

# Total and inelastic $pp$ cross sections at LHC in the light of a minijet model with soft gluon resummation

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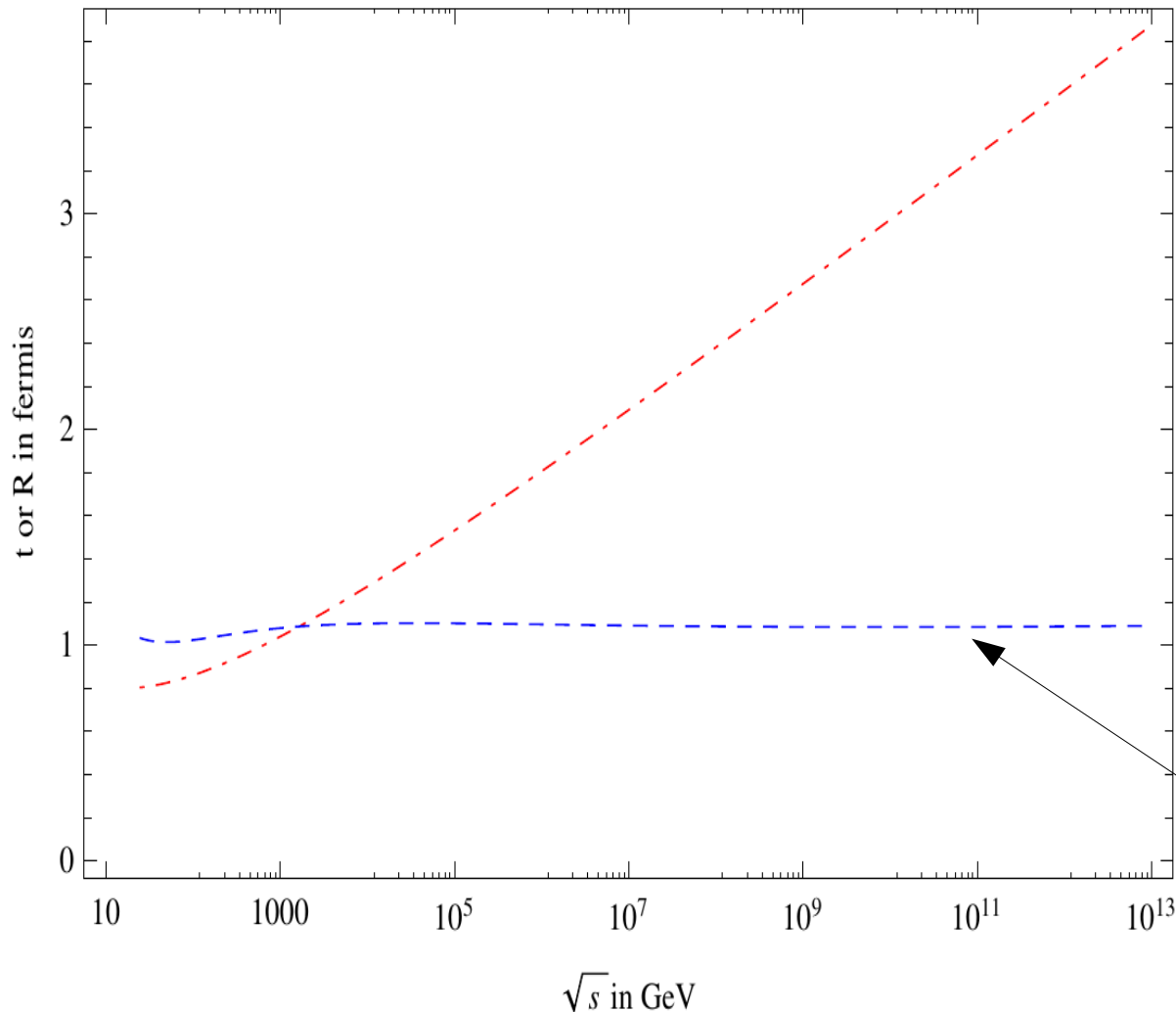
**International WE-Heraeus Physics School**  
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# Outline

- Soft edge in  $pp$  scattering
- One-channel model with soft gluon resummation
- Total and inelastic (uncorrelated) cross sections
- Elastic cross section – model vs empirical
- Inelastic diffraction and the 'full' inelastic
- Predictions for LHC13-14
- Summary

# Soft *edge* in $pp$ scattering

M.M. Block et al, Phys.Rev. D91 (2015) 1, 011501



$$\sigma^{TOT} - 2\sigma^{EL} = 4\pi \int_0^\infty \eta(1 - \eta) b db.$$

$$(\sigma^{TOT} - 2\sigma^{EL}) / \sqrt{(\pi/2)\sigma^{TOT}} \approx 4\mathcal{I} \rightarrow \text{constant}$$

$$\mathcal{I} = \frac{1}{4}t \quad R = \sqrt{\sigma^{TOT}/2\pi}$$

From 1 TeV onwards – an 'energy-invariant' edge

The energy behaviour of cross sections is behind the appearance of such 'edge'.

# pQCD and the *edge*

With the energy increases there will be more available energy for gluon emission and a nonzero probability for hard gluon-gluon scattering leading to final state partons with  $p_t \sim 1$  GeV or even larger. In this region we can apply pQCD to estimate the threshold energy where the scattering moves into a high luminosity regime. A typical scale to analyze is

$$1/x \sim \sqrt{s}/(2p_t)$$

Contributions from hard processes to the total cross section become sizable when

$$1/x = \sqrt{s}/2p_t \gg 1 \quad \text{and} \quad p_t > 1 \text{ GeV}$$

With the condition  $p_t > 1$  GeV and asymptotic freedom satisfied, the rise starts at  $x \ll 1$ , typically  $x \sim 0.1-0.2$  or

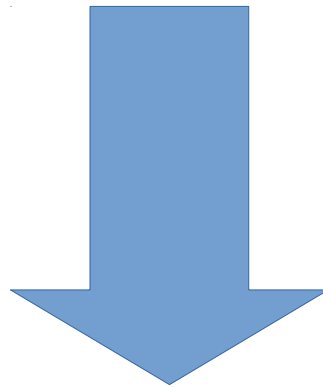
$$\sqrt{s} \gtrsim (2/x) \text{ GeV}$$

$$\sqrt{s} \sim (10 \div 20) \text{ GeV}$$

# Total cross section: a 1-channel model

In the impact parameter/eikonal representation, through the optical theorem:

$$\sigma_{total} = 2 \int d^2\mathbf{b} \Re e[1 - e^{i\chi(b,s)}] = 2 \int d^2\mathbf{b} [1 - \cos \Re \chi(b,s) e^{-\Im m \chi(b,s)}]$$



$$\Re \chi(b,s) \approx 0$$

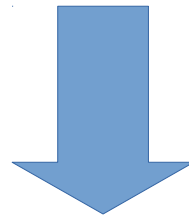
$$\bar{n}(b,s) = 2\Im m \chi(b,s)$$

$$\sigma_{tot}(s) = 2 \int d^2\mathbf{b} [1 - e^{-\bar{n}(b,s)/2}]$$

# Inelastic cross section: semiclassical interpretation

Multiple parton-parton interactions independently distributed → Poisson distribution

$$P(\{n, \bar{n}\}) = \frac{(\bar{n})^n e^{-\bar{n}}}{n!}$$



$$\sigma_{inel}(s) = \sum_{n=1} \int d^2\mathbf{b} P(\{n, \bar{n}\}) = \int d^2\mathbf{b} [1 - e^{-\bar{n}(b,s)}]$$

Average number of collisions at given  $\mathbf{b}$  and  $s$ .

