

Diffraction in the shockwave formalism

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[RB, A.Grabovsky, L.Szymanowski, S.Wallon (arXiv:1405.7676)]

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1 DGLAP regime vs BFKL regime

2 Diffractive DIS

- Rapidity gap events at HERA
- Collinear factorization approach
- k_T -factorization approach : two exchanged gluons
- Confrontation of the two approaches with HERA data

3 Diffractive production of jets : our approach

- $q\bar{q}$ production
- $q\bar{q}g$ production
- Linear approximation : 2 and 3 exchanged gluons

4 Conclusion

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2 Diffractive DIS

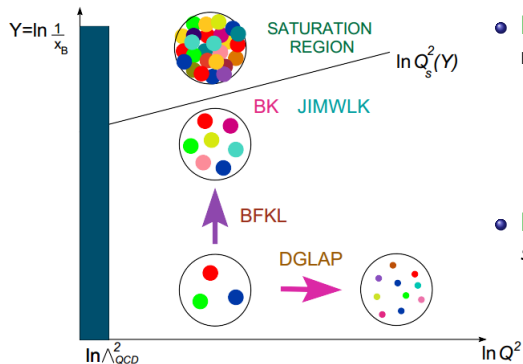
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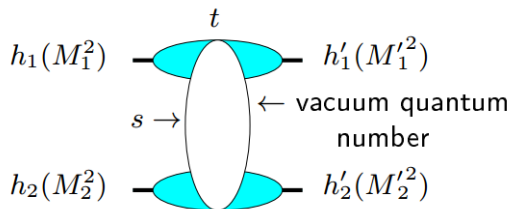
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Two regimes of perturbative QCD



- **DGLAP** dynamics : $Q^2 \rightarrow \infty$, moderate x_B
 - Governed by **collinear** dynamics
 - Resummation of Q^2 logs : $(\alpha_s \ln Q^2)^n, \alpha_s (\alpha_s \ln Q^2)^n \dots$
- **BFKL** dynamics (Regge limit) $s \gg Q^2 \gg \Lambda_{QCD}$ ($x_B \ll 1$)
 - Governed by **soft** dynamics
 - Resummation of $\frac{1}{x_B} \sim s$ logs : $(\alpha_s \ln s)^n, \alpha_s (\alpha_s \ln s)^n \dots$



$$\sigma = \frac{1}{s} \text{Im} \mathcal{A} \sim s^{\alpha(0)-1}$$

$$\alpha(0) - 1 = C \alpha_s$$

$$C > 0$$

\Rightarrow Violation of the Froissart bound

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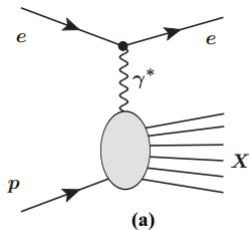
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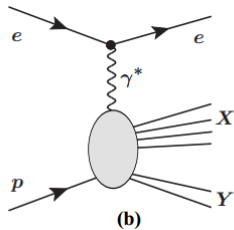
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Rapidity gap events at HERA

