The Bound on Inelastic Diffraction at the LHC

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Overview

Introduction

- The measurements by ALICE, ATLAS , CMS, LHCb and TOTEM at the LHC have confirmed an increase with energy of the total, elastic and inelastic cross–sections, the trend earlier observed at lower energies
- The analysis of the data on elastic scattering obtained by the TOTEM at $\sqrt{s} = 7$ TeV has revealed an existence of the new regime in strong interaction dynamics, related to transition to the new scattering mode
- The main issue of this talk is discussion of the reflective scattering mode, its influence and manifestation in the inelastic diffraction at the LHC. In particular, a new upper bound on the inelastic diffractive cross–section is obtained.

Reflective and Absorptive Scattering

Two Scattering Modes

Unitarity equation in the impact parameter representation assumes the two scattering modes, which can be designated as absorptive and reflective ones and the particular selection will be described below.

- Unitarity

$$\mathrm{Im} f(s, b) = |f(s, b)|^2 + h_{inel}(s, b)$$

Reflective Scattering: S-matrix

S-matrix elastic scattering element

Elastic scattering S-matrix element is related to the amplitude $f(s,b)$ by the relation $S(s,b) = 1 + 2if(s,b)$, it can be presented in the form

$$S(s,b) = \kappa(s,b) \exp[2i\delta(s,b)],$$

two real functions $\kappa(s,b)$ and $\delta(s,b)$.

The function κ ($0 \leq \kappa \leq 1$) is a transmission factor, its value $\kappa = 0$ corresponds to complete absorption. The real part of the scattering amplitude is small and can be neglected, hence the substitution $f \to if$.

Phase and Scattering Mode

- Elastic scattering mode: absorptive or reflective one, is governed by the phase $\delta(s, b)$.
- The common assumption is that $S(s, b) \to 0$ at the fixed impact parameter b and $s \to \infty$. It is called a black disk limit, and in this case the elastic scattering is completely absorptive. This implies the limitation $f(s, b) \le 1/2$.
- Another option: the function $S(s, b) \to -1$ at fixed b and $s \to \infty$, i.e. $\kappa \to 1$ and $\delta = \pi/2$. This limiting case is interpreted as a pure reflective scattering. The principal point here is that $1/2 < f(s, b) \le 1$, as allowed by unitarity .

Reflective Scattering Effects in the Inelastic Processes

- Appearance of the reflective scattering mode at the LHC energies is a key point for the derivation of the upper bounds for the anisotropic flows coefficients.
- Line $S(s,b)=0$ separates two regions: of a pure absorptive scattering and where reflective scattering is present: maximum value of $h_{inel}(s,b)=1/4$,

$$\frac{\partial h_{inel}(s,b)}{\partial b}=S(s,b)\frac{\partial f(s,b)}{\partial b},$$

  i.e. it equals to zero at $S(s,b)=0$.
- The central profile of the function $f(s,b)$ goes to a peripheral one of $h_{inel}(s,b)$, i.e. $h_{inel}(s,b)$ can be expressed as a product, i.e

$$h_{inel}(s,b)=f(s,b)[1-f(s,b)].$$

Beyond the Black Disk Limit

Black Disk Limit Exceeded at the LHC

The analysis of elastic scattering data has demonstrated that $f(s,b)$ is greater than black disk limit $1/2$ at $\sqrt{s}=7$ TeV, but the relative positive deviation α ($f(s,b)=1/2[1+\alpha(s,b)]$) is small at this energy. The value of α is about 0.08 at this energy and $b=0$. The most relevant objects to study starting deviation from the black disk limit are $f(s,b)$ and $h_{el}(s,b)$, but not $h_{inel}(s,b)$ since relative deviation in the latter function is of order α^2, namely $h_{inel}(s,b)=1/4[1-\alpha^2(s,b)]$, where $\alpha(s,b)$ is positive in the region $0\leq b<r(s)$.