$$\bar{n}(b, s) = \bar{n}_{soft}(b, s) + \bar{n}_{hard}(b, s)$$

# How do we model $n(b,s)$ ?

$$\bar{n}_{soft/hard}(b, s) = A_{soft/hard}(b, s)\sigma_{soft/hard}(s)$$

**Hard part** → drives the rise with energy of the total cross section, being dominated by parton-parton scattering with

$$p_t > p_{tmin}$$

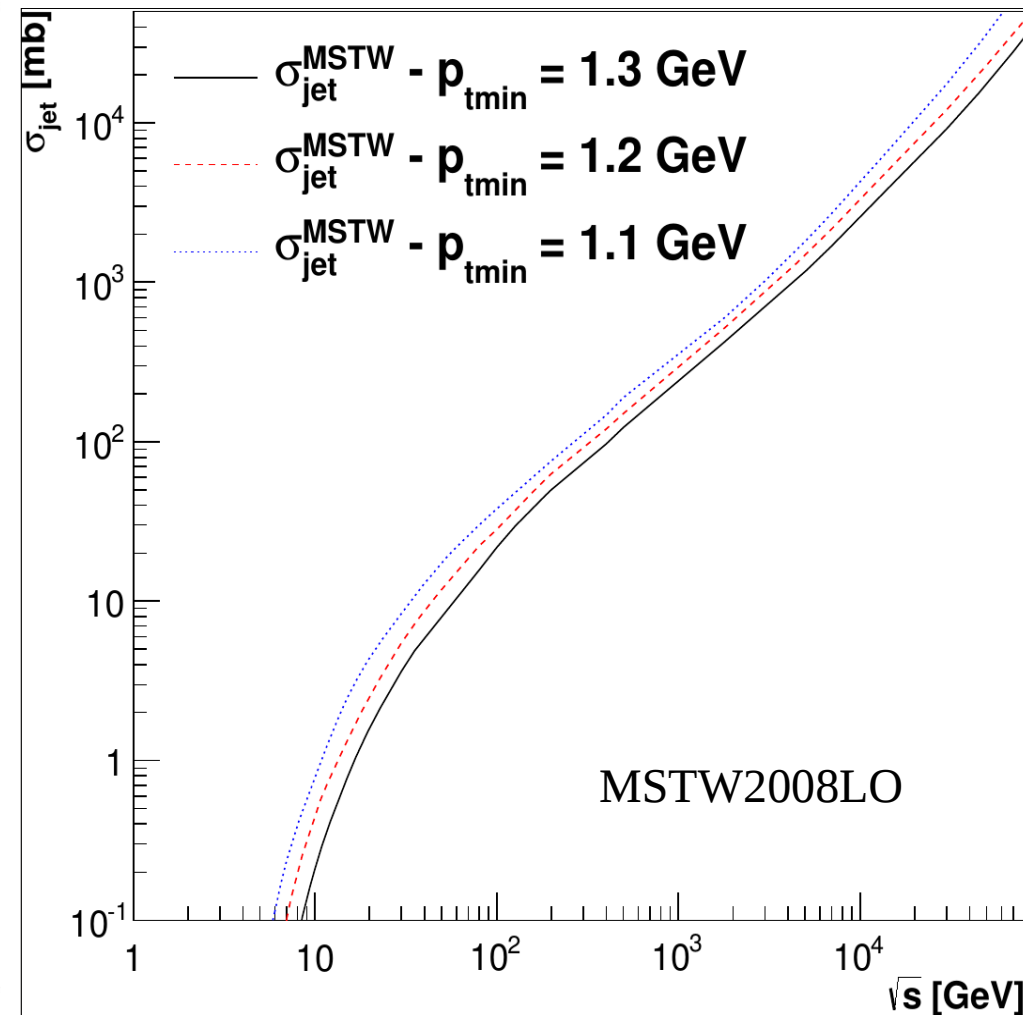
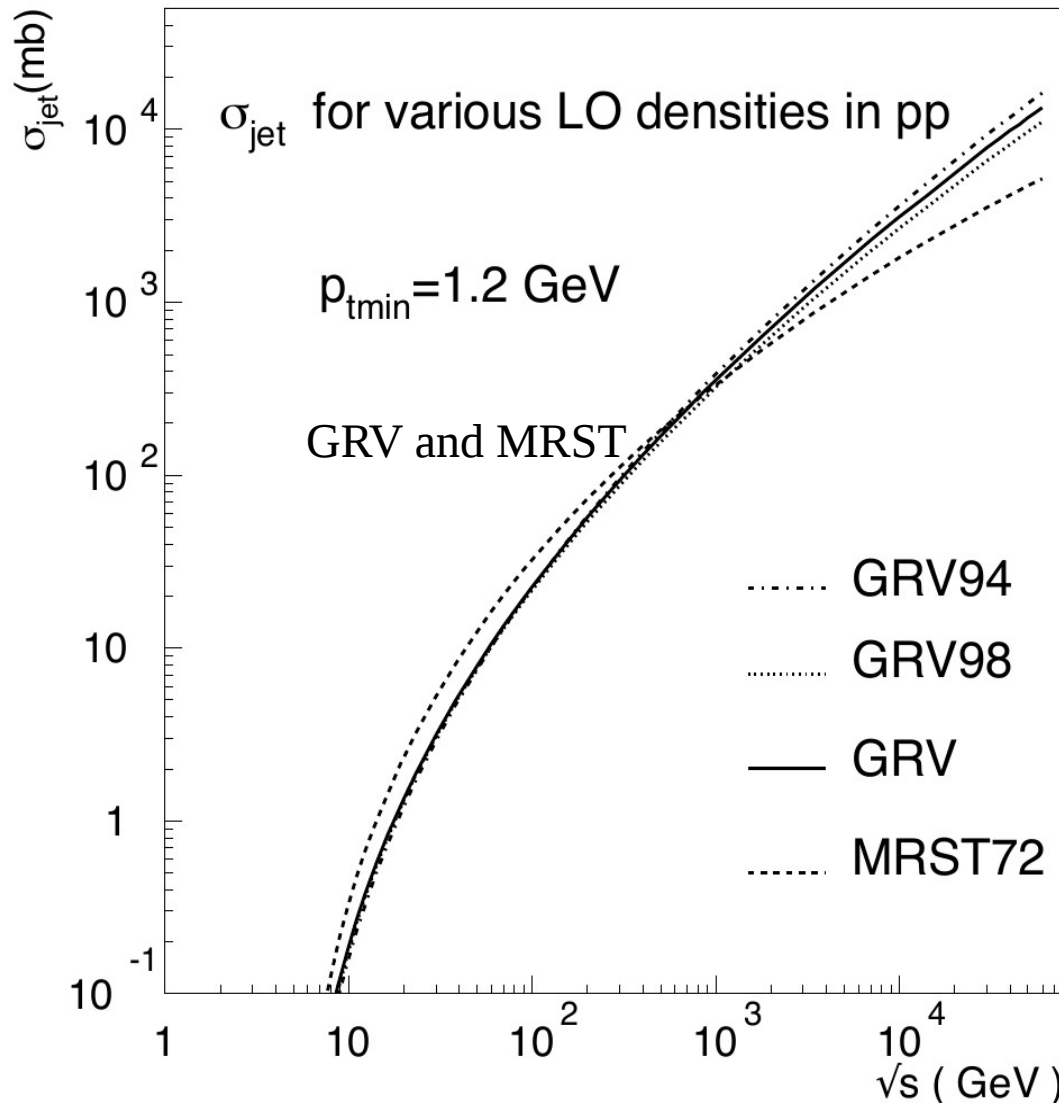
**Soft part** → contribution from processes with

$$p_t < p_{tmin}$$

is modeled phenomenologically with a form factor and low energy cross section.

# Hard processes: minijet cross section

$$\sigma_{\text{jet}}^{ab}(s) = \int_{p_{t\text{min}}}^{\sqrt{s}/2} dp_t \int_{4p_t^2/s}^1 dx_1 \int_{4p_t^2/(x_1 s)}^1 dx_2 \sum_{i,j,k,l} f_{i|a}(x_1) f_{j|b}(x_2) \frac{d\hat{\sigma}_{ij \rightarrow kl}(\hat{s})}{dp_t}$$



Threshold for minijets at  $\sim 10 \text{ GeV} \rightarrow$  cross section starts rising very fast!



# Soft gluon emission and resummation

G. Pancheri-Srivastava and Y.N. Srivastava, Phys. Rev. D15 (1977) 2915.

Y.L. Dokshitzer, D. Diakonov and S.I. Troian, Phys. Lett. B79 (1978) 269.