Experiments at HERA : about 10% of scattering events reveal a **rapidity gap**



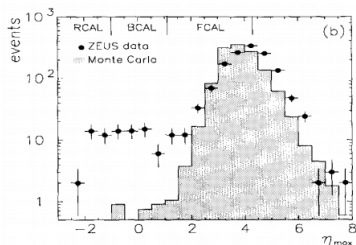
DIS events



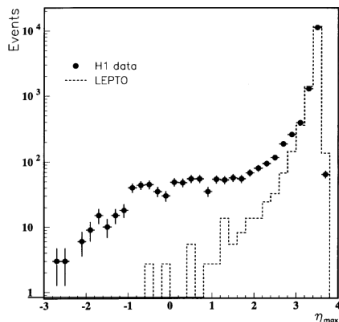
DDIS events

Rapidity gap events at HERA

Experiments at HERA : about 10% of events reveal a rapidity gap



ZEUS, 1993



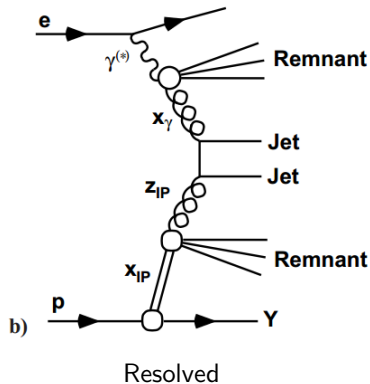
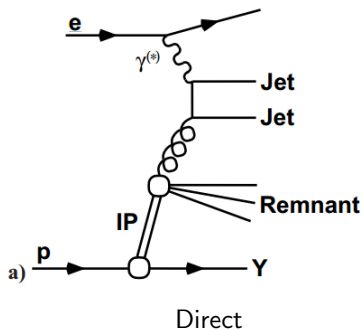
H1, 1994

Theoretical approaches for DDIS using pQCD

- **Collinear factorization** approach
 - Relies on QCD factorization theorem, using a hard scale such as the **virtuality** Q^2 of the incoming photon
 - One needs to introduce a **diffractive distribution function** for partons *within a pomeron*
- **k_T factorization** approach for two exchanged gluons
 - low-x QCD approach : $s \gg Q^2 \gg \Lambda_{QCD}$
 - The pomeron is described as a **two-gluon color-singlet** state

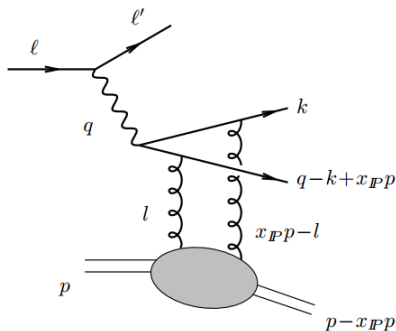
Theoretical approaches for DDIS using pQCD

Collinear factorization approach



Theoretical approaches for DDIS using pQCD

k_T -factorization approach : two gluon exchange

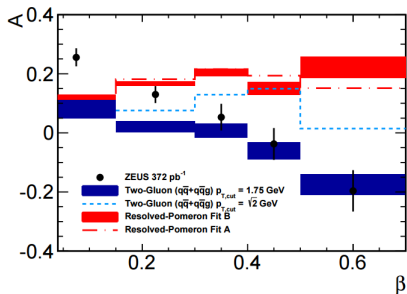


Bartels, Ivanov, Jung, Lotter, Wüsthoff

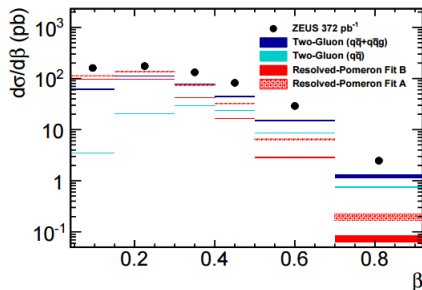
Braun and Ivanov developed a similar model in [collinear factorization](#)

Theoretical approaches for DDIS using pQCD

Confrontation of the two approaches with HERA data



ZEUS collaboration, 2015



ZEUS collaboration, 2015

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Assumptions

- Regge limit : $s \gg Q^2 \gg \Lambda_{QCD}$
- No approximation for the outgoing gluon, contrary to e.g. :
 - Collinear approximation [Wüsthoff, 1995]
 - Soft approximation [Bartels, Jung, Wüsthoff, 1999]
- Lightcone coordinates and lightcone gauge :

$$\begin{aligned}p^+ &= n_2 \cdot p = \frac{1}{2} (p^0 + p^3), \\p^- &= n_1 \cdot p = p^0 - p^3 \\n_2 \cdot \mathcal{A} &= 0 \\n_2^2 &= 0\end{aligned}$$

