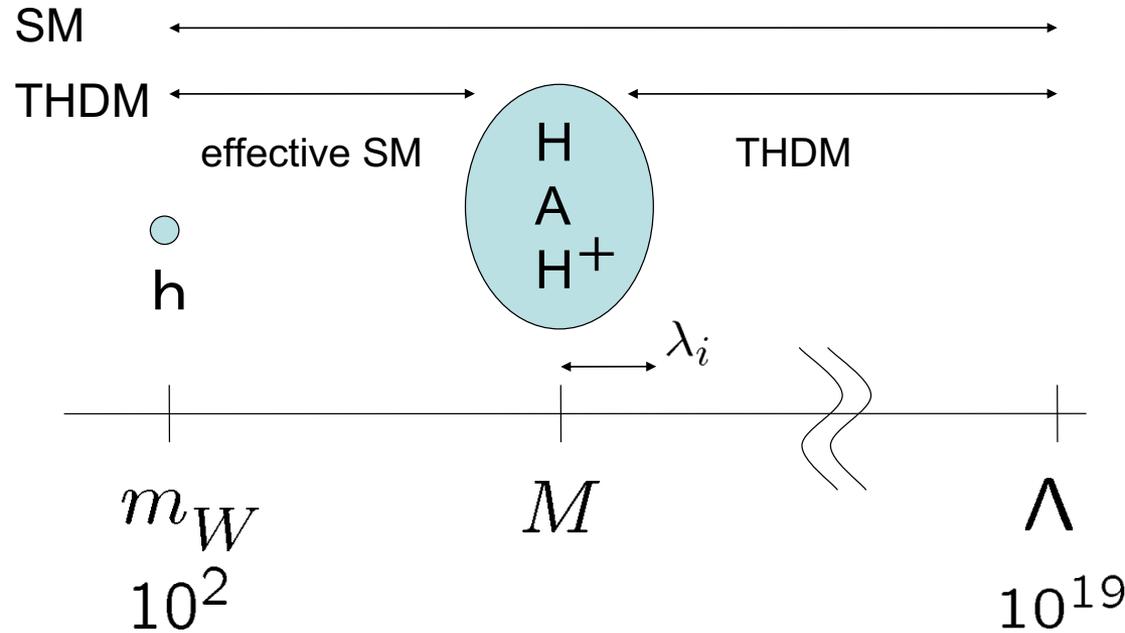
Lightest Higgs mass

$$m_h^2 = \lambda v^2$$

Additional Higgs masses

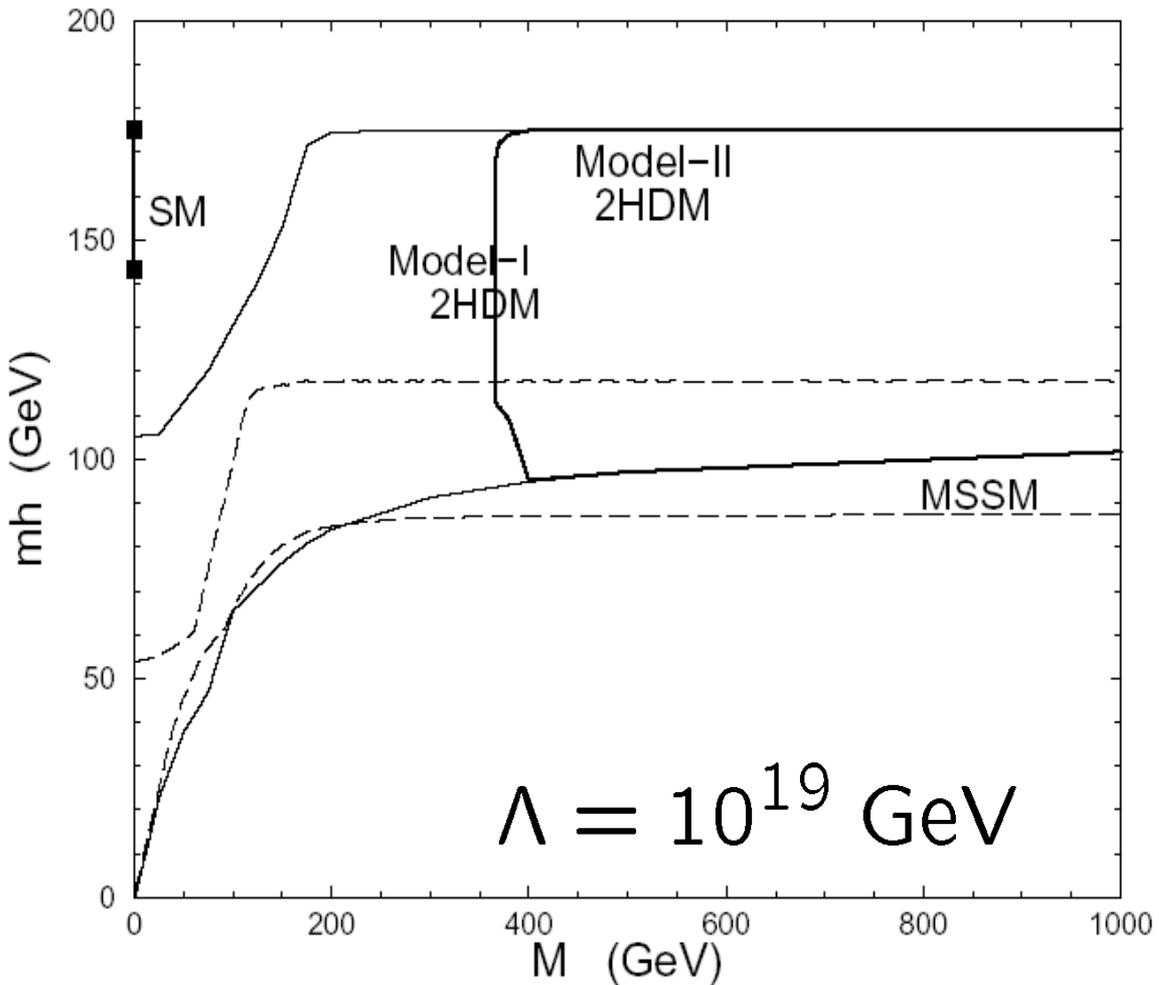
$$m_\phi^2 \simeq M^2 + \lambda' v^2$$

RGE 
$$16\pi^2 \mu \frac{d}{d\mu} \lambda = 24\lambda^2 - 6y_t^2 + A(\lambda', \lambda'', \dots)$$



# Mass of the lightest Higgs boson

- SM
- 2HDM type1
- 2HDM type 2
- MSSM



The predicted region of mass can be different even if all the other phenomena behave like the SM in the low energy.

Kanemura, Kasai, Okada  
1999

# Higgs singlet extension (HSM)

$$V_0 = -\mu_\Phi^2 |\Phi|^2 + \lambda_\Phi |\Phi|^4 + \mu_{\Phi S} |\Phi|^2 S + \frac{\lambda_{\Phi S}}{2} |\Phi|^2 S^2 + \mu_S^3 S + \frac{m_S^2}{2} S^2 + \frac{\mu'_S}{3} S^3 + \frac{\lambda_S}{4} S^4$$

$$\Phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v_\Phi + \phi_1 + iG^0) \end{pmatrix}, \quad S = v_S + \phi_2$$

Mass eigenstates and mixing angle

$$(\phi_1, \phi_2) \rightarrow (h, H) \text{ with } \theta$$

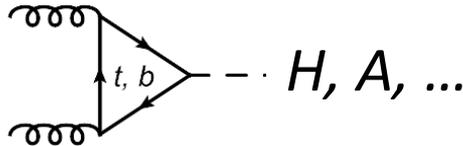
125GeV Higgs boson

# Fingerprinting

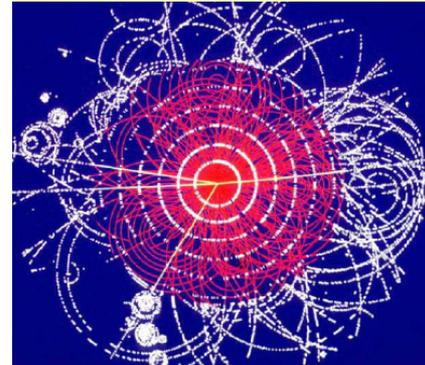
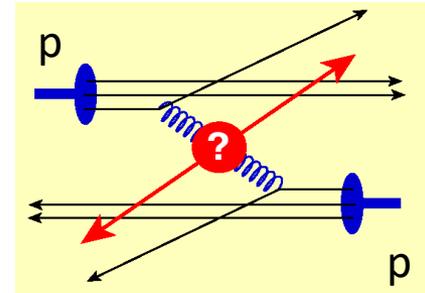
# Direct search and indirect tests

- Direct searches of additional Higgs bosons

$h(125), H, A, H^+, H^{++}, \dots$  **Machine for discovery!**



**Hadron Collider (LHC)**  
 Run1 7-8 TeV 20fb<sup>-1</sup>  
 Run 2,3 13-14 TeV 300fb<sup>-1</sup>  
 HL-LHC 13-14 TeV 3000fb<sup>-1</sup>



- Indirect test by finding deviations from SM

**EW parameters**  $m_W, S, T, U, Zff, Wff', WWV, \dots$

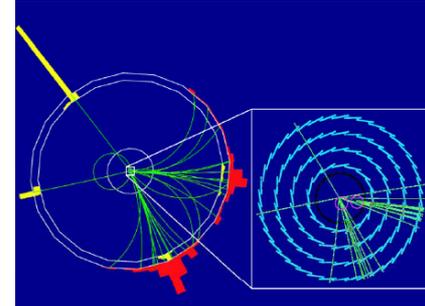
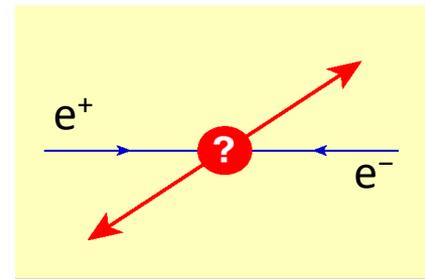
**Couplings of  $h(125)$**   $hWW, hZZ, h\gamma\gamma, hff, hhh, \dots$

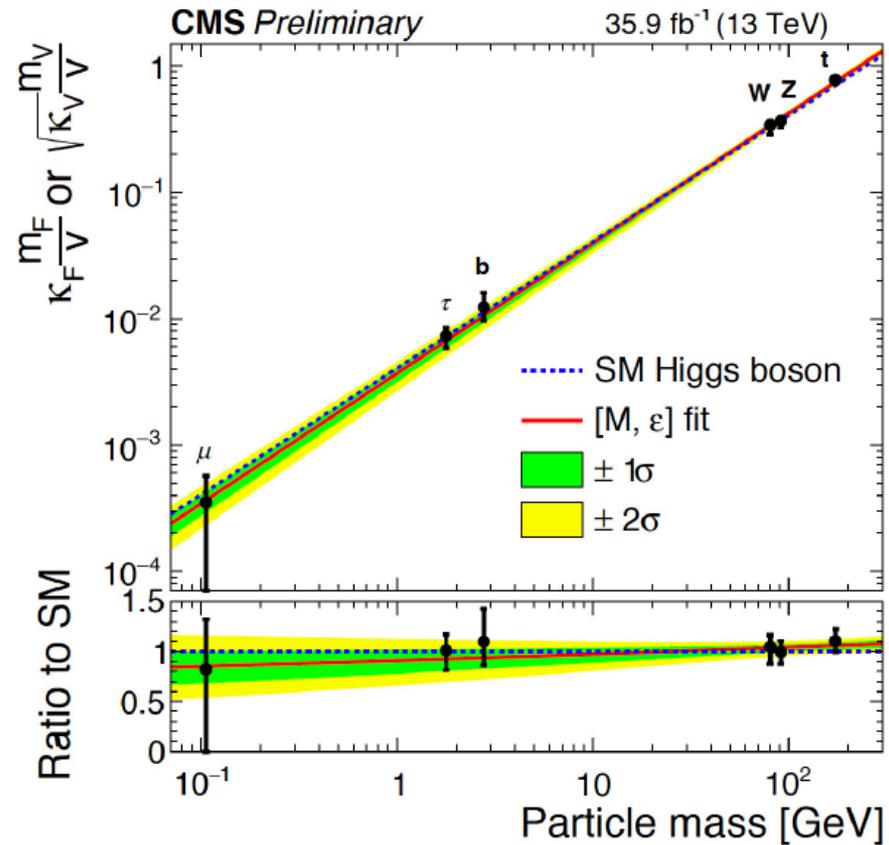
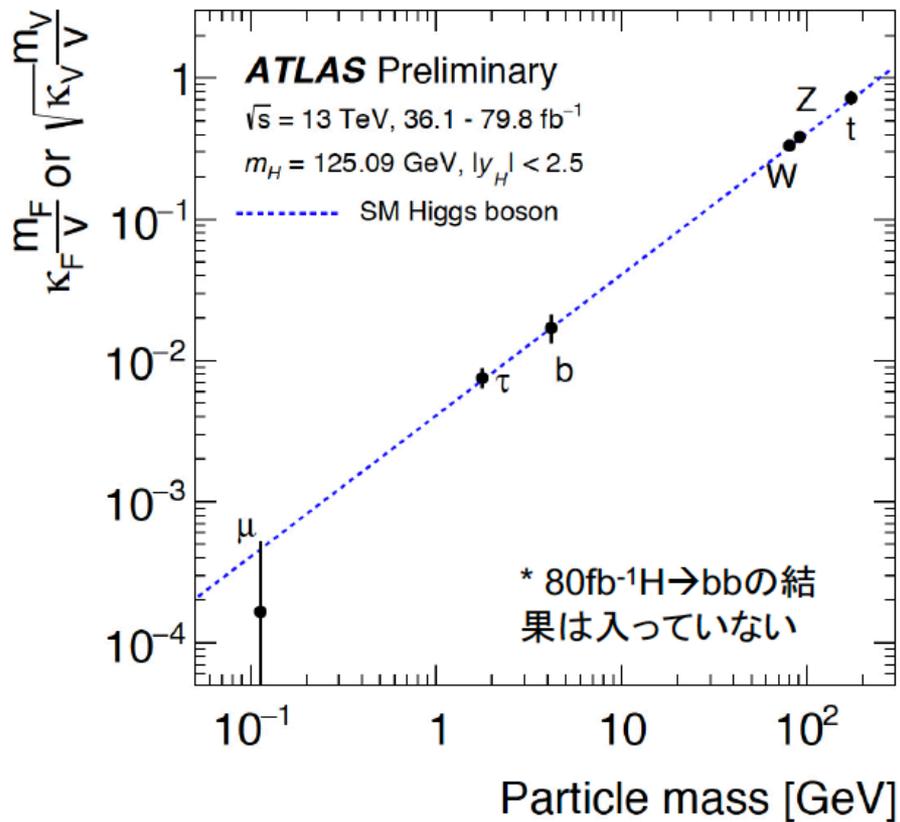
**Precision measurements!**

