Electroweak Structure of Baryons at Low & Intermediate Momentum Transfers

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Hadrons at Low Energies

Common methods for dealing with low-energy hadronic phenomena on the basis of QCD nowadays:

- Lattice QCD (works but faces computational limitations)
- Chiral perturbation theory (works at low energies, as a systematic expansion but is limited to a few terms)
- Effective field theories or functional methods (depend on assumptions and regularizations)
- Effective models, e.g. **constituent-quark models** (depend on assumptions and input parameters)

Here, results for the **electroweak structure of baryons**, from the **relativistic constituent-quark model** (RCQM) in comparison to experiments and results especially from lattice QCD.

Outline

Low-Energy QCD / Relevant Degrees of Freedom

Relativistic Constituent-Quark Model (RCQM)
- Interacting mass operator with
- Goldstone-boson-exchange dynamics

Baryon Spectroscopy
- Light, strange, charm, bottom

Baryon Structure
- Nucleon e.m. form factors, including flavor analysis
- Baryon electromagnetic form factors
- Nucleon and baryon axial form factors / charges
- Nucleon gravitational form factors

Summary and Conclusions

Low-Energy QCD

Low-energy QCD of N_f flavors is characterized by:

- spontaneous breaking of chiral symmetry ($SB\chi S$):

$$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$$

$\rightarrow$ appearance of $(N_f^2 - 1)$ **Goldstone bosons** $\vec{\phi}$
$\rightarrow$ generation of quasiparticles with dynamical mass, i.e. **constituent quarks** ψ

- thus (effective) interaction Lagrangian:

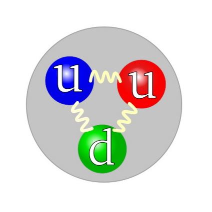
$$\mathcal{L}_{\rm int} \sim ig\bar{\psi}\gamma_5\vec{\lambda}^f \cdot \vec{\phi}\psi$$

A. Manohar and H. Georgi: Nucl. Phys. B 234 (1984) 189
E.V. Shuryak: Phys. Rep. **115**, 151 (1984)
L.Ya. Glozman and D.O. Riska: Phys. Rep. **268**, 263 (1996)
see also:
S. Weinberg: Phys. Rev. Lett. **105**, 261601 (2010)

Baryons

Baryons are considered as colorless bound states of three constituent quarks.

Here the proton:



- 'Constituent' quarks are quasiparticles with dynamical mass, NOT the original QCD d.o.f. (i.e. 'current' quarks).
- 'Constituent' quarks are confined and interact via hyperfine interactions associated with $SB\chi S$, i.e. Goldstone-boson exchange.

Aspects Suggestive for Quark Models

Search for a non-perturbative tool to describe/understand

- in a consistent manner
- on the microscopic level
- in accordance with the properties of low-energy QCD such phenomena like

- **hadron spectra**: ground states & excitations
- **hadron structure**: $r_E, \mu, g_A\,;\ G_E, G_M, G_A, G_P$, ...
  i.e. electroweak form factors etc.
- **resonance excitations**: $\gamma N \to N^*$, $e^- N \to N^*$, ...
- **resonance decays**:
  $\rho \to \pi\pi$, $\omega \to \pi\pi\pi$, $N^* \to N\pi$, $\Delta \to N\pi$, $\Lambda^* \to KN$, ...
- **meson-baryon interactions**: $\pi - N$, $K - N$, ...
- **hyperon-hyperon interactions**: $N - N$, $N - Y$, ...

etc. etc.

Relativistic quantum mechanics (RQM)

i.e. **Hamiltonian quantum theory** respecting

Poincaré invariance

(theory on a Hilbert space $\mathcal{H}$ corresponding to a finite number of particles, not a field theory)

Invariant mass operator

$$\hat{M} = \hat{M}_{free} + \hat{M}_{int}$$

Eigenvalue equations

$$\hat{M}\,|P, J, \Sigma\rangle = M\,|P, J, \Sigma\rangle \quad , \qquad \hat{M}^2 = \hat{P}^\mu \hat{P}_\mu$$

$$\hat{P}^\mu\,|P, J, \Sigma\rangle = P^\mu\,|P, J, \Sigma\rangle \quad , \qquad \hat{P}^\mu = \hat{M}\hat{V}^\mu$$

Interacting mass operator

$$
\begin{aligned}
\hat{M} &= \hat{M}_{free} + \hat{M}_{int} \\
\hat{M}_{free} &= \sqrt{\hat{H}^2_{free} - \hat{\vec{P}}^2_{free}} \\
\hat{M}^{rest\ frame}_{int} &= \sum_{i<j}^{3} \hat{V}_{ij} = \sum_{i<j}^{3} [\hat{V}^{conf}_{ij} + \hat{V}^{hf}_{ij}]
\end{aligned}
$$

fulfilling the **Poincaré algebra**

$$
\begin{array}{lll}
[\hat{P}_i, \hat{P}_j] = 0, & [\hat{J}_i, \hat{H}] = 0, & [\hat{P}_i, \hat{H}] = 0, \\
[\hat{K}_i, \hat{H}] = -i\hat{P}_i & [\hat{J}_i, \hat{J}_j] = i\epsilon_{ijk}\hat{J}_k & [\hat{J}_i, \hat{K}_j] = i\epsilon_{ijk}\hat{K}_k, \\
[\hat{J}_i, \hat{P}_j] = i\epsilon_{ijk}\hat{P}_k, & [\hat{K}_i, \hat{K}_j] = -i\epsilon_{ijk}\hat{J}_k, & [\hat{K}_i, \hat{P}_j] = -i\delta_{ij}\hat{H}
\end{array}
$$

$\hat{H}, \hat{P}_i$ ... time and space translations,
$\hat{J}_i$ ... rotations, $\hat{K}_i$ ... Lorentz boosts

Universal GBE RCQM

Phenomenologically, baryons with 5 flavors: u, d, s, c, b

$$\Rightarrow \quad H_{free} = \sum_{i=1}^{3} \sqrt{m_i^2 + \vec{k}_i^2}$$

$$V^{conf}(\vec{r}_{ij}) = B + C\, r_{ij}$$

$$V^{hf}(\vec{r}_{ij}) = \left[V_{24}(\vec{r}_{ij}) \sum_{f=1}^{24} \lambda_i^f \lambda_j^f + V_0(\vec{r}_{ij}) \lambda_i^0 \lambda_j^0 \right] \vec{\sigma}_i \cdot \vec{\sigma}_j$$

- i.e., for $N_f = 5$, we have the exchange of a **24-plet** plus a **singlet** of Goldstone bosons.

L.Ya. Glozman, W. Plessas, K. Varga, and R.F. Wagenbrunn: Phys. Rev. D **58**, 094030 (1998)

J.P. Day, K.-S. Choi, and W. Plessas: arXiv:1205.6918

J.P. Day, K.-S. Choi, and W. Plessas: Few-Body Syst. **54**, 329 (2013)

Universal GBE RCQM Parametrization

$$V^{conf}(\vec{r}_{ij}) = B + C\,r_{ij}$$

$$V_\beta(\vec{r}_{ij}) = \frac{g_\beta^2}{4\pi}\frac{1}{12m_im_j}\left\{\mu_\beta^2\frac{e^{-\mu_\beta r_{ij}}}{r_{ij}} - 4\pi\delta(\vec{r}_{ij})\right\}$$

$$= \frac{g_\beta^2}{4\pi}\frac{1}{12m_im_j}\left\{\mu_\beta^2\frac{e^{-\mu_\beta r_{ij}}}{r_{ij}} - \Lambda_\beta^2\frac{e^{-\Lambda_\beta r_{ij}}}{r_{ij}}\right\}$$

$$B = -402\text{ MeV}, \quad C = 2.33\text{ fm}^{-2}$$

$$\beta = 24: \quad \frac{g_{24}^2}{4\pi} = 0.7, \quad \mu_{24} = \mu_\pi = 139\text{ MeV}, \quad \Lambda_{24} = 700.5\text{ MeV}$$

$$\beta = 0: \quad \left(\frac{g_0}{g_{24}}\right)^2 = 1.5, \quad \mu_0 = \mu_{\eta'} = 958\text{ MeV}, \quad \Lambda_0 = 1484\text{ MeV}$$

$$m_u = m_d = 340\text{ MeV}, \quad m_s = 480\text{ MeV},$$

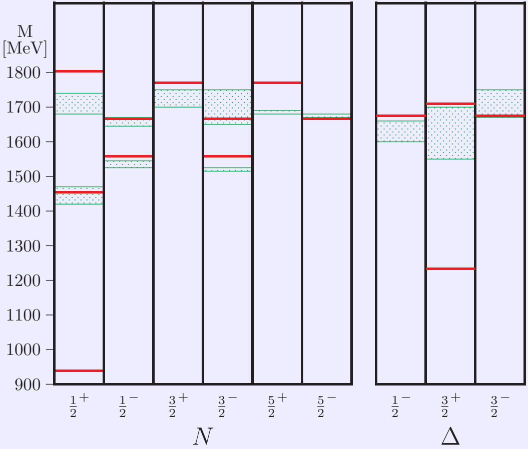
$$m_c = 1675\text{ MeV}, \quad m_b = 5055\text{ MeV}$$