- Shockwave (Wilson lines) approach [Balitsky, 1995]

The shockwave approach

One decomposes the gluon field \mathcal{A} into an **internal field** and an **external field** :

$$\mathcal{A}^\mu = A^\mu + b^\mu$$

The internal one contains the gluons with rapidity $p^+ > e^\eta = \sigma$ and the external one contains the gluons with rapidity $p^+ < \sigma$. One writes :

$$b^\mu(z) = \delta(z^+) B(\vec{z}) n_2^\mu$$

Intuitively, large boost λ along the $+$ direction :

$$b^+(x^+, x^-, \vec{x}) \rightarrow \frac{1}{\lambda} b^+ \left(\lambda x^+, \frac{1}{\lambda} x^-, \vec{x} \right)$$

$$b^-(x^+, x^-, \vec{x}) \rightarrow \lambda b^- \left(\lambda x^+, \frac{1}{\lambda} x^-, \vec{x} \right)$$

$$b^i(x^+, x^-, \vec{x}) \rightarrow b^i \left(\lambda x^+, \frac{1}{\lambda} x^-, \vec{x} \right)$$

Propagator through a shockwave

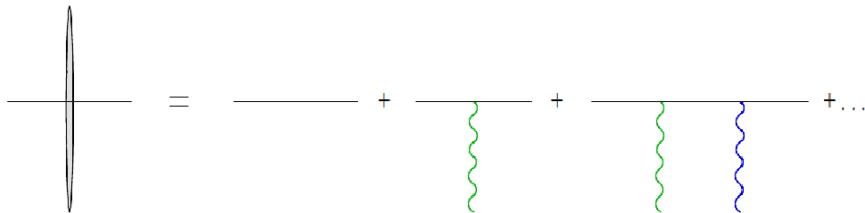
$$G(z_2, z_0) = - \int d^4 z_1 \theta(z_2^+) \delta(z_1^+) \theta(-z_0^+) G(z_2 - z_1) \gamma^+ G(z_1 - z_0) U_1$$

$$G(q, p) = (2\pi) \theta(p^+) \int d^D q_1 \delta(q + q_1 - p) G(q) \gamma^+ \tilde{U}_{\bar{q}_1} G(p)$$

Wilson lines :

$$U_i = U_{\bar{z}_i} = U(\bar{z}_i, \eta) = P \exp \left[ig \int_{-\infty}^{+\infty} b_{\eta}^{-}(z_i^+, \bar{z}_i) dz_i^+ \right]$$

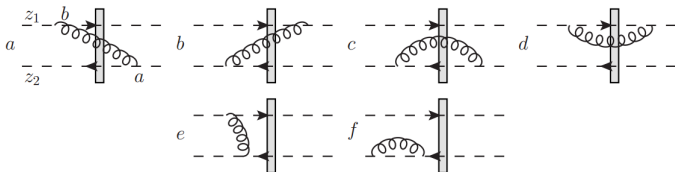
$$U_i = 1 + ig \int_{-\infty}^{+\infty} b_{\eta}^{-}(z_i^+, \bar{z}_i) dz_i^+ + (ig)^2 \int_{-\infty}^{+\infty} b_{\eta}^{-}(z_i^+, \bar{z}_i) b_{\eta}^{-}(z_j^+, \bar{z}_j) \theta(z_{ji}^+) dz_i^+ dz_j^+ + \dots$$



The BK equation

Dipole operator

$$\mathbf{U}_{12} = \frac{1}{N_c} \text{Tr} \left(U_1 U_2^\dagger \right) - 1$$

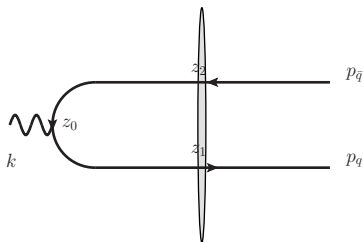


BK equation [Balitsky, 1995] [Kovchegov, 1999]

$$\frac{d\mathbf{U}_{12}}{d\ln\sigma} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \frac{\vec{z}_{12}^2}{\vec{z}_{13}^2 \vec{z}_{23}^2} [\mathbf{U}_{13} + \mathbf{U}_{32} - \mathbf{U}_{12} - \mathbf{U}_{13} \mathbf{U}_{32}]$$