**Advantage for lepton colliders**

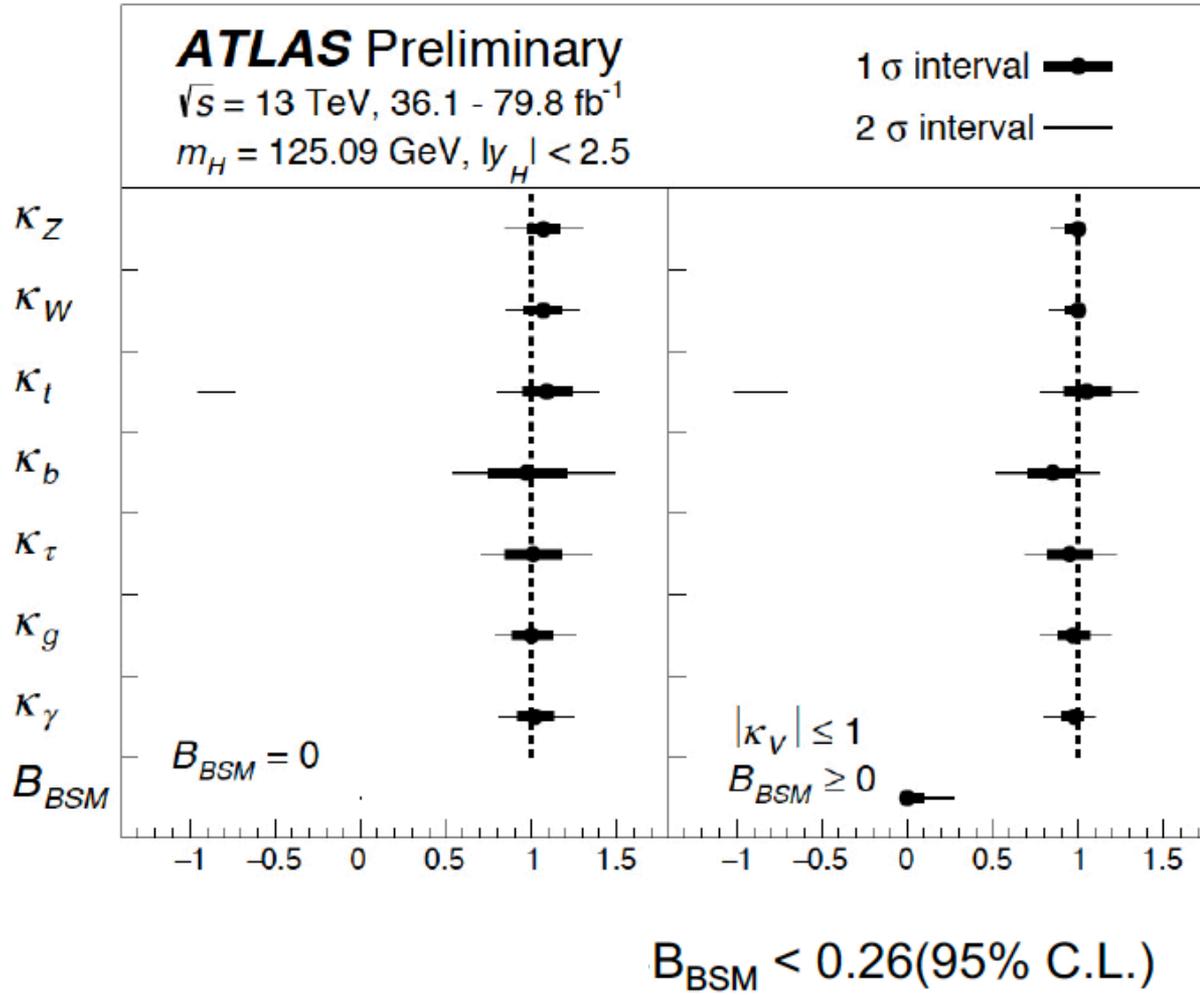
**Future International Collider (ILC), CEPC, FCCee,**

**$E = 240-250$  GeV, (500GeV, 1 TeV, ...)**





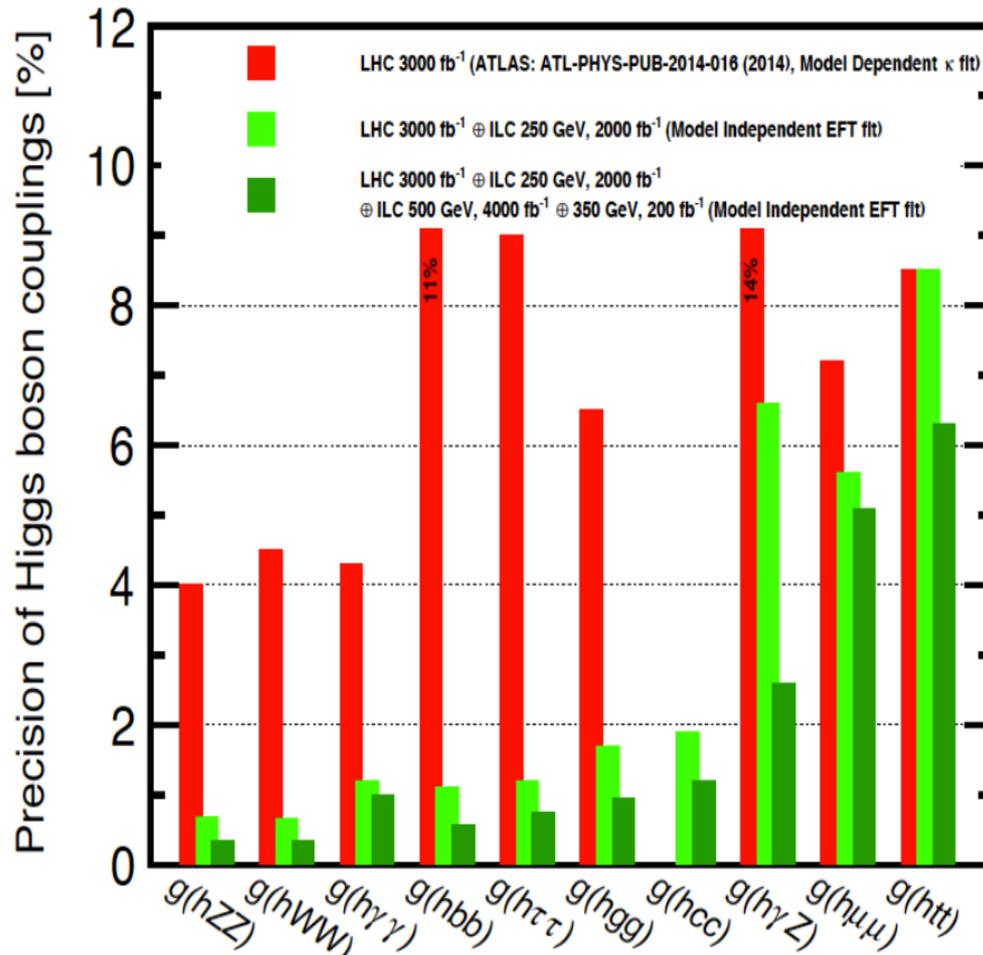
# Coupling Measurements



$$\frac{\Gamma_H}{\Gamma_H^{SM}} = \frac{\kappa_H^2}{1 - BR_{BSM}}$$

# Higgs Precision at HL-LHC, ILC250, ...

[K. Fujii, et al., arXiv:1710.07621]



**Future lepton colliders ILC, CEPC, FCCee, CLIC, ...**

# Deviation = New Physics scale

Scaling factor  $\kappa_i$  : factor of deviation from the SM value

Coupling of  $h(125)$  and weak bosons

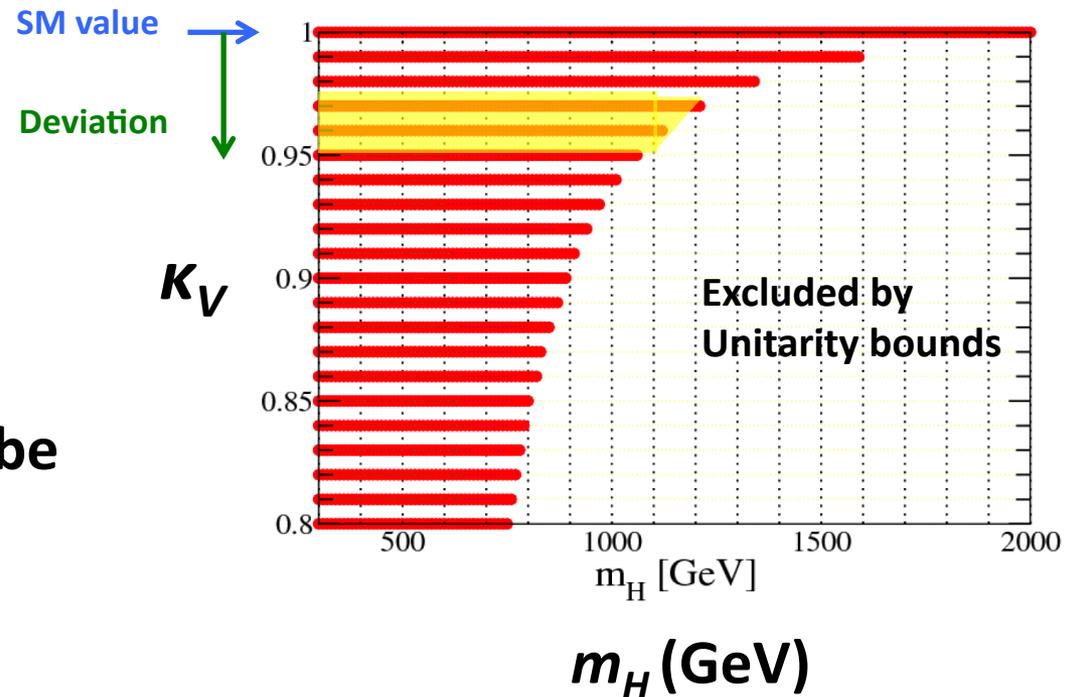
$V (=W, Z)$   $hVV$

$$\kappa_V = \sin(\beta - \alpha)$$

If a **3%** deviation in  $\kappa_V$



The second Higgs  $H$  should be lighter than **1200 GeV**



**Precision test has the similar power to the direct search**

# Scaling factors

$$\kappa_X = g_{hXX}^{EX} / g_{hXX}^{SM}$$

2HDM :

$$\kappa_V = \sin(\beta - \alpha)$$

$$\kappa_f = \sin(\beta - \alpha) + \xi_f \cos(\beta - \alpha)$$

HSM :

$$\kappa_V = \cos \alpha$$

$$\kappa_f = \cos \alpha$$

	Mixing factor		
	$\xi_u$	$\xi_d$	$\xi_e$
Type-I	$\cot \beta$	$\cot \beta$	$\cot \beta$
Type-II	$\cot \beta$	$-\tan \beta$	$-\tan \beta$
Type-X	$\cot \beta$	$\cot \beta$	$-\tan \beta$
Type-Y	$\cot \beta$	$-\tan \beta$	$\cot \beta$

$$\frac{\Gamma(h \rightarrow VV^*)_{EX.}}{\Gamma(h \rightarrow VV^*)_{SM}} \sim \kappa_V^2$$

$$\frac{\Gamma(h \rightarrow ff)_{EX.}}{\Gamma(h \rightarrow ff)_{SM}} \sim \kappa_f^2$$

# Pattern of deviations

Gauge couplings

Yukawa couplings

$hVV$

$h\tau\tau$

$hbb$

$hcc$

$\cos(\beta-\alpha) < 0$

	$K_V$	$K_\tau$	$K_b$	$K_c$
Type-I	↓	↓	↓	↓
Type-II	↓	↑	↑	↓
Type-X	↓	↑	↓	↓
Type-Y	↓	↓	↑	↓

Direction of deviation in each coupling

We can fingerprint extended Higgs models from the pattern of deviation in Higgs couplings

# Fingerprinting the 2HDM

$$\kappa_V \equiv \frac{g_{hVV(2HDM)}}{g_{hVV(SM)}} = \sin(\beta - \alpha)$$

$x = \cos(\beta - \alpha)$  **SM-like:  $|x| \ll 1$**

$$\kappa_V = 1 - (1/2)x^2 + \dots$$

When a Fermion couples to  $\Phi_1$

$$K_f = 1 + \cot\beta x + \dots$$

and if it couples to  $\Phi_2$

$$K_f = 1 - \tan\beta x + \dots$$

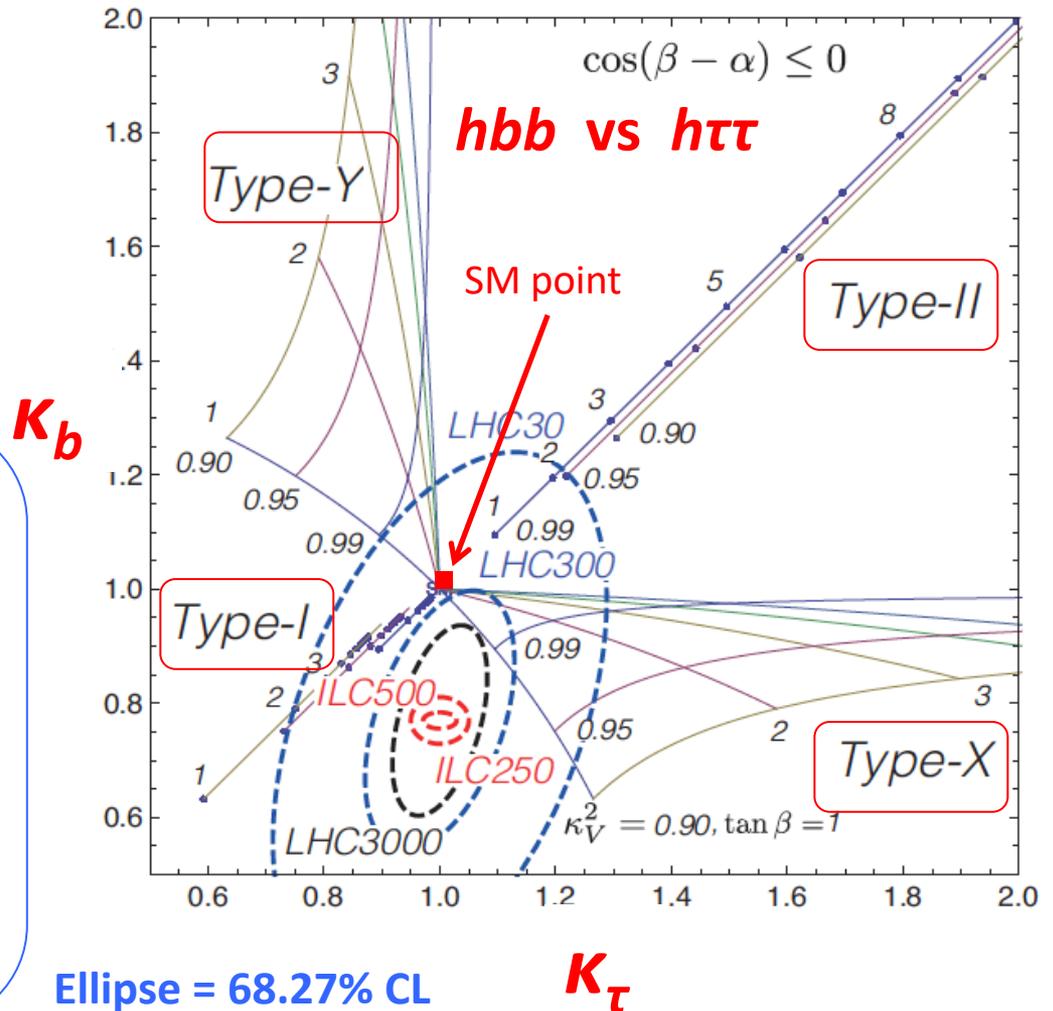
If deviation in  $\kappa_V^2$  can be large enough to be detected at future collider



4-models can be separated by looking at deviations in Yukawa couplings  $K_\tau, K_b, K_c,$

...