Baryon **Excitation Spectra**

and

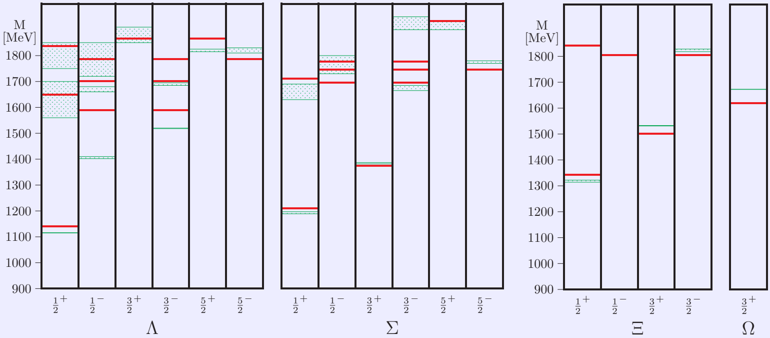
Mass-Operator **Eigenstates**

Light Baryon Spectra



red	Universal GBE RCQM
green	PDG 2013 (experiment)

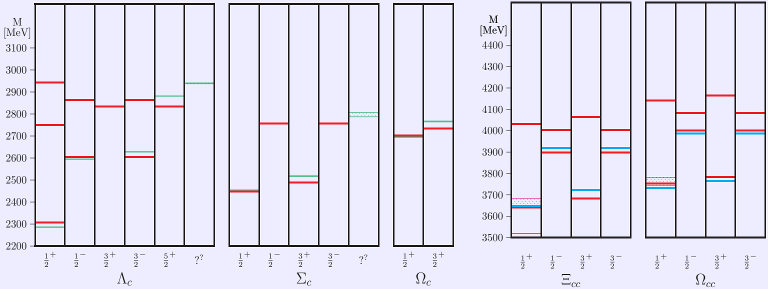
Strange Baryon Spectra



red Universal GBE RCQM

green PDG 2013 (experiment)

Charm Baryon Spectra



Left panel – single charm:

red Universal GBE RCQM prediction

green PDG 2013 (experiment)

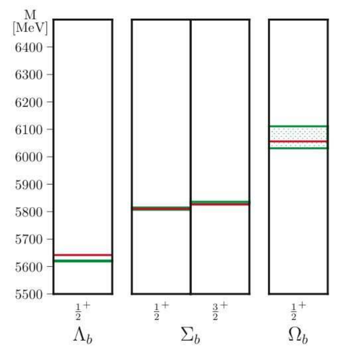
Right panel – double charm:

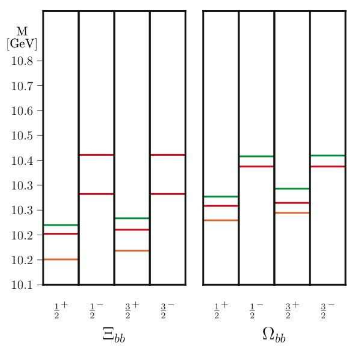
green M. Mattson et al.: Phys. Rev. Lett. 89 (2002) 112001 (SELEX experiment)

cyan S. Migura, D. Merten, B. Metsch, and H.-R. Petry: Eur. Phys. J. A 28 (2006) 41 (Bonn RCQM)

magenta L. Liu et al.: Phys. Rev. D 81 (2010) 094505 (Lattice QCD)

Bottom Baryon Spectra





Left panel – single bottom:

red Universal GBE RCQM prediction

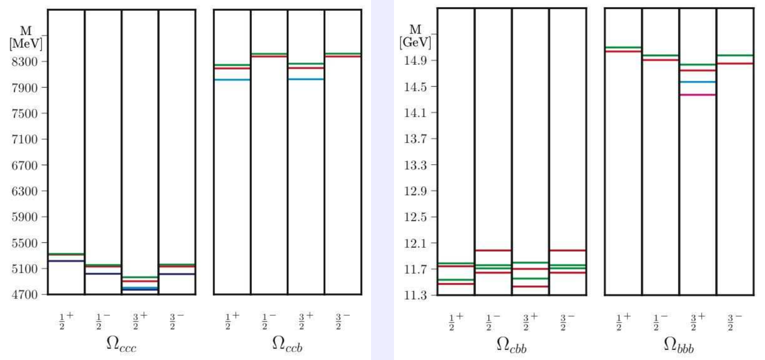
green PDG 2013 (experiment)

Right panel – double bottom:

green W. Roberts and M. Pervin: Int. J. Mod. Phys. A 23 (2008) 2817 (nonrel. one-gluon-exchange CQM)

orange D. Ebert, R.N. Faustov, V.O. Galkin, and A.P. Martynenko: Phys. Rev. D 66 (2002) 014008 (RCQM)

Triple-Heavy Baryon Spectra



red Universal GBE RCQM

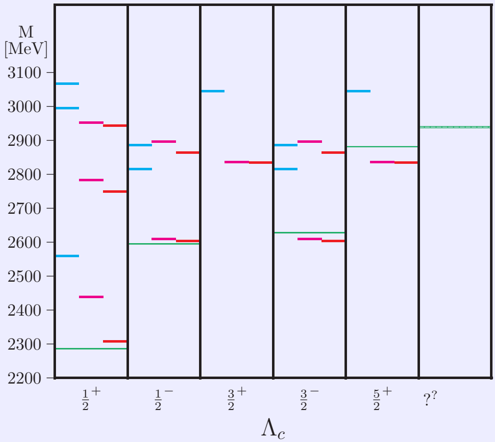
green W. Roberts and M. Pervin: Int. J. Mod. Phys. A 23 (2008) 2817 (nonrelativistic one-gluon-exchange CQM)

blue S. Migura, D. Merten, B. Metsch, and H.-R. Petry: Eur. Phys. J. A 28 (2006) 41 (Bonn RCQM)

cyan A.P. Martynenko: Phys. Lett. B 663 (2008) 317 (RCQM)

magenta S. Meinel: Phys. Rev. D 82 (2010) 114502 (lattice QCD)

Influence of Light-Heavy Q-Q Interaction



leftmost cyan levels	confinement only
middle magenta levels	including only light-light GBE
rightmost red levels	including full GBE RCQM

Rest-Frame Baryon States

Mass operator eigenstates

$$\hat{M}\,|P,J,\Sigma,T,M_T\rangle = M\,|P,J,\Sigma,T,M_T\rangle$$

represented in configuration space

$$\left\langle \vec{\xi},\vec{\eta}\,\middle|\,P,J,\Sigma,T,M_T\right\rangle = \Psi_{PJ\Sigma TM_T}(\vec{\xi},\vec{\eta})$$

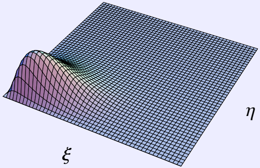
with $\vec{\xi}$ and $\vec{\eta}$ the usual Jacobi coordinates.

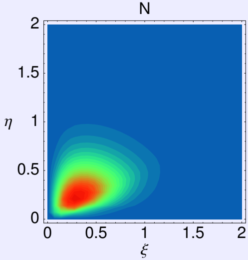
Picture the baryon wave functions through
spatial probability density distributions

$$\rho(\xi,\eta) = \xi^2\eta^2 \int d\Omega_\xi d\Omega_\eta \; \Psi^\star_{PJ\Sigma TM_T}(\xi,\Omega_\xi,\eta,\Omega_\eta)\Psi_{PJ\Sigma TM_T}(\xi,\Omega_\xi,\eta,\Omega_\eta)$$

Pictures of Baryons (rest frame)

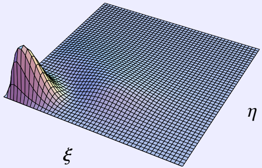
N GBE CQM

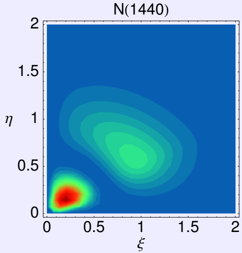




units are [*fm*]

N(1440) GBE CQM





units are [*fm*]