Non-linear term : **saturation**

Matrix element for EM current



$q\bar{q}$ production

$$M_0^\alpha = \int d\vec{z}_1 d\vec{z}_2 F(\vec{z}_1, \vec{z}_2)^\alpha N_c \mathbf{U}_{12}$$

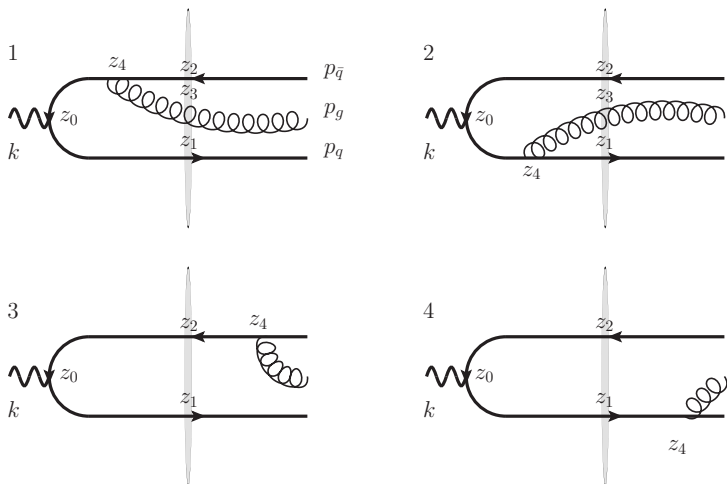
We recover the well-known results :

$$F(\vec{z}_1, \vec{z}_2)^\alpha \varepsilon_{L\alpha} = \theta(p_q^+) \theta(p_{\bar{q}}^+) \frac{\delta(k^+ - p_q^+ - p_{\bar{q}}^+)}{(2\pi)^2} e^{-i\vec{p}_q \cdot \vec{z}_1 - i\vec{p}_{\bar{q}} \cdot \vec{z}_2} \\ \times (-2i) \delta_{\lambda_q, -\lambda_{\bar{q}}} x_q x_{\bar{q}} Q K_0 \left(Q \sqrt{x_q x_{\bar{q}} \vec{z}_{12}^2} \right)$$

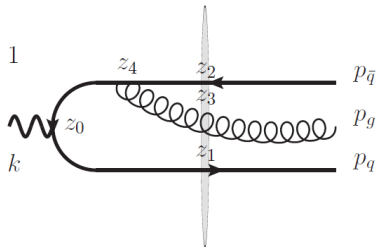
and

$$F(\vec{z}_1, \vec{z}_2)^j \varepsilon_{Tj} = \theta(p_q^+) \theta(p_{\bar{q}}^+) \frac{\delta(k^+ - p_q^+ - p_{\bar{q}}^+)}{(2\pi)^2} e^{-i\vec{p}_q \cdot \vec{z}_1 - i\vec{p}_{\bar{q}} \cdot \vec{z}_2} \delta_{\lambda_q, -\lambda_{\bar{q}}} \\ \times (x_q - x_{\bar{q}} + s\lambda_q) \frac{\vec{z}_{12} \cdot \vec{\varepsilon}_T}{\vec{z}_{12}^2} Q \sqrt{x_q x_{\bar{q}} \vec{z}_{12}^2} K_1 \left(Q \sqrt{x_q x_{\bar{q}} \vec{z}_{12}^2} \right).$$