SK, K. Tsumura, K. Yagyu, H. Yokoya, 2014



# **Radiative Correction Decoupling/Non-decoupling**

# Higgs discovery in 2012

The mass is 125 GeV

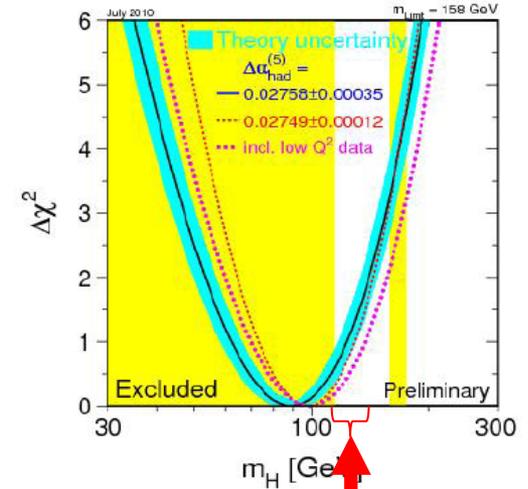
Spin/Parity  $0^+$

It couples to  $\gamma\gamma, ZZ, WW, bb, \tau\tau, \dots$

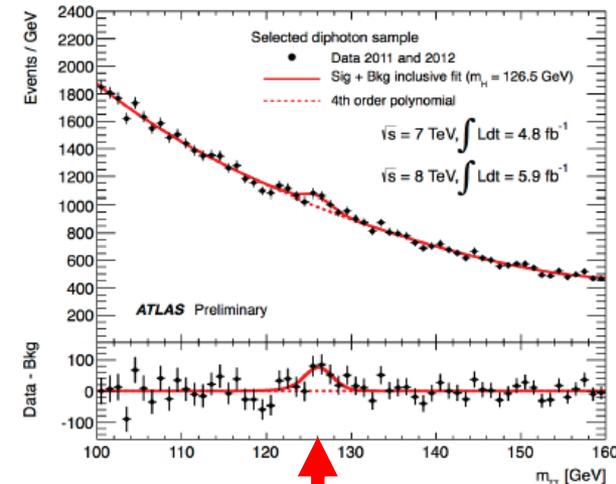
This is really a Higgs!



Measured couplings look consistent with the SM Higgs within the current errors



Higgs Mass indicated by LEP/SLC



ATLAS/CMS July 2012

New Particle !

# Radiative Corrections

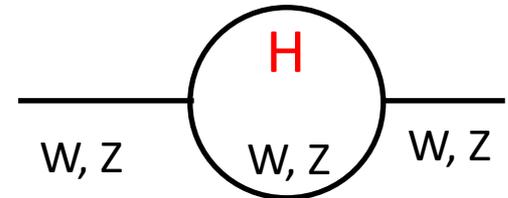
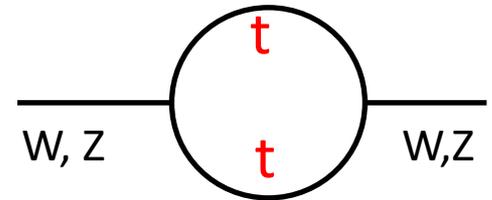
Rho parameter (unity in the SM)

$$\rho_{\text{exp}} = 1.0008^{+0.0017}_{-0.0007}$$

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} (= 1)$$

Loop corrections

$$\Delta\rho = 4\sqrt{2}G_F [\Pi_T^{33}(p^2 = 0) - \Pi_T^{11}(p^2 = 0)]$$



Loop effect of  $m_t$  and  $m_H$

$$\Delta\rho \simeq \frac{3G_F}{8\sqrt{2}\pi^2} \left( m_t^2 - M_Z^2 \sin^2 \theta_W \ln \frac{m_H^2}{m_W^2} \right)$$

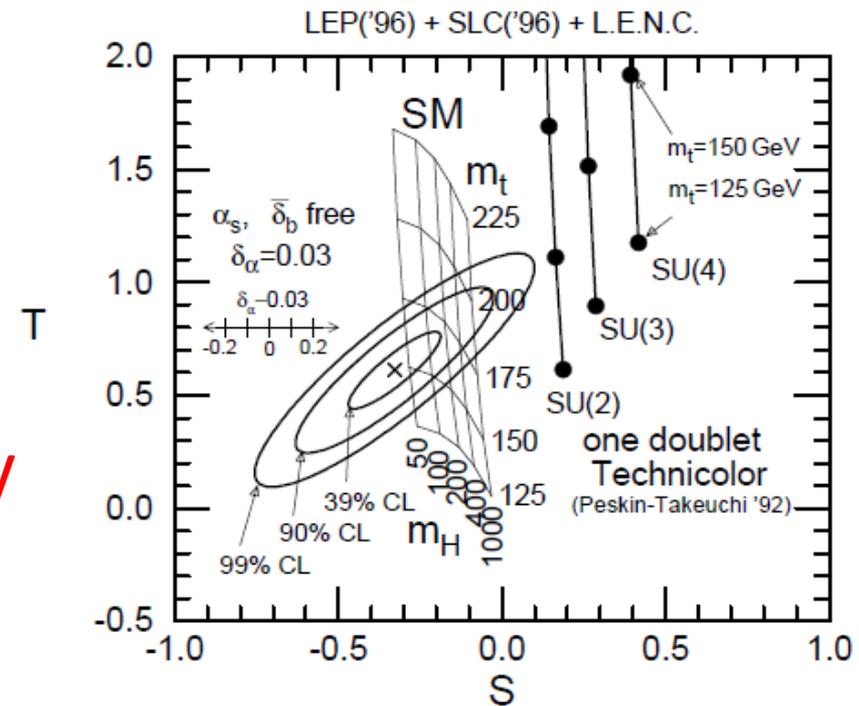
**Quadratic**

**Logarithmic**

# We knew the mass before discovery!

## Case of the top quark

- Quadratic mass dep. in  $\rho$  parameter (T parameter)
- Forget about  $m_H$  because it is only logarithmic
- LEP1 says  $m_t = 150-200 \text{ GeV}$
- Discovery at Tevatron (about  $175 \text{ GeV}$ )



Hagiwara, et al

$$\Delta\rho \simeq \frac{3G_F}{8\sqrt{2}\pi^2} \left( m_t^2 - M_Z^2 \sin^2 \theta_W \ln \frac{m_H^2}{m_W^2} \right)$$

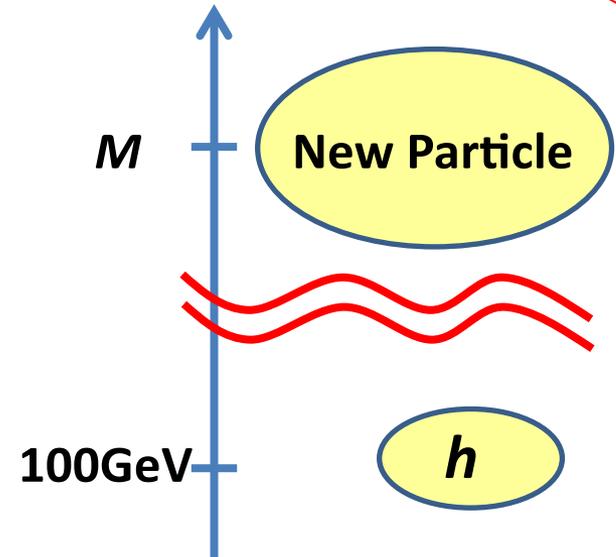
# Decoupling Theorem and its breaking

## Decoupling Theorem

Low energy observable  $O$

Renormalized quantity  $O$  is a function of  $M$  via loop contributions, but it decouples in the large mass limit

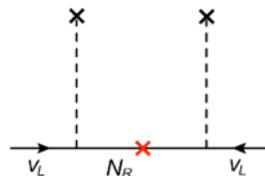
$$O(M) \sim \frac{1}{M^n}$$



Ex) GUT scale ( $10^{16}$  GeV) physics does not affect TeV scale physics

Ex) Seesaw Mechanism (Dim 5) at the tree-level

$$\mathcal{L} = \frac{c}{\Lambda} (\Phi^T \overline{\nu}_L^c) (\nu_L \Phi)$$



$$m_\nu \sim \frac{v^2}{M_{N_R}}$$

**QED**

Example of decoupling theorem

One-loop contributions to the two point functions

$$\mathcal{M}_{tree} \sim Qe \frac{1}{k^2} eQ' = \frac{QQ'}{\frac{1}{e^2}k^2}$$

$$\mathcal{M} \sim \frac{QQ'}{\frac{1}{e^2}k^2 - \Pi_{new}(k^2)}$$

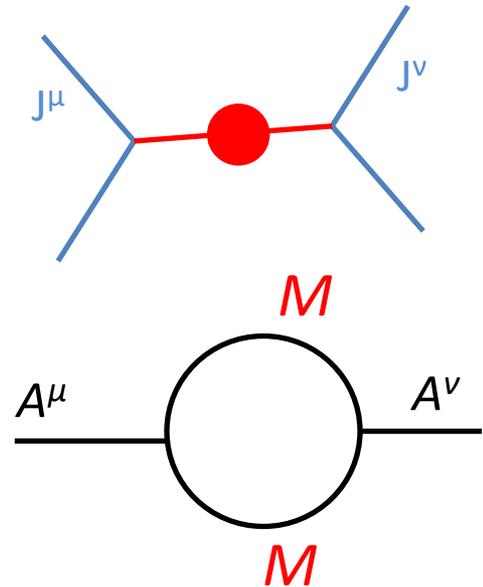
Self-Energy  $\Pi_{new}(k^2)$  has dim. 2, so that it can have  $M^2$  or  $\ln M$  dependence from power counting (non-decoupling effects)

$$\Pi_{new}(k^2) = \Pi_{new}(0) + k^2 \Pi'_{new}(0) + \dots$$

However from U(1) gauge symmetry  $\Pi_{new}(0)=0$ , and  $\Pi'_{new}(0)$  is absorbed by renormalization

$$\mathcal{M} \sim \frac{QQ'}{\left(\frac{1}{e^2} - \Pi'(0)_{New}\right)k^2 - \frac{(k^2)^2}{2}\Pi''_{new}(k^2)} = \frac{QQ'}{\frac{1}{e_R^2}k^2 - \frac{(k^2)^2}{2}\Pi''_{new}(0) + \dots}$$

Remaining  $\Pi''_{new}(0)$  is dim. -2, so that at most  $1/M^2$  (Decouple!)



# QED with spontaneously broken U(1)

$$\mathcal{M}_{tree} \sim Qe \frac{1}{k^2 - m_A^2} eQ' = \frac{QQ'}{\frac{1}{e^2}k^2 - v^2}$$

$$\mathcal{M} \sim \frac{QQ'}{\frac{1}{e^2}k^2 - v^2 - \Pi_{new}(k^2)}$$

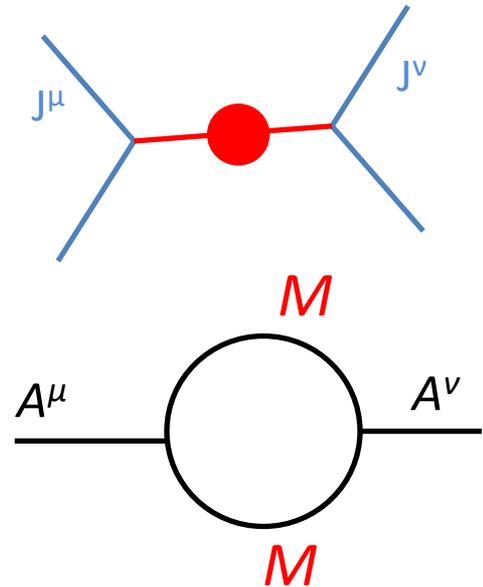
Self-Energy  $\Pi_{new}(k^2)$  has dim. 2, so that it can have  $M^2$  or  $\ln M$  dependence from power counting (non-decoupling effects)

$$\Pi_{new}(k^2) = \Pi_{new}(0) + k^2 \Pi'_{new}(0) + \dots$$

This time, U(1) is spontaneously broken, so that  $\Pi_{new}(0)$  is non-zero. But this time,  $\Pi_{new}(0)$  and  $\Pi'_{new}(0)$  are absorbed by  $v$  (or  $m_A$ ) and  $e$

$$\mathcal{M} \sim \frac{QQ'}{\left(\frac{1}{e^2} - \Pi'_{new}(0)\right)k^2 - (v^2 + \Pi_{new}(0)) - \frac{(k^2)^2}{2}\Pi''_{new}(0) + \dots} = \frac{QQ'}{\frac{1}{e_R^2}k^2 - v_R^2 - \frac{(k^2)^2}{2}\Pi''_{new}(0) + \dots}$$

Remaining  $\Pi''_{new}(0)$  is dim(-2), so that at most  $1/M^2$  (Decouple!)