**Momentum transverse distribution for soft gluon emission from parton-parton pair**

$$\frac{d^2 P(\mathbf{K}_\perp)}{d^2 \mathbf{K}_\perp} \equiv \Pi(K_\perp) = \int \frac{d^2 \vec{b}}{(2\pi)^2} e^{i\mathbf{K}_\perp \cdot \mathbf{b} - h(b)}$$

$$h(b) = \int d^3 \bar{n}_g(k) [1 - e^{-i\mathbf{k}_\perp \cdot \mathbf{b}}] = \int \frac{d^3 k}{2k_0} \sum_{i,j=\text{colors}} |j^{\mu,i}(k) j_{\mu,j}(k)| [1 - e^{-i\mathbf{k}_\perp \cdot \mathbf{b}}]$$

Distribution of single gluon emission

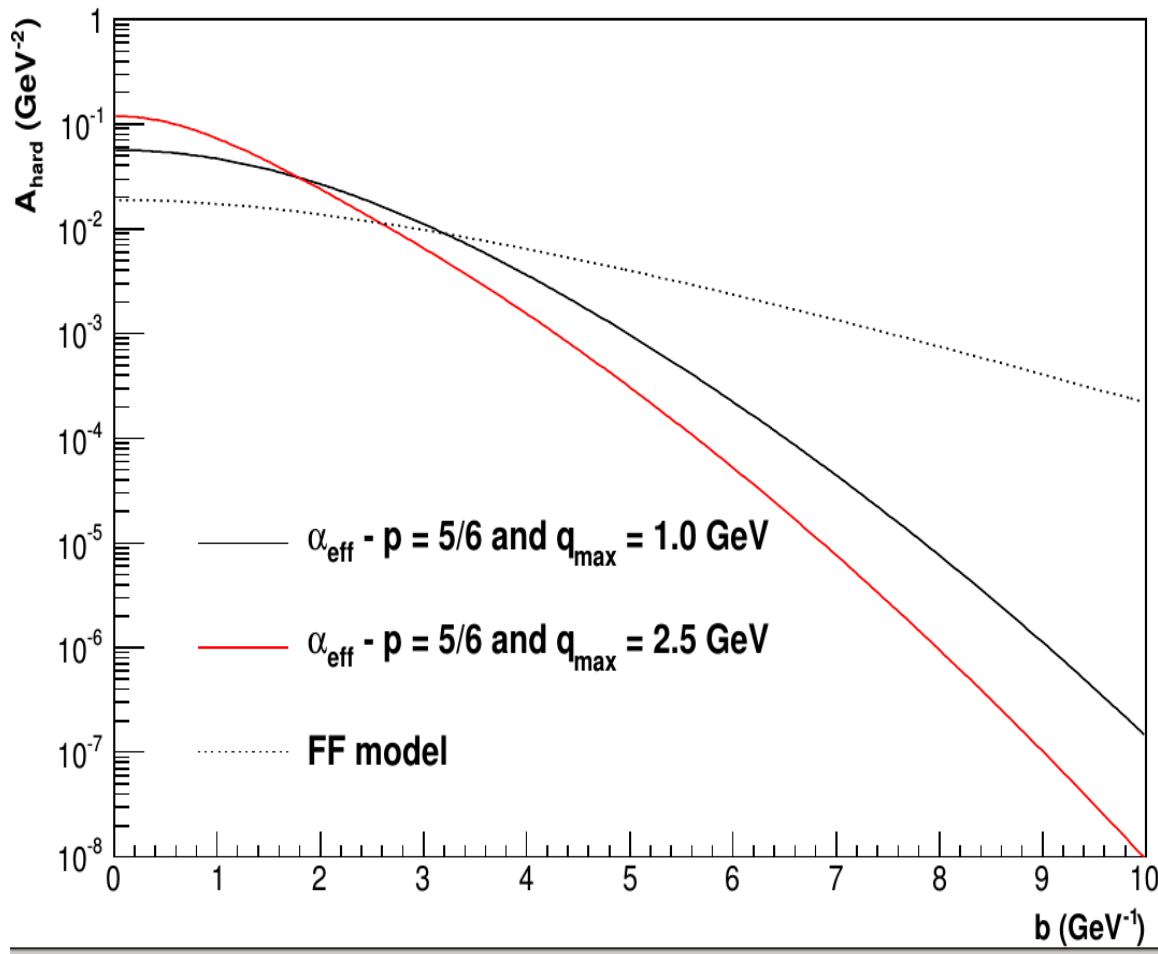
QCD current for emission

**Used in studies of initial state transverse momentum distribution of Drell-Yan and W-production.**

# Soft gluon emission and b-distribution

$$A_{BN}^{PP}(p, PDF; b, s) = \frac{e^{-h(p;b,s)}}{\int d^2\mathbf{b} e^{-h(p,b,s)}}$$

$$h(b, s; p) = \frac{16}{3\pi} \int_0^{q_{\max}} \frac{dk_t}{k_t} \alpha_{\text{eff}}(k_t) \log \frac{2q_{\max}}{k_t} [1 - J_0(bk_t)]$$



$$\alpha_{\text{eff}} = \frac{12\pi}{33 - 2N_f} \frac{p}{\log[1 + p(k_t/\Lambda_{\text{QCD}})^{2p}]}$$

**(Singular, but integrable)**

$$A_{\text{hard}}(b, s) \sim e^{-(b\bar{\Lambda})^{2p}} \quad b \rightarrow \infty$$

**Not exponential, not a gaussian.**

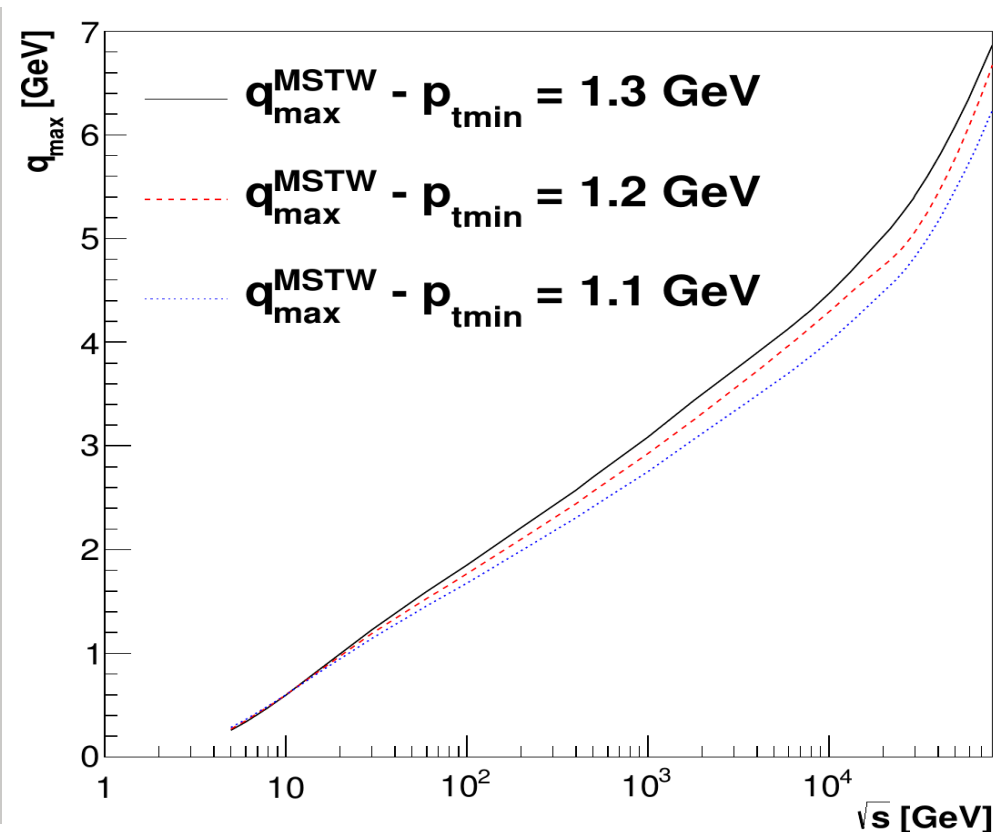
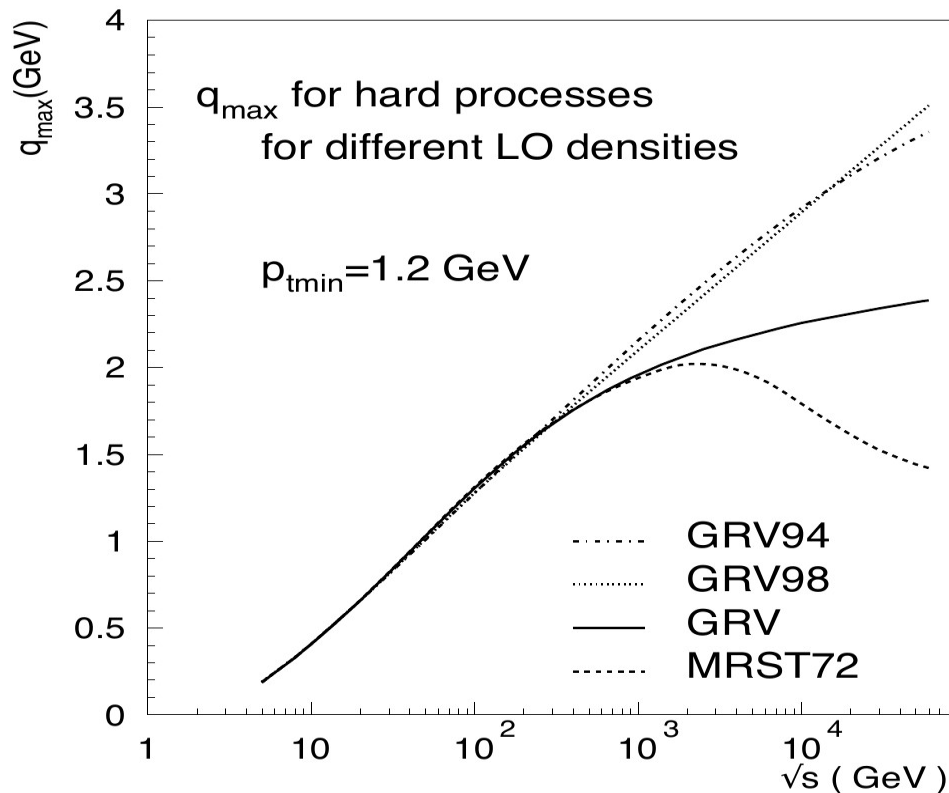
$$1/2 < p < 1$$