T. Melde, W. Plessas, and B. Sengl: Phys. Rev. D **77** (2008) 114002

Spatial Probability Density Distributions

Low-energy QCD
RCQM
GBE RCQM
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Baryon E.m.
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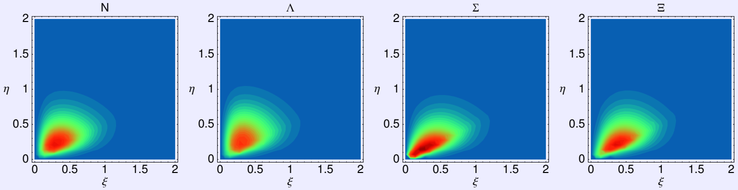
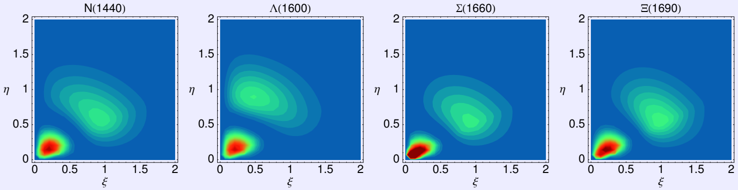
GBE vs. OGE
PFSM details
N scalar FF
πNN, $\pi N\Delta$ vertex FFs

$\rho(\xi,\eta)$ for the $\frac{1}{2}^+$ **octet** baryon ground states $N(939)$, $\Lambda(1116)$, $\Sigma(1193)$, $\Xi(1318)$:



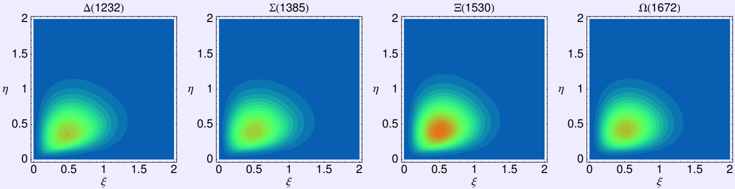
$\rho(\xi,\eta)$ for the $\frac{1}{2}^+$ **octet** baryon states $N(1440)$, $\Lambda(1600)$, $\Sigma(1660)$, $\Xi(1690)$:



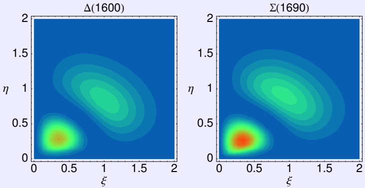
T. Melde, W. Plessas, and B. Sengl: Phys. Rev. D **77** (2008) 114002

Spatial Probability Density Distributions

$\rho(\xi,\eta)$ for the $\frac{3}{2}^+$ **decuplet** baryon states **$\Delta(1232)$, $\Sigma(1385)$, $\Xi(1530)$, $\Omega(1672)$**:



$\rho(\xi,\eta)$ for the $\frac{3}{2}^+$ **decuplet** baryon states $\Delta(1600)$, $\Sigma(1690)$:



T. Melde, W. Plessas, and B. Sengl: Phys. Rev. D **77** (2008) 114002

New Quark-Model Classification

multiplet	$(LS)J^P$				
octet	$(0\frac{1}{2})\frac{1}{2}^+$	$N(939)^{100}$	$\Lambda(1116)^{100}$	$\Sigma(1193)^{100}$	$\Xi(1318)^{100}$
octet	$(0\frac{1}{2})\frac{1}{2}^+$	$N(1440)^{100}$	$\Lambda(1600)^{96}$	$\Sigma(1660)^{100}$	$\Xi(1690)^{100}$
octet	$(0\frac{1}{2})\frac{1}{2}^+$	$N(1710)^{100}$		$\Sigma(1880)^{99}$	
octet	$(1\frac{1}{2})\frac{1}{2}^-$	$N(1535)^{100}$	$\Lambda(1670)^{72}$	$\Sigma(1560)^{94}$	
octet	$(1\frac{3}{2})\frac{1}{2}^-$	$N(1650)^{100}$	$\Lambda(1800)^{100}$	$\Sigma(1620)^{100}$	
octet	$(1\frac{1}{2})\frac{3}{2}^-$	$N(1520)^{100}$	$\Lambda(1690)^{72}$	$\Sigma(1670)^{94}$	$\Xi(1820)^{97}$
octet	$(1\frac{3}{2})\frac{3}{2}^-$	$N(1700)^{100}$		$\Sigma(1940)^{100}$	
octet	$(1\frac{3}{2})\frac{5}{2}^-$	$N(1675)^{100}$	$\Lambda(1830)^{100}$	$\Sigma(1775)^{100}$	$\Xi(1950)^{100}$
decuplet	$(0\frac{3}{2})\frac{3}{2}^+$	$\Delta(1232)^{100}$	$\Sigma(1385)^{100}$	$\Xi(1530)^{100}$	$\Omega(1672)^{100}$
decuplet	$(0\frac{3}{2})\frac{3}{2}^+$	$\Delta(1600)^{100}$	$\Sigma(1690)^{99}$		
decuplet	$(1\frac{1}{2})\frac{1}{2}^-$	$\Delta(1620)^{100}$	$\Sigma(1750)^{94}$		
decuplet	$(1\frac{1}{2})\frac{3}{2}^-$	$\Delta(1700)^{100}$			
singlet	$(1\frac{1}{2})\frac{1}{2}^-$	$\Lambda(1405)^{71}$			
singlet	$(1\frac{1}{2})\frac{3}{2}^-$	$\Lambda(1520)^{71}$			
singlet	$(0\frac{1}{2})\frac{1}{2}^+$	$\Lambda(1810)^{92}$			

T. Melde, W. Plessas, and B. Sengl: Phys. Rev. D **77**, 114002 (2008)

See also the PDG: Chin. Phys. C **38**, 090001 (2014)

$SU(3)$ Flavor Multiplets – New

Classification of baryon resonances by the PDG since **2010**

(results from the GBE relativistic CQM marked by asterisks)

J^P	$(D, L_N^P)\,S$	Octet members				Singlets
$1/2^+$	$(56,0_0^+)$ 1/2	$N(939)$	$\Lambda(1116)$	$\Sigma(1193)$	$\Xi(1318)$	
$1/2^+$	$(56,0_2^+)$ 1/2	$N(1440)$	$\Lambda(1600)$	$\Sigma(1660)$	$\Xi(1690)^\dagger$	
$1/2^-$	$(70,1_1^-)$ 1/2	$N(1535)$	$\Lambda(1670)$	$\Sigma(1620)$	$\Xi(?)$	$\Lambda(1405)$
				$\Sigma(1560)^\dagger$		
$3/2^-$	$(70,1_1^-)$ 1/2	$N(1520)$	$\Lambda(1690)$	$\Sigma(1670)$	$\Xi(1820)$	$\Lambda(1520)$
$1/2^-$	$(70,1_1^-)$ 3/2	$N(1650)$	$\Lambda(1800)$	$\Sigma(1750)$	$\Xi(?)$	
				$\Sigma(1620)^\dagger$		
$3/2^-$	$(70,1_1^-)$ 3/2	$N(1700)$	$\Lambda(?)$	$\Sigma(1940)^\dagger$	$\Xi(?)$	
$5/2^-$	$(70,1_1^-)$ 3/2	$N(1675)$	$\Lambda(1830)$	$\Sigma(1775)$	$\Xi(1950)^\dagger$	
$1/2^+$	$(70,0_2^+)$ 1/2	$N(1710)$	$\Lambda(1810)$	$\Sigma(1880)$	$\Xi(?)$	$\Lambda(1810)^\dagger$
$3/2^+$	$(56,2_2^+)$ 1/2	$N(1720)$	$\Lambda(1890)$	$\Sigma(?)$	$\Xi(?)$	
$5/2^+$	$(56,2_2^+)$ 1/2	$N(1680)$	$\Lambda(1820)$	$\Sigma(1915)$	$\Xi(2030)$	
$7/2^-$	$(70,3_3^-)$ 1/2	$N(2190)$	$\Lambda(?)$	$\Sigma(?)$	$\Xi(?)$	$\Lambda(2100)$
$9/2^-$	$(70,3_3^-)$ 3/2	$N(2250)$	$\Lambda(?)$	$\Sigma(?)$	$\Xi(?)$	
$9/2^+$	$(56,4_4^+)$ 1/2	$N(2220)$	$\Lambda(2350)$	$\Sigma(?)$	$\Xi(?)$	

PDG: J. Phys. G **37**, 075021 (2010); Phys. Rev. D **86**, 010001 (2012);
Chin. Phys. C **38**, 090001 (2014)

N and Δ Rest-Frame Wave Functions

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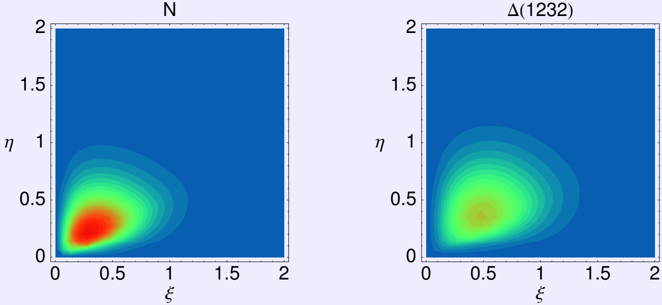
Rest-frame **spatial distribution of constituent quarks** in terms of 3-body Jacobi coordinates $\vec{\xi}$ and $\vec{\eta}$:

$$\rho(\xi,\eta) = \xi^2\eta^2 \int d\Omega_\xi d\Omega_\eta \; \Psi^\star_{PJ\Sigma TM_T}(\xi,\Omega_\xi,\eta,\Omega_\eta)\Psi_{PJ\Sigma TM_T}(\xi,\Omega_\xi,\eta,\Omega_\eta)$$



Units on abscissa and ordinates are [fm]

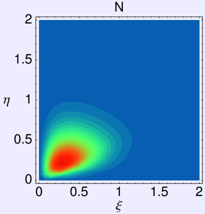
Root-Mean-Square Radii

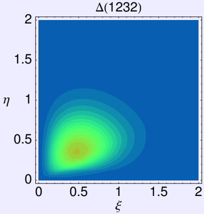
The **root-mean-square radius** (in the rest frame):

$$r_{\rm rms} = \sqrt{\langle r_i^2 \rangle} = \left(\int d^3 r_i \left\langle P = 0, J, \Sigma \left| \hat{r}_i^2 \right| P = 0, J, \Sigma \right\rangle \right)^{\frac{1}{2}}$$

Is NOT an **observable**! Is NOT **relativistically invariant**!