# SM: Electroweak Theory with SSB

Two point functions

6 nondec. d.o.f.

$$\begin{array}{c}
 \text{wavy } W \text{ line} \text{---} \text{circle} \text{---} \text{wavy } W \text{ line} \\
 = M_{\text{New}}^2 + p^2 \ln \frac{M_{\text{New}}^2}{p^2} + \dots
 \end{array}$$

$$\begin{array}{c}
 \text{wavy } Z \text{ line} \text{---} \text{circle} \text{---} \text{wavy } Z \text{ line} \\
 = M_{\text{New}}^2 + p^2 \ln \frac{M_{\text{New}}^2}{p^2} + \dots
 \end{array}$$

$$\begin{array}{c}
 \text{wavy } \gamma \text{ line} \text{---} \text{circle} \text{---} \text{wavy } Z \text{ line} \\
 = \cancel{M_{\text{New}}^2} + p^2 \ln \frac{M_{\text{New}}^2}{p^2} + \dots
 \end{array}$$

$$\begin{array}{c}
 \text{wavy } \gamma \text{ line} \text{---} \text{circle} \text{---} \text{wavy } \gamma \text{ line} \\
 = \cancel{M_{\text{New}}^2} + p^2 \ln \frac{M_{\text{New}}^2}{p^2} + \dots
 \end{array}$$

$$\Pi_T^{\gamma\gamma}(p^2) = e^2 \Pi_T^{QQ}(p^2),$$

$$\Pi_T^{\gamma Z}(p^2) = eg_Z [\Pi_T^{3Q}(p^2) - s_W^2 \Pi_T^{QQ}(p^2)],$$

$$\Pi_T^{ZZ}(p^2) = g_Z^2 [\Pi_T^{33}(p^2) - 2s_W^2 \Pi_T^{3Q}(p^2) + s_W^4 \Pi_T^{QQ}(p^2)],$$

$$\Pi_T^{WW}(p^2) = g^2 \Pi_T^{11}(p^2).$$

WT identity:  $\Pi_{QQ}(0) = \Pi_{3Q}(0) = 0$

$$\alpha_*(0) = \alpha [1 + e^2 \Pi'_{QQ}(0)]$$

$$G_F = \frac{1}{\sqrt{2}v^2} - \frac{4}{\sqrt{2}v^4} \Pi_{11}(0)$$

$$m_Z^2 = \frac{e_0^2 v^2}{s_0^2 c_0^2} \frac{1}{4} + \frac{e_0^2}{s_0^2 c_0^2} (\Pi_{33} - 2s_0^2 \Pi_{3Q} + s_0^4 \Pi_{QQ})|_{q^2=m_Z^2}$$

Input parameters ( $\alpha$ ,  $G_F$ ,  $M_Z$ ) can absorb 3 of 6 non-decoupling effects.

Still, there remain 3 non-vanishing non-decoupling effects

**S, T, U** (Peskin-Takeuchi)

$$S = 16\pi [\bar{\Pi}_T^{3Q'}(p^2=0) - \bar{\Pi}_T^{33'}(p^2=0)],$$

$$T = \frac{4\sqrt{2}G_F}{\alpha_{\text{EM}}} [\bar{\Pi}_T^{33}(p^2=0) - \bar{\Pi}_T^{11}(p^2=0)],$$

$$U = 16\pi [\bar{\Pi}_T^{33'}(p^2=0) - \bar{\Pi}_T^{11'}(p^2=0)],$$

# Non-decoupling effects

Non-decoupling effects on electroweak parameters

$$\Gamma_Z, \sin\theta_w, m_W, \rho, \dots$$

are all described by  $S, T, U$  (at the leading level)

$$\begin{aligned} S &= 16\pi [\bar{\Pi}_T^{3Q'}(p^2=0) - \bar{\Pi}_T^{33'}(p^2=0)], \\ T &= \frac{4\sqrt{2}G_F}{\alpha_{\text{EM}}} [\bar{\Pi}_T^{33}(p^2=0) - \bar{\Pi}_T^{11}(p^2=0)], \\ U &= 16\pi [\bar{\Pi}_T^{33'}(p^2=0) - \bar{\Pi}_T^{11'}(p^2=0)], \end{aligned}$$

Ex)

$$\Delta\rho \equiv \rho - 1 = \alpha T$$

$$m_W^2 = m_W^2(\text{ref}) + \frac{\alpha c^2}{c^2 - s^2} \left( -\frac{1}{2}S + c^2 T + \frac{c^2 - s^2}{4s^2} U \right)$$

# Non-decoupling effects

What kind of new physics can produce large non-decoupling effects?

- Chiral Fermion Loop

$$m_f = 0 \rightarrow m_f = y_f v$$

- Higgs Loop

$$m_h^2 = 2 \lambda v^2$$

- Scalar Loop

$$m_S^2 = \lambda v^2 + M_{inv}^2$$

SU(2) doublet (N,E), hypercharge Y, masses  $m_N, m_E$



$$S = \frac{1}{6\pi} \left[ 1 - Y \ln \frac{m_N^2}{m_E^2} \right]$$

$$T = \frac{1}{16\pi s^2 c^2 m_Z^2} \left[ m_N^2 + m_E^2 - \frac{2m_N^2 m_E^2}{m_N^2 - m_E^2} \ln \frac{m_N^2}{m_E^2} \right]$$

$$U = \frac{1}{6\pi} \left[ -\frac{5m_N^4 - 22m_N^2 m_E^2 + 5m_E^4}{3(m_N^2 - m_E^2)^2} + \frac{m_N^6 - 3m_N^4 m_E^2 - 3m_N^2 m_E^4 + m_E^6}{(m_N^2 - m_E^2)^3} \ln \frac{m_N^2}{m_E^2} \right]$$

$$\Delta m = |m_N - m_E| \ll m_N, m_E$$

$$S = \frac{1}{6\pi}$$

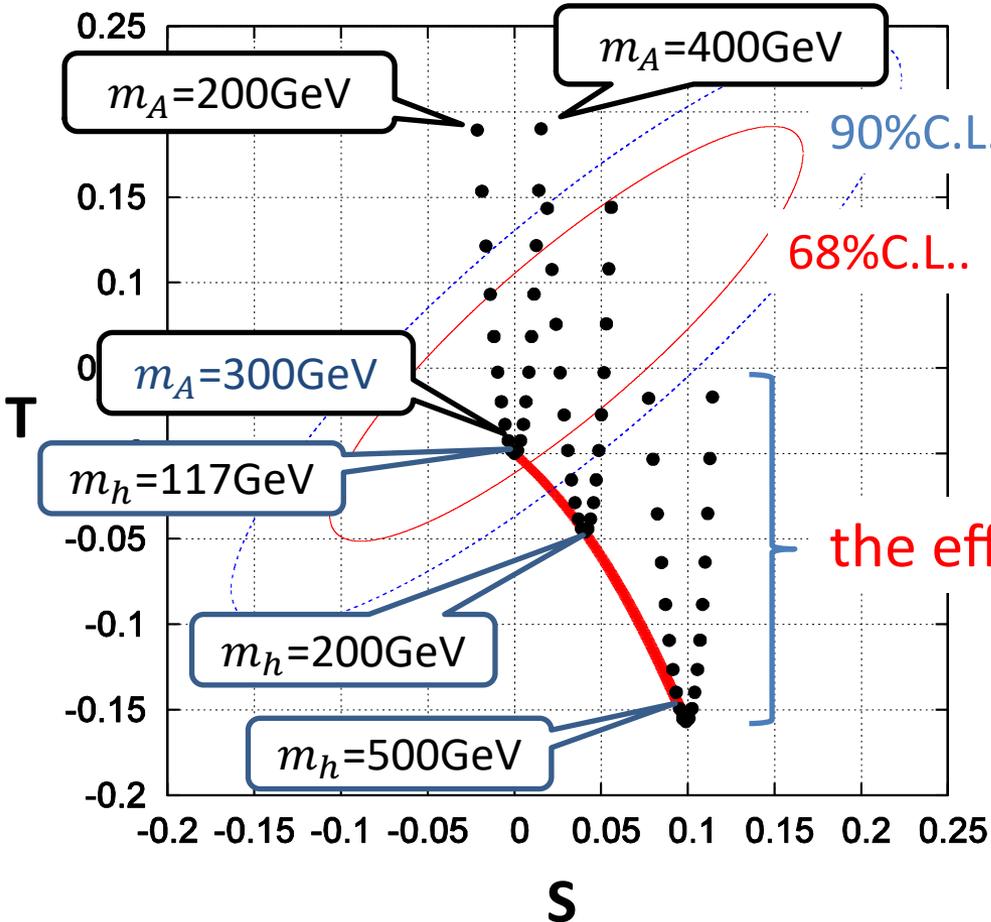
$$T = \frac{1}{12\pi s^2 c^2} \frac{(\Delta m)^2}{m_Z^2}$$

$$U = \frac{2}{15\pi} \frac{(\Delta m)^2}{m_N^2}$$

$$m_N = m_E \rightarrow T=U=0 \quad \text{Custodial Symmetric } O(4) \cong SU(2)_L \times SU(2)_R$$

# Effect of additional scalars in 2HDM

ST plot in the **THDM**



$$S_{\Phi} \approx -\frac{1}{12\pi} \ln \frac{m_{H^{\pm}}^2}{m_A^2},$$

$$T_{\Phi} \approx \frac{\sqrt{2}G_F}{12\pi^2\alpha_{EM}} (m_A - m_{H^{\pm}})^2$$

$$m_H = m_A, \sin(\beta - \alpha) = 1$$

$$T = T_{SM} + \underline{T_{\Phi}}$$

the effect of the THDM

$m_{H^{\pm}} = 300\text{GeV}$ ,  $m_A$  is varied from 200 to 400 GeV.

When  $m_A = m_H = m_{H^{\pm}}$ , we obtain  $S_{\Phi} = 0$  and  $T_{\Phi} = 0$ .

# **Radiative corrections to the Higgs boson couplings**

# All SM parameters are found

Next target is new physics!

- Importance of **Radiative Correction** calculation
- Future precision measurements
  - $S, T, U$  (Giga Z, Mega W)
  - Top (e.g. ttZ) couplings
  - Couplings of the discovered Higgs

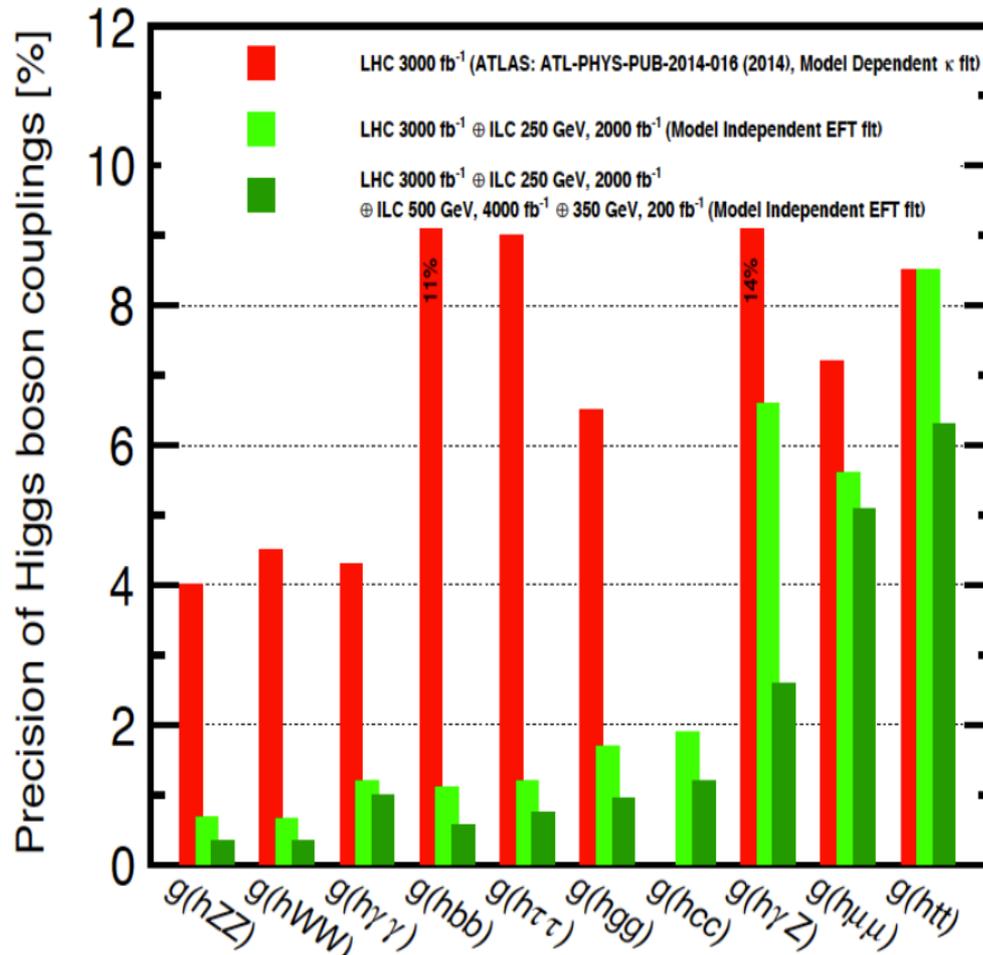
*hgg, hγγ, hWW, hZZ, htt, hbb, hττ, hμμ, hcc, ..., hhh*

At ILC, we may be able to distinguish models by detecting a **pattern of deviations** in the  **$h(125)$**  couplings from the SM values!

## Fingerprinting new physics models

# Higgs Precision at HL-LHC, ILC250, ...

[K. Fujii, et al., arXiv:1710.07621]



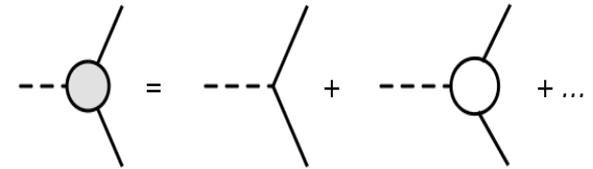
Future lepton colliders ILC, CEPC, FCCee, CLIC, ...