# q<sub>max</sub>: an s-dependent scale

Inspired in applications to Drell-Yan processes, but for pp scattering **not so simple**:

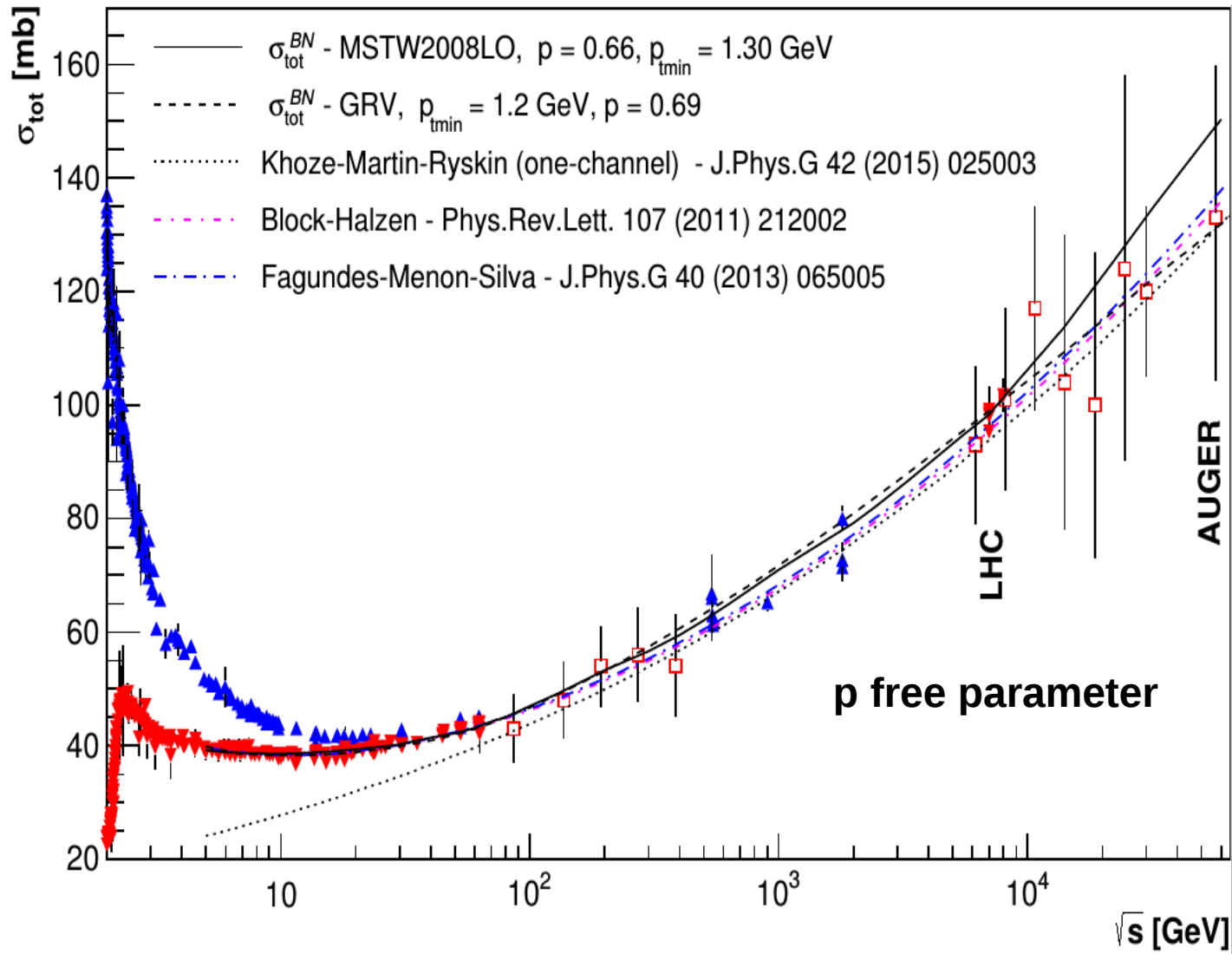
PDFs

$$q_{\max}(s; p_{\text{tmin}}) = \sqrt{\frac{s}{2}} \frac{\sum_{i,j} \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \int_{z_{\min}}^1 dz f_i(x_1) f_j(x_2) \sqrt{x_1 x_2} (1-z)}{\sum_{i,j} \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \int_{z_{\min}}^1 dz f_i(x_1) f_j(x_2)}$$



# Total cross section: results

D.A. Fagundes et al, Phys.Rev. D91 (2015) 11, 114011



Asymptotic behaviour:

$$\sigma_{\text{tot}}^{pp} \sim \frac{2\pi}{(\bar{\Lambda})^2} [\epsilon \log s]^{1/p}$$