$\rightarrow$ Idea about the spatial distribution of constituent quarks.





$r^N_{\rm rms}$=0.304 fm

r^p_E=0.905 fm, $(r^n_E)^2$=−0.128 fm^2

$r^\Delta_{\rm rms}$=0.390 fm

$r^{\Delta^{++}}_E$=$r^{\Delta^{+}}_E$=$r^{\Delta^{-}}_E$=0.656 fm, $r^{\Delta^{0}}_E$=0 fm

See: K. Berger, R.F. Wagenbrunn, and W. Plessas: Phys. Rev. D **70**, 094027 (2004)

Various Baryon Reactions

Matrix elements of a transition operator $\hat{O}$ between baryon eigenstates $|P, J, \Sigma, T, T_3, Y\rangle$

$$\langle P', J', \Sigma', T', T_3', Y'|\hat{O}|P, J, \Sigma, T, T_3, Y\rangle$$

$\hat{O}$ ... $\hat{J}^{\mu}_{\rm em}$ $\rightarrow$ electromagnetic FF's

... $\hat{A}^{\mu}_{\rm axial}$ $\rightarrow$ axial FF's

... $\hat{\Theta}^{\mu\nu}$ $\rightarrow$ gravitational/tensor FF's

... $\hat{S}$ $\rightarrow$ scalar FF

... $\hat{D}^{\mu}_{\lambda}$ $\rightarrow$ strong FF's and hadronic decays

To be calculated from microscopic three-quark ME's

$$\langle p_1', p_2', p_3'; \sigma_1', \sigma_2', \sigma_3'; f_{i_1'}, f_{i_2'}, f_{i_3'}|\hat{O}|p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3; f_{i_1}, f_{i_2}, f_{i_3}\rangle$$

$\Uparrow$ boosted 3-body states $\qquad$ $\Uparrow$ boosted 3-body states

Electromagnetic Nucleon Observables

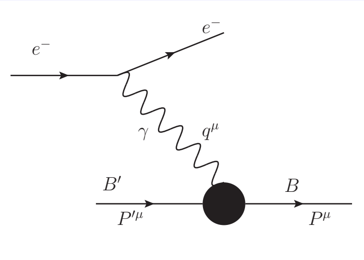
Covariant predictions for:

- **Electromagnetic** nucleon form factors
  $G_E^p(Q^2)$, $G_M^p(Q^2)$; $G_E^n(Q^2)$, $G_M^n(Q^2)$
- **Electric radii** and **magnetic moments**
  r_E^p, μ^p; r_E^n, μ^n

$\rightarrow$ Comparison to experiment

Electron Scattering and E.m. Form Factors

Elastic electron scattering:



Invariant form factors:

$$F^{\nu}_{\Sigma'\Sigma}(Q^2) = \langle P', J, \Sigma', T, M_T | \hat{J}^{\nu}_{\rm em} | P, J, \Sigma, T, M_T \rangle$$

$$\text{with } Q^2 = -q^2 \,; \qquad q^\mu = P^\mu - P'^\mu$$

Transition Matrix Elements in Point Form

Incoming baryon state: $|V, M, J, \Sigma\rangle \quad \hat{=} \, |P, J, \Sigma\rangle$

Outgoing baryon state: $|V', M', J', \Sigma'\rangle \quad \hat{=} \, |P', J', \Sigma'\rangle$

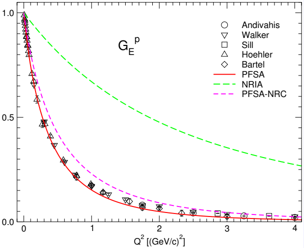
Transition operator: $\hat{O} = \hat{J}^{\mu}_{\mathrm{em}}$

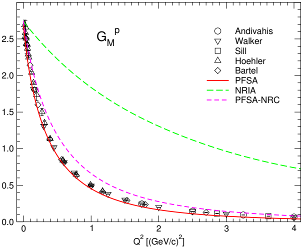
$$
\begin{aligned}
&\left\langle V', M', J', \Sigma' \right| \hat{J}^{\mu}_{\mathrm{em}} \left| V, M, J, \Sigma \right\rangle = \\
&= \frac{2}{MM'} \sum_{\sigma_i \sigma'_i} \sum_{\mu_i \mu'_i} \int d^3\vec{k}_2 d^3\vec{k}_3 d^3\vec{k}'_2 d^3\vec{k}'_3 \\
&\times \sqrt{\frac{\left(\sum_i \omega'_i\right)^3}{\prod_i 2\omega'_i}} \prod_{\sigma'_i} D^{\star \frac{1}{2}}_{\sigma'_i \mu'_i} \left\{ R_W \left[k'_i ; B\left(V'\right) \right] \right\} \Psi^{\star}_{M' J' \Sigma'} \left(\vec{k}'_1, \vec{k}'_2, \vec{k}'_3; \mu'_1, \mu'_2, \mu'_3 \right) \\
&\times \left\langle p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3 \right| \hat{J}^{\mu}_{rd} \left| p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \right\rangle \\
&\times \sqrt{\frac{\left(\sum_i \omega_i\right)^3}{\prod_i 2\omega_i}} \prod_{\sigma_i} D^{\frac{1}{2}}_{\sigma_i \mu_i} \left\{ R_W \left[k_i ; B\left(V\right) \right] \right\} \Psi_{M J \Sigma} \left(\vec{k}_1, \vec{k}_2, \vec{k}_3; \mu_1, \mu_2, \mu_3 \right) \\
&\times 2MV_0 \delta^3 \left(M\vec{V} - M'\vec{V}' - \vec{q} \right)
\end{aligned}
$$

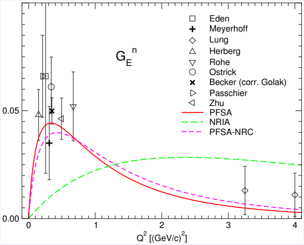
where $p_i = B_c(V)k_i$, $p'_i = B_c(V')k'_i$, and $\omega_i = \sqrt{\vec{k}_i^2 + m_i^2}$

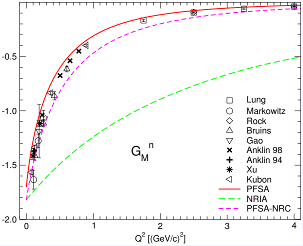
Electromagnetic Nucleon Form Factors

Covariant predictions of the GBE CQM:





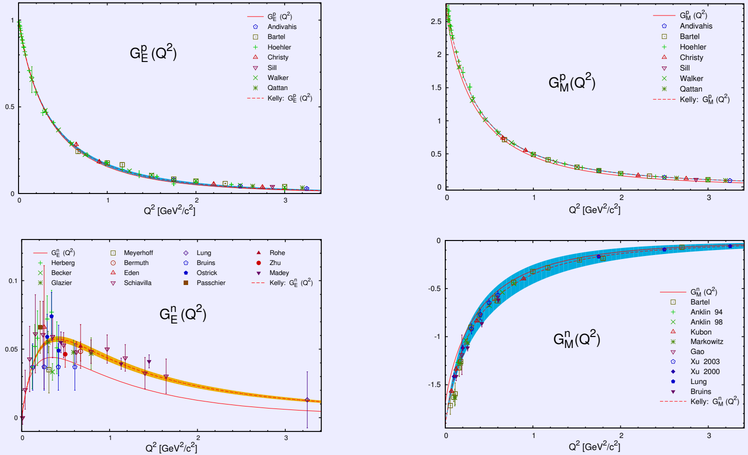




R.F. Wagenbrunn, S. Boffi, W. Klink, W. Plessas, and M. Radici: Phys. Lett. **B511** (2001) 33

Electromagnetic Nucleon Form Factors

Covariant predictions of the GBE CQM:



R.F. Wagenbrunn, S. Boffi, W. Klink, W. Plessas, and M. Radici: Phys. Lett. **B511** (2001) 33

M. Rohrmoser: Diploma Thesis, Univ. of Graz, 2013

Nucleon Electric Radii and Magnetic Moments

Electric radii r_E^2 [fm^2]

Baryon	GBE PFSM	Experiment
p	0.82	0.7692 ± 0.0123 [1]
		0.70870 ± 0.00113 [2]
n	-0.13	-0.1161 ± 0.0022

[1] CODATA value (PDG) [2] Pohl et al.: Nature **466** (2010) 213

Magnetic moments μ [n.m.]