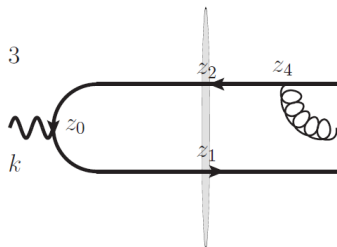
$q\bar{q}g$ production



$q\bar{q}g$ production



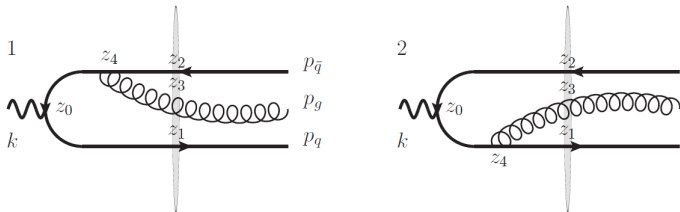
First kind



Second kind

$$\begin{aligned}
 M^\alpha &= \frac{N_c^2}{2} \int d\vec{z}_1 d\vec{z}_2 d\vec{z}_3 F_1(\vec{z}_1, \vec{z}_2, \vec{z}_3)^\alpha (\mathbf{U}_{13} + \mathbf{U}_{32} - \mathbf{U}_{12} + \mathbf{U}_{13}\mathbf{U}_{32}) \\
 &\quad + \int d\vec{z}_1 d\vec{z}_2 F_2(\vec{z}_1, \vec{z}_2)^\alpha (N_c^2 - 1) \mathbf{U}_{12}
 \end{aligned}$$

$q\bar{q}g$ production : first kind



$q\bar{q}g$ production : first kind

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Result for a longitudinal photon

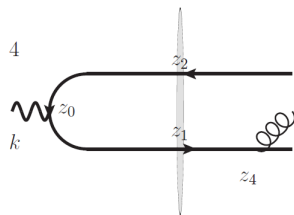
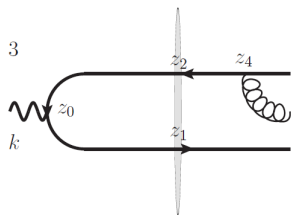
$$F_{1L}(\vec{z}_1, \vec{z}_2, \vec{z}_3) = \delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+) \theta(p_g^+ - \sigma) 2Qg \frac{e^{-i\vec{p}_q \cdot \vec{z}_1 - i\vec{p}_{\bar{q}} \cdot \vec{z}_2 - i\vec{p}_g \cdot \vec{z}_3}}{\pi \sqrt{2p_g^+}} K_0(QZ_{123})$$
$$\times \delta_{\lambda_q, -\lambda_{\bar{q}}} \left\{ (x_{\bar{q}} + x_g \delta_{-s_g \lambda_q}) x_q \frac{\vec{z}_{32} \cdot \vec{\epsilon}_g^*}{\vec{z}_{32}^2} - (x_q + x_g \delta_{-s_g \lambda_{\bar{q}}}) x_{\bar{q}} \frac{\vec{z}_{31} \cdot \vec{\epsilon}_g^*}{\vec{z}_{31}^2} \right\}$$

$$Z_{123} = \sqrt{x_q x_{\bar{q}} z_{12}^2 + x_q x_g z_{13}^2 + x_{\bar{q}} x_g z_{23}^2}$$