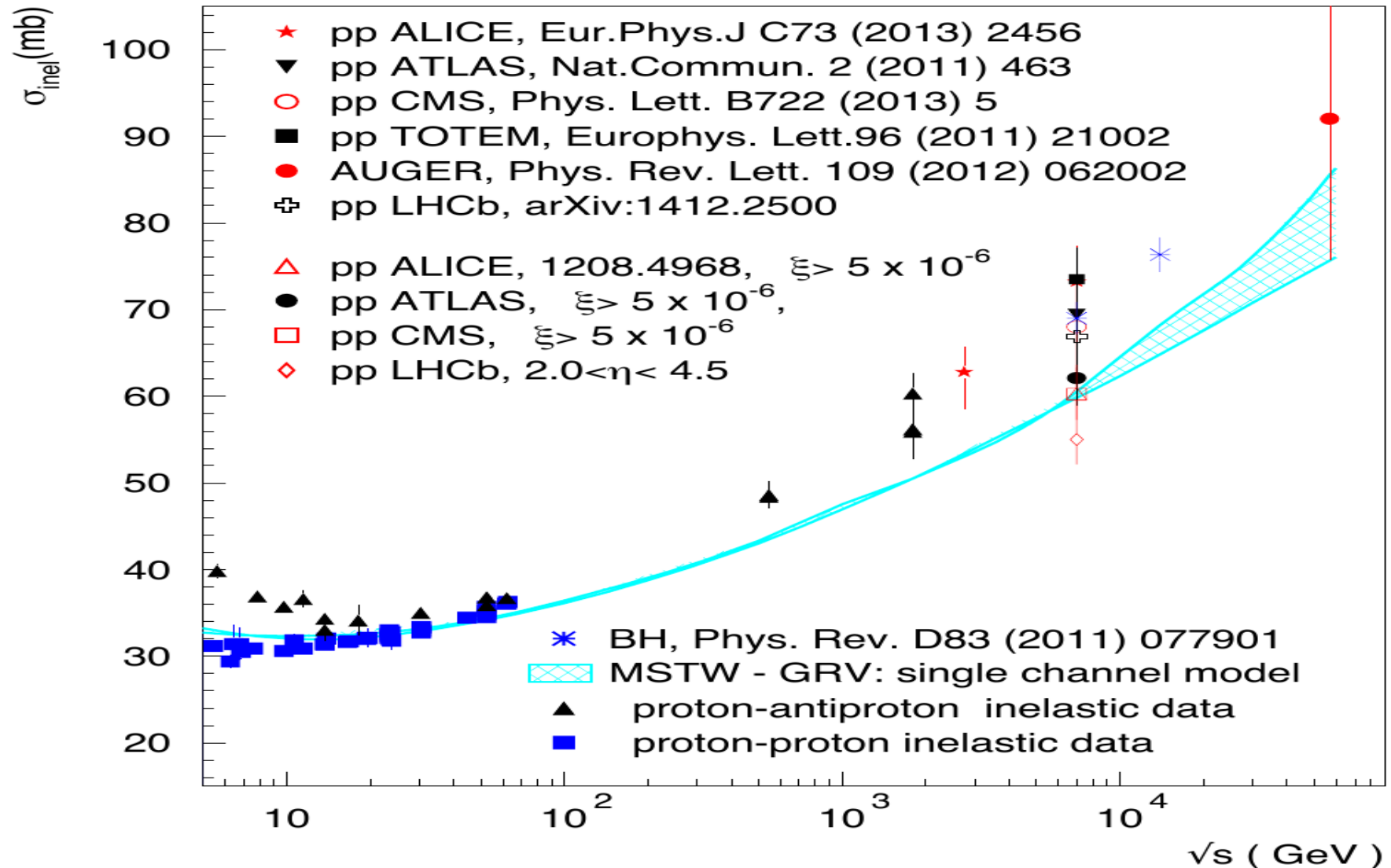
From analyticity:

$$1/2 < p < 1$$

Very nice description of data at LHC7 and LHC8 with  $p = 0.66-0.69$

# Inelastic cross section - results

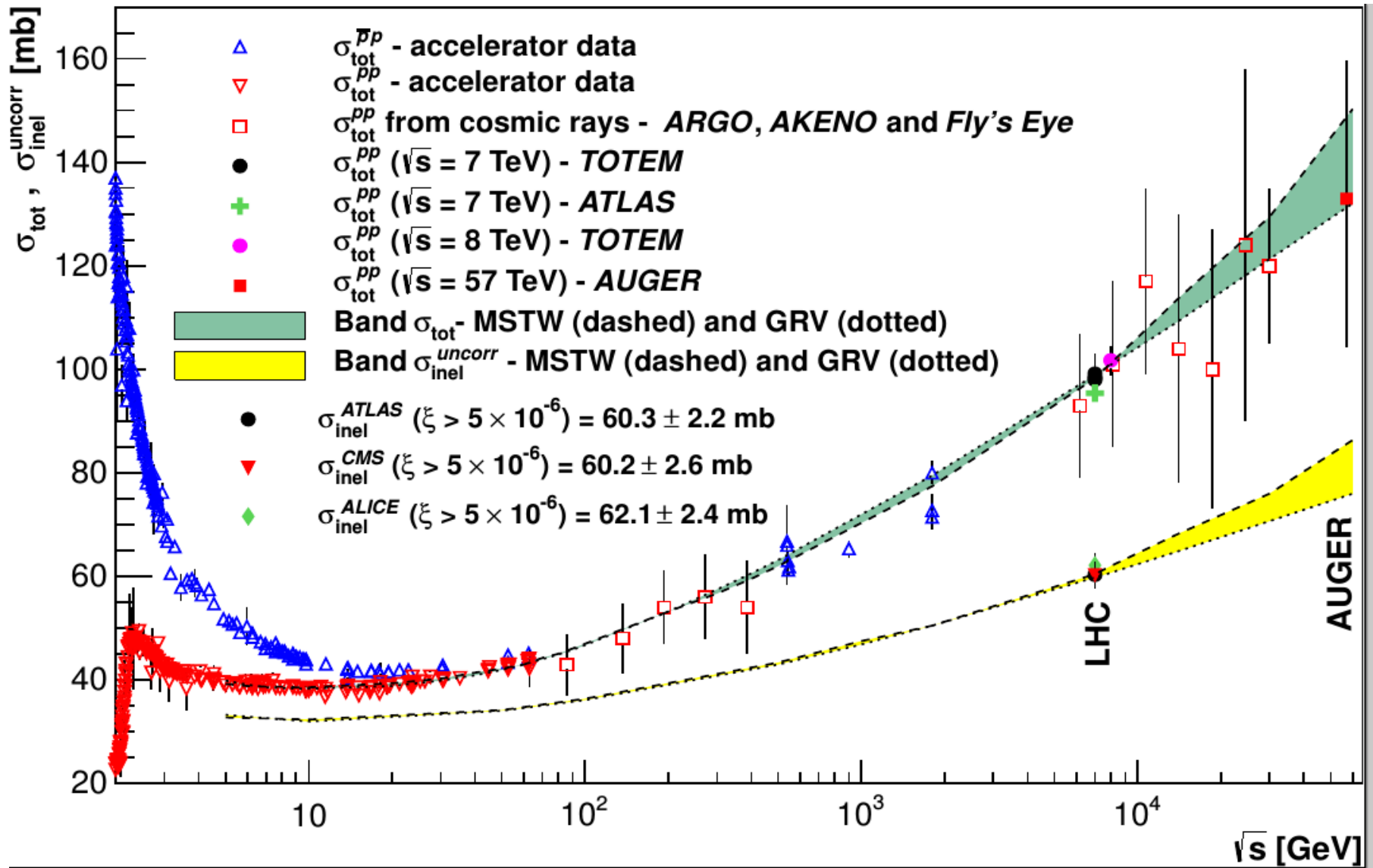
D.A. Fagundes et al, Phys.Rev. D91 (2015) 11, 114011