Baryon	GBE PFSM	Experiment
p	2.70	2.792847356
n	-1.70	-1.9130427

K. Berger, R.F. Wagenbrunn, and W. Plessas: Phys. Rev. D **70**, 094027 (2004)

Nucleon r_E^2 and μ – Nonrelativistic !!!

Electric radii r_E^2 [fm^2]

Baryon	GBE PFSM	GBE NRIA	Experiment
p	0.82	0.10	0.7692 ± 0.0123 [1)]
			0.70870 ± 0.00113 [2)]
n	-0.13	-0.01	-0.1161 ± 0.0022

1) CODATA value (PDG)   2) Pohl et al.: Nature **466** (2010) 213

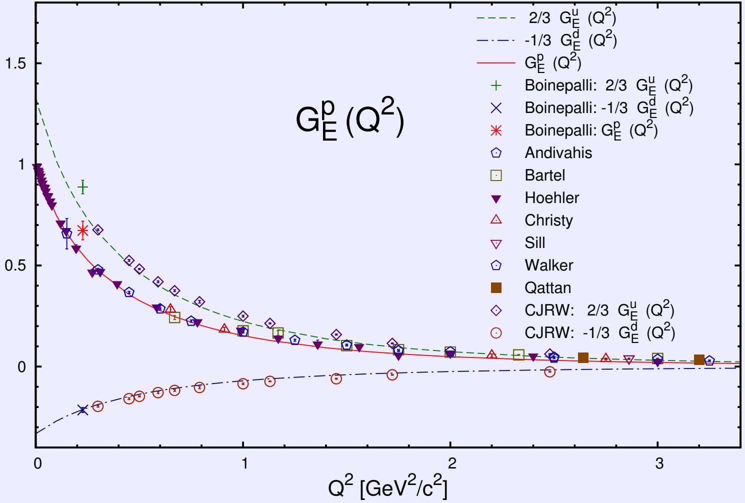
Magnetic moments μ [n.m.]

Baryon	GBE PFSM	GBE NRIA	Experiment
p	2.70	2.74	2.792847356
n	-1.70	-1.82	-1.9130427

K. Berger, R.F. Wagenbrunn, and W. Plessas: Phys. Rev. D **70**, 094027 (2004)

Nucleons N

$$G_E^p = \frac{2}{3} G_E^u - \frac{1}{3} G_E^d$$

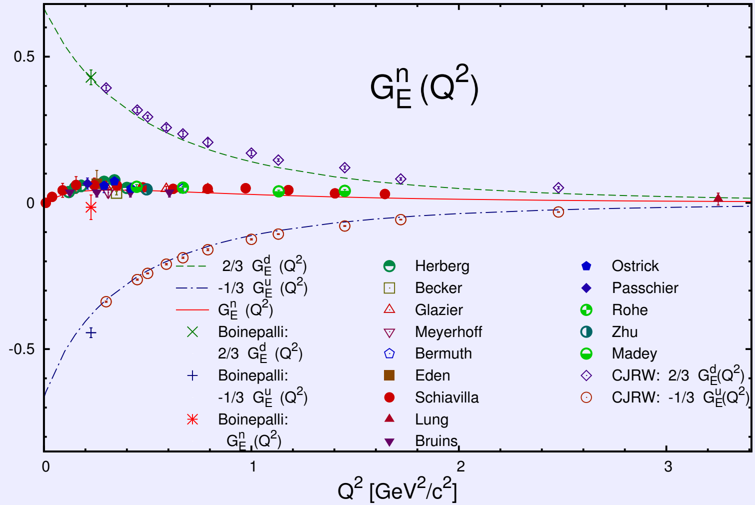


M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665

Low-energy QCD
RCQM
GBE RCQM
Spectroscopy
Light, strange, charm, bottom
Structure
Nucleon E.m.
Baryon E.m.
Axial FFs
Gravitational FF
Summary
Addenda
GBE RCQM & solution methods
GBE vs. OGE
PFSM details
N scalar FF
πNN, $\pi N\Delta$ vertex FFs

Neutron Electric Form Factor

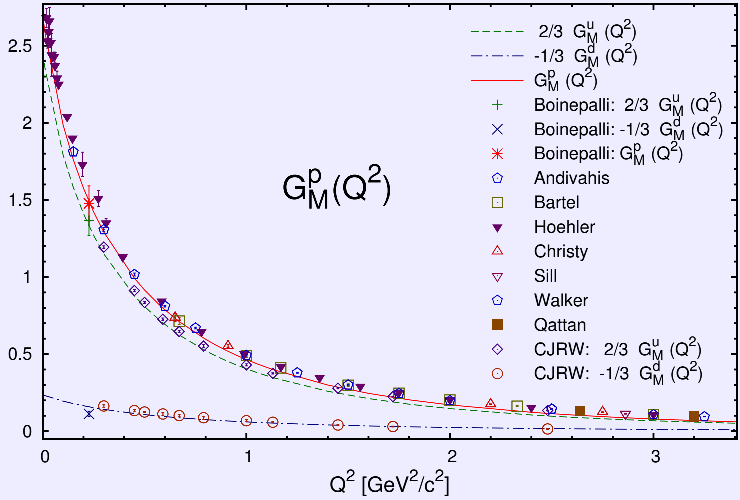
$$G_E^n = \frac{2}{3} G_E^d - \frac{1}{3} G_E^u$$



M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665

Proton Magnetic Form Factor

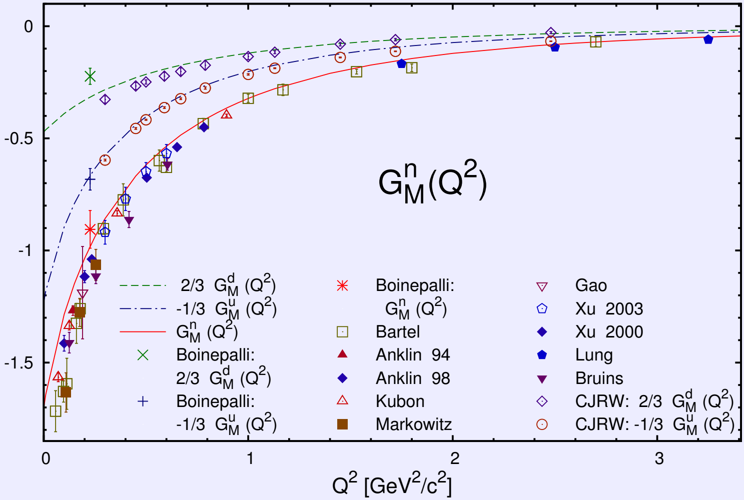
$$G_M^p = \frac{2}{3} G_M^u - \frac{1}{3} G_M^d$$



M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665

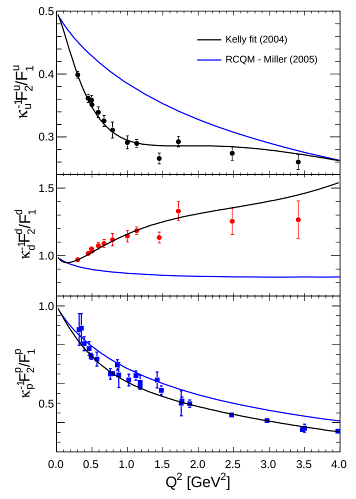
Neutron Magnetic Form Factor

$$G_M^n = \frac{2}{3} G_M^d - \frac{1}{3} G_M^u$$



M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665

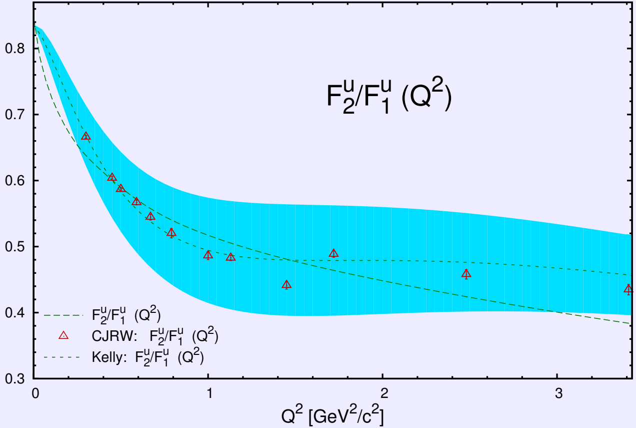
F_2/F_1 Ratios



3-Q vs. 5-Q components?