Result for a transverse photon

$$\begin{aligned}
 F_{1T}(\vec{z}_1, \vec{z}_2, \vec{z}_3) &= 2igQ\delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+)\theta(p_g^+ - \sigma) \frac{e^{-i\vec{p}_q \cdot \vec{z}_1 - i\vec{p}_{\bar{q}} \cdot \vec{z}_2 - i\vec{p}_g \cdot \vec{z}_3}}{\pi Z_{123} \sqrt{2p_g^+}} \\
 &\times \delta_{\lambda_q, -\lambda_{\bar{q}}} K_1(QZ_{123}) \left\{ -\frac{(\vec{z}_{23} \cdot \vec{\epsilon}_g^*)(\vec{z}_{13} \cdot \vec{\epsilon}_T)}{\vec{z}_{23}^2} x_q (x_q - \delta_{s\lambda_{\bar{q}}}) (x_{\bar{q}} + x_g \delta_{-s_g \lambda_q}) \right. \\
 &\quad \left. - \frac{(\vec{z}_{23} \cdot \vec{\epsilon}_g^*)(\vec{z}_{23} \cdot \vec{\epsilon}_T)}{\vec{z}_{23}^2} x_q x_{\bar{q}} (x_{\bar{q}} + x_g \delta_{-s_g \lambda_q} - \delta_{s\lambda_q}) \right\} - (q \leftrightarrow \bar{q})
 \end{aligned}$$

$q\bar{q}g$ production : second kind



$q\bar{q}g$ production : second kind

Result for a longitudinal photon

$$\begin{aligned} \tilde{F}_{2L}(\vec{z}_1, \vec{z}_2) &= 4ig Q \theta(p_g^+ - \sigma) \delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+) \frac{e^{-i\vec{p}_q \cdot \vec{z}_1 - i\vec{p}_{\bar{q}} \cdot \vec{z}_2}}{\sqrt{2p_g^+}} \\ &\times \delta_{\lambda_q, -\lambda_{\bar{q}}} \frac{x_q (x_g + x_{\bar{q}}) (\delta_{-s_g \lambda_q} x_g + x_{\bar{q}})}{x_{\bar{q}} x_g} \frac{\vec{P}_{\bar{q}} \cdot \vec{\epsilon}_g^*}{\vec{P}_{\bar{q}}^2} e^{-i\vec{p}_g \cdot \vec{z}_2} K_0(QZ_{122}) - (q \leftrightarrow \bar{q}) \end{aligned}$$

$$Z_{122} = \sqrt{x_q (1 - x_q)} \bar{z}_{12}^2$$

Result for a transverse photon

$$\begin{aligned} \tilde{F}_{2T}(\vec{z}_1, \vec{z}_2) &= -4g \theta(p_g^+ - \sigma) \delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+) \frac{e^{-i\vec{p}_q \cdot \vec{z}_1 - i\vec{p}_{\bar{q}} \cdot \vec{z}_2}}{\sqrt{2p_g^+}} \delta_{\lambda_q, -\lambda_{\bar{q}}} \\ &\times \frac{(\delta_{\lambda_{\bar{q}s}} - x_q) (\delta_{-s_g \lambda_q} x_g + x_{\bar{q}})}{x_{\bar{q}} x_g} \frac{\vec{P}_{\bar{q}} \cdot \vec{\epsilon}_g^*}{\vec{P}_{\bar{q}}^2} \frac{\vec{z}_{12} \cdot \vec{\epsilon}_T}{z_{12}^2} Q Z_{122} K_1(QZ_{122}) e^{-i\vec{p}_g \cdot \vec{z}_2} - (q \leftrightarrow \bar{q}) \end{aligned}$$

$$Z_{122} = \sqrt{x_q (1 - x_q)} z_{12}^2$$

Back to the general expression for the matrix element M^α :

$$M^\alpha = \frac{N_c^2}{2} \int d\vec{z}_1 d\vec{z}_2 d\vec{z}_3 F_1(\vec{z}_1, \vec{z}_2, \vec{z}_3)^\alpha (\mathbf{U}_{13} + \mathbf{U}_{32} - \mathbf{U}_{12} + \mathbf{U}_{13}\mathbf{U}_{32}) \\ + \int d\vec{z}_1 d\vec{z}_2 F_2(\vec{z}_1, \vec{z}_2)^\alpha (N_c^2 - 1) \mathbf{U}_{12}$$

For 2 or 3 gluon exchange, one can linearize this expression by neglecting the $\mathbf{U}_{13}\mathbf{U}_{32}$ term.