Illustration

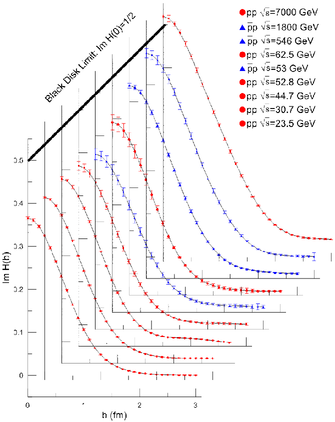


Figure : Energy Evolution from BEL to REL

Ratio of Elastic to Total Cross–Sections

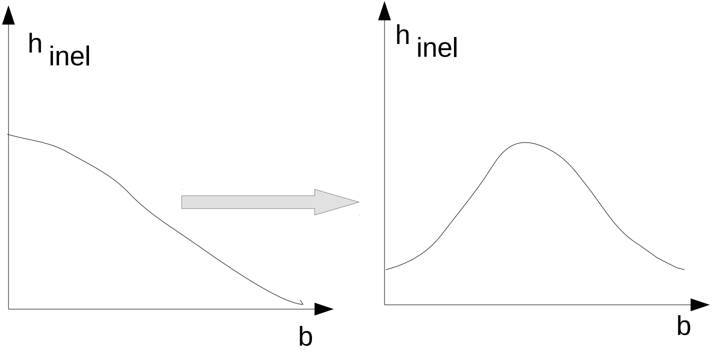
Ratio and Bound

$S(s,b) \to -1$ at fixed b and $s \to \infty$ – a pure reflective scattering by analogy with reflection in optics. Reflective scattering – increasing density of a scatterer with energy. Density goes beyond the critical value, corresponding to the black disk limit–the scatterer starts to reflect the initial wave in addition to its absorption. $1/2 < f(s,b) \leq 1$ and $0 > S(s,b) \geq -1$, as allowed by unitarity . The selection of absorptive or reflective scattering leads to the different values for the ratio $\sigma_{el}(s)/\sigma_{tot}(s)$ at the asymptotical energies.

$$\sigma_{tot}(s) \leq \frac{4\pi}{t_0}\left(\frac{\sigma_{el}(s)}{\sigma_{tot}(s)}\right)\left[\ln\left(\frac{s}{\sigma_{el}(s)}\right)\right]^2\left[1+\left(\frac{\mathrm{Re}F(s,t=0)}{\mathrm{Im}F(s,t=0)}\right)^2\right]^{-1}.$$

Several Details

The main role – the collision geometry and and the relation is applicable for the observables associated with the particle production processes at $s \to \infty$, where the reflective scattering being a dominating mode. The energy evolution of the inelastic overlap function:



Pumplin Bound

- Assuming an absorptive nature of scattering:

$$\sigma_{diff}(s,b) \leq \frac{1}{2}\sigma_{tot}(s,b) - \sigma_{el}(s,b),$$

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$$\sigma_{diff}(s,b) \equiv \frac{1}{4\pi}\frac{d\sigma_{diff}}{db^2}$$

is the total cross–section of all the inelastic diffractive processes in the impact parameter representation and, respectively,

$$\sigma_{tot}(s,b) \equiv \frac{1}{4\pi}\frac{d\sigma_{tot}}{db^2}\ ,\ \sigma_{el}(s,b) \equiv \frac{1}{4\pi}\frac{d\sigma_{el}}{db^2}.$$

Pumplin Bound Cont'd

The respective bound on the non-diffractive cross-section is the following :

$$\sigma_{ndiff}(s,b) \geq \frac{1}{2}\sigma_{tot}(s,b)$$

since $\sigma_{ndiff} = \sigma_{inel} - \sigma_{diff}$. Integrated over b:

$$\sigma_{diff}(s) \leq \frac{1}{2}\sigma_{tot}(s) - \sigma_{el}(s) \text{ and } \sigma_{ndiff}(s) \geq \frac{1}{2}\sigma_{tot}(s).$$

Experimental Status at the LHC

Conclusion on the large magnitude of the inelastic diffraction cross-section follows from comparison of the inelastic cross-section measurements performed by ATLAS and CMS with the TOTEM. In order to reconcile the data of all experiments one needs to assume large value for $\sigma_{diff}(s)$ and essential contribution from the low–mass region. As it was noted, an account for the contribution from this region would lead to a resolution of the inconsistency in the different experimental results.
Thus, the data obtained at the LHC demonstrate an approximate energy–independence of the ratio $\sigma_{diff}(s)/\sigma_{inel}(s)$. At $\sqrt{s} = 7$ TeV this ratio is about $1/3$. The ratio $\sigma_{diff}(s)/\sigma_{el}(s)$ is approximately equal to unity and

$$[\sigma_{el}(s) + \sigma_{diff}(s)]/\sigma_{tot}(s) = 0.495^{+0.05}_{-0.06}.$$

New Integrated Upper Bound

Pumplin bound in terms of $S(s,b)$:

$$\sigma_{diff}(s,b) \leq \frac{1}{4}S(s,b)[1-S(s,b)].$$

$S(s,b)$ is negative: $0 < b < r(s)$, $r(s)$ is the solution of the equation $S(s,b)=0$. There is no inelastic diffraction at the impact parameter value where the black disk limit is reached since $S(s,b)=0$. In this impact parameter range the obvious restriction

$$\sigma_{diff}(s,b) \leq \sigma_{inel}(s,b).$$

In case of reflective scattering this is not trivial since $\sigma_{inel}(s,b)$ has a peripheral impact parameter dependence. At $b \geq r(s)$ the scattering is absorptive and, therefore, the original bound on the inelastic diffractive cross–section should be valid.

Cont'd

$$\bar{\sigma}_{diff}(s) \leq \frac{1}{2}\bar{\sigma}_{tot}(s) - \bar{\sigma}_{el}(s),$$

$\bar{\sigma}_i(s)$:

$$\bar{\sigma}_i(s) \equiv \sigma_i(s) - 8\pi \int_0^{r(s)} bdb\sigma_i(s,b),$$

$$\sigma_{diff}(s) \leq \sigma_{inel}(s) - 2\pi \int_{r(s)}^{\infty} bdb[1 - S(s,b)]$$

and

$$\sigma_{ndiff}(s) \geq 2\pi \int_{r(s)}^{\infty} bdb[1 - S(s,b)].$$

$S(s,b)$ - from data on $d\sigma/dt$. Bound on $\sigma_{diff}(s)$ is 25.6 mb, i.e. 5% increase. At $\sqrt{s} = 13$ TeV the bound on $\sigma_{diff}(s)$ is 28.2 mb and $(6-8)$% increase, $r(s)$ is 0.3 fm.

Model Estimates

Unitary model for the $S(s,b)$:

$$S(s,b) = \frac{1-U(s,b)}{1+U(s,b)},$$

$$U(s,b) = g(s)\exp(-\mu b),$$

$$g(s) \sim s^\lambda$$

The $r(s)$ and $\sigma_{inel}(s)$ are

$$r(s) = \frac{1}{\mu}\ln g(s) \text{ and } \sigma_{inel}(s) = \frac{8\pi}{\mu^2}\ln(1+g(s)).$$

$$\sigma_{tot}(s) \sim \ln^2 s,\ \ \sigma_{el}(s) \sim \ln^2 s,\ \ \sigma_{inel}(s) \sim \ln s \text{ and } r(s) \sim \ln s$$

$$\frac{\sigma_{diff}(s)}{\sigma_{inel}(s)} \le 1 - \frac{2\pi}{\sigma_{inel}(s)}\int_{r(s)}^{\infty} bdb[1-S(s,b)].$$

Cont'd

$$\sigma_{diff,ndiff,inel}(s) \sim \ln s$$

Both parts of $\sigma_{inel}(s)$ would have similar asymptotical energy dependencies, which are proportional to $\ln s$, while the ratio of the inelastic diffractive to elastic cross–sections would decrease asymptotically like $1/\ln s$, i.e. the relation

$$\sigma_{diff}(s)/\sigma_{el}(s) \to 0 \qquad (1)$$

will take place at $s \to \infty$.
It would be also interesting to speculate further and assume the saturation of the bound. It would mean that an asymptotic equipartition of the inelastic cross-section on diffractive and non-diffractive ones occurs.

Conclusion

No inconsistency between saturation of the unitarity limit and the bound on the inelastic diffractive cross–section in the case of reflective scattering, i.e. the reflective scattering limit and the ratio

$$\sigma_{diff}(s)/\sigma_{inel}(s) \to const.$$

at $s \to \infty$ can easily be reconciled. The energy-independent ratio $\sigma_{diff}(s)/\sigma_{inel}(s)$ is also consistent with the commonly accepted definition of the inelastic diffraction as a result of the Pomeron exchanges and account for the recent experimental trends found at the LHC.
If one assumes mechanism resulting in saturation of the black disk limit at the asymptotic energies, this is not the case.
The new LHC experiments at higher energies would be definitely helpful for resolving the asymptotical dynamics of the inelastic diffraction and elastic scattering.

s- and t-channel approaches to inelastic diffraction can be reconsiled.

The End