From: G. D. Cates, C. W. de Jager, S. Riordan, B. Wojtsekhowski: Phys. Rev. Lett. **106**, 252003 (2011)

Ratio F_2^u / F_1^u of u-Flavor Contr. to F_1 and F_2

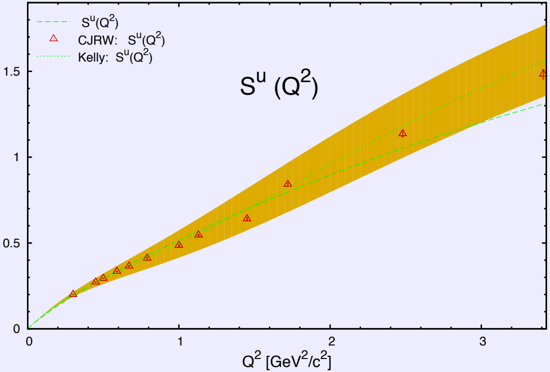


- - - - - - dashed green line: GBE RCQM ······· dotted line and blue area: Kelly fit with $\frac{1}{2}*$error

No indication for 5-Q components in the nucleons!

Ratio of u-Flavor Contr. to F_1 and F_2 by S^q

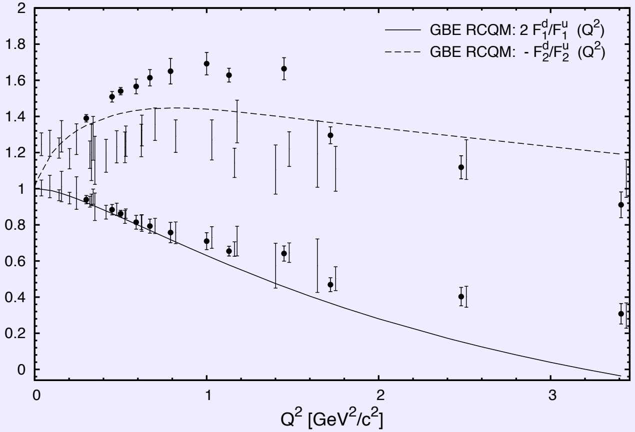
$$S^u(Q^2) = Q^2 \frac{F_2^u(Q^2)}{F_1^u(Q^2)}$$



- - - - - - dashed green line: GBE RCQM   ······· dotted line and orange area: Kelly fit with $\frac{1}{2}*$error

No indication for 5-Q components in the nucleons!

Ratios F_i^d/F_i^u of Flavor Contr. to F_1 and F_2



Fall-off is **no indication for diquark clustering** in the nucleons!

GBE RCQM prediction: M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665

Phenomenology:
- ● G. D. Cates et al.: Phys. Rev. Lett. **106**, 252003 (2011)
- I M. Diehl and P. Kroll: Eur. Phys. J. A **73**, 2397 (2013)

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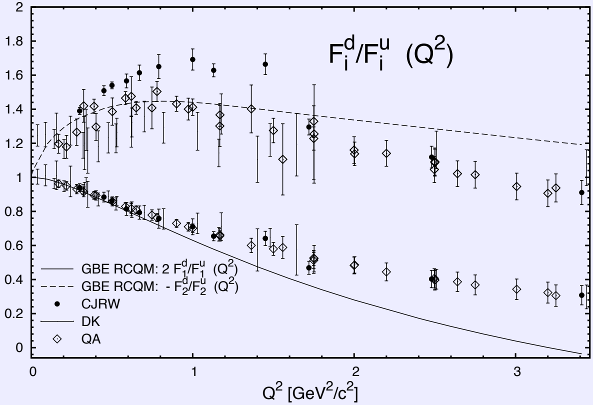


Problems between three different phenomenological analyses!

GBE RCQM prediction: M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665

Phenomenology:

- ● G. D. Cates et al.: Phys. Rev. Lett. **106**, 252003 (2011)
- Ī M. Diehl and P. Kroll: Eur. Phys. J. A **73**, 2397 (2013)
- ◇ I.A. Qattan and J. Arrington: Phys. Rev. C **86**, 065210 (2012)

Conclusions from Nucleon Flavor Analysis

- **Flavor analysis of nucleon e.m. form factors** in a relativistically invariant framework (point form).
- The **GBE RCQM** predicts flavor contributions in reasonable agreement with **experimental data**.
- The GBE RCQM relies on $\{QQQ\}$ degrees of freedom only; no explicit $\{QQQQ\bar{Q}\}$ etc.
- No explicit **meson-cloud effects** are included.
- No **strangeness content** in the nucleon for the low momentum transfers considered here.
- With respect to F_2^d/F_2^u three different phenomenological analyses give **distinct answers**.
- Details:

  M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665

  W. Plessas: Mod. Phys. Lett. A **28**, 136022 (2013)

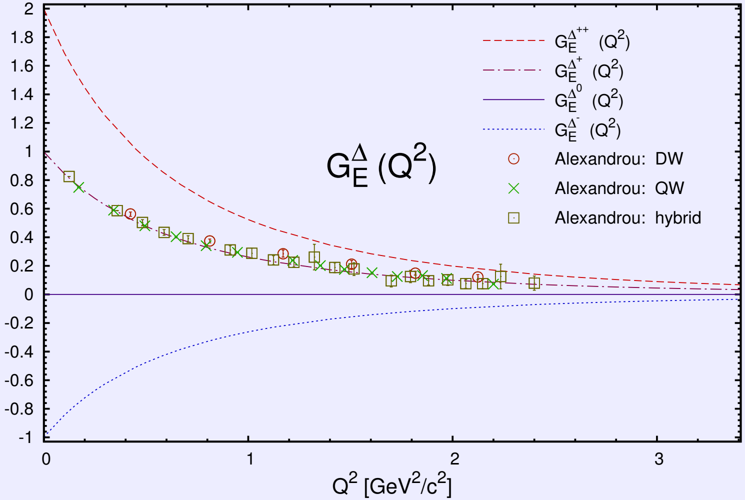
Δ and Hyperon E.m. Form Factors

$$\Delta$$

$$\Lambda, \Sigma, \Xi$$

$$\Sigma^*, \Xi^*, \Omega$$

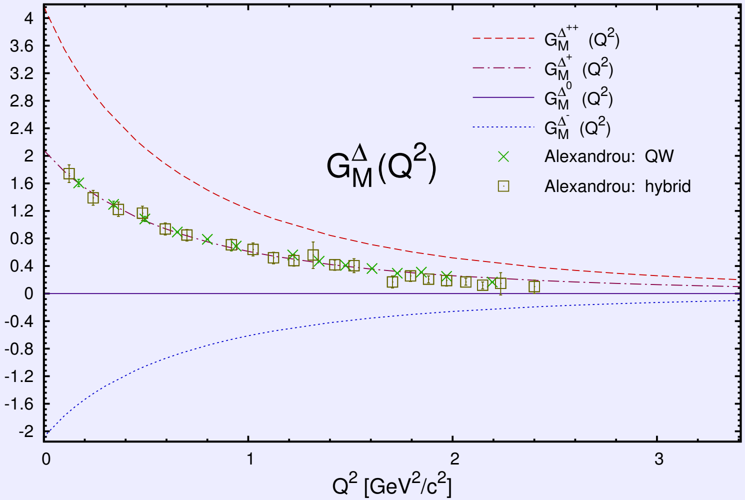
Electric Δ Form Factors



GBE RCQM: Ki-Seok Choi: PhD Thesis, Univ. Graz, 2011

Lattice QCD: C. Alexandrou et al. Phys. Rev. D **79** (2009) 014507

Magnetic Δ Form Factors



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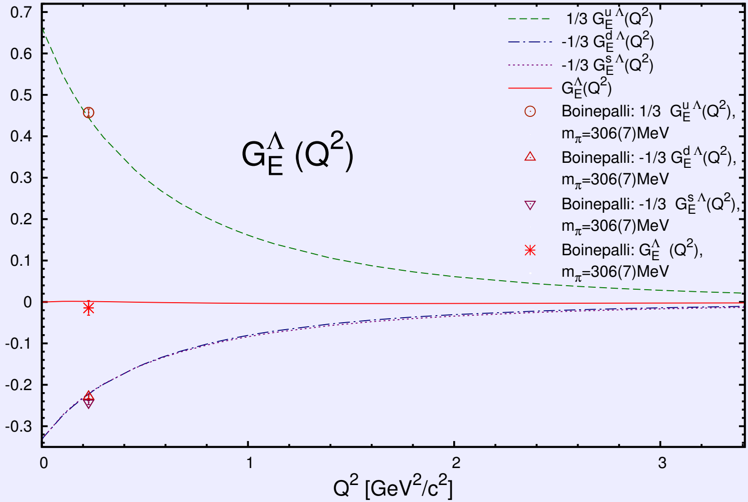
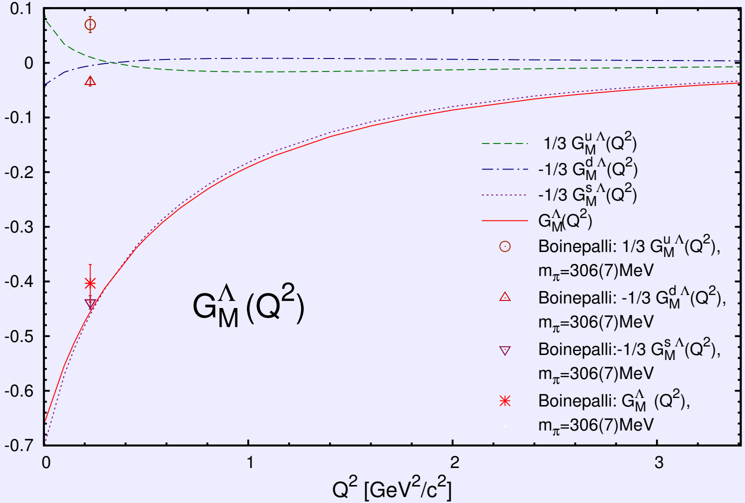
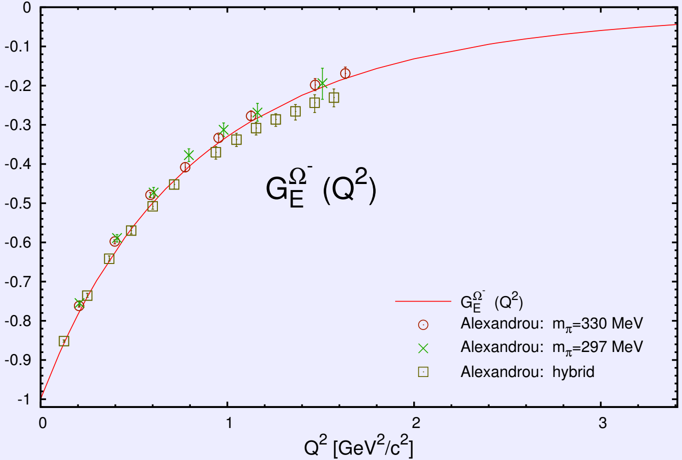
Octet $\Lambda(uds)$ Electric Form Factor



Octet $\Lambda(uds)$ Magnetic Form Factor

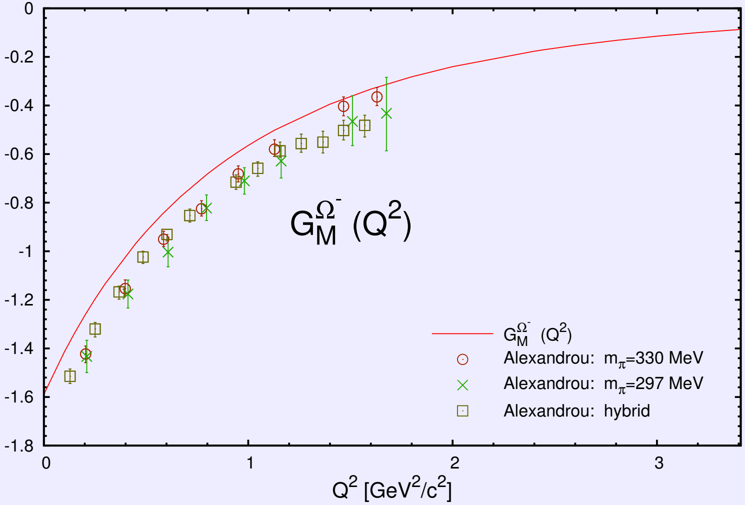


Decuplet $\Omega^-(sss)$ Electric Form Factor



Lattice-QCD: C. Alexandrou et al.: Phys. Rev. D**82** (2010) 034504

Decuplet $\Omega^-(sss)$ Magnetic Form Factor

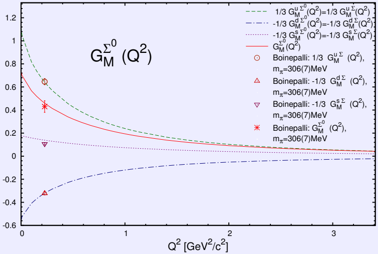


Lattice-QCD: C. Alexandrou et al.: Phys. Rev. D**82** (2010) 034504

Octet $\Sigma^0(dds)$ vs. Decuplet $\Sigma^{*0}(dds)$

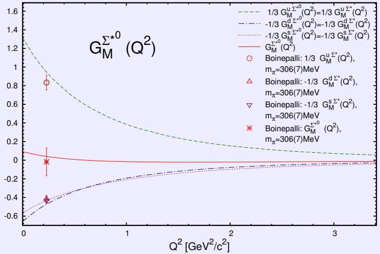
Octet

$$G_M^{\Sigma^0} = \frac{1}{3} G_M^{u,\Sigma} - \frac{1}{3} G_M^{d,\Sigma} - \frac{1}{3} G_M^{s,\Sigma}$$



Decuplet

$$G_M^{\Sigma^{*0}} = \frac{1}{3} G_M^{u,\Sigma^*} - \frac{1}{3} G_M^{d,\Sigma^*} - \frac{1}{3} G_M^{s,\Sigma^*}$$

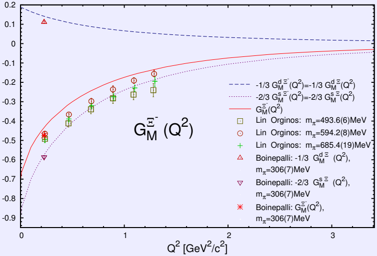


Lattice-QCD: S. Boinepalli et al.: Phys. Rev. D **74**, 093005 (2006)
S. Boinepalli et al.: Phys. Rev. D **80**, 054505 (2009)

Octet $\Xi^-(dss)$ vs. Decuplet Octet $\Xi^{*-}(dss)$

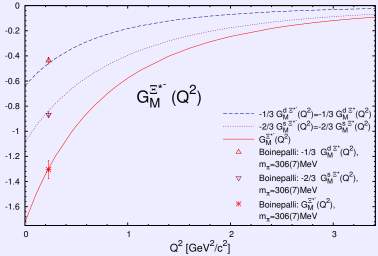
Octet

$$G_M^{\Xi^-} = -\frac{1}{3} G_M^{d,\Xi} - \frac{2}{3} G_M^{s,\Xi}$$



Decuplet

$$G_M^{\Xi^{*-}} = -\frac{1}{3} G_M^{d,\Xi^*} - \frac{2}{3} G_M^{s,\Xi^*}$$



Lattice-QCD: S. Boinepalli et al.: Phys. Rev. D **74**, 093005 (2006)

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p	0.82	0.7692 ± 0.0123
n	-0.13	-0.1161 ± 0.0022
Σ^-	0.72	$0.61 \pm 0.12 \pm 0.09$

Magnetic moments μ [n.m.]

Baryon	GBE PFSM	Experiment
p	2.70	2.792847356
n	-1.70	-1.9130427
Λ	-0.64	-0.613 ± 0.004
Σ^+	2.38	2.458 ± 0.010
Σ^-	-0.93	-1.160 ± 0.025
Ξ^0	-1.25	-1.250 ± 0.014
Ξ^-	-0.70	-0.6507 ± 0.0025
Δ^+	2.08	$2.7^{+1.0}_{-1.3} \pm 1.5 \pm 3$
Δ^{++}	4.17	$3.7 - 7.5$
Ω^-	-1.59	-2.020 ± 0.05

K. Berger, R.F. Wagenbrunn, and W. Plessas: Phys. Rev. D **70**, 094027 (2004)

Axial **Charges** and Axial **Form Factors**

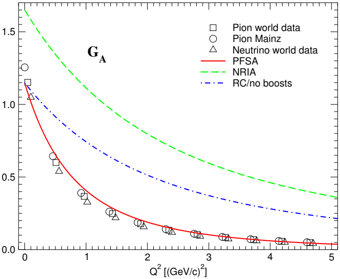
of

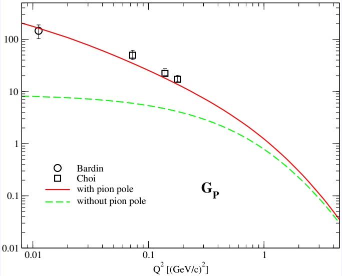
N Ground State and N^* Resonances

as well as

$\Delta, \Sigma, \Xi, \Sigma^*, \Xi^*$

Covariant predictions of the GBE RCQM:





$$g_A^{GBE} = 1.15 \quad \text{vs.} \quad g_A^{exp} = 1.2695 \pm 0.0029$$

L.Ya. Glozman, M. Radici, R.F. Wagenbrunn, S. Boffi, W. Klink, and W. Plessas: Phys. Lett. B **516**, 183 (2001)