Linear approximation

After linearization one gets :

$$M^\alpha \stackrel{\text{eg.3}}{=} \frac{1}{2} \int d\vec{z}_1 d\vec{z}_2 \mathbf{U}_{12} \left\{ \tilde{F}_1(\vec{z}_1, \vec{z}_2)^\alpha + (N_c^2 - 1) \tilde{F}_2(\vec{z}_1, \vec{z}_2)^\alpha \right\}$$

$$\tilde{F}_1(\vec{z}_1, \vec{z}_2)^\alpha = \int d\vec{z}_3 \left[N_c^2 F_1(\vec{z}_1, \vec{z}_3, \vec{z}_2)^\alpha + N_c^2 F_1(\vec{z}_3, \vec{z}_2, \vec{z}_1)^\alpha - F_1(\vec{z}_1, \vec{z}_2, \vec{z}_3)^\alpha \right]$$

⇒ One has to integrate the previously derived expressions

Linear approximation

No analytical expression for most of the integrals. BUT :

- For **null transverse momenta** \vec{p}_q , $\vec{p}_{\bar{q}}$ and \vec{p}_g they can be performed analytically
- In any case, they can be reduced to **convergent integrals** over a real parameter in $[0,1]$ so a numerical calculation can be done.

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Further analysis from our results

- The calculation of **virtual correction** for the cross section is still to be performed (**Work in progress**)
- A lot of phenomenology can be done from HERA data :
 - From $q\bar{q}g$ production to **jet** production
 - Is our approach better than the **2-gluon** approximation for H1 and ZEUS data?
- The same calculation can be done again for **massive quarks**
- One could adapt those results for the study of hard diffractive events in **ultraperipheral collisions** at LHC

Backup slides

First kind, longitudinal photon

$$\begin{aligned}
 F_{1L}(\vec{p}_1, \vec{p}_2, \vec{p}_3) &= \delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+) \delta(\vec{p}_{1q} + \vec{p}_{2\bar{q}} + \vec{p}_{3g}) \theta(p_g^+ - \sigma) \\
 &\times \frac{\delta_{\lambda_q, -\lambda_{\bar{q}}}}{\sqrt{2p_g^+}} \frac{4iQ g(x_q + x_g \delta_{-s_g \lambda_{\bar{q}}}) ((\vec{p}_{2\bar{q}} \cdot \vec{\epsilon}_g^*) x_q + (\vec{p}_{1q} \cdot \vec{\epsilon}_g^*) (1 - x_{\bar{q}}))}{(1 - x_{\bar{q}}) x_g x_q \left(Q^2 + \frac{\vec{p}_{2\bar{q}}^2}{x_{\bar{q}}(1-x_{\bar{q}})}\right) \left(Q^2 + \frac{\vec{p}_{1q}^2}{x_q} + \frac{\vec{p}_{2\bar{q}}^2}{x_{\bar{q}}} + \frac{\vec{p}_{3g}^2}{x_g}\right)} - (q \leftrightarrow \bar{q}),
 \end{aligned}$$

Second kind, longitudinal photon

$$\begin{aligned} \tilde{F}_{2L}(\vec{p}_1, \vec{p}_2) &= \theta(p_g^+ - \sigma) \delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+) \delta(\vec{p}_{1q} + \vec{p}_{2\bar{q}} - \vec{p}_g) \\ &\times \frac{4igQ}{\sqrt{2p_g^+}} \delta_{\lambda_q, -\lambda_{\bar{q}}} \frac{x_{\bar{q}} + \delta_{-s_g \lambda_q} x_g}{x_{\bar{q}} x_g} \frac{2\pi}{Q^2 + \frac{\vec{p}_{1q}^2}{x_q(1-x_q)}} \frac{\vec{P}_{\bar{q}} \cdot \vec{\epsilon}_g^*}{\vec{P}_{\bar{q}}^2} - (q \leftrightarrow \bar{q}). \end{aligned}$$