# Radiative Corrections

Higgs couplings  $h\gamma\gamma$ ,  $hgg$ ,  $hWW$ ,  $hZZ$ ,  $h\tau\tau$ ,  $hbb$ ,  $htt$ , ...

will be measured thoroughly in the future

**Analyses with radiative corrections are necessary**



Accurate Theory Predictions

×

Future Precision Measurements

**New Physics!**

**H-COUP Project** *SK, Kikuchi, Sakurai, Yagyu (2017)*

Full set of Fortran codes to systematically calculate quantum corrections to Higgs couplings in various extended Higgs models

Program H-COUP ver. 1 completed and released  
[Manual arXiv: 1710.04603]

Models

Additional Singlet

2HDM (I)

2HDM (II)

2HDM (X)

2HDM (Y)

Inert doublet/singlet

Triplet model,

GM model

# Non-decoupling effect on the Higgs couplings

Top-loop contribution in the SM

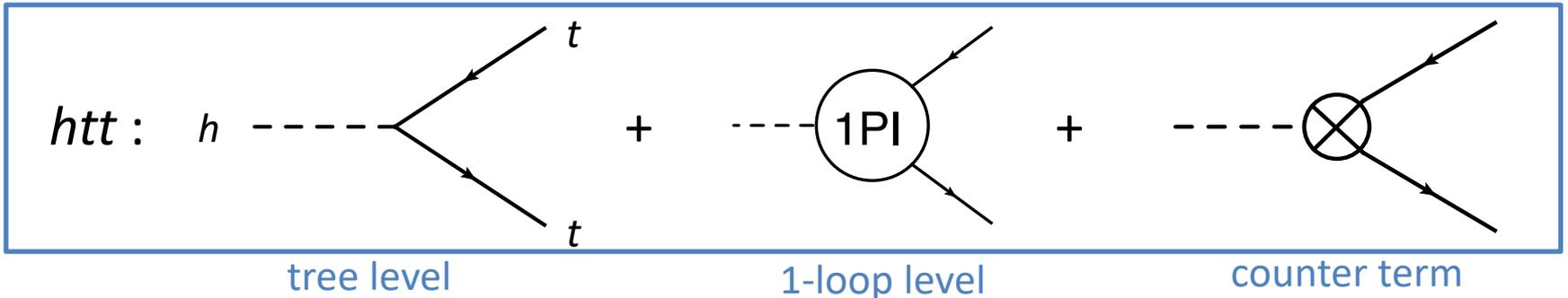
$$g_{hWW}^R \sim \frac{2m_W^2}{v} \left( 1 - \frac{5N_c}{96\pi^2} \frac{m_t^2}{v^2} \right)$$

---

$$y_{hff}^R \sim \frac{\sqrt{2}m_f}{v} \left( 1 - \frac{N_c}{12\pi^2} \frac{m_t^2}{v^2} \right)$$

How about the new physics loop contributions?

# For example: renormalized $htt$ coupling



tree level

$$h \text{---} \begin{array}{l} \nearrow t \\ \searrow t \end{array} = -\frac{m_t}{v} \xi_h^u$$

$$\xi_h^u = \sin(\beta - \alpha) + \cot\beta \cos(\beta - \alpha)$$

$\alpha$  : mixing angle for CP-even Higgs  $H, h$   
 $\beta$  : mixing angle for CP-odd Higgs  $A, G^0$

1-loop level

$$\text{---} \text{1PI} \text{---} = h \text{---} \begin{array}{c} \text{S} \\ \circlearrowleft \\ \text{S} \end{array} \begin{array}{l} \nearrow t \\ \searrow t \end{array} + \text{---} \begin{array}{c} \text{V} \\ \circlearrowleft \\ \text{V} \end{array} \begin{array}{l} \nearrow t \\ \searrow t \end{array} + \text{---} \begin{array}{c} \text{F} \\ \circlearrowleft \\ \text{F} \end{array} \begin{array}{l} \nearrow t \\ \searrow t \end{array} + \dots$$

counter term

$$\text{---} \text{X} \text{---} = -\frac{m_t}{v} \xi_h^u \left[ \frac{\delta m_t}{m_t} - \frac{\delta v}{v} + \frac{1}{2} \delta Z_h + \delta Z_t + \frac{\delta \xi_h^u}{\xi_h^u} + \frac{\xi_H^u}{\xi_h^u} (\delta C_h + \delta \alpha) \right]$$

counter term parameters :  $\delta m_t, \delta v, \delta \alpha, \delta \xi_h^u, \delta Z_h, \delta Z_t, \delta C_h$

these are determined by relevant renormalization conditions.

# Scale Factors (**1-loop level**) in 2HDM

Mixing parameter  $x = \cos(\beta - \alpha)$   $\left[ \sin(\beta - \alpha) = 1 - \frac{x^2}{2} \right]$  **SM-like**  
 $x \ll 1$

Scale Factor  
of the ***hVV*** Couplings

$$\Delta\kappa_X = \kappa_X - 1$$

$$\Delta\hat{\kappa}_V \simeq \underbrace{-\frac{1}{2}x^2}_{\text{mixing}} - \underbrace{\frac{A(m_\Phi^2, M^2)}{}}_{\text{loop}}$$

Loop Effect

$$A(m_\Phi, M) = \frac{1}{16\pi^2} \frac{1}{6} \sum_{\Phi} c_\Phi \frac{m_\Phi^2}{v^2} \left( 1 - \frac{M^2}{m_\Phi^2} \right)^2$$

$$m_\Phi^2 = M^2 + \lambda_i v^2$$

( $\Phi = H^\pm, A, H$ )

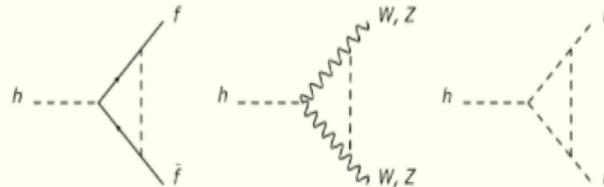
where

$$m_\Phi^2 \left( 1 - \frac{M^2}{m_\Phi^2} \right)^2 \begin{cases} \propto \frac{1}{m_\Phi^2} & (M \gg v) & \text{Decoupling} \\ \propto m_\Phi^2 & (M \sim v) & \text{Non-decoupling} \end{cases}$$

# H-COUP

## Website of H-COUP

You can download  
the program  
and the manual



H-COUP is a calculation tool composed of a set of Fortran codes to compute the renormalized Higgs boson couplings with radiative corrections in various non-minimal Higgs models, such as the Higgs singlet model, four types of two Higgs doublet models and the inert doublet model. The involved on-shell renormalization scheme is adopted, where the gauge dependence is eliminated.

Authors: Shinya Kanemura, Mariko Kikuchi, Kodai Sakurai and Kei Yagyu

The manual for H-COUP version 1.0 can be taken on [arXiv:1710.04603 \[hep-ph\]](https://arxiv.org/abs/1710.04603).

## Downloads

- H-COUP version 1.0 : [\[HCOUP-1.0.zip\]](#) [The manual is [here](#)]

In order to run H-COUP version 1.0, you need to install LoopTools ([www.feynarts.de/looptools/](http://www.feynarts.de/looptools/)).

## History

## Contact

# Example for the application of H-COUP

## H-COUP:

provides the EW (and Higgs) one-loop correction to the Higgs vertex functions in various extended Higgs sectors

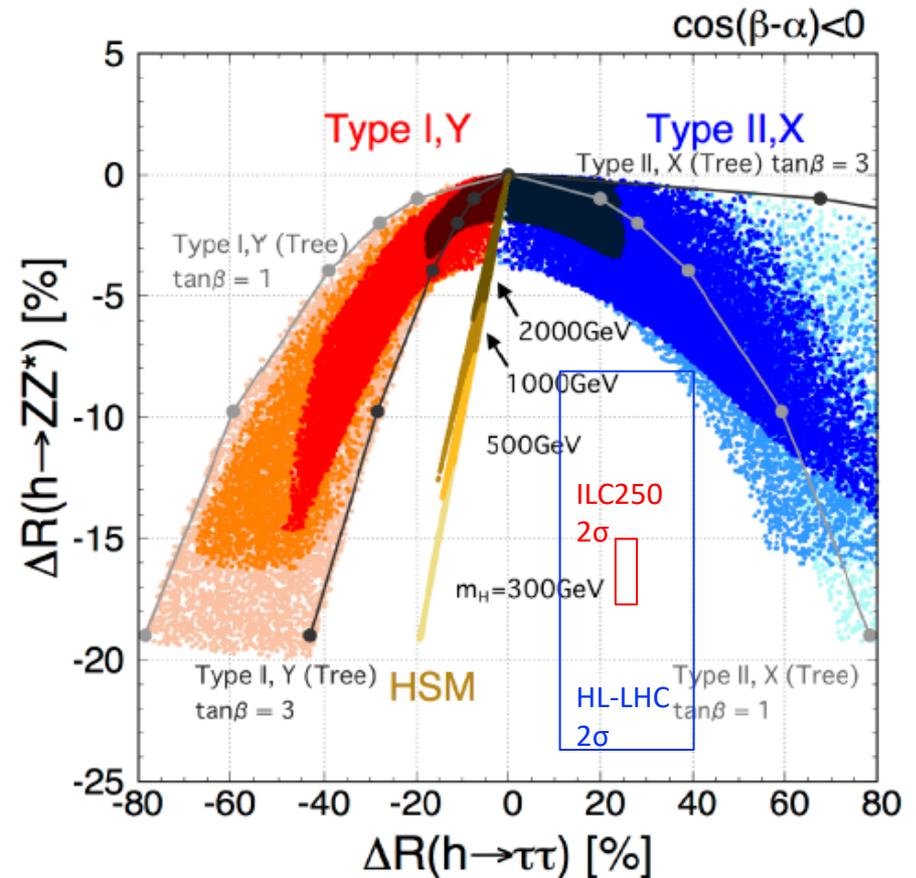
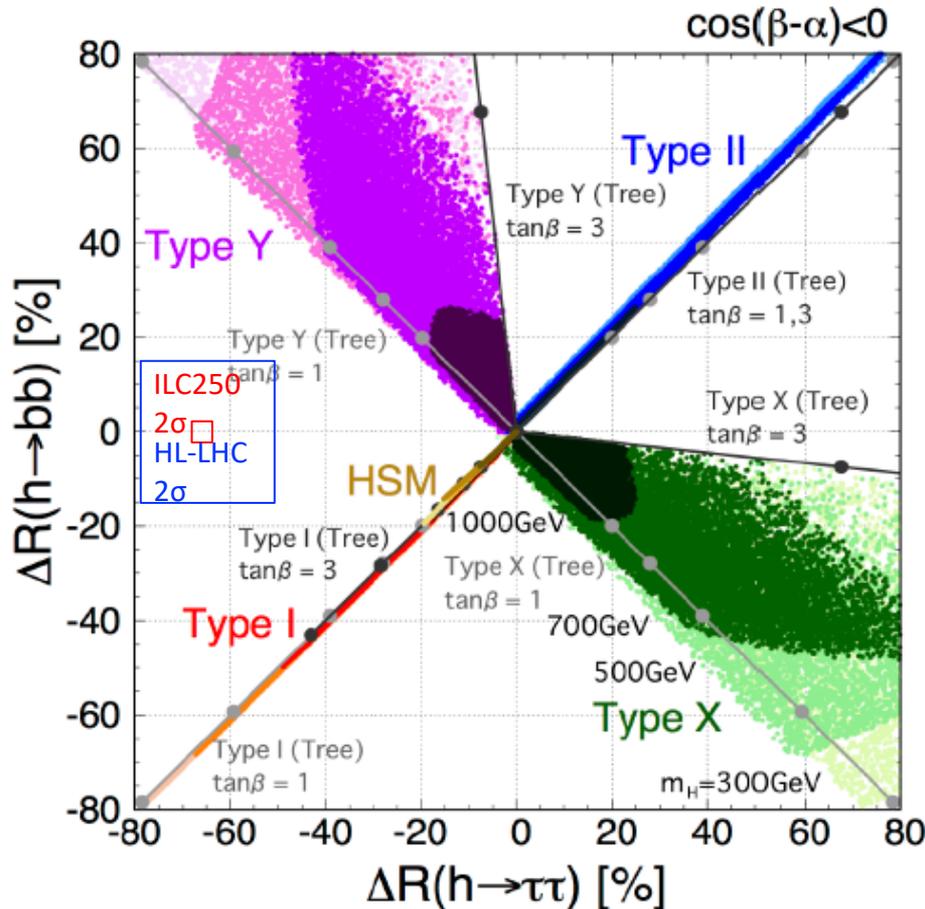
**Using H-COUP,**  
**the decay rates of the SM-like Higgs boson**  
**with EW (Higgs) and QCD corrections are**  
**Calculated in the HSM and 2HDM (type I, II, X, Y)**

$$\Gamma(h \rightarrow f\bar{f}), \Gamma(h \rightarrow ZZ^* \rightarrow Zf\bar{f}), \Gamma(h \rightarrow \gamma\gamma), \Gamma(h \rightarrow Z\gamma), \Gamma(h \rightarrow gg)$$

$$\Delta R(h \rightarrow XX) = \frac{\Gamma_{\text{NP}}(h \rightarrow XX)}{\Gamma_{\text{SM}}(h \rightarrow XX)} - 1$$