**Model suitable for events of nondiffractive nature**

# PDFs at low-x - uncertainties

D.A. Fagundes et al, Phys.Rev. D91 (2015) 11, 114011



Bands for total and inelastic (uncorrelated) cross section with same p

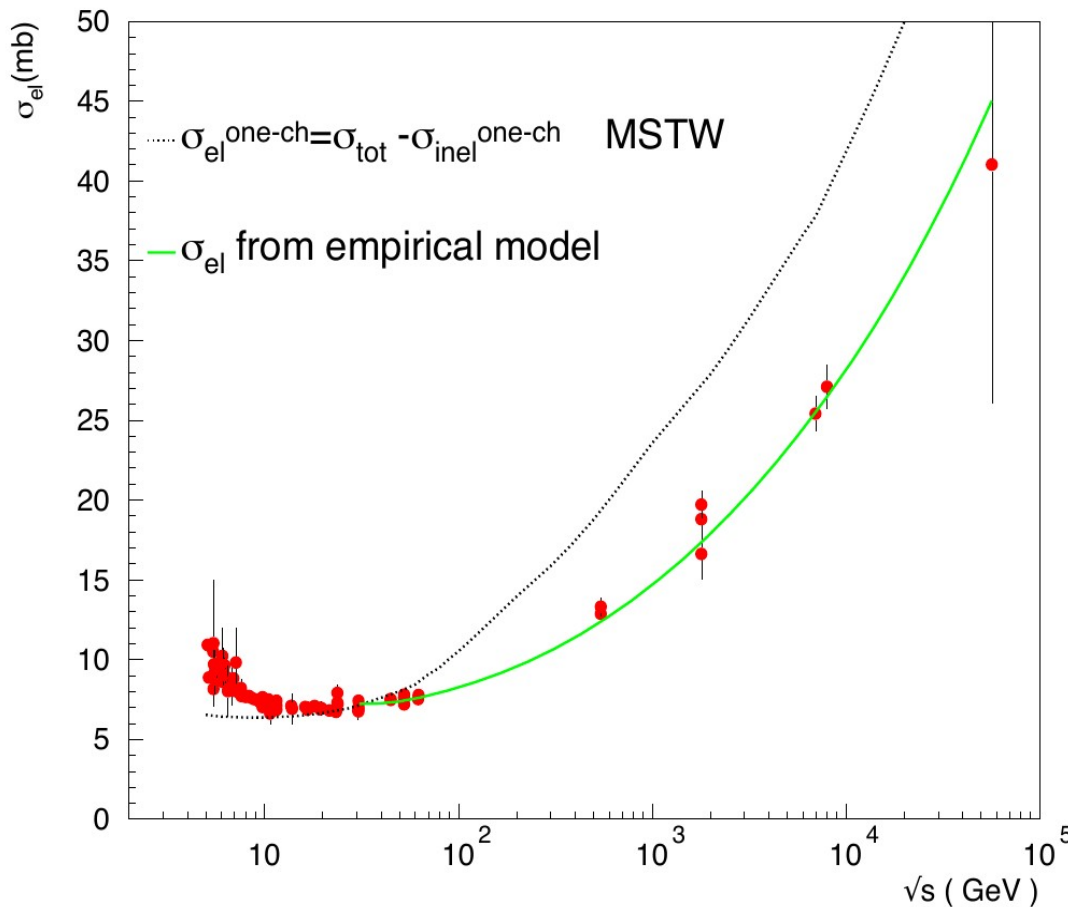
# Elastic cross section – results

D.A. Fagundes et al, Phys.Rev. D91 (2015) 11, 114011

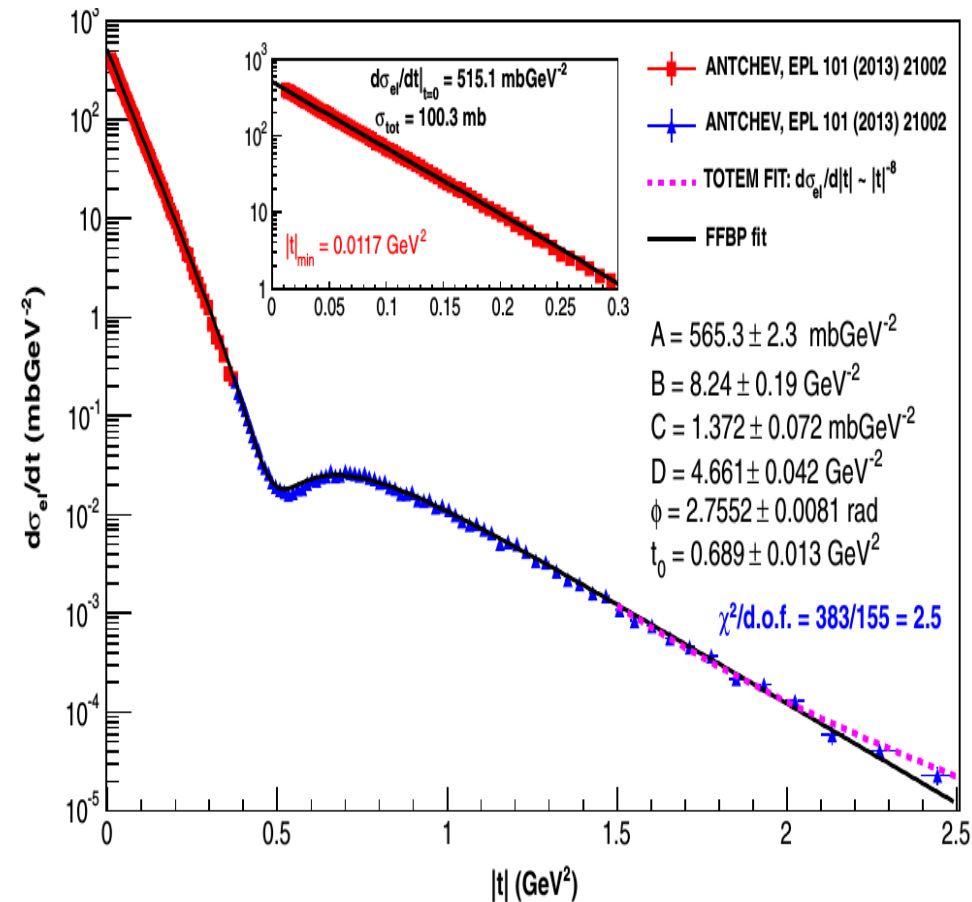
D.A. Fagundes et al, Phys.Rev. D88 (2013) 9, 094019

$$\sigma_{\text{elastic}}^{\text{one-ch}} = \int d^2\vec{b} |1 - e^{-\chi_I(b,s)}|^2$$

$$\sigma_{\text{elastic}} = At_0 e^{Bt_0} E_8(Bt_0) + \frac{C}{D} + 2\sqrt{AC} \cos \phi t_0 e^{(B+D)t_0/2} E_4\left(\frac{(B+D)t_0}{2}\right)$$



Model – elastic integrated ~30% above data



Empirical – Barger-Phillips amplitude

# Break up of the inelastic cross section

Obviously, the model treats pure elastic scattering inconsistently, as it apparently encompass inelastic correlated processes in it. Our strategy is then subtract this contribution from it:

$$\sigma_{\text{elastic}} = \sigma_{\text{elastic}}^{\text{one-ch}} - \sigma_{\text{Diff}}$$

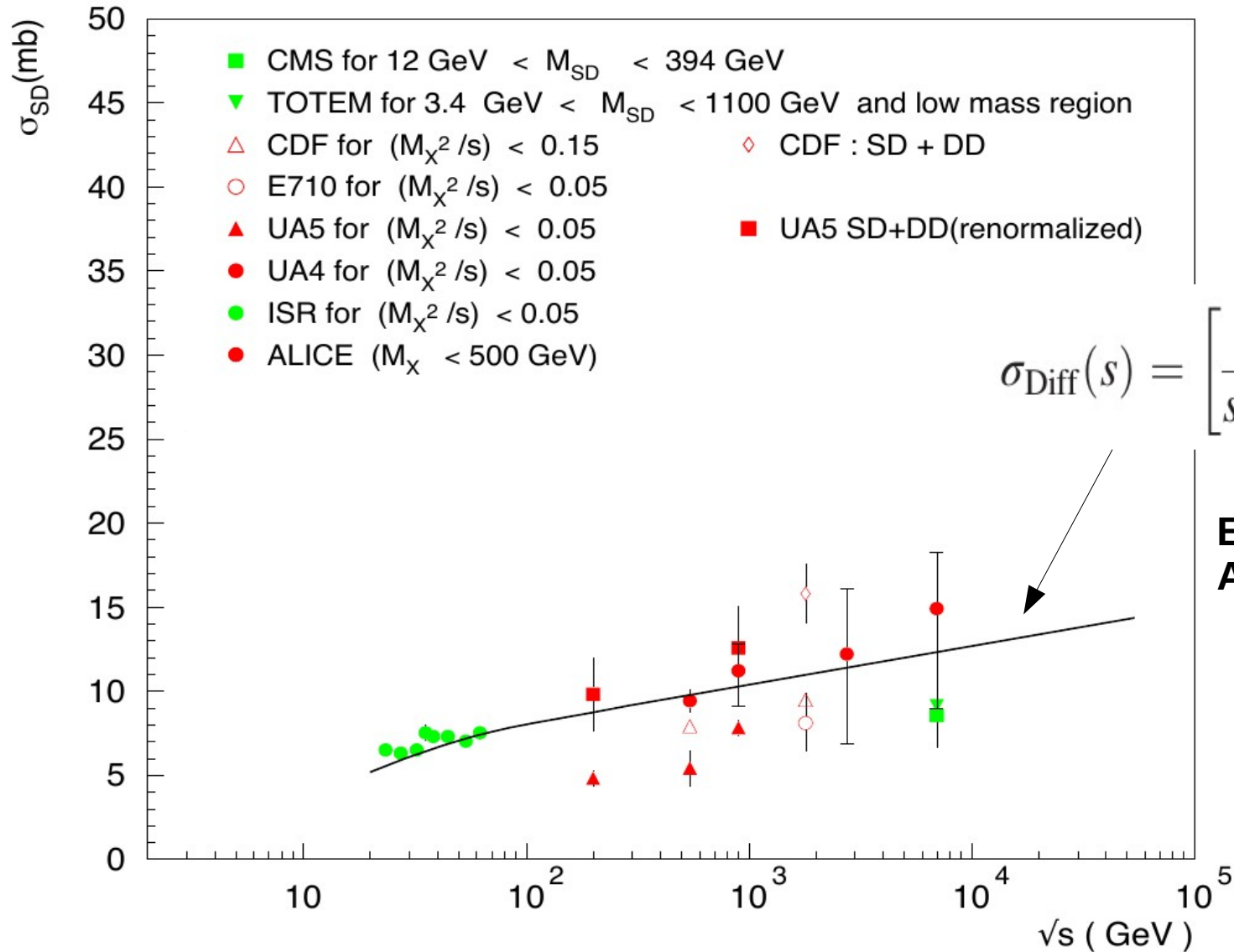
At the same time, to preserve s-channel unitarity

$$\sigma_{\text{inel}} = \sigma_{\text{inel}}^{\text{one-ch}} + \sigma_{\text{Diff}}$$

This is just the standard decomposition of inelastic processes into nondiff + diff.