First kind, transverse photon

$$\begin{aligned}
 F_{1T}(\vec{p}_1, \vec{p}_2, \vec{p}_3) = & \frac{2ig}{\sqrt{2\rho_g^+}} \frac{\delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+) \delta(\vec{p}_{1q} + \vec{p}_{2\bar{q}} + \vec{p}_{3g}) \theta(p_g^+ - \sigma) \delta - \lambda_{\bar{q}} \lambda_q}{Q^2 (1 - x_q) \left(\frac{\vec{p}_{1q}^2}{x_q} + \frac{\vec{p}_{2\bar{q}}^2}{x_{\bar{q}}} + \frac{\vec{p}_{3g}^2}{x_g} + Q^2 \right)} \left\{ \delta_{ssg} \delta_s \lambda_q \right. \\
 & \left. + 2(\vec{p}_{1q} \cdot \vec{\epsilon}_T)((\vec{p}_{2\bar{q}} \cdot \vec{\epsilon}_g^*)(x_{\bar{q}} + x_g) + (\vec{p}_{1q} \cdot \vec{\epsilon}_g^*)x_{\bar{q}}) \frac{(x_q - \delta_s \lambda_{\bar{q}})(x_g \delta - s_g \lambda_q + x_{\bar{q}})}{(1 - x_q) x_q x_{\bar{q}} x_g \left(Q^2 + \frac{\vec{p}_{1q}^2}{(1 - x_q) x_q} \right)} \right\} - (q \leftrightarrow \bar{q}).
 \end{aligned}$$

Second kind, transverse photon

$$\begin{aligned} \tilde{F}_{2T}(\vec{p}_1, \vec{p}_2) = & -\theta(p_g^+ - \sigma) \delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+) \delta(\vec{p}_{1q} + \vec{p}_{2\bar{q}} - \vec{p}_g) \frac{\delta_{\lambda_q, -\lambda_{\bar{q}}}}{\sqrt{2p_g^+}} \\ & \times 4g \frac{(\delta_{\lambda_{\bar{q}}s} - x_q)(\delta_{-s_g, \lambda_q} x_g + x_{\bar{q}})}{x_{\bar{q}} x_g} \frac{2\pi i (\vec{p}_{1q} \cdot \vec{\epsilon}_T)}{x_q (1 - x_q) Q^2 + \vec{p}_{1q}^2} \frac{\vec{P}_{\bar{q}} \cdot \vec{\epsilon}_g^*}{\vec{P}_{\bar{q}}^2} - (q \leftrightarrow \bar{q}) \end{aligned}$$

Example of an integral for linear approximation calculations

$$\begin{aligned}
 & \int d\vec{z}_3 e^{-i\vec{p}_g \cdot \vec{z}_3} \frac{\vec{z}_{32}}{\vec{z}_{32}^2} K_0(QZ_{123}) \\
 &= -\frac{\pi e^{-i\vec{p}_g \cdot \vec{z}_2}}{(1-x_g)x_g} \int_0^1 d\alpha e^{\alpha \frac{i x_q (\vec{z}_{21} \cdot \vec{p}_g)}{x_{\bar{q}} + x_q}} \left(\frac{i\vec{p}_g Z_{q\bar{q}g}}{Q_g(\alpha)} K_1(Q_g(\alpha) Z_{q\bar{q}g}) + x_g x_q \vec{z}_{21} K_0(Q_g(\alpha) Z_{q\bar{q}g}) \right) \\
 & \int d\vec{z}_3 \frac{\vec{z}_{32}}{\vec{z}_{32}^2} K_0(QZ_{123}) = -\frac{2\pi}{x_g x_q Q} \frac{\vec{z}_{21}}{\vec{z}_{21}^2} (Z_{q\bar{q}} K_1(QZ_{q\bar{q}}) - Z_{122} K_1(QZ_{122}))
 \end{aligned}$$