Axial Charges of N and N^* Resonances

State	J^P	EGBE	Lattice QCD	GN	NR
N(939)	$\frac{1}{2}^+$	1.15	1.18∼1.31	1.66	1.65
N(1440)	$\frac{1}{2}^+$	1.16	?	1.66	1.61
N(1535)	$\frac{1}{2}^-$	0.02	∼0.00	-0.11	-0.20
N(1710)	$\frac{1}{2}^+$	0.35	?	0.33	0.42
N(1650)	$\frac{1}{2}^-$	0.51	∼0.55	0.55	0.64

EGBE — **E**xtended **GBE** RCQM covariant result

Lattice — **Lattice QCD** calculations by LHPC Collaboration and Takahashi-Kunihiro (Kyoto)

GN — **G**lozman-**N**efediev $SU(6) \times O(3)$ nonrelativistic QM

NR — **N**on-**R**elativistic EGBE result

K.-S. Choi, W. Plessas, and R.F. Wagenbrunn: Phys. Rev. C **81**, 028201 (2010)

Axial Charges of N and N^* Resonances

		EGBE		psGBE		OGE	
State	J^P	Mass	g_A	Mass	g_A	Mass	g_A
N(939)	$\frac{1}{2}^+$	939	1.15	939	1.15	939	1.11
N(1520)	$\frac{3}{2}^-$	1524	-0.64	1519	-0.21	1520	-0.15
N(1440)	$\frac{1}{2}^+$	1464	1.16	1459	1.13	1578	1.10
N(1535)	$\frac{1}{2}^-$	1498	0.02	1519	0.09	1520	0.13
N(1680)	$\frac{5}{2}^+$	1689	0.89	1728	0.83	1858	0.70
N(1675)	$\frac{5}{2}^-$	1676	0.84	1647	0.83	1690	0.80
N(1710)	$\frac{1}{2}^+$	1757	0.35	1776	0.37	1860	0.32
N(1650)	$\frac{1}{2}^-$	1581	0.51	1647	0.46	1690	0.44
N(1720)	$\frac{3}{2}^+$	1746	0.35	1728	0.34	1858	0.25
N(1700)	$\frac{3}{2}^-$	1608	-0.10	1647	-0.50	1690	-0.47

K.-S. Choi, W. Plessas, and R.F. Wagenbrunn: Phys. Rev. C **81**, 028201 (2010)

Axial Form Factor of the Δ

Low-energy QCD
RCQM
GBE RCQM
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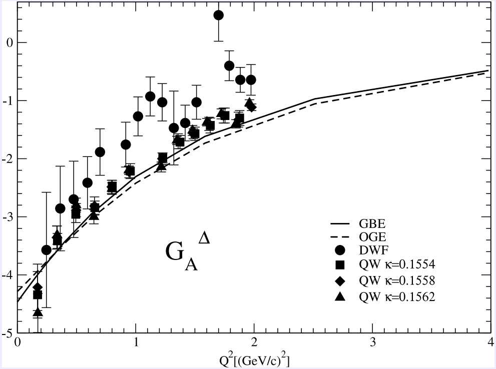
Covariant predictions of the GBE and OGE RCQMs:

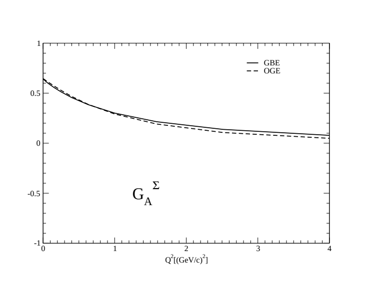


K.-S. Choi and W. Plessas: Few-Body Syst. **54**, 1055 (2013)

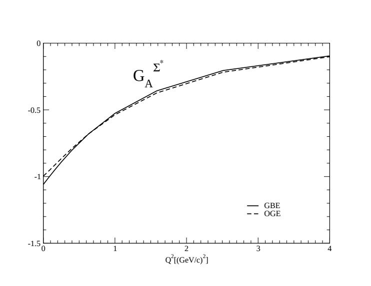
(Lattice QCD data from C. Alexandrou et al., PoS LATTICE2010, 141 (2010))

Axial Form Factors of Σ and Σ^*

Covariant predictions of the GBE and OGE RCQMs:

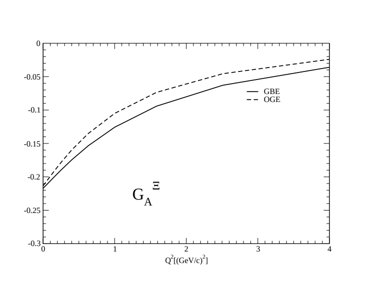


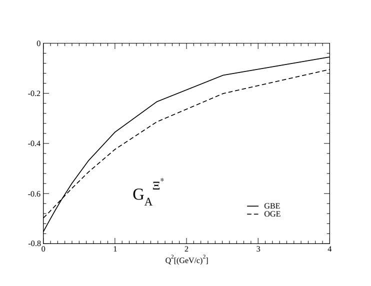
Octet Σ



Decuplet Σ^*

Covariant predictions of the GBE and OGE RCQMs:





Octet Ξ

Decuplet Ξ*

Axial Charges of $\Delta, \Sigma, \Xi, \Sigma^*, \Xi^*$

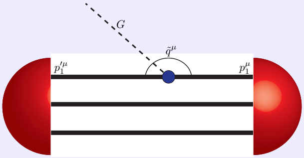
	J^P	Exp	EGBE	LO	EOT	JT	NR
N	$\frac{1}{2}^+$	1.2695	1.15	1.18	1.314	1.18	1.65
Σ	$\frac{1}{2}^+$	-	0.65	0.636	0.686	0.73	0.93
Ξ	$\frac{1}{2}^+$	-	-0.21	-0.277	-0.299	-0.23	-0.32
Δ	$\frac{3}{2}^+$	-	-4.48	-	-	$\sim$-4.5	-6.00
Σ^*	$\frac{3}{2}^+$	-	-1.06	-	-	-	-1.41
Ξ^*	$\frac{3}{2}^+$	-	-0.75	-	-	-	-1.00

EGBE — **E**xtended **GBE** RCQM covariant result
LO — **L**in and **O**rginos lattice-QCD calculation
EOT — **E**rkol, **O**ka, and **T**akahashi lattice-QCD calculation
JT — **J**iang and **T**iburzi χPT calculation
NR — **N**on-**R**elativistic EGBE result

K.-S. Choi, W. Plessas, and R.F. Wagenbrunn: Phys. Rev. D **82**, 014007 (2010)

Gravitational Form Factors of the Nucleon

Gravitational Form Factors

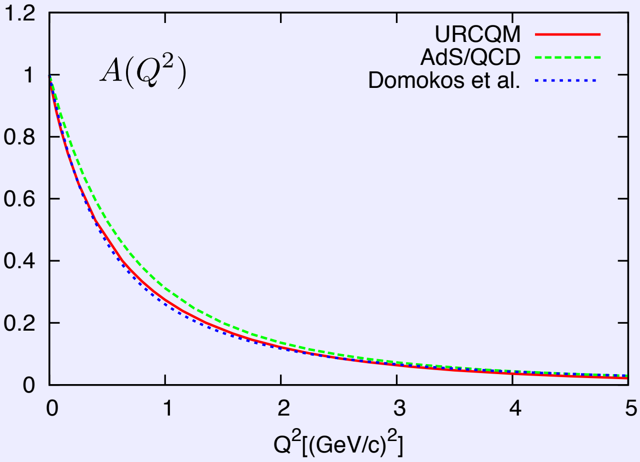


Invariant ME of **energy-momentum tensor** $\hat{\Theta}^{\mu\nu}$:

$$\langle P' J \Sigma' | \hat{\Theta}^{\mu\nu} | P J \Sigma \rangle = \bar{U}(P') \left[\gamma^{(\mu} \bar{P}^{\nu)} A(Q^2) + \frac{i}{2M} \bar{P}^{(\mu} \sigma^{\nu)} B(Q^2) + \frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M} C(Q^2) \right] U(P)$$

$$A(Q^2) \sim \langle P' J \Sigma' | \Theta^{00} | P J \Sigma \rangle$$

Nucleon Gravitational Form Factor $A(Q^2)$



J.P. Day: PhD Thesis, Univ. Graz, August 2013

Summary and Conclusions

- Surprisingly **good agreement** of predictions by the $\{QQQ\}$ GBE RCQM with experimental data (wherever such data are available)
- **Small deviations** left in some observables, such as electric radii and magnetic moments
- Surprisingly **good agreement** of predictions by the GBE RCQM with lattice-QCD results
- Most important symmetries of the GBE RCQM:
  - **SBχS**
  - **Lorentz invariance**
  - **time-reversal invariance**
  - **current conservation**
- The **non-relativistic** quark model **does not work** in any instance

Collaborators

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Thank you very much

for

your attention!