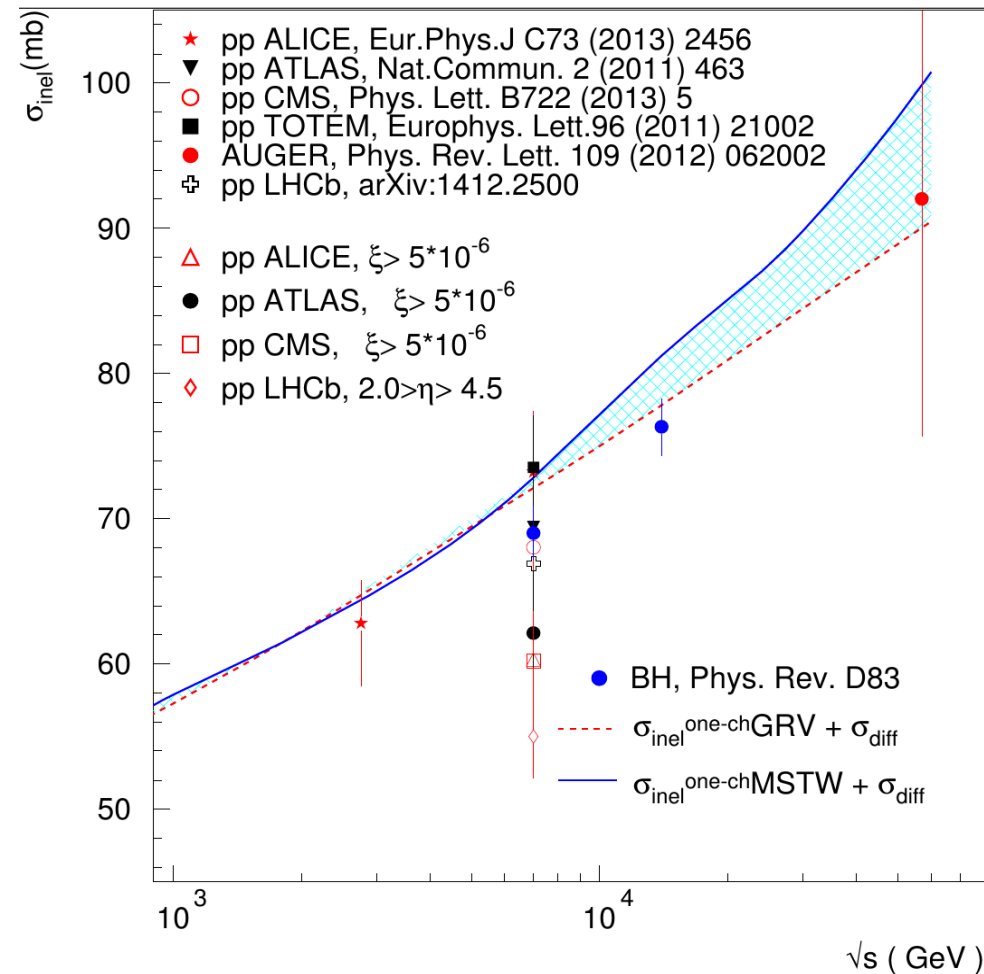
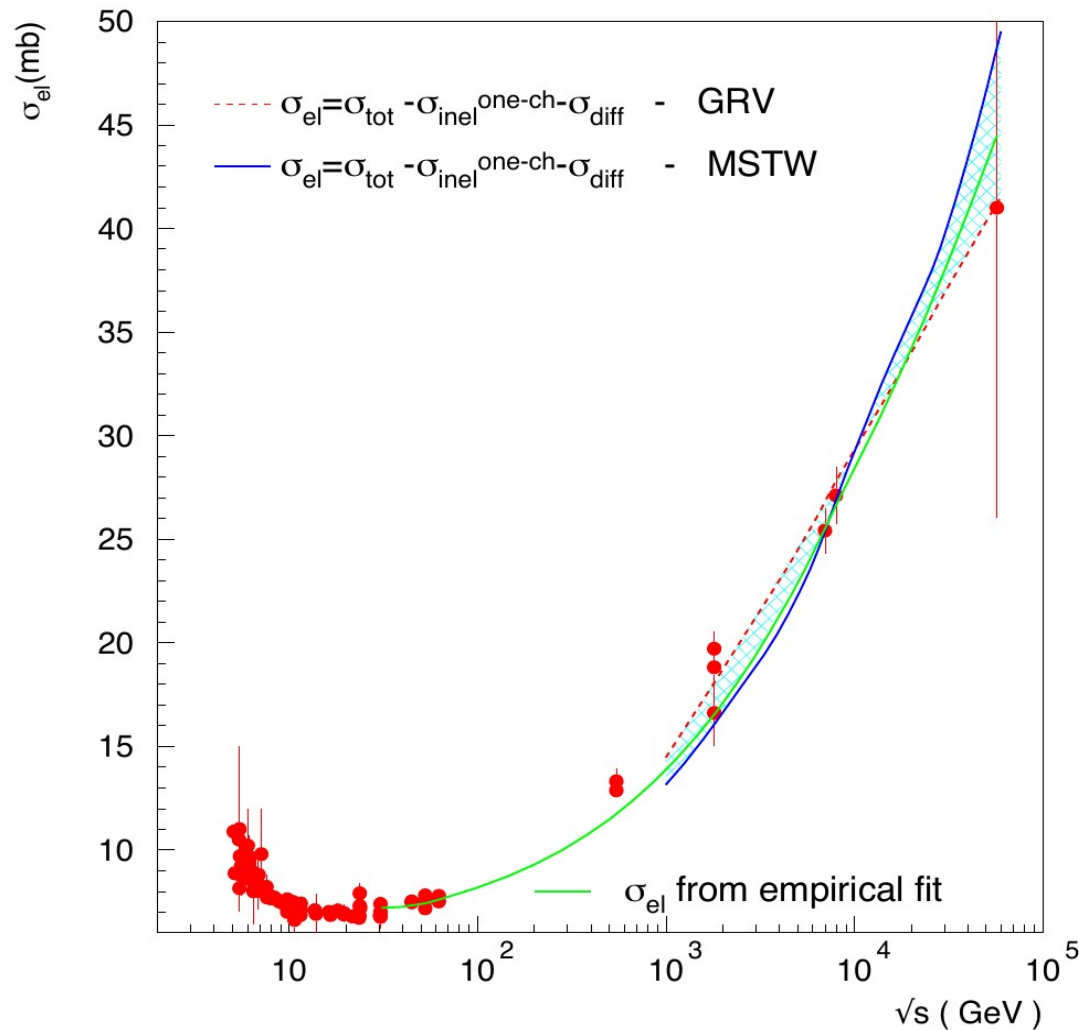
# Inelastic diffraction – from ISR to LHC



$$\sigma_{\text{Diff}}(s) = \left[ \frac{(0.5 \text{ mb}) s}{s + (10 \text{ GeV})^2} \right] \log \left( \frac{10^3 s}{\text{GeV}^2} \right)$$

**Empirical fit by Engel-Ulrich  
Auger GAP NOTE 2012**

# Elastic & inelastic cross sections: reanalysis



**Soft inelastic diffraction is important at high energies!!**

# Predictions for LHC13

TABLE I. Total cross section values in mb, from the minijet model with two different PDFs sets.