## H-COUP Project

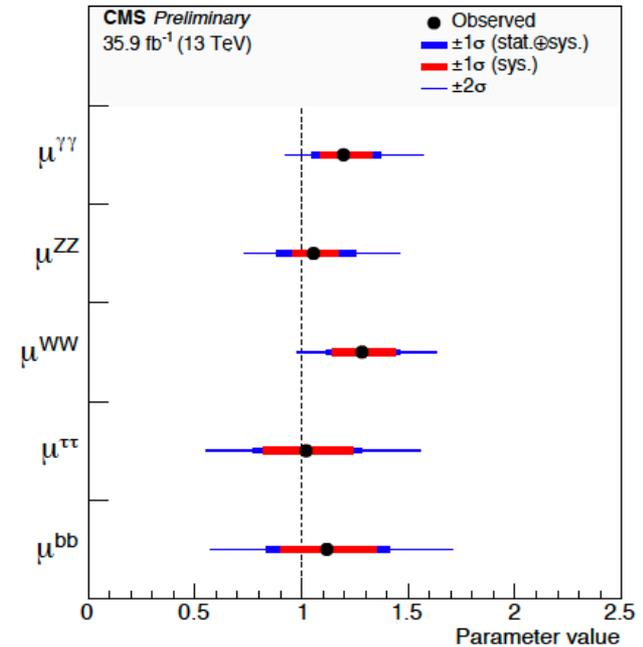
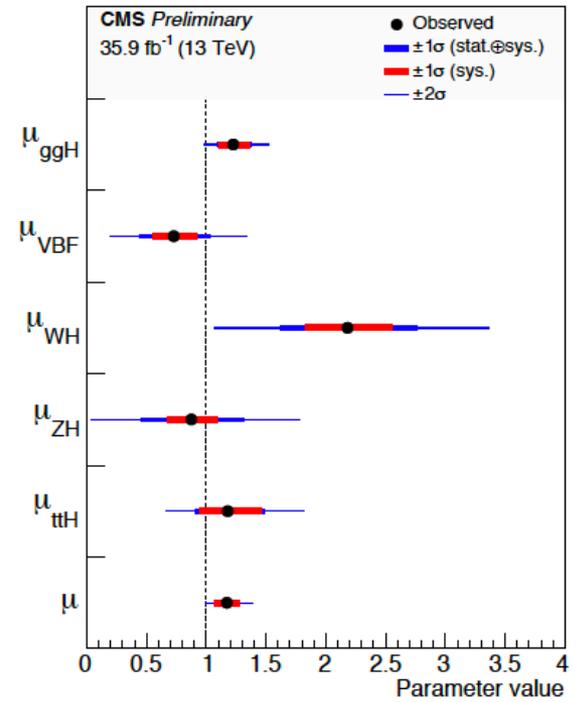
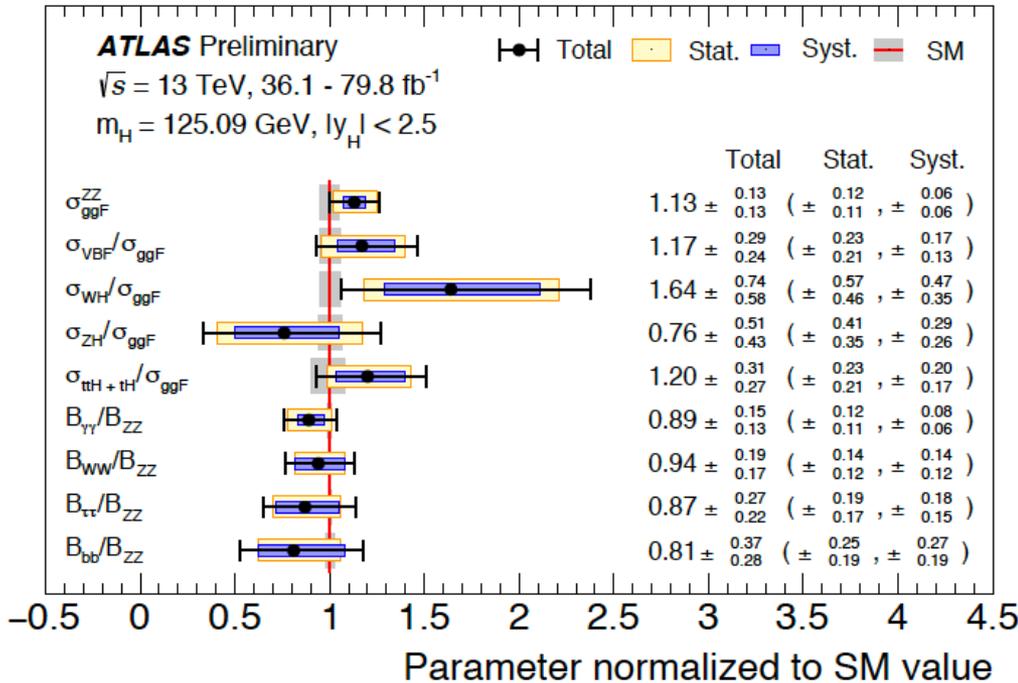
SK, Kikuchi, Mawatari, Sakurai, Yagyu 2018



Full set of 1-loop corrections (EW + QCD + Higgs) to the decay rates in various Higgs sectors and future precision measurements at ILC250 make us possible to fingerprint models and also to get information of inner parameters such as mass of the second Higgs boson

# By Production and Decay

- Several possible choices for model parameters allowed to account for SM deviations in combinations
- Ratios of cross sections and branching ratios cancel out some uncertainties



**Another example of non-  
decoupling effects  
Higgs potential**

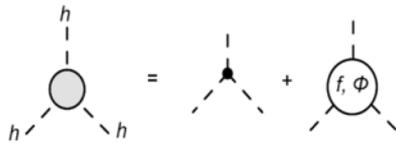
# Self-Coupling Constant

It is very important to know  $hhh$  coupling to reconstruct the Higgs potential

$$V_{\text{Higgs}} = \frac{1}{2} \underline{m_h^2} h^2 + \frac{1}{3!} \underline{\lambda_{hhh}} h^3 + \frac{1}{4!} \lambda_{hhhh} h^4 + \dots$$

**Effective Potential**  $V_{\text{eff}}(\varphi) = -\frac{\mu_0^2}{2} \varphi^2 + \frac{\lambda_0}{4} \varphi^4 + \sum_f \frac{(-1)^{2s_f} N_{C_f} N_{S_f}}{64\pi^2} m_f(\varphi)^4 \left[ \ln \frac{m_f(\varphi)^2}{Q^2} - \frac{3}{2} \right]$

**Renormalization**  $\left. \frac{\partial V_{\text{eff}}}{\partial \varphi} \right|_{\varphi=v} = 0, \quad \left. \frac{\partial^2 V_{\text{eff}}}{\partial \varphi^2} \right|_{\varphi=v} = m_h^2, \quad \left. \frac{\partial^3 V_{\text{eff}}}{\partial \varphi^3} \right|_{\varphi=v} = \lambda_{hhh}$



**Top loop Effect  
in the SM**

$$\lambda_{hhh}^{\text{SMloop}} \sim \frac{3m_h^2}{v} \left( 1 - \frac{N_c m_t^4}{3\pi^2 v^2 m_h^2} + \dots \right)$$

**Non-decoupling effect**

Tree level coupling  $\lambda_{hhh} = \frac{3m_h^2}{v_0}$

Effective Potential

$$V_{\text{eff}}(\varphi) = V_{\text{tree}}(\varphi) + \frac{1}{64\pi^2} N_{c_i} N_{s_i} (-1)^{2s_i} (M_i(\varphi))^4 \left[ \ln \left( \frac{(M_i(\varphi))^2}{Q^2} - \frac{3}{2} \right) \right]$$

Top quark effect  $M_\varphi = \frac{y_t \varphi}{\sqrt{2}}$

Expand the  $V_{\text{eff}}$  by  $h$   $\varphi = v_0 + h$

$$V_{\text{eff}} = -\frac{\mu^2}{2}(v_0 + h) + \frac{1}{4}\tilde{\lambda}(v_0 + h)^4 - \frac{N_c}{16\pi^2} \frac{y_t^4}{2} v_0^4 \left( \frac{h}{v_0} + \frac{7}{2} \frac{h^2}{v_0^2} + \frac{13}{3} \frac{h^3}{v_0^3} + \dots \right)$$

$$\tilde{\lambda} = \lambda - \frac{N_c}{16\pi^2} y_t^4 \left( \ln \frac{y_t^2 v_0^2}{2Q^2} - \frac{3}{2} \right)$$

Renormalization  
conditions

$$\left. \frac{\partial V}{\partial \varphi} \right|_{\varphi=v} = 0, \quad \left. \frac{\partial^2 V}{\partial h^2} \right|_{\varphi=v} = m_h^2, \quad \left. \frac{\partial^3 V}{\partial h^3} \right|_{\varphi=v} = \lambda_{hhh}^R$$

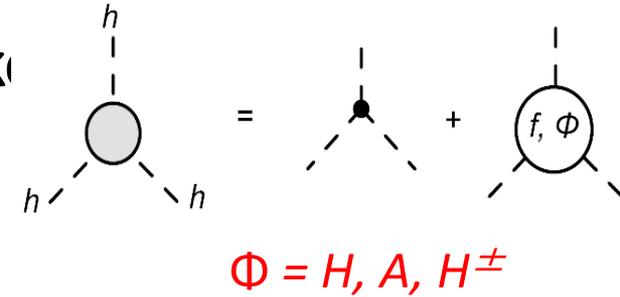
$$\begin{aligned} \frac{\partial V_{\text{eff}}}{\partial h} &= -\mu^2 v_0 + \tilde{\lambda} v_0^3 - \frac{1}{2} A v_0^3 = 0, \\ \frac{\partial^2 V_{\text{eff}}}{\partial^2 h} &= -\mu^2 + 3\tilde{\lambda} v_0^2 - \frac{7}{2} A v_0^2 = m_h^2, \\ \frac{\partial^3 V_{\text{eff}}}{\partial^3 h} &= 6\tilde{\lambda} v_0 - 13A v_0 = \lambda_{hhh}^R, \end{aligned} \quad A = \frac{N_c y_t^4}{16\pi^2}$$

Eliminating  $\mu^2$  and  $\tilde{\lambda}$ , and using  $y_t = \frac{\sqrt{2}m_t}{v_0}$

$$\lambda_{hhh}^R = \frac{3m_h^2}{v_0} \left( 1 - \frac{N_c}{3\pi^2} \frac{m_t^4}{v_0^2 m_h^2} \right)$$

# Case of Non-SUSY 2HDM

- Consider when the lightest  $h$  is SM-like [ $\sin(\beta-\alpha)=1$ ]
- At tree, the  $hhh$  coupling takes the same form as in the SM
- At 1-loop, non-decoupling effect  $m_\Phi^4$  (If  $M < v$ )



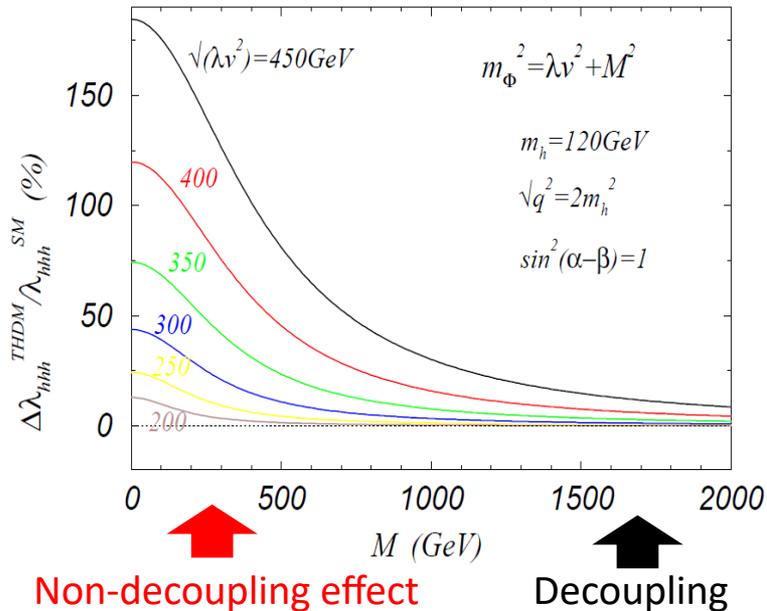
SK, Kiyoura, Okada, Senaha, Yuan, PLB558 (2003)

$$\lambda_{hhh}^{2\text{HDM}} \simeq \frac{3m_h^2}{v} \left[ 1 + \frac{m_\Phi^4}{12\pi^2 m_h^2} \left( 1 - \frac{M^2}{m_\Phi^2} \right)^3 - \frac{m_t^4}{\pi^2 v^2 m_h^2} \right]$$

$$m_\Phi^2 = M^2 + \lambda_i v^2$$

( $\Phi = H, A, H^\pm$ )

Extra scalar loop (green)  
Top loop (red)



**Correction can be huge  $\sim 100\%$**

# Part II Summary

- A Higgs boson was found, but the Higgs sector remains unknown
- Possibility of Extended Higgs sectors
- Direct Searches at LHC
- Indirect test of the Higgs sector via precision measurements for couplings of  $h(125)$  at future lepton colliders (ILC, CEPC, FCCee, CLIC, ...)
- Study with radiative corrections is important



# Numerical calculations

We discuss a possibility of discrimination among various extended Higgs models with the deviations from the SM in the decay width.

- Model

HSM, THDM Type-I, THDM Type-II, THDM Type-X, THDM Type-Y

- Scan region of input parameter in the THDMs

$$0.9 < \sin(\beta - \alpha) < 1, \quad 1 < \tan\beta < 3,$$

$$m_\Phi = 300, 500, 700, 1000 \text{ GeV}, \quad 0 < M^2 < m_\Phi, \quad m_H = m_A = m_{H^\pm}$$

( $\Phi = H, A, H^\pm$ )