$\sqrt{s}$ (GeV)	$\sigma_{\text{total}}^{\text{GRV}}$ (mb)	$\sigma_{\text{total}}^{\text{MSTW}}$ (mb)
5	39.9	39.21
10	38.2	38.6
50	41.9	42.2
500	63.2	62.0
1800	79.5	77.5
2760	85.4	83.6
7000	98.9	98.3
8000	100.9	101.3
13000	108.3	113.3
14000	109.3	113.7
57000	131.1	144.5

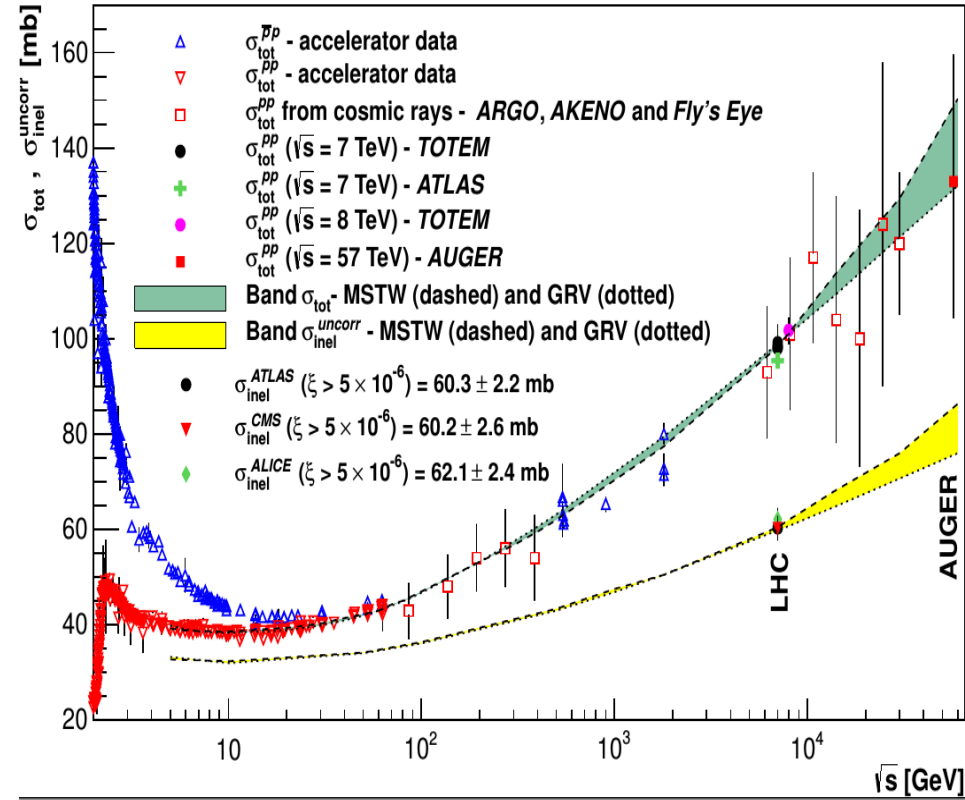
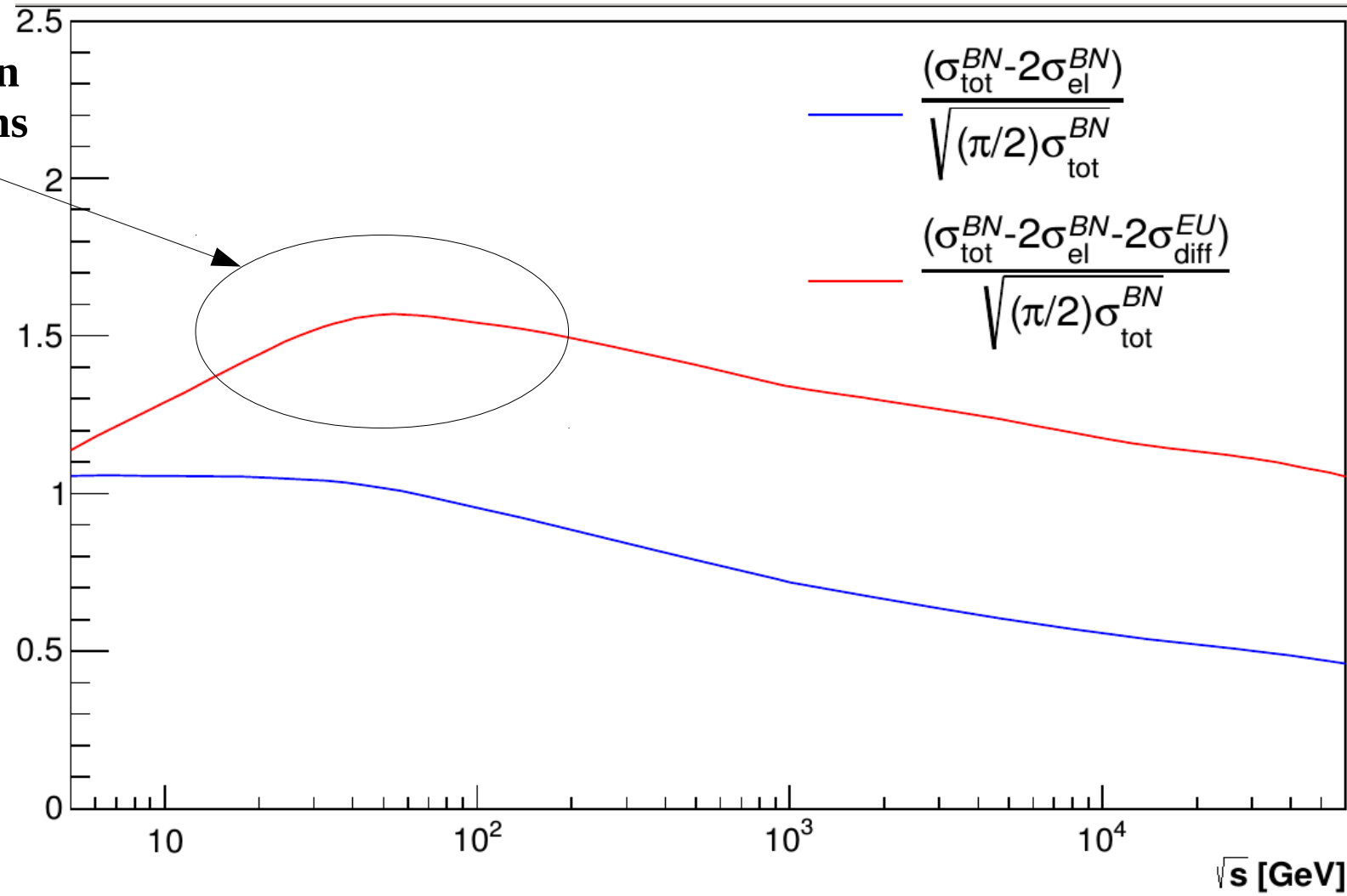


TABLE II. Minijet model predictions for the inelastic cross section at  $\sqrt{s} = 13$  TeV. Predictions of  $\sigma_{\text{inel}}$  in the full phase space were obtained by adding  $\sigma_{\text{diff}}(13 \text{ TeV}) = 12.9$  mb to  $\sigma_{\text{inel}}^{\text{uncorr}} \equiv \sigma_{\text{inel}}^{\text{one-ch}}$ .

PDF	$\sigma_{\text{inel}}^{\text{uncorr}}$ (mb)	$\sigma_{\text{inel}}$ (mb)
GRV	64.3	77.2
MSTW	68.1	81.0

# Back to the *soft edge*

Diffractive dissociation changes regime  $\rightarrow \sim \ln s$



We do not see such a constant edge, mainly because diffraction affects the peripheral interaction region changing the shape of the *edge*.

# Summary

- Concept of *soft edge* in pp has been discussed
- It is intimately related to the energy behaviour of total cross sections
- At the same time, to understand their energy behaviour is far from being a trivial task, especially diffractive dissociation.
- We give here our perspective on this problem, through an s-channel approach
- It can be regarded as good model for the total cross section and for uncorrelated inelastic processes, typically of nondiffractive nature
- Once inelastic diffraction is incorporated it is possible to account for each of the components making the total cross section, namely elastic and 'full' inelastic
- Predictions of this model have been given for LHC13-14
- Improvements are on the way, as we want to incorporate inelastic diffraction in more natural way via Good-Walker or multichannel schemes like Miettinen-Thomas/Lipari-Lusginoli

# Thank you!