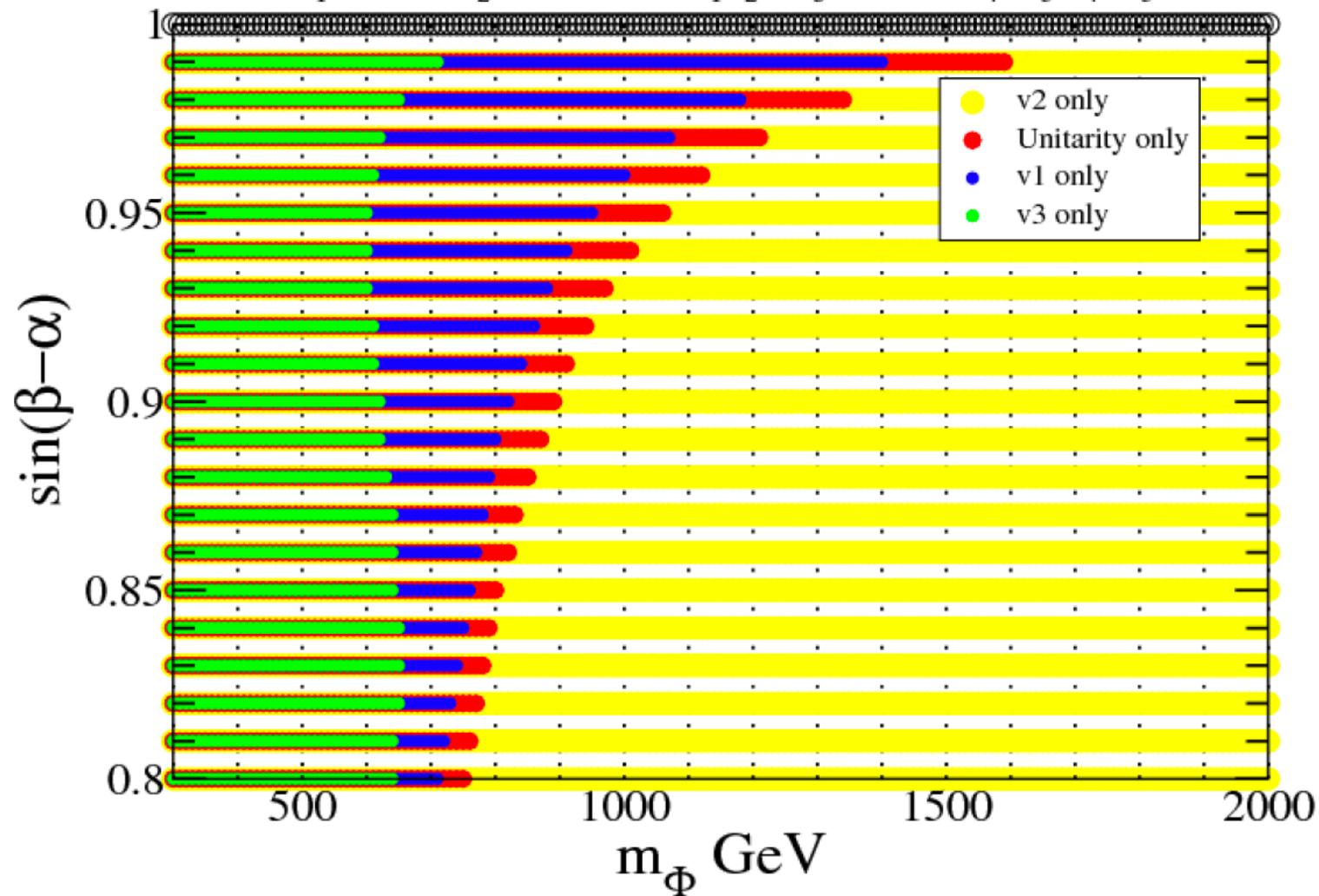
- Constraint

Perturbative unitarity, Vacuum stability,

Wrong vacuum condition (for HSM),

S, T parameters

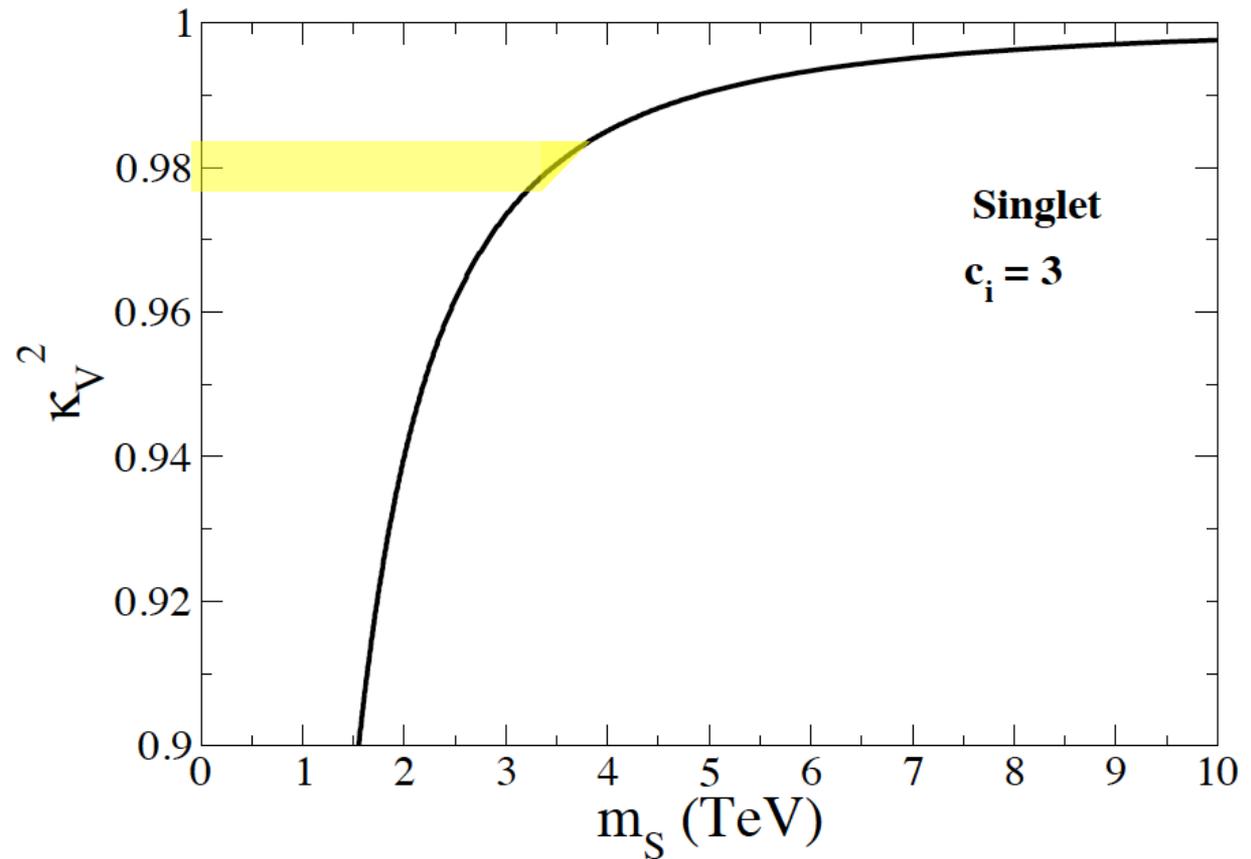
Allowed regions under theoretical constraints  
 $v1: \lambda_1 > 0$ ,  $v2: \lambda_2 > 0$ ,  $v3: \sqrt{\lambda_1 \lambda_2} + \lambda_3 + \text{MIN}(0, \lambda_4 + \lambda_5, \lambda_4 - \lambda_5) > 0$



# Unitarity in Non-SUSY 2HDM

In Higgs Singlet model ( $\Phi+S$ )

$$K_V^2 = \cos^2\theta$$



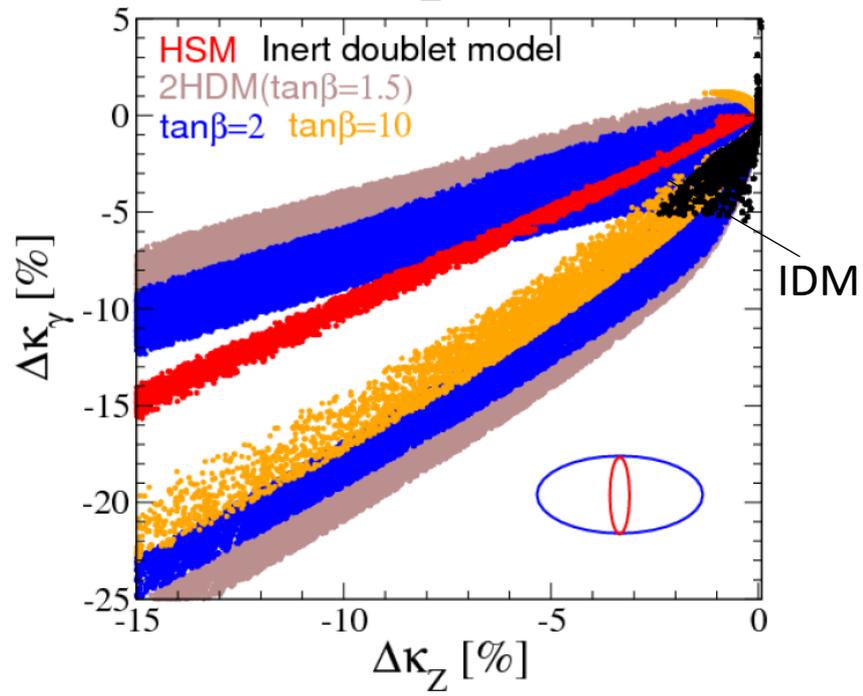
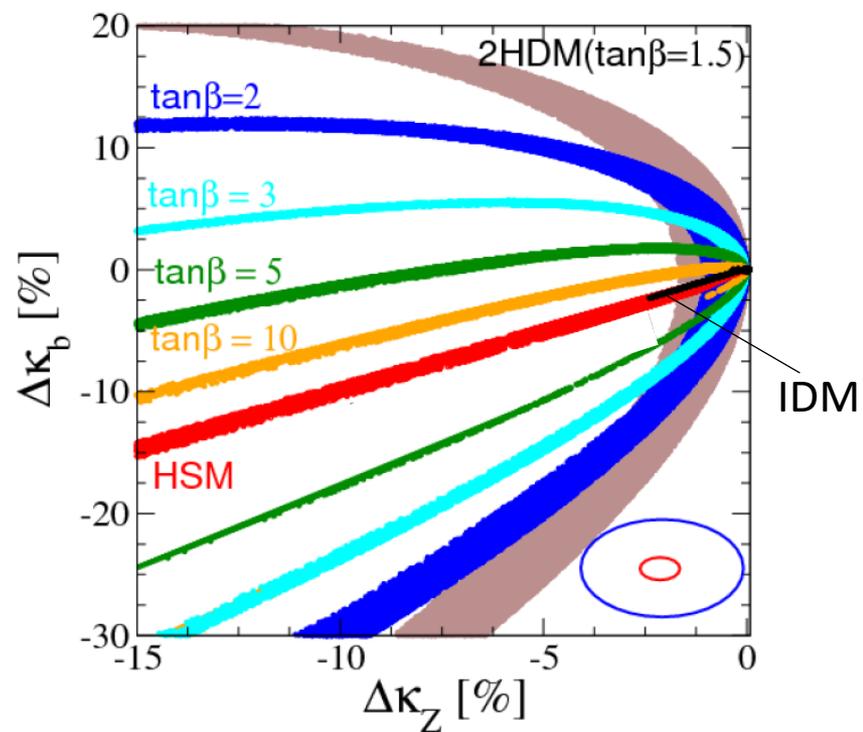
# Comparison of

1. 2HDM-I
2. Doublet-Singlet Model (HSM)
3. Inert Doublet Model (IDM)

Scan of inner parameters (mass, mixing angles) under the theoretical conditions of  
Perturbative unitarity  
Vacuum stability  
Condition for avoiding wrong vacuum (HSM)

These models may be distinguished, as long as a deviation in  $\kappa_Z$  is detected

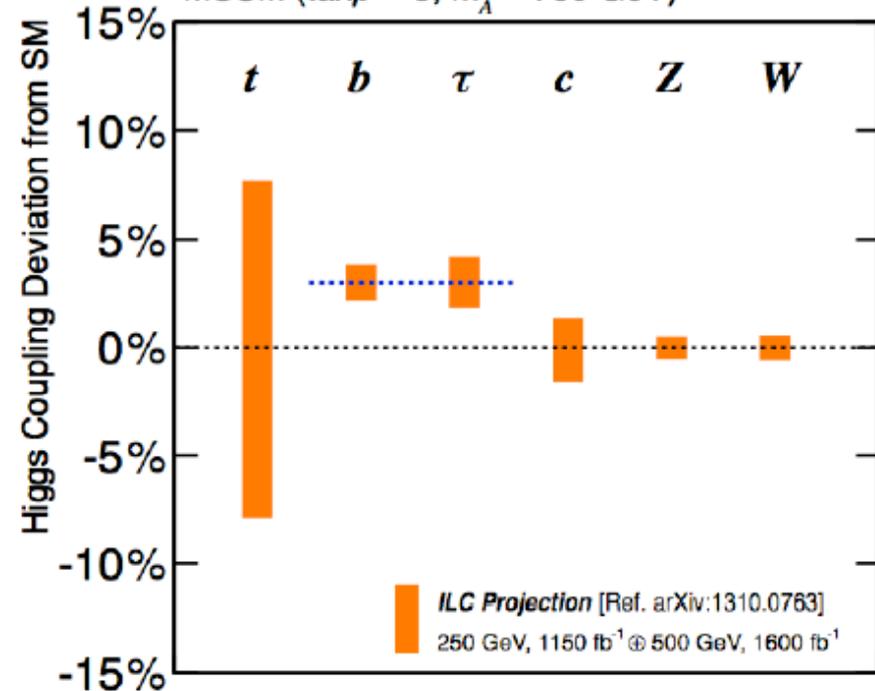
Ellipse,  $\pm 1\sigma$  at LHC3000 and ILC500



# Fingerprinting SUSY model and Composite Higgs models

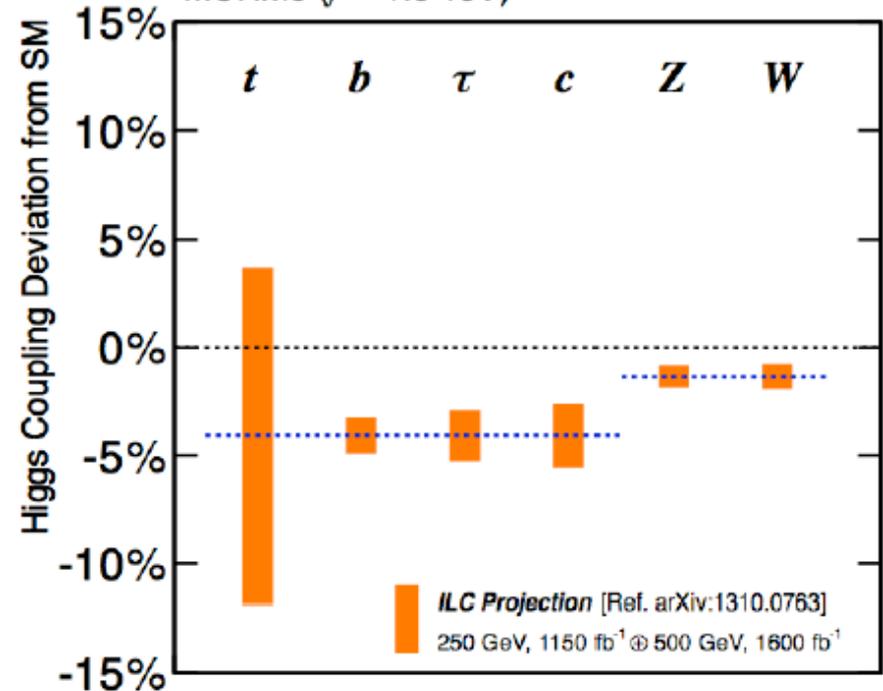
## SUSY

MSSM ( $\tan\beta = 5$ ,  $M_A = 700$  GeV)



## Composite Higgs

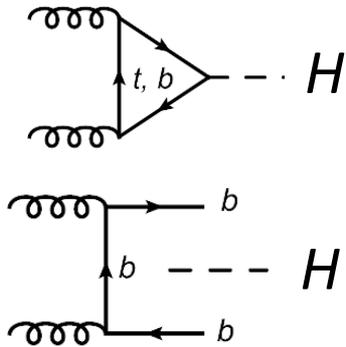
MCHM5 ( $f = 1.5$  TeV)



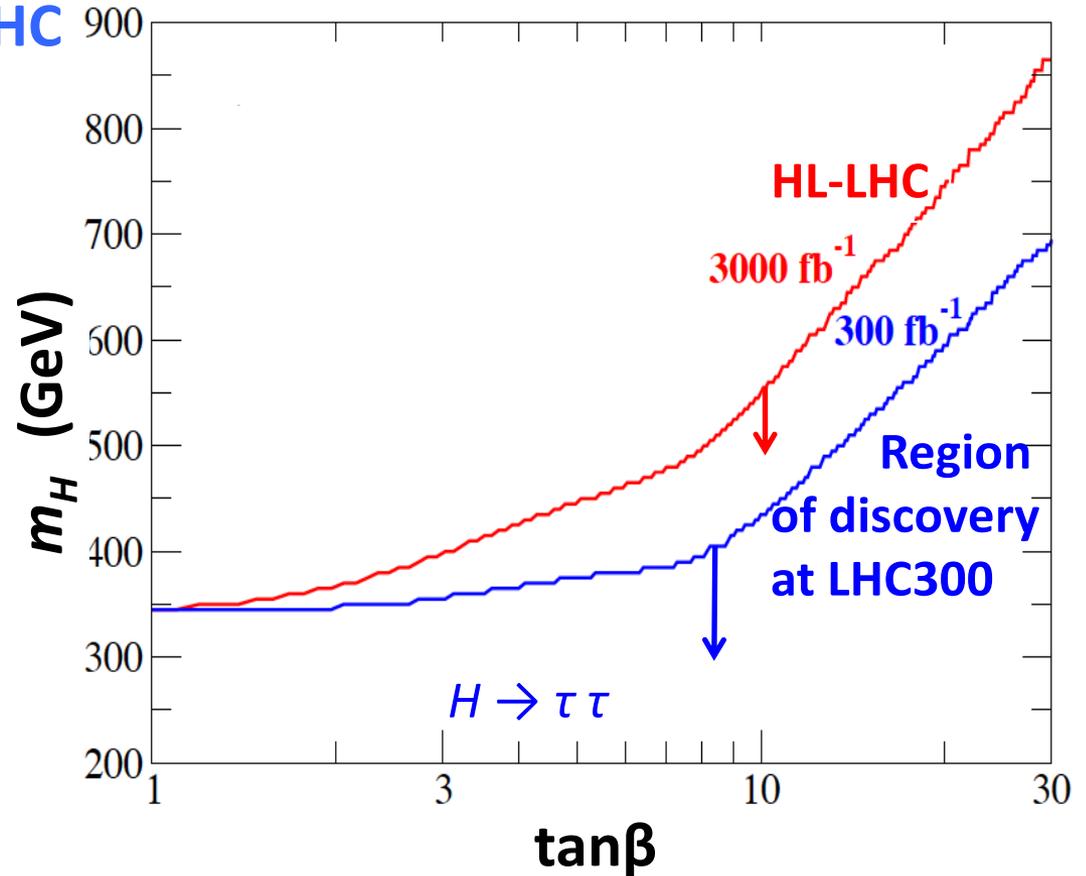
Fingerprinting models by precision study at ILC

# Complementarity

Direct detection of the heavier Higgs boson  $H$  at LHC



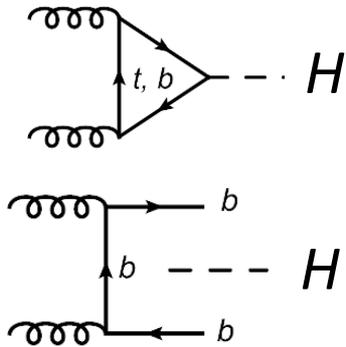
Type-II 2HDM



SK, Tsumura, Yagyū, Yokoya, 2014

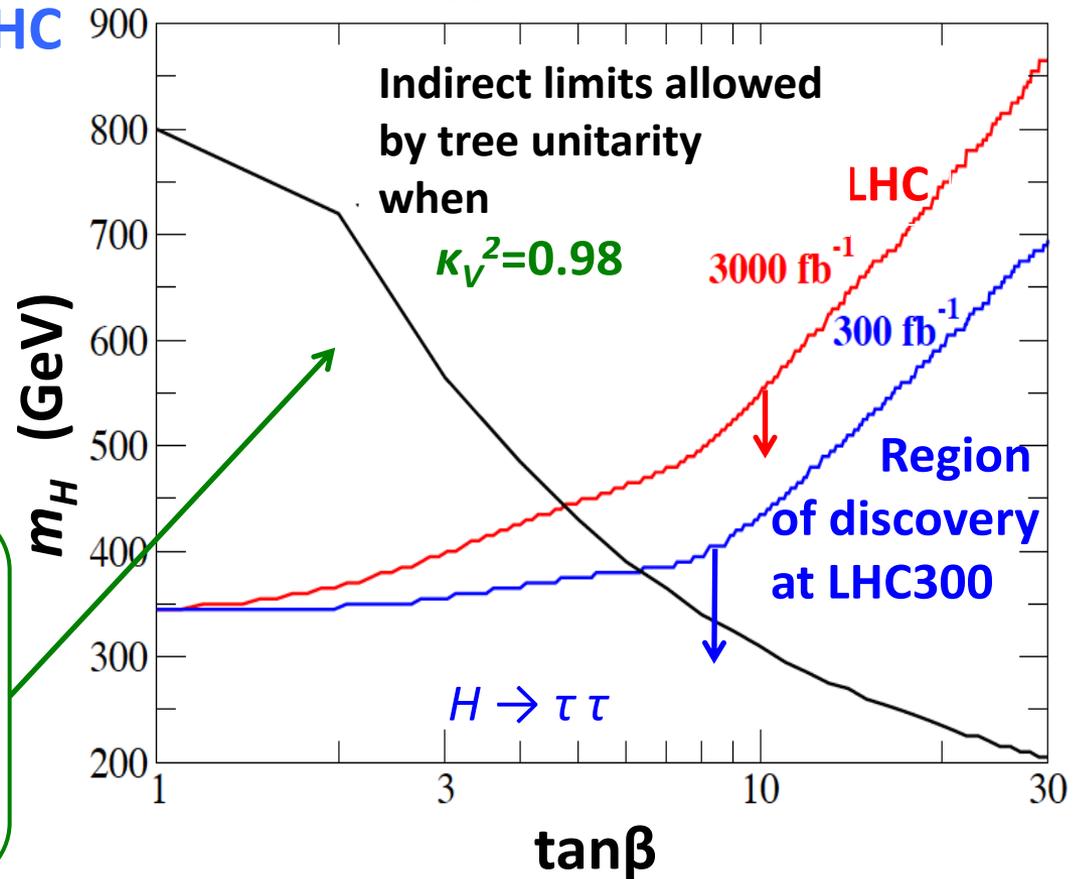
# Complementarity

Direct detection of the heavier Higgs boson  $H$  at LHC



Indirectly, new physics can be surveyed by detecting deviations even out of the direct search regions

Type-II 2HDM



# Fingerptinting the model (Exotics)

SK, K. Tsumura, K. Yagyu, H. Yokoya 2014

Universal Fermion  
Coupling ( $\kappa_F$ )

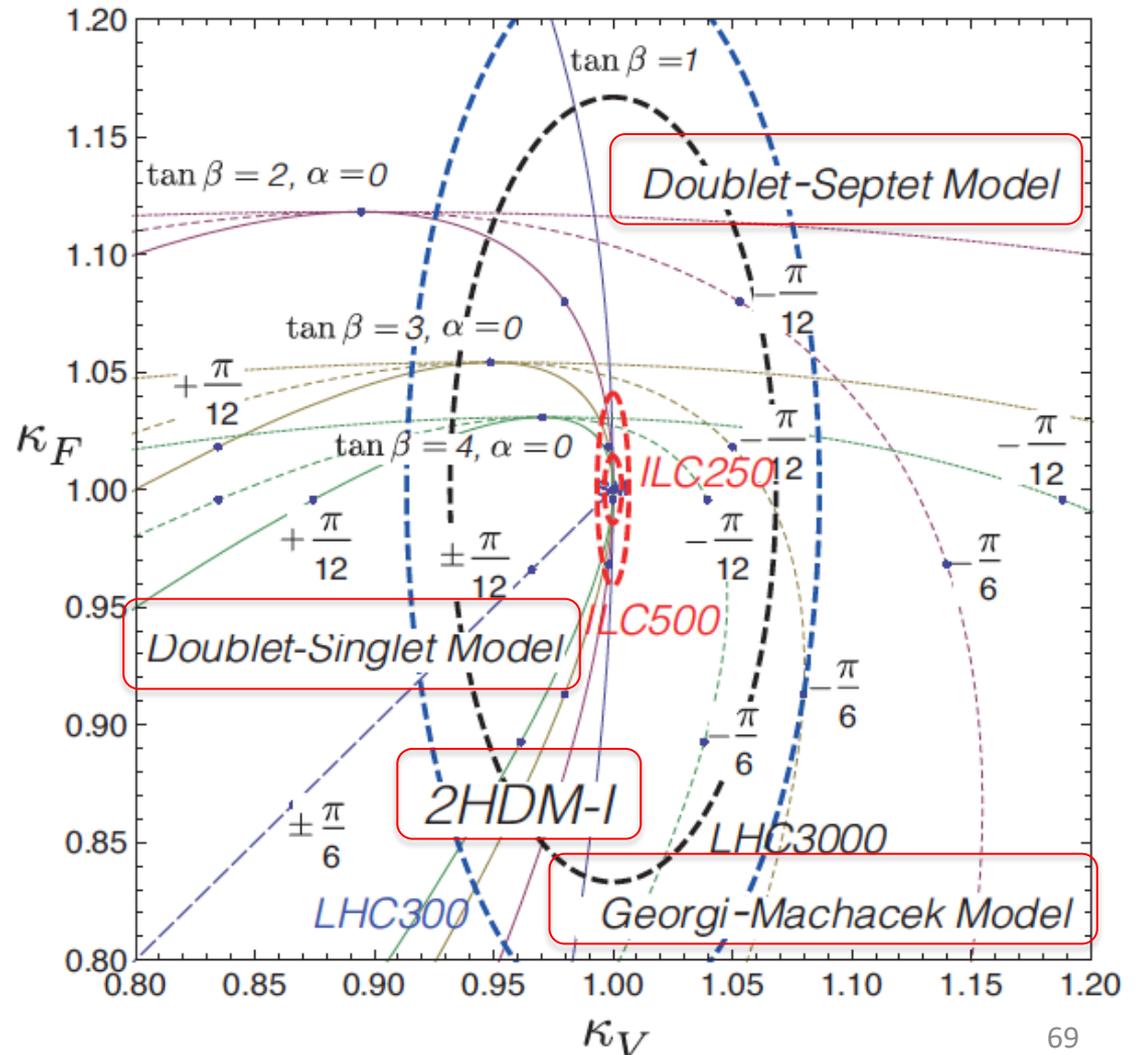
VS

$hVV$  coupling ( $\kappa_V$ )

Exotic models  
predict  $\kappa_V > 1$

We can discriminate  
Exotic models

Ellipse = 68.27% CL



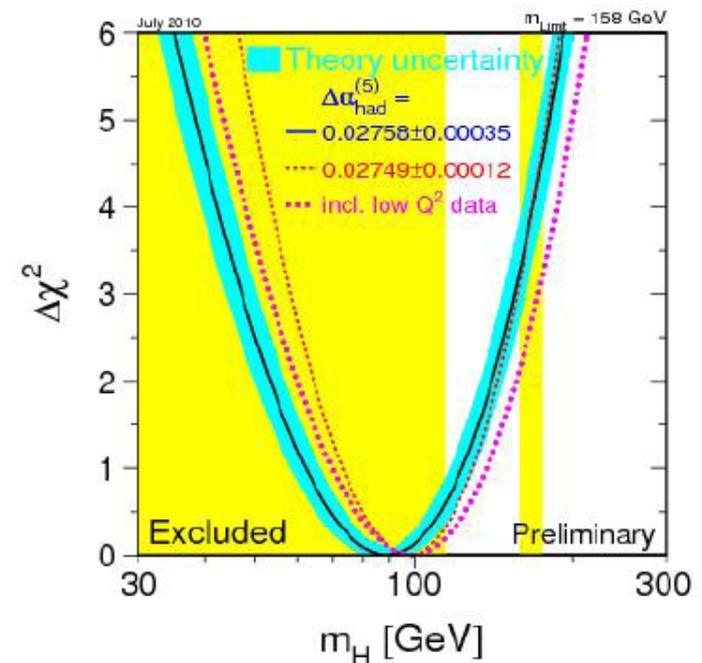
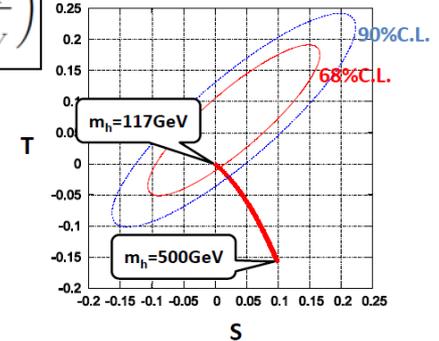
# It was repeated for Higgs at LEP2

## Case of Higgs boson

- Now we know top mass
- Rho is a function of only  $m_H$
- Precision measurement at LEP2
- **$114\text{GeV} < m_H < 150\text{ GeV}!$**
- LHC found new boson at  **$126\text{GeV}$**  (Higgs boson!)

Victory of precision measurements and theory calculations  
(VIVA! SM)

$$\Delta\rho \simeq \frac{3G_F}{8\sqrt{2}\pi^2} \left( m_t^2 - M_Z^2 \sin^2 \theta_W \ln \frac{m_H^2}{m_W^2} \right)$$



# Deviation in $hff$

Singlet, Exotics,

$$\Delta\kappa_u = - (1/2) x^2, \quad \Delta\kappa_d = - (1/2) x^2, \quad \Delta\kappa_\tau = - (1/2) x^2$$

If  $\Delta\kappa_V = 1\%$

$O(1)\%$

Type I 2HDM

$$\Delta\kappa_u = - \cot\beta |x|, \quad \Delta\kappa_d = - \cot\beta |x|, \quad \Delta\kappa_\tau = - \cot\beta |x|$$

$O(10)\%$

Type X (Lepton Specific) 2HDM

$$\Delta\kappa_u = - \cot\beta |x|, \quad \Delta\kappa_d = - \cot\beta |x|, \quad \Delta\kappa_\tau = + \tan\beta |x|$$

$O(10)\%$

MSSM (Type II 2HDM)

$$\Delta\kappa_u = + \cot\beta |x|, \quad \Delta\kappa_d = - \tan\beta |x|, \quad \Delta\kappa_\tau = - \tan\beta |x|$$

$O(10)\%$

MCHM4

$$\Delta\kappa_u = - (1/2) x^2, \quad \Delta\kappa_d = - (1/2) x^2, \quad \Delta\kappa_\tau = - (1/2) x^2$$

$O(1)\%$

MCHM5

$$\Delta\kappa_u = - (3/2) x^2, \quad \Delta\kappa_d = - (3/2) x^2, \quad \Delta\kappa_\tau = - (3/2) x^2$$

$O(1)\%$